Negotiation protocols and dynamic social networks

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1 Introduction

Multi-agent systems (MAS) represent a powerful framework to analyze complex systems, including realistic agent behavior (such as learning mechanisms). The analysis can be made both at a micro level to study individual scenarii and interpret results and at a macro (global) level to study emergence and structural evolution (beyond agent complexity and local analysis) [1]. MAS make it also possible to study systems with complex interaction protocols, particulary useful to simulate more realistic systems with minimum information transmission between the agents (e.g. pricing without preference transmission). Such complex interaction systems are particulary important in a field such as social networks where links, which are the heart of the problem, support these interactions.

This paper investigates a socio-economic MAS, focusing on three specific aspects: i) agents heterogeneity (each agent follows its individual preferences modeled after a parameterized long term utility function); ii) dynamic social network (the agent activity is shaped after its social network, which evolves as a result of the agent activity); iii) computational interaction design (covering the rules of interaction and the information revelation among agents).

The presented MAS considers a loan-granting scenario where each agent can borrow/lend money to its neighbors and/or consume it. We define six interaction protocols, ranging from fixed equilibrium rate loans to double-free auctions, and we study their impact on the network structure and the global welfare of the economy. Further, the agent fitness is investigated in relation with its connectivity (number of neighbors) and eccentricity (longest path to the other agents).

After describing the state of the art and the problem tackled in this paper (section 2), we present a loan granting game played by a society of rational agents with a long-term utility function, conditioned by and shaping their social network (section 3). Section 4 reports on the simulation results; the main contributions of the approach are discussed together with perspectives for further studies in section 5.

2 State of the Art and Goal of the Study

Introduced by Epstein and Axtell [2], Agent-based Computational Economics established two major results (the interested reader is referred to [3, 1] for a comprehensive presentation). Firstly, the lack of a centralized walrasian auctioneer does not prevent a society of 0th-intelligence agents from converging towards an economical equilibrium when agents interact and exchange in a decentralized manner; secondly, this result does not hold any longer if the agents can die or evolve.

The importance of interaction protocol in artificial markets using MAS has been since widely studied, especially in the popular automated auction domain (see [4] for a comprehensive study). In particular, the theoretical Pareto-optimality of the English and Vickrey [5] auction mechanism has been shown despite their known limitation (e.g. preference elicitation for the English auction and trust problems for the Vickrey auction). From a reverse engineering perspective, Chevaleyre et al. [6] have investigated the conditions on the rationality criteria enforcing an egalitarian welfare.

Meanwhile, the structure of social networks is acknowledged a major factor of economics efficiency. A framework for analyzing social network economics was defined [7], and exploited through either analytical approaches, or various simulation-based extensions [8, 9, 10, 11].

In this paper, the goal of this study is to investigate the protocol impact on the economic structure and welfare. One originality of this study compared with MAS work [4, 6] is to deal with a dynamic social network environment, the evolution of the network being as important as the macro variables.

Contrasting with the standard framework [7], the social network considers interdependent periods: the money borrowed at time t must be payed back at time t+1. Decisions at each period are no longer independent. Therefore, this study examines the impact of the agent models and strategies on the global welfare in a long term perspective. This contrasts with e.g. [7] considering the immediate network efficiency, and neglecting the negative impact of current decisions in the long run.

Another contribution compared to social network economic analysis [7, 8] is to study several different interaction protocols involving different information sharing levels, whereas economic studies usually assume a perfect preference knowledge. This assumption is required for many game-theory analysis. In the framework of ACE, a relaxed assumption is done (such as [2] to compute the exchange price as the average of preference values).

3 Overview

This section presents the agent model, the interaction setting and the observed variables of the system.

3.1 Agent model

The agent utility function models the intertemporal choice of the consumer after the standard economic theory [12]. Formally, agent A_i maximizes the sum over all time steps of its weighted instant utilities. The utility weight at time t, set to p_i^t $(0 < p_i < 1)$, reflects the agent preference toward the present (parameter p_i). The instant utility reflects the current consumption level $C_{i,t}$, with a diminishing marginal utility modeled through parameter b_i $(0 < b_i < 1)$, standing for the fact that the agent satisfaction is sublinear with its consumption level [13]. Letting M_i denote the lifelength of agent A_i, it comes:

$$U_i = \sum_{t=0}^{M_i} (p_i^t C_{i,t}^{b_i}) \tag{1}$$

The instant neighborhood of agent A_i, noted $V_{i,t}$ involving all agents A_j such that link (i, j) belongs to the social network at time t (subscript t is omitted when clear from the context). Additional agent parameters and variables are described in Appendix A.

The social network comes at a cost, i.e. every link (i, j) must be paid by agents A_i or A_j or both. Agents accept to pay for a link iff it was profitable during the last time steps (if the utility increase due to this link offsets the link cost). Under mild assumptions (see Appendix A), agent A_i can compute its **threshold interest rate** r_i (lower bound for loan granting activities and upper bound for money borrowing activities). Note that this rate needs be updated after every elementary transaction as it depends on the agent current and expected capital (see Equation 2 in Appendix A).

Agent life is viewed as a sequence of time steps, where each step involves four phases: i) salary and loans payback, ii) negotiation, iii) consumption, iv) social activity (link creation/deletion).

During the first phase, agent A_i receives its salary R_i, reimburses the money borrowed (plus interests) and is reimbursed for the money lended (plus interests). The negotiation phase involves a variable number of transactions, depending on one of the protocols defined below. During the consumption phase, the agent computes its optimal fraction of consumption (see Appendix B) and scores the corresponding utility. During the social phase, each agent decides whether it maintains its links whether the link has been profitable in the last time steps. Link (i, j) is either maintained by agents A_i and/or A_j, or deleted. Independently, A_i creates a new link (i, j) with probability s_i (its sociability factor), where j is uniformly randomly selected. If A_i has no neighbors, some link (i, j) is automatically created where j is selected uniformly.

After M_i steps, agent A_i dies. It is then replaced by a new agent (reinitializing all agent parameters) *with same neighborhood.*

3.2 Interaction protocols and computational design

To emphasize the importance and impact of the protocol and information transmission, we define 6 exchange protocols ranging from a complete and perfect knowledge of others preferences to minimum preference sharing between agents.

- **EQU**: Equilibrium rate exchange. Each agent knows the theoretical equilibrium rate of the economy (see Appendix D for the mathematical details). Each agent proposes to its neighbors to borrow/lend money at this rate depending on its individual preferences. Every accepted proposal is immediately enacted. The negotiation proceed until no more transactions are realized.
- **AVG**: Average rate exchange. Each agent knows the threshold rate of its neighbors. Each agent proposes to its neighbors to borrow/lend money at a rate equal to the average between its own rate and its neighbor's. Every accepted proposal is immediately concluded. The transactions proceed until no more transactions are realized.
- **AUCs**: Auctions. Iteratively, agent A_i proposes to every neighbor A_j whether A_j would be willing to borrow money, and which rate r_j A_j would be willing to accept. A_i determines the best transaction, with A_j^* such that r^*=max(r_j). If $r^* > r_i$, A_i grants a loan. The loan rate depends on the specific protocol:

  - **AUCAVG**: Average auction. The rate is the average of both threshold rates: $r^* = \frac{1}{2}(r_i + r_j)$.
  - **AUCVIC**: Vickrey auction (second-price sealed-bid auction). The rate is the second best offer of the auction.
  - **AUCSIM**: Standard (simple) auction (first-price sealed-bid auction). The best bid is selected, the rate is thus set to the borrower threshold rate r_j.

  The negotiation proceed until no more transactions are realized.
- **DOUBLE**: Double sided transaction. Auctions are asymmetrical. The objective of this mechanism is to minimize preference revelation in a symmetric protocol:
  Iteratively, agent A_i achieves its best possible borrowing and lending rate and concludes at most one loan granting and one borrowing(corresponding to one negotiation cycle); it maintains its estimation $r_{i,j}$ of the interest rate for a transaction (borrow or grant) with agent A_j, and proposes a transaction for one currency unit at rate $r_{i,j}$. Depending on whether this transaction is accepted, $r_{i,j}$ is updated (Algorithm 1). Agent A_j accepts a borrow transaction if the proposed rate is lower than i) its limit rate r_j and ii) its last borrow rate during its negotiation cycle (similar conditions hold for lend transactions).
  The negotiation proceed until no more transactions are realized. It is important to note that preferences are never transmitted in this protocol (in contrast with all AUC auction systems), agents only accept or refuse proposals.

```
BestRate r*=0;
who=i;
foreach A_j ∈ V_i such that r_ij > r_i do
    Propose Loan(rate=r_ij);
    if accepted then
        if r_ij > r* then r* = r_ij; who=j;
        Increase(r_ij)
    else
        Decrease(r_ij)
    end
end
if r* > 0 then Lend one currency unit at rate r* to A_who
```

Algorithm 1: Lending transactions (borrowing transactions proceed likewise). r_{ij} is uniformly initialized during the first time step.

3.3 Fitness and Global Welfare

The socio-economical system will be assessed from the global welfare of the agents. The difficulty is that the agent utility function depends on its individual preferences. While this setting is more realistic form a modeling viewpoint, it prevents from directly comparing the agents utilities. Therefore, an original normalization procedure is used, considering the canonical consumer-only alternative strategy. Each A_i, would it have adopted the consumer-only strategy, would get utility:

$$U_i^* = \sum_{t=0}^{M_i}(p_i^t R_i^{b_i}) = R_i^{b_i}\frac{1-p_i^{M_i+1}}{1-p_i}$$

Accordingly, the normalized fitness of A_i is defined as:

$$F_i = \left(\frac{U_i}{U_i^*}\right)^{\frac{1}{b_i}} - 1$$

Note that if A_i spends a fixed fraction α of its salary at each time step (without engaging in any borrowing or lending transactions), it scores a normalized fitness $\alpha - 1$. In brief, agent A_i benefits from the social network iff its fitness F_i is positive. The efficiency of the socio-economic system is thus assessed as the global welfare of the economy, i.e. the average normalized fitness of the individuals, and its standard deviation. An alternative welfare function is discussed in section 4.3 to take consider the inequalities involved by this measure: by convention, the egalitarian welfare is the minimum over all agent fitness.

4 Results

After the description of the experimental setting, this section reports on the impact of the network and agent dynamics on the network structure and on the global efficiency of the system.

4.1 Experimental settings

The socio-economical game is implemented and simulated within the Moduleco framework [14]. The initial structure of the social network is a ring, where each agent is connected to its two neighbors. Agents are initialized by independently drawing their parameters using Gaussian or uniform laws (see Appendix C for detailed values). Experiments were conducted with a population size ranging from 25 to 100, with similar results. All reported results are averaged over 1000 independent experiments conducted with 25 agents over 1000 time steps. The global fitness is computed by averaging the normalized fitness at their death instant of agents that died before the 1000th. step (we observed similar results after this step).

4.2 Structural analysis

Interest rate evolution

The theoretical interest rate τ at the equilibrium can be derived from the utility functions (see Appendix D for demonstration). As this rate depends on the current agent population, it fluctuates along time due to agents death and birth. Still, the position of the MAS with respect to the equilibrium can be assessed by comparing the rate of each loan and the rate of the last loan for each link with the equilibrium rate of the period (Fig 1). While EQU protocol enforces the equilibrium rate by construction, all other protocols converge toward the equilibrium rate at each time step (final distance to equilibrium is much lower than average distance). DOUBLE protocol enforces a fast convergence toward the equilibrium rate (the average difference being .3%, and the final rate average difference .2%). Other protocols have a much higher difference, ranging from 2% (AVG) to 11% (AUCSIM). Within each period, the standard deviation analysis confirms the previous remarks. The DOUBLE protocol appears again to have the most stable rates (after EQU) with a low standard deviation of 2%, while rates of the four other protocols are much more instable.

Even if the market mechanism leads near the theoretical equilibrium situation at the end of each period, the protocol has a clear impact on the individual rate evolution. Protocols relying on the preference sharing (AVG, AUCAVG) enforces a faster convergence. Meanwhile, asymmetrical protocols (in term of information sharing) AUCVIC and AUCSIM, lead to higher interest rate on average, favoring the granter, that collect information. Still, the DOUBLE protocol, which is both symmetrical and does not imply preference sharing, leads to a stable and fair market (both the lowest rate dispersion and difference with equilibrium).

Network evolution

Another structural variable which can explain the (non)convergence of the rate is the number of links (or equivalently the average connectivity

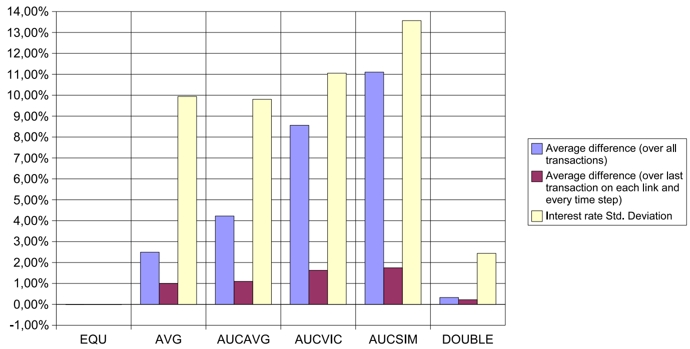

Fig. 1. Convergence toward equilibrium interest rate

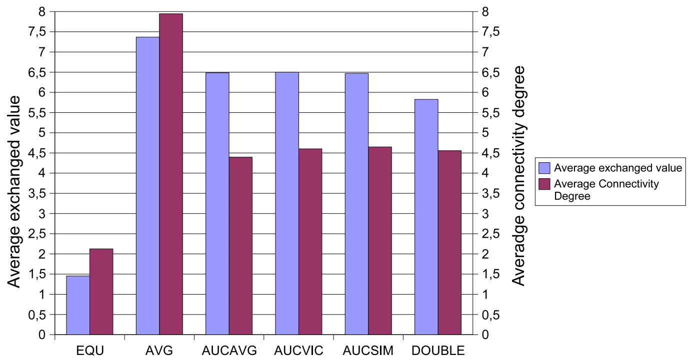

Fig. 2. Average exchanged value (for one agent at one time step) and average connectivity degree

of agents) and the number of transactions (measured by the average amount exchanged by one agent during one time step) achieved with each protocol (fig. 2). We have again three distinct groups: The EQU protocol with a very low number of links (average degree less than 2) and few exchanges (an average amount inferior to 2, representing less than 10% of its salary); The AVG protocol with a very high number of links (more than 6 average degree) and a high number of exchanges; And the four others with an average number of links and a high number of exchanges. Interestingly, while each link supports an average amount of less than one unit of transaction with EQU, it support more than 1.3 with AUC* and DOUBLE, which suggests that the social network is more effective in the latter case. A tentative explanation for this fact is the efficiency of the auction mechanism: only selecting the best loan at each cycle of proposal (vs every accepted loan in EQU and AVG).

4.3 Welfare analysis

Global welfare

The population global welfare associated to every mechanism is assessed from the average aggregated agent fitness (see section 3.3). The comparison of welfare, displayed on Figure 3 shows that AVG and EQU protocols results on a negative welfare, which can be linked to the bad link selection observed in 4.2, followed by DOUBLE. The best system appears to be the AUCSIM protocol, followed by the AUCVIC.

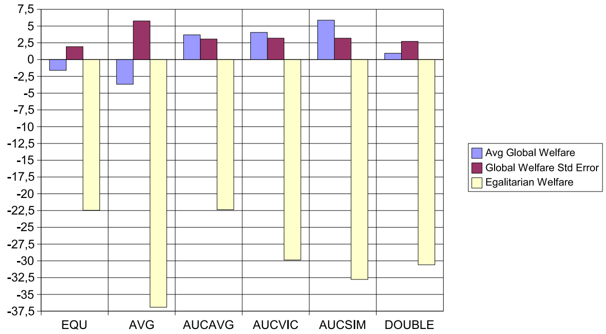

Fig. 3. Average Global welfare, standard error and global welfare for each protocol

The egalitarian welfare gives very different results: AVG is still the worse, but it is followed by AUCSIM (the best for global welfare), DOUBLE and AUCVIC. AUCAVG and EQU (worst for global welfare) are the best egalitarian systems. The welfare distribution can also be analyzed w.r.t. the standard deviation: while EQU is relatively egalitarian, AUCVIC and AUCSIM respectively follow a heavy tailed distribution.

There appears to be a real choice between equitable (global) and egalitarian (both in value and standard error) systems. The equality objective clearly requires common knowledge, but this knowledge can be limited to neighbors preferences (with AUCAVG). Market efficiency seems to rely on inequalities between agents. Finally, minimizing information revelation (DOUBLE) does not allow the system to reach either maximum global or egalitarian welfare.

Connectivity degree effect

Fig. 4 shows how the agent fitness is related to its position within the social network, depending on the interaction mechanism. The sociability is indifferent with the AUCAVG protocol; a high sociability is positively correlated to the agent fitness for the EQU protocol, whereas it is negatively related to the others.

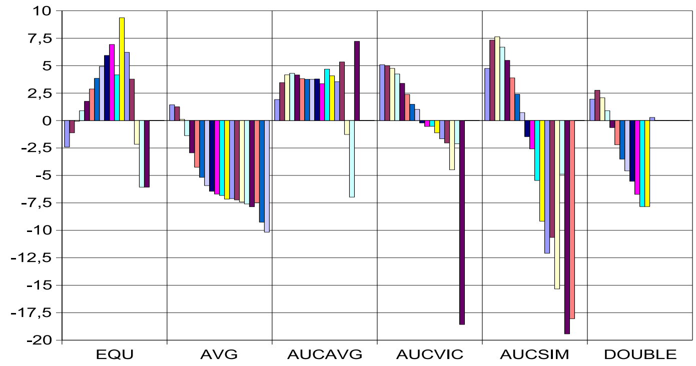

Fig. 4. Average Fitness for agents with a connectivity ranging from 1 to 20

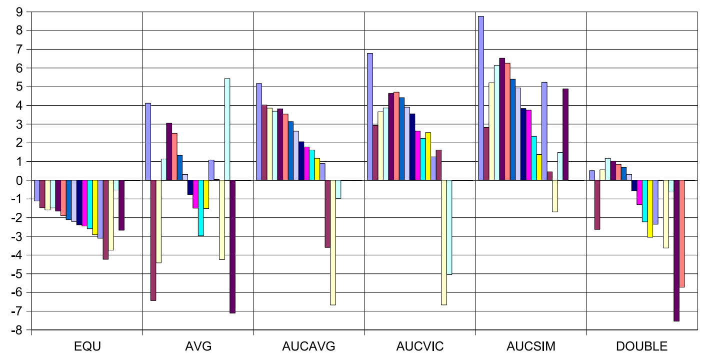

Fig. 5. Average Fitness for agents with an eccentricity ranging from 1 to 20

Another measure for the agent position is its eccentricity (longest path to the other agents in the network, see Fig. 5). While eccentricity is negatively correlated to the agent fitness for EQU (which is consistent with the fact that EQU rewards agent sociability), it has no clear impact within the AVG setting; and high eccentricity adversely affects the agent fitness on all other settings. In the latter case (AUC* and DOUBLE), it appears that the best agent position is to have few neighbors that are well connected, enforcing both a low eccentricity and a low connectivity.

Salary effect

Fig. 6 depict the impact of the agent salary on its fitness. As could have been expected, the higher the salary, the higher the fitness is. Nevertheless, this correlation is quite mysterious, as the normalized fitness was precisely designed to cancel out the salary level (see section 3.3). This unanticipated effect can be interpreted in several ways. Firstly, the fact that transactions concern a currency unit would marginally favor the agent with higher salary. Another interpretation, related to the "Matthew Effect"[15], suggests that the gain opportunities increase over with the salary activity.

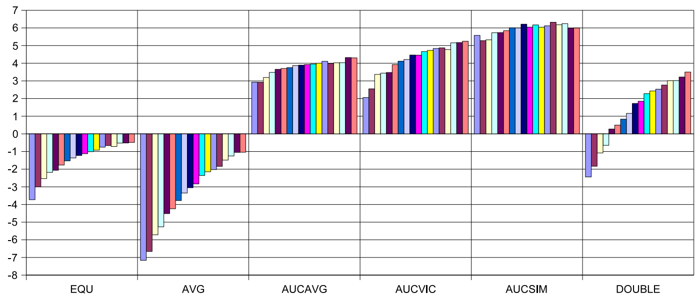

Fig. 6. Average Fitness for agents with increasing salary values

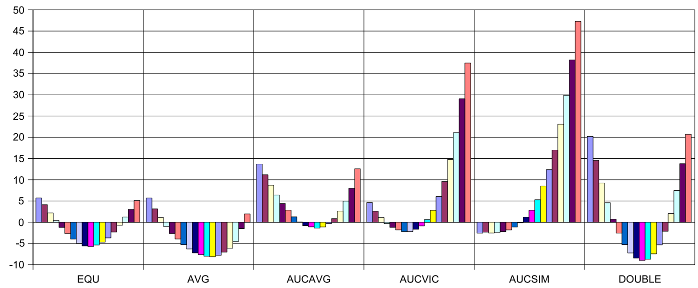

Fig. 7. Average Fitness for agents with increasing time preference values

Time preference effect

Despite the fact that Time preference value (parameter p_i) is compensated for in the fitness normalization (see section 3.3), Fig. 7 shows that the time preference has a strong impact on the agent fitness. Specifically, extreme values (very low and very high) appear to be much more profitable than medium values. The transaction benefit for an agent with extreme time preference can be much higher compared to that for an agent with average time preference (the difference between the interest thresholds increases when one of the agents has an extreme time preference). The impact of the time preference also depends on the protocol. reminding that AUC* frameworks are asymmetrical (transactions are driven by granting agents), we see that AUCSIM specifically favors agents with high p_i value, i.e. low preference toward the present. As these agents will propose many loans, and select the most profitable ones, these transactions will be all the more profitable as the interest rate is set to the borrower's threshold rate. It is interesting to see that the Vickrey mechanism (AUCVIC), using the second-best price instead of the best one, decreases the loan-granter advantage (because it allows the receiver to pay less than its threshold price). In contrast, AUCAVG tends to favor agents with high preference toward the present.

5 Conclusion and Perspectives

The complex system investigated in this paper involves heterogeneous agents engaged in a socio-economic game, following different interaction protocols. Several trends have been empirically demonstrated, showing the interdependency of the agents individual preferences and the interaction rules. Clearly, the considered settings correspond to quite different socio-economic organizations. The EQU depicts a context with little sociability and exchanges, with negative global welfare, where the most well off agents are the cental ones (high connectivity, low eccentricity). The AVG setting does not appear to be very efficient: the most well off agents have few neighbors and the global welfare is the worst one among all considered settings. The AUC* are rather efficient w.r.t. global welfare. The AUCAVG is particularly efficient in terms of egalitarian welfare. The main difference among the AUC* settings regards the impact of the agent time preference: while AUCAVG rewards agents with high time preferences, AUCSIM instead rewards the patient ones. Lastly, the DOUBLE framework enforces the convergence of the market toward the equilibrium interest rate with a parsimonious preference revelation; it leads to a well connected social network, though the global welfare is less that with the AUC* settings. Agents with limited sociability and low eccentricity are rewarded.

Further research will consider more complex agents, capable of learning and controlling (for some of them) the preferences of their neighbors. The merits of learning agents will be to study the robustness of the interactions settings, as far as learning abilities would allow agents to detect and exploit the "holes" and biases in the settings. In the long run, the possibility for agents to control some individual characteristics of their neighbors will allow us to model socio-economic games in term of dynamic systems.

6 Acknowledgments

We warmly thank D. Phan for making available his Moduleco Framework. This work was supported in part by the IST Programme of the European Community, under the PASCAL Network of Excellence, IST-2002-506778.

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Appendix

A Agent parameters

Agent is defined by some additional parameters:

- **Salary** R_i: A_i receives a fixed salary R_i at the beginning of each time step, and uses it to grant or pay back loans, to buy links, or for consumption.
- **Sociability factor** s_i ($0 < s_i < 1$): A_i creates a new link (i, j) (where j is uniformly chosen) with a probability s_i at each time step; in case A_i is isolated, a new link is automatically created.

The following variables can be defined using agents parameters:

- **Capital** K_i: The capital of an agent is the money it can use at a given time. At the beginning of each time step, it is equal to its salary, minus the amount it has to pay back (loans contracted at the previous step), plus the payback of the loans it granted. The capital may change during a time step (it increases when money is borrowed, decreases when loans are granted).
- **Expected capital** N_i: It is the amount of money that will be available at the next time step if the agent consumes all his current capital and does not realize any more transaction. If no transactions were done during the time step, it is equal to the salary R_i.
- **Limit rate** r_i: It is possible to compute the rate r_i so that agent i refuses to borrow at a higher rate and refuses to grant loans at a lower rate. Agent i will accept to grant a loan if the loan increases his utility:

$$K_i^{b_i} + p_i N_i^{b_i} < (K_i - q)^{b_i} + p_i(N_i + (1 + r_i)q)^{b_i}$$

$$\Rightarrow r_i > \frac{p_i^{-\frac{1}{b_i}} (K_i^{b_i} + p_i N_i^{b_i} - (K_i - q)^{b_i})^{\frac{1}{b_i}} - N_i}{q} - 1 \qquad (2)$$

B Optimal saving

Each agent increases its utility by consuming a part or the totality of his capital. Through a maximization of the utility function, we compute the optimal saving amount until the next time step (this value is most often zero: it is better for agents to lend money than to save it):

$$\hat{x} = \frac{K_i - p_i^{\frac{1}{b_i-1}} N_i}{1 + p_i^{\frac{1}{b_i-1}}}$$

C Experimental conditions

Agents parameters (see section 3.1) are defined with truncated gaussian variables (restrained to the interval $[\mu-2\sigma, \mu+2\sigma]$). Parameter analysis (Fig 6 and 7) use 18 uniformly distributed classes on this interval.

- Time preference $p_i \sim \mathcal{N}(0.8, 0.075)$
- Utility factor $b_i \sim \mathcal{N}(0.5, 0.1)$,
- Link cost $c = 0.1$ and $c = 0.2$ alternatively
- Sociability $s_i \sim \mathcal{N}(0.01, 0.05), (0.025, 0.05), (0.04, 0.05)$ alternatively,
- Salary $R_i \sim \mathcal{N}(20, 5)$,
- Life expectancy $M_i \sim U(20, 100)$,

D Theoretical interest rate equilibrium

The term of each loan is the next time step. Marginal utility of agent A_i can thus be deduced:

$$\frac{dU}{dq_i} = b_i(K_i + q_i)^{b_i-1} - b_i p_i(1+\tau)(R_i - (1+\tau)q_i)^{b_i-1}$$

Since every agent maximizes its utility, one has at the equilibrium:

$$\frac{dU}{dq_i} = 0 \Leftrightarrow (K_i + q_i)^{b_i-1} = p_i(1+\tau)(R_i - (1+\tau)q_i)^{b_i-1}$$

$$q_i = \frac{(p_i(1+\tau))^{\frac{1}{b_i-1}} R_i - K_i}{1 + (p_i(1+\tau)^{b_i})^{\frac{1}{b_i-1}}}$$

The equilibrium rate is thus the solution of the equation:

$$\sum_i q_i = \sum_i \frac{(p_i(1+\tau))^{\frac{1}{b_i-1}} R_i - K_i}{1 + (p_i(1+\tau)^{b_i})^{\frac{1}{b_i-1}}} = 0$$