RAPPORT DE RECHERCHE

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HOL-TestGen Version 1.8
USER GUIDE

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HOL-TestGen 1.8.0
User Guide

http://www.brucker.ch/projects/hol-testgen/

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Note:
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Contents

1. Introduction

Today, essentially two validation techniques for software are used: *software verification* and *software testing*. Whereas verification is rarely used in "real" software development, testing is widely-used, but normally in an ad-hoc manner. Therefore, the attitude towards testing has been predominantly negative in the formal methods community, following what we call *Dijkstra's verdict* [13, p.6]:

> "Program testing can be used to show the presence of bugs, but never to show their absence!"

More recently, three research areas, albeit driven by different motivations, converge and result in a renewed interest in testing techniques:

Abstraction Techniques: model-checking raised interest in techniques to abstract infinite to finite models. Provided that the abstraction has been proven sound, testing may be sufficient for establishing correctness [3, 12].

Systematic Testing: the discussion over *test adequacy criteria* [26], i. e. criteria solving the question "when did we test enough to meet a given test hypothesis," led to more systematic approaches for *partitioning* the space of possible test data and the choice of representatives. New systematic testing methods and abstraction techniques can be found in [16, 14].

Specification Animation: constructing counter-examples has raised interest also in the theorem proving community, since combined with animations of evaluations, they may help to find modelling errors early and to increase the overall productivity [2, 17, 11].

The first two areas are motivated by the question "are we building the program right?" the latter is focused on the question "are we specifying the right program?" While the first area shows that Dijkstra's Verdict is no longer true under all circumstances, the latter area shows, that it simply does not apply in practically important situations. In particular, if a formal model of the environment of a software system (e. g. based among others on the operation system, middleware or external libraries) must be reverse-engineered, testing ("experimenting") is without alternative (see [7]).

Following standard terminology [26], our approach is a *specification-based unit test*. In general, a test procedure for such an approach can be divided into:

Test Case Generation: for each operation the pre/postcondition relation is divided into sub-relations. It assumes that all members of a sub-relation lead to a similar behavior of the implementation.

Test Data Generation: (also: Test Data Selection) for each test case (at least) one representative is chosen so that coverage of all test cases is achieved. From the resulting test data, test input data processable by the implementation is extracted.

Test Execution: the implementation is run with the selected test input data in order to determine the test output data.

Test Result Verification: the pair of input/output data is checked against the specification of the test case.

The development of HOL-TestGen [8] has been inspired by [15], which follows the line of specification animation works. In contrast, we see our contribution in the development of techniques mostly on the first and to a minor extent on the second phase.

Building on QuickCheck [11], the work presented in [15] performs essentially random test, potentially improved by hand-programmed external test data generators. Nevertheless, this work also inspired the development of a random testing tool for Isabelle [2]. It is well-known that random test can be ineffective in many cases; in particular, if preconditions of a program based on recursive predicates like "input tree must be balanced" or "input must be a typable abstract syntax tree" rule out most of randomly generated data. HOL-TestGen exploits these predicates and other specification data in order to produce adequate data, combining automatic data splitting, automatic constraint solving, and manual deduction.

As a particular feature, the automated deduction-based process can log the underlying test hypothesis made during the test; provided that the test hypothesis is valid for the program and provided the program passes the test successfully, the program must guarantee correctness with respect to the test specification, see [6, 9] for details.

2. Preliminary Notes on Isabelle/HOL

2.1. Higher-order logic — HOL

Higher-order logic(HOL) [10, 1] is a classical logic with equality enriched by total polymorphic[1] higher-order functions. It is more expressive than first-order logic, since e. g. induction schemes can be expressed inside the logic. Pragmatically, HOL can be viewed as a combination of a typed functional programming language like Standard ML (SML) or Haskell extended by logical quantifiers. Thus, it often allows a very natural way of specification.

2.2. Isabelle

Isabelle [21, 18] is a *generic* theorem prover. New object logics can be introduced by specifying their syntax and inference rules. Among other logics, Isabelle supports first order logic (constructive and classical), Zermelo-Fränkel set theory and HOL, which we chose as the basis for the development of HOL-TestGen.

Isabelle consists of a logical engine encapsulated in an abstract data type *thm* in Standard ML; any *thm* object has been constructed by trusted elementary rules in the kernel. Thus Isabelle supports user-programmable extensions in a logically safe way. A number of generic proof procedures (*tactics*) have been developed; namely a simplifier based on higher-order rewriting and proof-search procedures based on higher-order resolution.

We use the possibility to build on top of the logical core engine own programs performing symbolic computations over formulae in a logically safe (conservative) way: this is what HOL-TestGen technically is.

[1] to be more specific: *parametric polymorphism*

3. Installation

3.1. Prerequisites

HOL-TestGen is built on top of Isabelle/HOL, version 2013-2, thus you need a working installation of *Isabelle 2013-2*. To install Isabelle, follow the instructions on the Isabelle web-site:

`http://isabelle.in.tum.de/website-Isabelle2013-2/index.html`

3.2. Installing HOL-TestGen

In the following we assume that you have a running Isabelle 2013-2 environment. The installation of HOL-TestGen requires the following steps:

1. Unpack the HOL-TestGen distribution, e. g.:

   ```
   tar zxvf hol-testgen-1.8.0.tar.gz
   ```

   This will create a directory `hol-testgen-1.8.0` containing the HOL-TestGen distribution.

   ```
   cd hol-testgen-1.8.0
   ```

   and build the HOL-TestGen heap image for Isabelle by calling

   ```
   isabelle build -d . -b HOL-TestGen
   ```

3.3. Starting HOL-TestGen

HOL-TestGen can now be started using the `isabelle` command:[1]

```
isabelle jedit -d . -l HOL-TestGen "examples/unit/List/List_test.thy"
```

After a few seconds you should see an jEdit window similar to the one shown in Figure 3.1.

Alternatively, the example can be run in batch mode, e. g.,

```
isabelle build -d . HOL-TestGen-List
```

[1]Note that the `isabelle` command must be provided by Isabelle 2013-2.

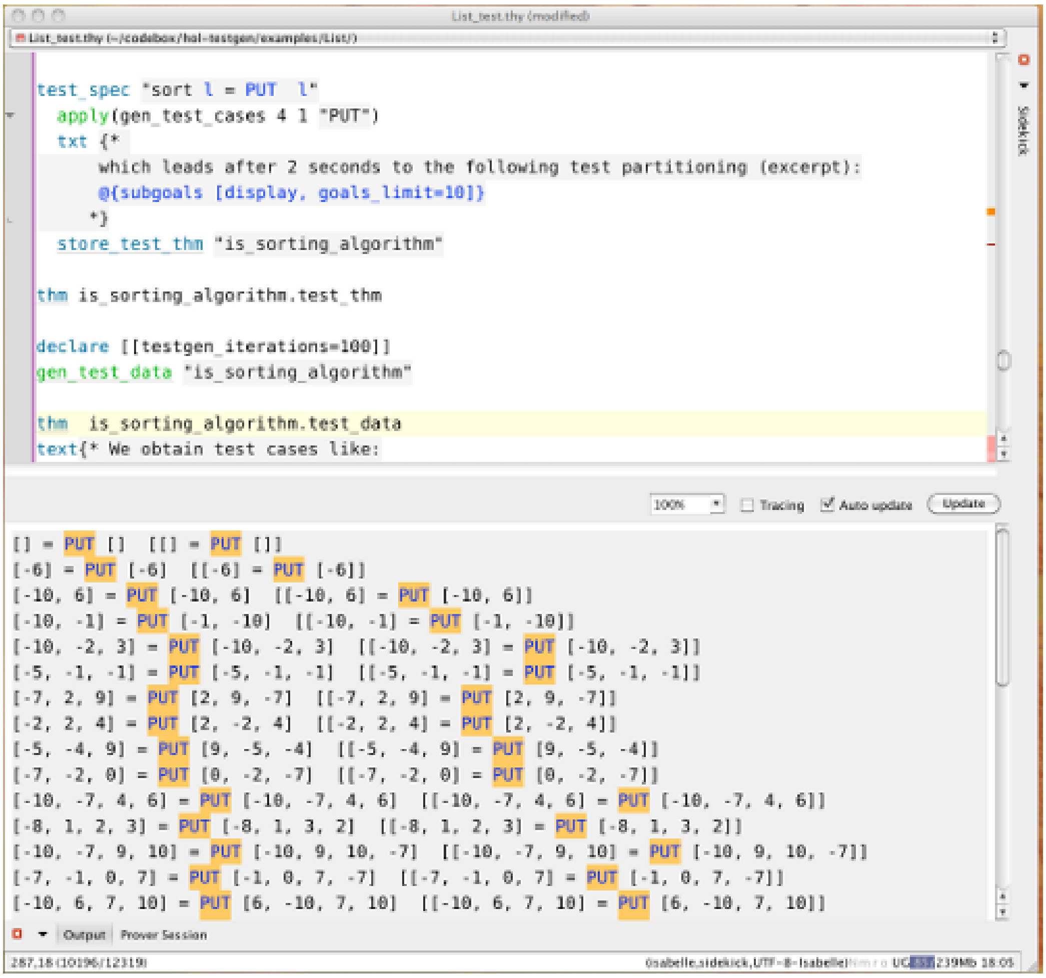


Figure 3.1.: A HOL-TestGen session Using the jEdit Interface of Isabelle

4. Using HOL-TestGen

4.1. HOL-TestGen: An Overview

HOL-TestGen allows one to automate the interactive development of test cases, refine them to concrete test data, and generate a test script that can be used for test execution and test result verification. The test case generation and test data generation (selection) is done in an Isar-based [25] environment (see Figure 4.1 for details). The test executable (and the generated test script) can be built with any SML-system.

4.2. Test Case and Test Data Generation

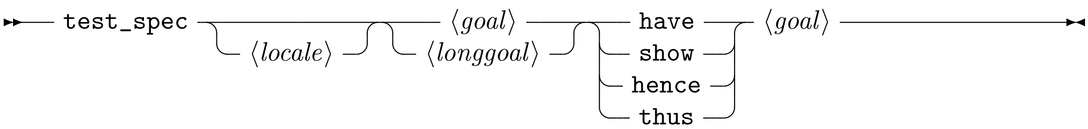
In this section we give a brief overview of HOL-TestGen related extension of the Isar [25] proof language. We use a presentation similar to the one in the *Isar Reference Manual* [25], e. g. "missing" non-terminals of our syntax diagrams are defined in [25]. We introduce the HOL-TestGen syntax by a (very small) running example: assume we want to test a function that computes the maximum of two integers.

Starting your own theory for testing: For using HOL-TestGen you have to build your Isabelle theories (i. e. test specifications) on top of the theory Testing instead of Main. A sample theory is shown in Table 4.1.

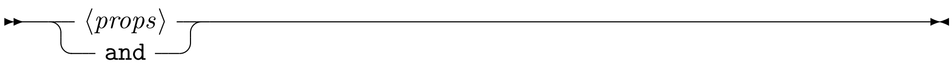
Defining a test specification: Test specifications are defined similar to theorems in Isabelle, e. g.,

test_spec "prog a b = max a b"

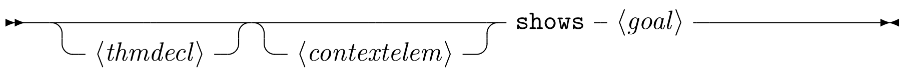
would be the test specification for testing a simple program computing the maximum value of two integers. The syntax of the keyword `test_spec` : *theory* $\rightarrow$ *proof* (*prove*) is given by:



⟨*goal*⟩ ::=



⟨*longgoal*⟩ ::=



Please look into the Isar Reference Manual [25] for the remaining details, e. g. a description of ⟨*contextelem*⟩.

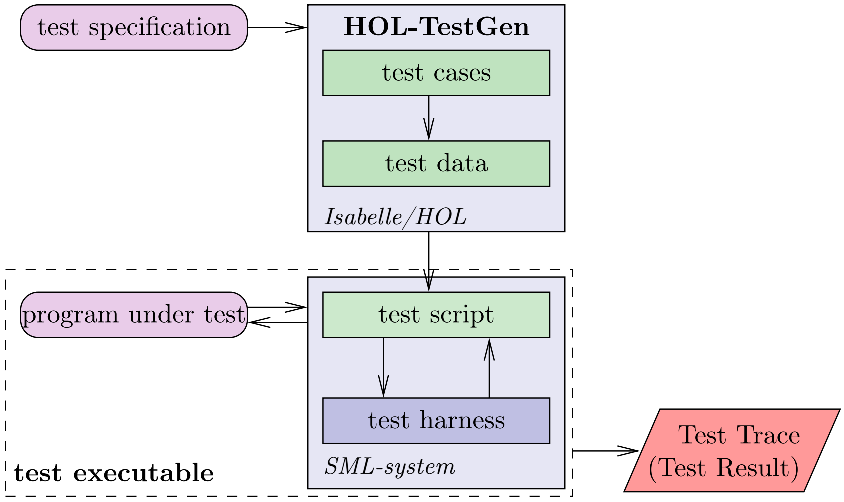


Figure 4.1.: Overview of the system architecture of HOL-TestGen

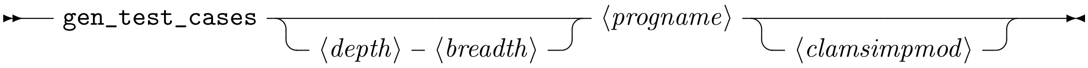
```
theory max_test
imports Testing
begin

test_spec "prog a b = max a b"
  apply(gen_test_cases "prog" simp: max_def)
  mk_test_suite "max_test"

gen_test_data "max_test"

thm max_test.concrete_tests

generate_test_script "max_test"
thm max_test.test_script

text {* Testing an SML implementation: *}
export_code max_test.test_script in SML   module_name TestScript file "impl/sml/max_test_script.sml"

text {* Finally , we export the raw test data in an XML-like format: *}
export_test_data "impl/data/max_data.dat" max_test

end
```

Table 4.1.: A simple Testing Theory

Generating symbolic test cases: Now, abstract test cases for our test specification can (automatically) be generated, e. g. by issuing

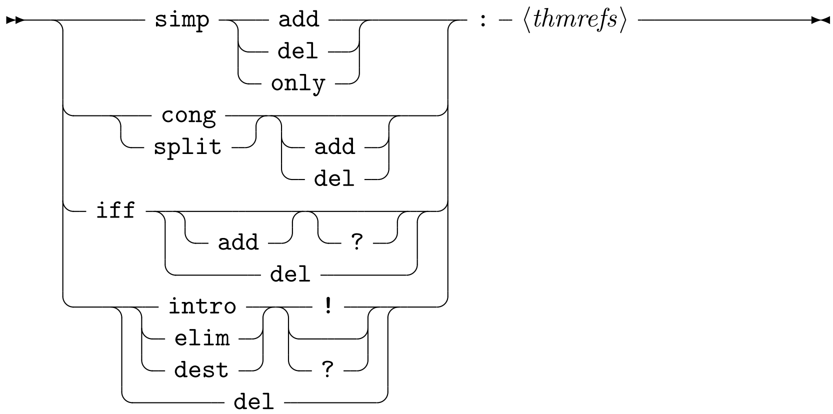
apply(gen_test_cases "prog" simp: max_def)

The **gen_test_cases** : *method* tactic allows to control the test case generation in a fine-granular manner:



where ⟨*depth*⟩ is a natural number describing the depth of the generated test cases and ⟨*breadth*⟩ is a natural number describing their breadth. Roughly speaking, the ⟨*depth*⟩ controls the term size in data separation lemmas in order to establish a regularity hypothesis (see [6] for details), while the ⟨*breadth*⟩ controls the number of variables occurring in the test specification for which regularity hypotheses are generated. The default for ⟨*depth*⟩ and ⟨*breadth*⟩ is 3 resp. 1. ⟨*progname*⟩ denotes the name of the program under test. Further, one can control the classifier and simplifier sets used internally in the **gen_test_cases** tactic using the optional ⟨*clasimpmod*⟩ option:

⟨*clamsimpmod*⟩ ::=

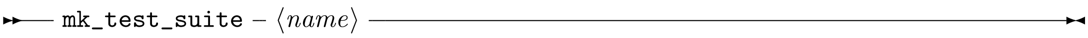


The generated test cases can be further processed, e. g., simplified using the usual Isabelle/HOL tactics.

Creating a test suite: HOL-TestGen provides a kind of container, called *test-suites*, which store all relevant logical and configuration information related to a particular test-scenario. Test-suites were initially created after generating the test cases (and test hypotheses); you should store your result of the derivation, usually the test-theorem which is the output of the test-generation phase, in a test suite by:

mk_test_suite "max_test"

for further processing. This is done using the **mk_test_suite** : *proof*(*prove*) → *proof*(*prove*) | *theory* command which also closes the actual "proof state" (or *test state*. Its syntax is given by:



where ⟨*name*⟩ is a fresh identifier which is later used to refer to this test state. This name is even used at the very end of the test driver generation phase, when test-executions are performed (externally to HOL-TestGen in a shell). Isabelle/HOL can access the corresponding test theorem using the identifier ⟨*name*⟩.test_thm, e. g.:

thm max_test.test_thm

Generating test data: In a next step, the test cases can be refined to concrete test data:

gen_test_data "max_test"

The *gen_test_data* : *theory*|*proof* → *theory*|*proof* command takes only one parameter, the name of the test suite for which the test data should be generated:

▸▸— gen_test_data – ⟨*name*⟩ ——————————————————————▸◂

After the successful execution of this command Isabelle can access the test hypothesis using the identifier ⟨*name*⟩.test_hyps and the test data using the identifier ⟨*name*⟩.test_data

thm max_test.test_hyps
thm max_test.concrete_test

In our concrete example, we get the output:

THYP ((∃x xa. x ≤xa ∧prog x xa = xa) ⟶(∀x xa. x ≤xa ⟶prog x xa = xa))
THYP ((∃x xa. ¬x ≤xa ∧ prog x xa = x) ⟶(∀x xa. ¬ x ≤xa ⟶prog x xa = x))

as well as :

prog −9 −3 = −3
prog −5 −8 = −5

By default, generating test data is done by calling the *random solver*. This is fine for such a simple example, but as explained in the introduction, this is far incomplete when the involved data-structures become more complex. To handle them, HOL-TestGen also comes with a more advanced data generator based on *SMT solvers* (using their integration in Isabelle, see e. g. [4]).

To turn on SMT-based data generation, use the following option:

declare [[testgen_SMT]]

(which is thus set to false by default). It is also recommenced to turn off the random solver:

declare [[testgen_iterations =0]]

In order for the SMT solver to know about constant definitions and properties, one needs to feed it with these definitions and lemmas. For instance, if the test case involves some inductive function foo, you can provide its definition to the solver using:

declare foo.simps [testgen_smt_facts]

as well as related properties (if needed).

A complete description of the configuration options can be found below.

Exporting test data: After the test data generation, HOL-TestGen is able to export the test data into an external file, e. g.:

export_test_data "test_max.dat" "max_test"

exports the generated test data into a file `text_max.dat`. The generation of a test data file is done using the *export_test_data* : *theory*|*proof* → *theory*|*proof* command:

`export_test_data` – ⟨*filename*⟩ – ⟨*name*⟩ [⟨*smlprogname*⟩]

where ⟨*filename*⟩ is the name of the file in which the test data is stored and ⟨*name*⟩ is the name of a collection of test data in the test environment.

Generating test scripts: After the test data generation, HOL-TestGen is able to generate a test script, e. g.:

gen_test_script "test_max.sml" "max_test" "prog"
"myMax.max"

produces the test script shown in Table 4.2 that (together with the provided test harness) can be used to test real implementations. The generation of test scripts is done using the *generate_test_script* : *theory*|*proof* → *theory*|*proof* command:

`gen_test_script` – ⟨*filename*⟩ – ⟨*name*⟩ – ⟨*progname*⟩ [⟨*smlprogname*⟩]

where ⟨*filename*⟩ is the name of the file in which the test script is stored, and ⟨*name*⟩ is the name of a collection of test data in the test environment, and ⟨*progname*⟩ the name of the program under test. The optional parameter ⟨*smlprogname*⟩ allows for the configuration of different names of the program under test that is used within the test script for calling the implementation.

Alternatively, the code-generator can be configured to generate test-driver code in other progamming languages, see below.

Configure HOL-TestGen: The overall behavior of test data and test script generation can be configured, e. g.

declare [[testgen_iterations =15]]

The parameters (all prefixed with testgen_) have the following meaning:

depth:	Test-case generation depth. Default: 3.
breadth:	Test-case generation breadth. Default: 1.
bound:	Global bound for data statements. Default: 200.
case_breadth:	Number of test data per case, weakening uniformity. Default: 1.
iterations:	Number of attempts during random solving phase. Default: 25. Set to 0 to turn off the random solver.
gen_prelude:	Generate datatype specific prelude. Default: true.
gen_wrapper:	Generate wrapper/logging-facility (increases verbosity of the generated test script). Default: true.
SMT:	If set to "true" external SMT solvers (e.g., Z3) are used during test-case generation. Default: false.

```
structure TestDriver : sig end = struct
  val return    = ref ~63;
  fun eval x2 x1 = let
                       val ret = myMax.max x2 x1
                   in
                      ((return := ret);ret)
                   end
  fun retval () = SOME(!return);
  fun toString a = Int.toString a;
  val testres   = [];

  val pre_0  = [];
  val post_0 = fn () => ( (eval ~23 69 = 69));
  val res_0  = TestHarness.check retval pre_0 post_0;
  val testres = testres@[res_0];

  val pre_1  = [];
  val post_1 = fn () => ( (eval ~11 ~15 = ~11));
  val res_1  = TestHarness.check retval pre_1 post_1;
  val testres = testres@[res_1];

  val _ = TestHarness.printList toString testres;
end
```

Table 4.2.: Test Script

smt_facts:	Add a theorem to the SMT-based data generator basis.
toString:	Type-specific SML-function for converting literals into strings (e.g., `Int.toString`), used for generating verbose output while executing the generated test script. Default: `""`.
setup_code:	Customized setup/initialization code (copied verbatim to generated test script). Default: `""`.
dataconv_code:	Customized code for converting datatypes (copied verbatim to generated test script). Default: `""`.
type_range_bound:	Bound for choosing type instantiation (effectively used elements type grounding list). Default: 1.
type_candidates:	List of types that are used, during test script generation, for instantiating type variables (e.g., α list). The ordering of the types determines their likelihood of being used for instantiating a polymorphic type. Default: [int, unit, bool, int set, int list]

Configuring the test data generation: Further, an attribute *test* : *attribute* is provided, i. e.:

```
lemma max_abscase [test "maxtest"]:"max 4 7 = 7"
```

or

```
structure myMax = struct
  fun max x y = if (x < y) then y else x
end
```

Table 4.3.: Implementation in SML of max

declare max_abscase [test "maxtest"]

that can be used for hierarchical test case generation:

test – ⟨*name*⟩

4.3. Test Execution and Result Verification

In principle, any SML-system, e. g. [24, 22, 23, 19, 20], should be able to run the provided test-harness and generated test-script. Using their specific facilities for calling foreign code, testing of non-SML programs is possible. For example, one could test

- implementations using the .Net platform (more specific: CLR IL), e. g. written in C# using sml.net [23],
- implementations written in C using, e. g. the foreign language interface of sml/NJ [24] or MLton [20],
- implementations written in Java using mlj [19].

Also, depending on the SML-system, the test execution can be done within an interpreter (it is even possible to execute the test script within HOL-TestGen) or using a compiled test executable. In this section, we will demonstrate the test of SML programs (using SML/NJ or MLton) and ANSI C programs.

4.3.1. Testing an SML-Implementation

Assume we have written a max-function in SML (see Table 4.3) stored in the file `max.sml` and we want to test it using the test script generated by HOL-TestGen. Following Figure 4.1 we have to build a test executable based on our implementation, the generic test harness (`harness.sml`) provided by HOL-TestGen, and the generated test script (`test_max.sml`), shown in Table 4.2.

If we want to run our test interactively in the shell provided by sml/NJ, we just have to issue the following commands:

```
use "harness.sml";
use "max.sml";
use "test_max.sml";
```

After the last command, sml/NJ will automatically execute our test and you will see a output similar to the one shown in Table 4.4.

If we prefer to use the compilation manager of sml/NJ, or compile our test to a single test executable using MLton, we just write a (simple) file for the compilation manager of sml/NJ (which is understood both, by MLton and sml/NJ) with the following content:

```
Test Results:
=============
Test 0 -     SUCCESS, result:  69
Test 1 -     SUCCESS, result: ~11


Summary:
--------
Number successful tests cases:  2 of 2 (ca. 100%)
Number of warnings:             0 of 2 (ca. 0%)
Number of errors:               0 of 2 (ca. 0%)
Number of failures:             0 of 2 (ca. 0%)
Number of fatal errors:         0 of 2 (ca. 0%)

Overall result: success
===============
```

Table 4.4.: Test Trace

```
Group  is
harness.sml
max.sml
test_max.sml

#if(defined(SMLNJ_VERSION))
  $/basis.cm
  $smlnj/compiler/compiler.cm
#else
#endif
```

and store it as `test.cm`. We have two options, we can

- use sml/NJ: we can start the sml/NJ interpreter and just enter

  ```
  CM.make("test.cm")
  ```

  which will build a test setup and run our test.

- use MLton to compile a single test executable by executing

  ```
  mlton test.cm
  ```

  on the system shell. This will result in a test executable called `test` which can be directly executed.

In both cases, we will get a test output (test trace) similar to the one presented in Table 4.4.

```
int max (int x, int y) {
        if (x < y) {
                return y;
        }else{
                return x;
        }
}
```

Table 4.5.: Implementation in ANSI C of max

4.3.2. Testing Non-SML Implementations

Suppose we have an ANSI C implementation of max (see Table 4.5) that we want to test using the foreign language interface provided by MLton. First we have to import the max method written in C using the `_import` keyword of MLton. Further, we provide a "wrapper" function doing the pairing of the curried arguments:

```
structure myMax = struct
  val cmax   = _import "max": int * int -> int ;
  fun max a b = cmax(a,b);
end
```

We store this file as `max.sml` and write a small configuration file for the compilation manager:

```
Group  is
harness.sml
max.sml
test_max.sml
```

We can compile a test executable by the command

```
mlton -default-ann 'allowFFI true' test.cm max.c
```

on the system shell. Again, we end up with an test executable `test` which can be called directly. Running our test executable will result in trace similar to the one presented in Table 4.4.

4.4. Profiling Test Generation

HOL-TestGen includes support for profiling the test procedure. By default, profiling is turned off. Profiling can be turned on by issuing the command

```
profiling_on
```

Profiling can be turned off again with the command

```
profiling_off
```

When profiling is turned on, the time consumed by gen_test_cases and gen_test_data is recorded and associated with the test theorem. The profiling results can be printed by

```
print_clocks
```

A LaTeX version of the profiling results can be written to a file with the command

►►— `write_clocks` – ⟨*filename*⟩ —►◄

Users can also record the runtime of their own code. A time measurement can be started by issuing

►►— `start_clock` – ⟨*name*⟩ —►◄

where ⟨*name*⟩ is a name for identifying the time measured. The time measurement is completed by

►►— `stop_clock` – ⟨*name*⟩ —►◄

where ⟨*name*⟩ has to be the name used for the preceding start_clock. If the names do not match, the profiling results are marked as erroneous. If several measurements are performed using the same name, the times measured are added. The command

►►— `next_clock` —►◄

proceeds to a new time measurement using a variant of the last name used.

These profiling instructions can be nested, which causes the names used to be combined to a path. The `Clocks` structure provides the tactic analogues `start_clock_tac`, `stop_clock_tac` and `next_clock_tac` to these commands. The profiling features available to the user are independent of HOL-TestGen's profiling flag controlled by profiling_on and profiling_off.

5. Examples

5.1. List

5.1.1. Testing List Properties

theory
  List-test
imports
  List
    ../../../src/codegen-fsharp/Code-Integer-Fsharp
  ../../../src/Testing

begin

In this example we present the current main application of HOL-TestGen: generating test data for black box testing of functional programs within a spec ification based unit test. We use a simple scenario, developing the test theory for testing sorting algorithms over lists, develop test specifications (elsewhere called test targets or test goals), and explore the different possibilities.

A First Model and a Quick Walk Through

In the following we give a first impression of how the testing process using HOL-TestGen looks like. For brevity we stick to default parameters and explain possible decision points and parameters where the testing can be improved in the next section.

Writing the Test Specification We start by specifying a primitive recursive predicate describing sorted lists:

primrec *is-sorted*:: *int list* $\Rightarrow$ *bool*
  where *is-sorted* [] = *True* |
    is-sorted $(x\#xs)$ = (*case xs of*
      [] $\Rightarrow$ *True*
      | $y\#ys \Rightarrow x \leq y \wedge$ *is-sorted xs*)

We will use this HOL predicate for describing our test specification, i.e. the properties our implementation should fulfill:

test-spec *is-sorted*$(PUT\ l)$

where *prog* is a "placeholder" for our program under test.

However, for the code-generation necessary to generate a test-driver and actually *run* the test of an external program, the *program under test* or `PUT` for short, it is sensible to represent the latter as an un-interpreted constant; the code-generation will later on tweaked such that the place-holder in the test-driver code is actually linked to the real, external program which is a black box from the point of view of this model (the testing procedure needs actually only executable code).

consts *SUT* :: *'a list* ⇒ *'a list*

Note that any other name would do the trick as well.

Generating test cases Now we can automatically generate *test cases*. Using the default setup, we just apply our *gen-test-cases*:

declare *PO-def* [*simp del*] **apply**(*gen-test-cases 3 1 SUT*)

which leads to the test partitioning one would expect:

1. *is-sorted* (*SUT* [])
2. *THYP* (*is-sorted* (*SUT* []) ⟶ *is-sorted* (*SUT* []))
3. *is-sorted* (*SUT* [*??X8X31*])
4. *THYP* ((∃ *x*. *is-sorted* (*SUT* [*x*])) ⟶ (∀ *x*. *is-sorted* (*SUT* [*x*])))
5. *is-sorted* (*SUT* [*??X6X25*, *??X5X24*])
6. *THYP*
   ((∃ *x xa*. *is-sorted* (*SUT* [*xa*, *x*])) ⟶ (∀ *x xa*. *is-sorted* (*SUT* [*xa*, *x*])))
7. *is-sorted* (*SUT* [*??X3X17*, *??X2X16*, *??X1X15*])
8. *THYP*
   ((∃ *x xa xb*. *is-sorted* (*SUT* [*xb*, *xa*, *x*])) ⟶
   (∀ *x xa xb*. *is-sorted* (*SUT* [*xb*, *xa*, *x*])))
9. *THYP* (*3* < *length l* ⟶ *is-sorted* (*SUT l*))

Now we bind the test theorem to a particular named *test environment*.

mk-test-suite *is-sorted-result*

The current test theorem contains holes, that correspond to the concrete data of the test that have not been generated yet

thm *is-sorted-result.test-thm*

Generating test data Now we want to generate concrete test data, i.e. all variables in the test cases must be instantiated with concrete values. This involves a random solver which tries to solve the constraints by randomly choosing values.

thm *is-sorted-result.test-thm*
gen-test-data *is-sorted-result*
thm *is-sorted-result.test-inst-thm*

Which leads to the following test data:

is-sorted (*SUT* [])
is-sorted (*SUT* [*10*])
is-sorted (*SUT* [*3*, *10*])
is-sorted (*SUT* [−*8*, −*3*, −*3*])

Note that by the following statements, the test data, the test hypotheses and the test theorem can be inspected interactively.

thm *is-sorted-result.concrete-tests*
thm *is-sorted-result.test-hyps*

The generated test data can be exported to an external file:

export-test-data *impl/data/test-data.data is-sorted-result*

Test Execution and Result Verification In principle, any SML-system should be able to run the provided test-harness and generated test-script. Using their specific facilities for calling foreign code, testing of non-SML programs is possible. For example, one could test implementations written

- for the.Net platform, e.g., written in C# using sml.net [23],
- in C using, e.g. the foreign language interface of sml/NJ [24] or MLton [20],
- in Java using MLj [19].

Depending on the SML-system, the test execution can be done within an interpreter or using a compiled test executable. Testing implementations written in SML is straight-forward, based on automatically generated test scripts. This generation is based on the internal code generator of Isabelle and must be set up accordingly.

The the following, we show the general generation of test-scripts (part of the finally generated test-driver) in different languages; finally, we will concentrate on the test-generation scenario for C.

code-printing
  constant *SUT* => (*Fsharp*) *myList.sort*
    and (*SML*) *myList.sort*
    and (*Scala*) *myList.sort*

generate-test-script *is-sorted-result*
thm *is-sorted-result.test-script*

Testing an SML implementation:

export-code *is-sorted-result.test-script* **in** *SML*
module-name *TestScript* **file** *impl/sml/is-sorted-test-script.sml*

We use the SML test script also for testing an implementation written in C:

export-code *is-sorted-result.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/is-sorted-test-script.sml*

Testing an F# implementation:

export-code *is-sorted-result.test-script* **in** *Fsharp*
module-name *TestScript* **file** *impl/fsharp/is-sorted-test-script.fs*

We use the F# test script also for testing an implementation written in C#:

export-code *is-sorted-result.test-script* **in** *Fsharp*
module-name *TestScript* **file** *impl/csharp/is-sorted-test-script.fs*

Testing a Scala implementation:

export-code *is-sorted-result.test-script* **in** *Scala*

module-name *TestScript* **file** *impl/scala/is-sorted-test-script.scala*

We use the Scala script also for testing an implementation written in Java:

export-code *is-sorted-result.test-script* **in** *Scala*
module-name *TestScript* **file** *impl/java/is-sorted-test-script.scala*

Finally, we export the raw test data in an XML-like format:

export-test-data *impl/data/is-sorted-test-data.dat is-sorted-result*

which generates the following test harness:

In the following, we assume an ANSI C implementation of our sorting method for sorting C arrays that we want to test. (In our example setup, it is contained in the file `impl/c/sort.c`.) Using the foreign language interface provided by the SML compiler MLton we first have to import the sort method written in C using the `_import` keyword of MLton and further, we provide a "wrapper" doing some datatype conversion, e.g. converting lists to arrays and vice versa:

```
structure myList = struct
  val csort = _import "sort": int array * int -> int array;
              (* this is the link to the external, "black-box" program *)
  fun ArrayToList a = Array.foldl (op ::) [] a;
  fun sort_list list = ArrayToList (csort(Array.fromList(list),(length list)));
  fun sort list = map IntInf.fromInt (sort_list (map IntInf.toInt list))
end
```

That's all, now we can build the test executable using MLton and end up with a test executable which can be called directly. In `impl/c`, the process of

1. compiling the generated `is_sorted_test_script.sml`, the test harness (`harness.sml`), a main routine (`main.sml`) and this wrapper `myList` (contained in the generated `List.sml`) to to a combined test-driver in C,

2. compiling the C test-driver and linking it to the program under test `sort.c`, and

3. executing the test

is captured in a `Makefile`. So: executes the test and displays a test-statistic as shown in Table 5.1 on the facing page.

A Refined Model and Improved Test-Results

Obviously, in reality one would not be satisfied with the test cases generated in the previous section: for testing sorting algorithms one would expect that the test data somehow represents the set of permutations of the list elements. We have already seen that the test specification used in the last section "only" enumerates lists up to a specific length without any ordering constraints on their elements. What is missing, is a test that input and output sequence are in fact *permutations* of each other. We could state for example :

fun *del-member* :: $'a \Rightarrow\ 'a\ list \Rightarrow\ 'a\ list\ option$
where *del-member* $x\ [] = None$

```
>make
mlton -default-ann 'allowFFI true' is_sorted_test.mlb sort.c
./is_sorted_test


Test Results:
=============
Test 0 -     SUCCESS
Test 1 -     SUCCESS
Test 2 -     SUCCESS
Test 3 -     SUCCESS
Test 4 -     SUCCESS
Test 5 -     SUCCESS
Test 6 -     SUCCESS


Summary:
--------
Number successful tests cases: 7 of 7 (ca. 100%)
Number of warnings:             0 of 7 (ca. 0%)
Number of errors:               0 of 7 (ca. 0%)
Number of failures:             0 of 7 (ca. 0%)
Number of fatal errors:         0 of 7 (ca. 0%)

Overall result: success
===============
```

Table 5.1.: A Sample Test Trace: The ascending property tested.

$|$*del-member* x $(y \# S) = ($*if* $x = y$ *then Some* S
else case del-member x S *of*
None $\Rightarrow$ *None*
$|$ *Some* S' $\Rightarrow$ *Some*$(y \# S'))$

fun *is-permutation* :: $'a$ *list* $\Rightarrow$ $'a$ *list* $\Rightarrow$ *bool*
where *is-permutation* [] [] = *True*
$|$*is-permutation* $(a\#S)(a'\#S')$ =(*if* $a = a'$ *then is-permutation* S S'
else case del-member a S' *of*
None $\Rightarrow$ *False*
$|$ *Some* S'' $\Rightarrow$ *is-permutation* S $(a'\#S''))$
$|$*is-permutation* - - = *False*

fun *is-perm* :: $'a$ *list* $\Rightarrow$ $'a$ *list* $\Rightarrow$ *bool*
where *is-perm* [] [] = *True*
$|$*is-perm* [] T = *False*
$|$*is-perm* $(a\#S)$ T = (*if length* T = *length* $S + 1$
then is-perm S (*remove1* a T)
else False)

value *is-perm* [*1*,*2*,*3*::*int*] [*3*,*1*,*2*]

A test for permutation, that not is hopelessly non-constructive like "the existence of a bijection on the indexes [0 .. n-1], that is pairwise mapped to the list" or the like, is obviously quite complex; the apparent "mathematical specification" is not always the easiest. We convince ourselves that the predicate *is-permutation* indeed captures our intuition by animations of the definition:

value *is-permutation* [*1*,*2*,*3*] [*3*,*2*,*1*::*nat*]
value ¬ *is-permutation* [*1*,*2*,*3*] [*3*,*1*::*nat*]
value ¬ *is-permutation* [*2*,*3*] [*3*,*2*,*1*::*nat*]
value ¬ *is-permutation* [*1*,*2*,*1*,*3*] [*3*,*2*,*1*::*nat*]
value *is-permutation* [*2*,*1*,*3*] [*1*::*nat*,*3*,*2*]

value *is-perm* [*1*,*2*,*3*] [*3*,*2*,*1*::*nat*]
value ¬ *is-perm* [*1*,*2*,*3*] [*3*,*1*::*nat*]
value ¬ *is-perm* [*2*,*3*] [*3*,*2*,*1*::*nat*]
value ¬ *is-perm* [*1*,*2*,*1*,*3*] [*3*,*2*,*1*::*nat*]
value *is-perm* [*2*,*1*,*3*] [*1*::*nat*,*3*,*2*]

... which are all executable and thus were compiled and all evaluated to true.

Based on these concepts, a test-specification is straight-forward and easy:

declare [[*goals-limit*=*5*]]
apply(*gen-test-cases 5 1 SUT*)
mk-test-suite *ascending-permutation-test*

A quick inspection of the test theorem reveals that there are in fact no relevant constraints to solve, so test-data selection is easy:

declare [[*testgen-iterations*=*100*]]
gen-test-data *ascending-permutation-test*
thm *ascending-permutation-test.concrete-tests*

Again, we convert this into test-scripts that can be compiled to a test-driver.

generate-test-script *ascending-permutation-test*
thm *ascending-permutation-test.test-script*

We use the SML implementation also for testing an implementation written in C:

export-code *ascending-permutation-test.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/ascending-permutation-test-script.sml*

Try `make run_ascending_permutation` in directory `impl/c` to compile and execute the generated test-driver.

A Test-Specification based on a Comparison with a Reference Implementation

We might opt for an alternative modeling approach: Thus we decide to try a more ''descriptive'' test specification that is based on the behavior of an insertion sort algorithm:

fun $ins :: ('a::linorder) \Rightarrow 'a\ list \Rightarrow 'a\ list$
where $ins\ x\ []=[x]$
$|ins\ x\ (y\#ys) = (if\ (x < y)\ then\ x\#y\#ys\ else\ (y\#(ins\ x\ ys)))$
fun $sort:: ('a::linorder)\ list \Rightarrow 'a\ list$
where $sort\ [] = []$
$|sort\ (x\#xs) = ins\ x\ (sort\ xs)$

Now we state our test specification by requiring that the behavior of the program under test PUT is identical to the behavior of our specified sorting algorithm $sort$:

Based on this specification `gen_test_cases` produces test cases representing all permutations of lists up to a fixed length n. Normally, we also want to configure up to which length lists should be generated (we call this the *depth* of test case), e.g. we decide to generate lists up to length 3. Our standard setup

declare [[*goals-limit=100*]]
test-spec $sort\ l = PUT\ \ l$
apply(*gen-test-cases PUT*)
mk-test-suite *is-sorting-algorithm0*

generates 9 test cases describing all permutations of lists of length $1, 2$ and 3. "Permutation" means here that not only test cases (i.e. I/O-partitions) are generated for lists of length 0, 1, 2 and 3; the partitioning is actually finer: for two-elementary lists, for example, the case of a list with the first element larger or equal and the dual case are distinguished. The entire test-theorem looks as follows:

$[\![[] = PUT\ []; THYP\ ([] = PUT\ [] \longrightarrow [] = PUT\ []); [??X31X177] = PUT\ [??X31X177]; THYP\ ((\exists x.\ [x] = PUT\ [x]) \longrightarrow (\forall x.\ [x] = PUT\ [x])); PO\ (??X29X169 < ??X28X168); [??X29X169,\ ??X28X168] = PUT\ [??X29X169,\ ??X28X168]; THYP\ ((\exists x\ xa.\ xa < x \wedge [xa, x] = PUT\ [xa, x]) \longrightarrow (\forall x\ xa.\ xa < x \longrightarrow [xa, x] = PUT\ [xa, x])); PO\ (\neg\ ??X26X158 < ??X25X157); [??X25X157,\ ??X26X158] = PUT\ [??X26X158,\ ??X25X157]; THYP\ ((\exists x\ xa.\ \neg\ xa < x \wedge [x, xa] = PUT\ [xa, x]) \longrightarrow (\forall x\ xa.\ \neg\ xa < x \longrightarrow [x, xa] = PUT\ [xa, x])); PO\ ((??X22X144 < ??X21X143 \wedge ??X23X145 < ??X21X143) \wedge ??X23X145 < ??X22X144); [??X23X145,\ ??X22X144,\ ??X21X143] = PUT\ [??X23X145,\ ??X22X144,\ ??X21X143]; THYP\ ((\exists x\ xa\ xb.\ xa < x \wedge xb < x \wedge xb < xa \wedge [xb,\ xa,\ x] = PUT\ [xb,\ xa,\ x]) \longrightarrow (\forall x\ xa\ xb.\ xa < x \longrightarrow xb < x \longrightarrow xb < xa \longrightarrow [xb,\ xa,\ x] = PUT\ [xb,\ xa,\ x])); PO\ ((\neg$

$??X18X127 < ??X17X126 \wedge ??X19X128 < ??X17X126) \wedge ??X19X128 < ??X18X127)$; $[??X19X128, ??X17X126, ??X18X127] = PUT\ [??X19X128, ??X18X127, ??X17X126]$; $THYP\ ((\exists\, x\ xa\ xb.\ \neg\ xa < x \wedge xb < x \wedge xb < xa \wedge [xb, x, xa] = PUT\ [xb, xa, x]) \longrightarrow (\forall\, x\ xa\ xb.\ \neg\ xa < x \longrightarrow xb < x \longrightarrow xb < xa \longrightarrow [xb, x, xa] = PUT\ [xb, xa, x]))$; $PO\ ((\neg\ ??X14X110 < ??X13X109 \wedge \neg\ ??X15X111 < ??X13X109) \wedge ??X15X111 < ??X14X110)$; $[??X13X109, ??X15X111, ??X14X110] = PUT\ [??X15X111, ??X14X110, ??X13X109]$; $THYP\ ((\exists\, x\ xa\ xb.\ \neg\ xa < x \wedge \neg\ xb < x \wedge xb < xa \wedge [x, xb, xa] = PUT\ [xb, xa, x]) \longrightarrow (\forall\, x\ xa\ xb.\ \neg\ xa < x \longrightarrow \neg\ xb < x \longrightarrow xb < xa \longrightarrow [x, xb, xa] = PUT\ [xb, xa, x]))$; $PO\ ((??X10X93 < ??X9X92 \wedge ??X11X94 < ??X9X92) \wedge \neg\ ??X11X94 < ??X10X93)$; $[??X10X93, ??X11X94, ??X9X92] = PUT\ [??X11X94, ??X10X93, ??X9X92]$; $THYP\ ((\exists\, x\ xa\ xb.\ xa < x \wedge xb < x \wedge \neg\ xb < xa \wedge [xa, xb, x] = PUT\ [xb, xa, x]) \longrightarrow (\forall\, x\ xa\ xb.\ xa < x \longrightarrow xb < x \longrightarrow \neg\ xb < xa \longrightarrow [xa, xb, x] = PUT\ [xb, xa, x]))$; $PO\ ((??X6X76 < ??X5X75 \wedge \neg\ ??X7X77 < ??X5X75) \wedge \neg\ ??X7X77 < ??X6X76)$; $[??X6X76, ??X5X75, ??X7X77] = PUT\ [??X7X77, ??X6X76, ??X5X75]$; $THYP\ ((\exists\, x\ xa\ xb.\ xa < x \wedge \neg\ xb < x \wedge \neg\ xb < xa \wedge [xa, x, xb] = PUT\ [xb, xa, x]) \longrightarrow (\forall\, x\ xa\ xb.\ xa < x \longrightarrow \neg\ xb < x \longrightarrow \neg\ xb < xa \longrightarrow [xa, x, xb] = PUT\ [xb, xa, x]))$; $PO\ ((\neg\ ??X2X59 < ??X1X58 \wedge \neg\ ??X3X60 < ??X1X58) \wedge \neg\ ??X3X60 < ??X2X59)$; $[??X1X58, ??X2X59, ??X3X60] = PUT\ [??X3X60, ??X2X59, ??X1X58]$; $THYP\ ((\exists\, x\ xa\ xb.\ \neg\ xa < x \wedge \neg\ xb < x \wedge \neg\ xb < xa \wedge [x, xa, xb] = PUT\ [xb, xa, x]) \longrightarrow (\forall\, x\ xa\ xb.\ \neg\ xa < x \longrightarrow \neg\ xb < x \longrightarrow \neg\ xb < xa \longrightarrow [x, xa, xb] = PUT\ [xb, xa, x]))$; $THYP\ (3 < length\ l \longrightarrow List\text{-}test.sort\ l = PUT\ l)]\!] \Longrightarrow (List\text{-}test.sort\ l = PUT\ l)$

A more ambitious setting is:

test-spec *sort l = SUT l*

apply(*gen-test-cases 5 1 SUT*)

which leads after 2 seconds to the following test partitioning (excerpt):

1. $[] = SUT\ []$
2. $THYP\ ([] = SUT\ [] \longrightarrow [] = SUT\ [])$
3. $[??X871X8301] = SUT\ [??X871X8301]$
4. $THYP\ ((\exists\, x.\ [x] = SUT\ [x]) \longrightarrow (\forall\, x.\ [x] = SUT\ [x]))$
5. $PO\ (??X869X8293 < ??X868X8292)$
6. $[??X869X8293, ??X868X8292] = SUT\ [??X869X8293, ??X868X8292]$
7. $THYP$
   $((\exists\, x\ xa.\ xa < x \wedge [xa, x] = SUT\ [xa, x]) \longrightarrow$
   $(\forall\, x\ xa.\ xa < x \longrightarrow [xa, x] = SUT\ [xa, x]))$
8. $PO\ (\neg\ ??X866X8282 < ??X865X8281)$
9. $[??X865X8281, ??X866X8282] = SUT\ [??X866X8282, ??X865X8281]$
10. $THYP$
    $((\exists\, x\ xa.\ \neg\ xa < x \wedge [x, xa] = SUT\ [xa, x]) \longrightarrow$
    $(\forall\, x\ xa.\ \neg\ xa < x \longrightarrow [x, xa] = SUT\ [xa, x]))$

A total of 461 subgoals...

mk-test-suite *permutation-test*

thm *permutation-test.test-thm*

In this scenario, 39 test cases are generated describing all permutations of lists of length $1, 2, 3$ and 4. "Permutation" means here that not only test cases (i.e. I/O-partitions) are generated for lists of length 0, 1, 2, 3, 4; the partitioning is actually finer: for two-elementary lists, take one case for the lists with the first element larger or equal.

The case for all lists of depth 5 is feasible, however, it will already take 8 minutes. The resulting constraints for the test cases are complex and require more intensive effort in resolving.

There are several options for the test-data selection. On can either use the (very old) random solver or the more modern smt interface. (One day, we would also have a nitpick-interface to constsraint solving via bitblasting sub-models of the constraints to SAT.) The random solver, however, finds only 67 instances out of 150 abstract test cases, while smt instantiates all of them:

```
Test theorem (gen_test_data) 'permutation_test': 67 test cases in 2.951 seconds
```

declare [[*testgen-iterations=0*]]
declare [[*testgen-SMT*]]
gen-test-data *permutation-test*
thm *permutation-test.concrete-tests*

generate-test-script *permutation-test*
thm *permutation-test.test-script*

We use the SML implementation also for testing an implementation written in C:

export-code *permutation-test.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/permutation-test-script.sml*

We obtain test cases like:

$[] = SUT\ []$
$[-3] = SUT\ [-3]$
$[-1, 0] = SUT\ [-1, 0]$
$[0, 0] = SUT\ [0, 0]$
$[-2, -1, 0] = SUT\ [-2, -1, 0]$
$[0, 1, 1] = SUT\ [0, 1, 1]$
$[0, 0, 1] = SUT\ [0, 1, 0]$
$[-1, -1, 0] = SUT\ [-1, -1, 0]$
$[-1, 0, 0] = SUT\ [0, -1, 0]$
$[0, 0, 0] = SUT\ [0, 0, 0]$
$[-3, -2, -1, 0] = SUT\ [-3, -2, -1, 0]$
$[-1, 0, 1, 1] = SUT\ [-1, 0, 1, 1]$
$[0, 1, 1, 2] = SUT\ [0, 1, 2, 1]$
$[0, 0, 1, 2] = SUT\ [0, 1, 2, 0]$
$[-2, -2, -1, 0] = SUT\ [-2, -2, -1, 0]$
$[0, 0, 1, 1] = SUT\ [0, 0, 1, 1]$
$[0, 1, 1, 2] = SUT\ [1, 0, 2, 1]$
$[0, 0, 0, 1] = SUT\ [0, 0, 1, 0]$
$[-2, -1, -1, 0] = SUT\ [-2, -1, -1, 0]$
$[-2, -1, 0, 0] = SUT\ [-2, 0, -1, 0]$
$[0, 1, 1, 1] = SUT\ [0, 1, 1, 1]$
$[0, 0, 1, 1] = SUT\ [0, 1, 1, 0]$
$[-2, -1, -1, 0] = SUT\ [-1, -2, -1, 0]$

$[-2,\ -1,\ 0,\ 0] = SUT\ [0,\ -2,\ -1,\ 0]$
$[0,\ 1,\ 1,\ 1] = SUT\ [1,\ 0,\ 1,\ 1]$
$[0,\ 0,\ 1,\ 1] = SUT\ [1,\ 0,\ 1,\ 0]$
$[-2,\ -2,\ -1,\ 0] = SUT\ [-2,\ -1,\ -2,\ 0]$
$[-1,\ -1,\ 0,\ 0] = SUT\ [-1,\ 0,\ -1,\ 0]$
$[-1,\ 0,\ 0,\ 1] = SUT\ [0,\ 1,\ -1,\ 0]$
$[0,\ 0,\ 0,\ 1] = SUT\ [0,\ 1,\ 0,\ 0]$
$[-1,\ -1,\ -1,\ 0] = SUT\ [-1,\ -1,\ -1,\ 0]$
$[-1,\ -1,\ 0,\ 0] = SUT\ [0,\ -1,\ -1,\ 0]$
$[-1,\ 0,\ 0,\ 0] = SUT\ [0,\ 0,\ -1,\ 0]$
$[0,\ 0,\ 0,\ 0] = SUT\ [0,\ 0,\ 0,\ 0]$
$[-4,\ -3,\ -2,\ -1,\ 0] = SUT\ [-4,\ -3,\ -2,\ -1,\ 0]$
$[-2,\ -1,\ 0,\ 1,\ 1] = SUT\ [-2,\ -1,\ 0,\ 1,\ 1]$
$[-1,\ 0,\ 1,\ 1,\ 2] = SUT\ [-1,\ 0,\ 1,\ 2,\ 1]$
$[0,\ 1,\ 1,\ 2,\ 3] = SUT\ [0,\ 1,\ 2,\ 3,\ 1]$
$[0,\ 0,\ 1,\ 2,\ 3] = SUT\ [0,\ 1,\ 2,\ 3,\ 0]$
$[-3,\ -3,\ -2,\ -1,\ 0] = SUT\ [-3,\ -3,\ -2,\ -1,\ 0]$
$[-1,\ -1,\ 0,\ 1,\ 1] = SUT\ [-1,\ -1,\ 0,\ 1,\ 1]$
$[0,\ 0,\ 1,\ 1,\ 2] = SUT\ [0,\ 0,\ 1,\ 2,\ 1]$
$[0,\ 1,\ 1,\ 2,\ 3] = SUT\ [1,\ 0,\ 2,\ 3,\ 1]$
$[0,\ 0,\ 0,\ 1,\ 2] = SUT\ [0,\ 0,\ 1,\ 2,\ 0]$
$[-3,\ -2,\ -2,\ -1,\ 0] = SUT\ [-3,\ -2,\ -2,\ -1,\ 0]$
$[-1,\ 0,\ 0,\ 1,\ 1] = SUT\ [-1,\ 0,\ 0,\ 1,\ 1]$
$[-1,\ 0,\ 1,\ 1,\ 2] = SUT\ [-1,\ 1,\ 0,\ 2,\ 1]$
$[0,\ 1,\ 1,\ 1,\ 2] = SUT\ [0,\ 1,\ 1,\ 2,\ 1]$
$[0,\ 0,\ 1,\ 1,\ 2] = SUT\ [0,\ 1,\ 1,\ 2,\ 0]$
$[-3,\ -2,\ -2,\ -1,\ 0] = SUT\ [-2,\ -3,\ -2,\ -1,\ 0]$
$[-1,\ 0,\ 0,\ 1,\ 1] = SUT\ [0,\ -1,\ 0,\ 1,\ 1]$
$[-1,\ 0,\ 1,\ 1,\ 2] = SUT\ [1,\ -1,\ 0,\ 2,\ 1]$
$[0,\ 1,\ 1,\ 1,\ 2] = SUT\ [1,\ 0,\ 1,\ 2,\ 1]$
$[0,\ 0,\ 1,\ 1,\ 2] = SUT\ [1,\ 0,\ 1,\ 2,\ 0]$
$[-3,\ -3,\ -2,\ -1,\ 0] = SUT\ [-3,\ -2,\ -3,\ -1,\ 0]$
$[0,\ 0,\ 1,\ 2,\ 2] = SUT\ [0,\ 1,\ 0,\ 2,\ 2]$
$[0,\ 0,\ 1,\ 1,\ 2] = SUT\ [0,\ 1,\ 0,\ 2,\ 1]$
$[0,\ 1,\ 1,\ 2,\ 3] = SUT\ [1,\ 2,\ 0,\ 3,\ 1]$
$[0,\ 0,\ 0,\ 1,\ 2] = SUT\ [0,\ 1,\ 0,\ 2,\ 0]$
$[-2,\ -2,\ -2,\ -1,\ 0] = SUT\ [-2,\ -2,\ -2,\ -1,\ 0]$
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If we scale down to only 10 iterations, this is not sufficient to solve all conditions, i.e. we obtain many test cases with unresolved constraints where RSF marks unsolved cases. In these cases, it is unclear if the test partition is empty. Analyzing the generated test data reveals that all cases for lists with length up to (and including) 3 could be solved. From the 24 cases for lists of length 4 only 9 could be solved by the random solver (thus, overall 19 of the 34 cases were solved). To achieve better results, we could interactively increase the number of iterations which reveals that we need to set iterations to 100 to find all solutions reliably.

iterations	5	10	20	25	30	40	50	75	100
solved goals (of 34)	13	19	23	24	25	29	33	33	34

Instead of increasing the number of iterations one could also add other techniques such as

1. deriving new rules that allow for the generation of a simplified test theorem,
2. introducing abstract test cases or
3. supporting the solving process by derived rules.

Running the test (in the current setup: `make run_permutation_test`)against our sample C-program under `impl/c` yields the following result:

```
> make run_permutation_test
mlton -default-ann 'allowFFI true' permutation_test.mlb sort.c
./permutation_test


Test Results:
=============
Test 0 -     SUCCESS
Test 1 -     SUCCESS
Test 2 - *** FAILURE: post-condition false
Test 3 - *** FAILURE: post-condition false
Test 4 - *** FAILURE: post-condition false
Test 5 - *** FAILURE: post-condition false
Test 6 - *** FAILURE: post-condition false
Test 7 - *** FAILURE: post-condition false
Test 8 - *** FAILURE: post-condition false
Test 9 - *** FAILURE: post-condition false
Test 10 - *** FAILURE: post-condition false
Test 11 - *** FAILURE: post-condition false
Test 12 - *** FAILURE: post-condition false
Test 13 - *** FAILURE: post-condition false
Test 14 - *** FAILURE: post-condition false
Test 15 - *** FAILURE: post-condition false
Test 16 - *** FAILURE: post-condition false
Test 17 - *** FAILURE: post-condition false
Test 18 - *** FAILURE: post-condition false
Test 19 - *** FAILURE: post-condition false
Test 20 - *** FAILURE: post-condition false
Test 21 - *** FAILURE: post-condition false
Test 22 -     SUCCESS
Test 23 -     SUCCESS
Test 24 - *** FAILURE: post-condition false
Test 25 - *** FAILURE: post-condition false
Test 26 - *** FAILURE: post-condition false
Test 27 - *** FAILURE: post-condition false
Test 28 - *** FAILURE: post-condition false
Test 29 - *** FAILURE: post-condition false
Test 30 - *** FAILURE: post-condition false
Test 31 - *** FAILURE: post-condition false
Test 32 - *** FAILURE: post-condition false


Summary:
--------
Number successful tests cases: 4 of 33 (ca. 12%)
Number of warnings:             0 of 33 (ca. 0%)
Number of errors:               0 of 33 (ca. 0%)
Number of failures:             29 of 33 (ca. 87%)
Number of fatal errors:         0 of 33 (ca. 0%)

Overall result: failed
===============
```

Table 5.2.: A Sample Test Trace for the Permutation Test Scenario

Summary A comparison of the three scenarios reveals that albeit a reasonable degree of automation in the test generation process, the essence of model-based test case generation remains an *interactive process* that is worth to be documented in a formal test-plan with respect to various aspects: the concrete modeling that is chosen, the precise formulation of the test-specifications (or: test-goals), the configuration and instrumentation of the test-data selection process, the test-driver synthesis and execution. This process can be complemented by proofs establishing equivalences allowing to convert initial test-specifications into more executable ones, or more 'symbolically evaluatable' ones, or that help to reduce the complexity of the constraint- resolution in the test-data selection process.

But the most important aspect remains: what is a good testing model ? Besides the possibility that the test specification simply does not test what the tester had in mind, the test theory and test-specification have a crucial importance on the quality of the generated test data that seems to be impossible to capture automatically.

Non-Inherent Higher-order Testing

HOL-TestGen can use test specifications that contain higher-order operators — although we would not claim that the test case generation is actually higher-order (there are no enumeration schemes for the function space, so function variables are untreated by the test case generation procedure so far).

Just for fun, we reformulate the problem of finding the maximal number in a list as a higher-order problem:

test-spec *foldr max l (0::int) = PUT2 l*
apply(*gen-test-cases PUT2 simp:max-def*)
mk-test-suite *maximal-number*

declare [[*testgen-iterations = 200*]]
gen-test-data *maximal-number*

thm *maximal-number.concrete-tests*

end

5.2. Bank

5.2.1. A Simple Deterministic Bank Model

theory
Bank
imports

../../../src/codegen-fsharp/Code-Integer-Fsharp
../../../src/Testing
begin

The Bank Example: Test of a Distributed Transaction Machine

declare [[*testgen-profiling*]]

The intent of this little example is to model deposit, check and withdraw operations of a little Bank model in pre-postcondition style, formalize them in a setup for HOL-TestGen test sequence generation and to generate elementary test cases for it. The test scenarios will be restricted to strict sequence checking; this excludes aspects of account creation which will give the entire model a protocol character (a create-operation would create an account number, and then all later operations are just refering to this number; thus there would be a dependence between system output and input as in reactive sequence test scenarios.).

Moreover, in this scenario, we assume that the system under test is deterministic.

The theory of Proof-based Sequence Test Methodology can be found in [9].

The state of our bank is just modeled by a map from client/account information to the balance.

type-synonym *client* = *string*

type-synonym *account-no* = *int*

type-synonym *data-base* = (*client* × *account-no*) ⇀ *int*

Operation definitions: Concept A standard, JML or OCL or VCC like interface specification might look like:

```
Init:  forall (c,no) : dom(data_base). data_base(c,no)>=0

op deposit (c : client, no : account_no, amount:nat) : unit
pre  (c,no) : dom(data_base)
post data_base'=data_base[(c,no) := data_base(c,no) + amount]

op balance (c : client, no : account_no) : int
pre  (c,no) : dom(data_base)
post data_base'=data_base and result = data_base(c,no)

op withdraw(c : client, no : account_no, amount:nat) : unit
pre  (c,no) : dom(data_base) and data_base(c,no) >= amount
post data_base'=data_base[(c,no) := data_base(c,no) - amount]
```

Operation definitions: The model as ESFM Interface normalization turns this interface into the following input type:

datatype *in-c* = *deposit client account-no nat*
  | *withdraw client account-no nat*
  | *balance client account-no*

typ *Bank.in-c*

datatype *out-c* = *depositO*| *balanceO nat* | *withdrawO*

fun *precond* :: *data-base* ⇒ *in-c* ⇒ *bool*
where *precond* σ (*deposit c no m*) = ((*c*,*no*) ∈ *dom* σ)
| *precond* σ (*balance c no*) = ((*c*,*no*) ∈ *dom* σ)
| *precond* σ (*withdraw c no m*) = ((*c*,*no*) ∈ *dom* σ ∧ (*int m*) ≤ *the*(σ(*c*,*no*)))

fun *postcond* :: *in-c* ⇒ *data-base* ⇒ (*out-c* × *data-base*) *set*
where *postcond* (*deposit c no m*) σ =
{ (*n*,σ'). (*n* = *depositO* ∧ σ'=σ((*c*,*no*)↦ *the*(σ(*c*,*no*)) + *int m*))}
| *postcond* (*balance c no*) σ =
{ (*n*,σ'). (σ=σ' ∧ (∃ *x*. *balanceO x* = *n* ∧ *x* = *nat*(*the*(σ(*c*,*no*)))))}
| *postcond* (*withdraw c no m*) σ =
{ (*n*,σ'). (*n* = *withdrawO* ∧ σ'=σ((*c*,*no*)↦ *the*(σ(*c*,*no*)) − *int m*))}

definition *init* :: *data-base* ⇒ *bool*
where *init* σ ≡ ∀ *x* ∈ *dom* σ. *the*(σ *x*) ≥ *0*

Constructing an Abstract Program Using the Operators `impl` and `strong_impl`, we can synthesize an abstract program right away from the specification, i.e. the pair of pre- and postcondition defined above. Since this program is even deterministic, we will derive a set of symbolic execution rules used in the test case generation process which will produce symbolic results against which the PUT can be compared in the test driver.

lemma *precond-postcond-implementable*:
implementable precond postcond
apply(*auto simp*: *implementable-def*)
apply(*case-tac* ι, *simp-all*)
done

Based on this input-output specification, we construct the system model as the canonical completion of the (functional) specification consisting of pre- and post-conditions. *Canonical completion* means that the step function explicitely fails (returns *None*) if the precondition fails; this makes it possible to to treat sequential execution failures in a uniform way. The system *SYS* can be seen as the step function in an input-output automata or, alternatively, a kind of Mealy machine over symbolic states, or, as an extended finite state machine.

definition *SYS* :: *in-c* ⇒(*out-c*, *data-base*)MON_{SE}
where *SYS* = (*strong-impl precond postcond*)

The combinator *strong-impl* turns the pre-post pair in a suitable step functions with the aforementioned characteristics for failing pre-conditions.

Prerequisites

Proving Symbolic Execution Rules for the Abstractly Program The following lemmas reveal that this "constructed" program is actually (due to determinism of the spec):

lemma *Eps-split-eq'* : (*SOME* (x', y'). x'= *x* ∧ y'= *y*) = (*SOME* (x', y'). *x* = x' ∧ *y* = y')
by(*rule arg-cong*[*of* - - *Eps*], *auto*)

deposit

interpretation *deposit* : *efsm-det*
$\quad$ *precond postcond SYS* (*deposit c no m*) λ-. *depositO*
$\quad$ $\lambda\, \sigma.\ \sigma((c,\ no) \mapsto\ (the(\sigma(c,\ no)) + int\ m))\ \lambda\, \sigma.\ ((c,\ no) \in dom\ \sigma)$
$\quad$ **by** *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

find-theorems *name*:*deposit*

withdraw

interpretation *withdraw* : *efsm-det*
$\quad$ *precond postcond SYS* (*withdraw c no m*) λ-. *withdrawO*
$\quad$ $\lambda\, \sigma.\ \sigma((c,\ no) \mapsto\ (the(\sigma(c,\ no)) - int\ m))\ \lambda\, \sigma.((c,\ no) \in dom\ \sigma) \wedge (int\ m) \leq the(\sigma(c,no))$
$\quad$ **by** *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

balance

interpretation *balance* : *efsm-det*
$\quad$ *precond postcond SYS* (*balance c no*) $\lambda\sigma.$ (*balanceO* (*nat*(*the*(σ(*c*, *no*)))))
$\quad$ $\lambda\, \sigma.\ \sigma\ \lambda\, \sigma.\ ((c,\ no) \in dom\ \sigma)$
$\quad$ **by** *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

Now we close the theory of symbolic execution by *exluding* elementary rewrite steps on $mbind_{FailSave}$, i.e. the rules $mbind_{FailSave}$ [] *?iostep* $?\sigma$ = *Some* ([], $?\sigma$) $mbind_{FailSave}$ (*?a* # *?S*) *?iostep* $?\sigma$ = (*case ?iostep ?a* $?\sigma$ *of None* $\Rightarrow$ *Some* ([], $?\sigma$) | *Some* (*out*, σ') $\Rightarrow$ *case* $mbind_{FailSave}$ *?S ?iostep* σ' *of None* $\Rightarrow$ *Some* ([*out*], σ') | *Some* (*outs*, σ'') $\Rightarrow$ *Some* (*out* # *outs*, σ''))

declare *mbind.simps*(*1*) [*simp del*]
$\quad$ *mbind.simps*(*2*) [*simp del*]

Here comes an interesting detail revealing the power of the approach: The generated sequences still respect the preconditions imposed by the specification - in this case, where we are talking about a client for which a defined account exists and for which we will never produce traces in which we withdraw more money than available on it.

Restricting the Test-Space by Test Purposes We introduce a constraint on the input sequence, in order to limit the test-space a little and eliminate logically possible, but irrelevant test-sequences for a specific test-purpose. In this case, we narrow down on test-sequences concerning a specific client c with a specific bank-account number *no*.

We make the (in this case implicit, but as constraint explicitly stated) test hypothesis, that the *SUT* is correct if it behaves correct for a single client. This boils down to the assumption that they are implemented as atomic transactions and interleaved processing does not interfere with a single thread.

fun *test-purpose* :: [*client*, *account-no*, *in-c list*] $\Rightarrow$ *bool*
where
$\quad$ *test-purpose c no* [*balance c' no'*] = ($c=c' \wedge no=no'$)
| *test-purpose c no* ((*deposit c' no' m*)#*R*) = ($c=c' \wedge no=no' \wedge$ *test-purpose c no R*)
| *test-purpose c no* ((*withdraw c' no' m*)#*R*) = ($c=c' \wedge no=no' \wedge$ *test-purpose c no R*)
| *test-purpose c no* - = *False*

lemma [*simp*] : *test-purpose c no* [*a*] = (*a* = *balance c no*)
by(*cases a*, *auto*)

lemma [*simp*] : $R \neq [] \Longrightarrow$ *test-purpose c no* $(a\#R) =$
$(((\exists\, m.\ a = (\mathit{deposit}\ c\ \mathit{no}\ m)) \vee (\exists\, m.\ a = (\mathit{withdraw}\ c\ \mathit{no}\ m)))$
$\wedge$ *test-purpose c no R*)
apply(*simp add*: *List.neq-Nil-conv*, *elim exE*,*simp*)
by(*cases a*, *auto*)

The TestGen Setup The default configuration of `gen_test_cases` does *not* descend into sub-type expressions of type constructors (since this is not always desirable, the choice for the default had been for "non-descent"). This case is relevant here since *in-c list* has just this structure but we need ways to explore the input sequence type further. Thus, we need configure, for all test cases, and derivation descendants of the relusting clauses during splitting, again splitting for all parameters of input type *in-c*:

set-pre-safe-tac⟪
(*fn ctxt* => *TestGen.ALLCASES*(
TestGen.CLOSURE (
TestGen.case-tac-typ ctxt [*Bank.in-c*])))
⟫

Preparation: Miscellaneous We construct test-sequences for a concrete client (implicitely assuming that interleaving actions with other clients will not influence the system behaviour. In order to prevent HOL-TestGen to perform case-splits over names, i. e., list of characters—we define it as constant.

definition c_0 :: *string* **where** $c_0 = ''meyer''$

consts *PUT* :: (*in-c* ⇒(*out-c*, $'\sigma$)MON_{SE})

lemma *HH* : $(A \wedge (A \longrightarrow B)) = (A \wedge\ B)$ **by** *auto*

Small, rewriting based Scenarios including standard code-generation

Exists in two formats : General Fail-Safe Tests (which allows for scenarios with normal *and* exceptional behaviour; and Fail-Stop Tests, which generates Tests only for normal behaviour and correspond to inclusion test refinement.

In the following, we discuss a test-scenario with failsave error semantics; i. e. in each test-case, a sequence may be chosen (by the test data selection) where the client has several accounts. In other words, tests were generated for both *standard* **and *exceptional behaviour*. The splitting technique is general exploration of the type *in-c list*.**

test-spec *test-balance*:
assumes *account-def* : $(c_0,no) \in dom\ \sigma_0$
and *accounts-pos* : *init* σ_0
and *test-purpose* : *test-purpose* c_0 *no S*
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*rule rev-mp*[*OF sym-exec-spec*])
apply(*rule rev-mp*[*OF account-def*])
apply(*rule rev-mp*[*OF accounts-pos*])
apply(*rule rev-mp*[*OF test-purpose*])
apply(*rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process.

apply(*gen-test-cases 5 1 PUT*)

apply(*simp-all add*: *init-def HH split*: *HOL.split-if-asm*)

mk-test-suite *bank-simpleSNXB*
thm *bank-simpleSNXB.test-thm*

And now the Fail-Stop scenario — this corresponds exactly to inclusion tests for normal-behaviour tests: any transition in the model is only possible iff the pre-conditions of the transitions in the model were respected.

declare *Monads.mbind'-bind* [*simp del*]
test-spec *test-balance2*:
assumes *account-def* : $(c_0, no) \in dom\ \sigma_0$
and *accounts-pos* : $init\ \sigma_0$
and *test-purpose* : $test\text{-}purpose\ c_0\ no\ S$
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*rule rev-mp*[*OF sym-exec-spec*])
apply(*rule rev-mp*[*OF account-def*])
apply(*rule rev-mp*[*OF accounts-pos*])
apply(*rule rev-mp*[*OF test-purpose*])
apply(*rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process - variant without uniformity generation.

using[[*no-uniformity*]]
apply(*gen-test-cases 4 1 PUT*)

So lets go for a more non-destructive approach:

using[[*goals-limit=20*]]
apply(*simp-all add*: *init-def HH split*: *HOL.split-if-asm*)

using[[*no-uniformity=false*]]
apply(*tactic TestGen.ALLCASES*(*TestGen.uniformityI-tac* @{*context*} [*PUT*]))
mk-test-suite *bank-simpleNB*
thm *bank-simpleNB.test-thm*

Test-Data Generation

Configuration

declare [[*testgen-iterations=0*]]

declare [[*testgen-SMT*]]

declare c_0*-def* [*testgen-smt-facts*]
declare *mem-Collect-eq* [*testgen-smt-facts*]
declare *Collect-mem-eq* [*testgen-smt-facts*]
declare *dom-def* [*testgen-smt-facts*]
declare *the.simps* [*testgen-smt-facts*]

Test Data Selection for the Normal and Exceptional Behaviour Test Scenario

gen-test-data *bank-simpleSNXB*
thm *bank-simpleSNXB.test-thm*
thm *bank-simpleSNXB.test-inst-thm*
thm *bank-simpleSNXB.concrete-tests*

Test Data Selection for the Normal Behaviour Test Scenario

declare [[*testgen-iterations=0*]]
declare [[*testgen-SMT*]]
gen-test-data *bank-simpleNB*

thm *bank-simpleNB.concrete-tests*
thm *bank-simpleNB.test-inst-thm*

Generating the Test-Driver for an SML and C implementation

The generation of the test-driver is non-trivial in this exercise since it is essentially two-staged: Firstly, we chose to generate an SML test-driver, which is then secondly, compiled to a C program that is linked to the actual program under test. Recall that a test-driver consists of four components:

- `../../../../../harness/sml/main.sml` the global controller (a fixed element in the library),
- `../../../../../harness/sml/main.sml` a statistic evaluation library (a fixed element in the library),
- `bank_simple_test_script.sml` the test-script that corresponds merely one-to-one to the generated test-data (generated)
- `bank_adapter.sml` a hand-written program; in our scenario, it replaces the usual (black-box) program under test by SML code, that calls the external C-functions via a foreign-language interface.

On all three levels, the HOL-level, the SML-level, and the C-level, there are different representations of basic data-types possible; the translation process of data to and from the C-code under test has therefore to be carefully designed (and the sheer space of options is sometimes a pain in the neck). Integers, for example, are represented in two ways inside Isabelle/HOL; there is the mathematical quotient construction and a "numerals" representation providing 'bit-string-representation-behind- the-scene" enabling relatively efficient symbolic computation. Both representations can be compiled "natively" to data types in the SML level. By an appropriate configuration, the code-generator can map "int" of HOL to three different implementations: the SML standard library `Int.int`, the native-C interfaced by `Int32.int`, and the `IntInf.int` from the multi-precision library `gmp` underneath the polyml-compiler.

We do a three-step compilation of data-reresentations model-to-model, model-to-SML, SML-to-C.

A basic preparatory step for the initializing the test-environment to enable code-generation is:

generate-test-script *bank-simpleSNXB*
thm *bank-simpleSNXB.test-script*

generate-test-script *bank-simpleNB*
thm *bank-simpleNB.test-script*

In the following, we describe the interface of the SML-program under test, which is in our scenario an *adapter* to the C code under test. This is the heart of the model-to-SML translation. The the SML-level stubs for the program under test are declared as follows:

consts *balance-stub* :: *string* ⇒ *int* ⇒ (*int*, $'\sigma$)MON_{SE}
code-printing
  constant *balance-stub* => (*SML*) *BankAdapter.balance*

consts *deposit-stub* :: *string* ⇒ *int* ⇒ *int* ⇒ (*unit*, $'\sigma$)MON_{SE}
code-printing
  constant *deposit-stub* => (*SML*) *BankAdapter.deposit*

consts *withdraw-stub*:: *string* ⇒ *int* ⇒ *int* ⇒ (*unit*, $'\sigma$)MON_{SE}
code-printing
  constant *withdraw-stub* => (*SML*) *BankAdapter.withdraw*

Note that this translation step prepares already the data-adaption; the type `nat` is seen as an predicative constraint on integer (which is actually not tested). On the model-to-model level, we provide a global step function that distributes to individual interface functions via stubs (mapped via the code generation to SML ...). This translation also represents uniformly nat by int's.

fun *my-nat-conv* :: *int* ⇒ *nat*
where *my-nat-conv x* =(*if x* <= *0 then 0 else Suc* (*my-nat-conv*(*x* − *1*)))

fun *stepAdapter* :: (*in-c* ⇒(*out-c*, $'\sigma$)MON_{SE})
where
  stepAdapter(*balance name no*) =
      (*x* ← *balance-stub name no*; *return*(*balanceO* (*my-nat-conv x*)))
  | *stepAdapter*(*deposit name no amount*) =
      (- ← *deposit-stub name no* (*int amount*); *return*(*depositO*))
  | *stepAdapter*(*withdraw name no amount*)=
      (- ← *withdraw-stub name no* (*int amount*); *return*(*withdrawO*))

The *stepAdapter* function links the HOL-world and establishes the logical link to HOL stubs which were mapped by the code-generator to adapter functions in SML (which call internally to C-code inside `bank_adapter.sml` via a foreign language interface)

... We configure the code-generator to identify the PUT with the generated SML code implicitely defined by the above *stepAdapter* definition.

code-printing
  constant *PUT* => (*SML*) *stepAdapter*

And there we go and generate the `bank_simple_test_script.sml`:

export-code *stepAdapter bank-simpleSNXB.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/bank-simpleSNXB-test-script.sml*

export-code *stepAdapter bank-simpleNB.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/bank-simpleNB-test-script.sml*

More advanced Test-Case Generation Scenarios

Exploring a bit the limits ...

Rewriting based approach of symbolic execution ... FailSave Scenario

test-spec *test-balance*:
assumes *account-def* : $(c_0,no) \in dom\ \sigma_0$
and *accounts-pos* : *init* σ_0
and *test-purpose* : *test-purpose* c_0 *no S*
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*insert account-def test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1,rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process.

apply(*gen-test-cases 5 1 PUT*)

Symbolic Execution:

apply(*simp-all add*: *HH split*: *HOL.split-if-asm*)

mk-test-suite *bank-large*

gen-test-data *bank-large*
thm *bank-large.concrete-tests*

Rewriting based approach of symbolic execution ... FailSave Scenario

test-spec *test-balance*:
assumes *account-def* : $(c_0,no) \in dom\ \sigma_0$
and *accounts-pos* : *init* σ_0
and *test-purpose* : *test-purpose* c_0 *no S*
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*insert account-def test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1,rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process.

apply(*gen-test-cases 3 1 PUT*)

Symbolic Execution:

apply(*simp-all add*: *HH split*: *HOL.split-if-asm*)

mk-test-suite *bank-large′*

gen-test-data *bank-large′*
thm *bank-large′.concrete-tests*

And now, to compare, elimination based procedures ...

declare *deposit.exec-mbindFSave-If*[*simp del*]
declare *balance.exec-mbindFSave-If*[*simp del*]
declare *withdraw.exec-mbindFSave-If*[*simp del*]
declare *deposit.exec-mbindFStop* [*simp del*]
declare *balance.exec-mbindFStop*[*simp del*]
declare *withdraw.exec-mbindFStop*[*simp del*]

thm *deposit.exec-mbindFSave-E withdraw.exec-mbindFSave-E balance.exec-mbindFSave-E*

test-spec *test-balance*:
assumes *account-defined*: $(c_0, no) \in dom\ \sigma_0$
and *accounts-pos* : *init* σ_0
and *test-purpose* : *test-purpose* c_0 *no S*
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$
apply(*insert account-defined test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1,rule-tac x=x* **in** *spec*[*OF allI*])
using [[*no-uniformity*]]
apply(*gen-test-cases*
3 1 PUT)

apply(*tactic ALLGOALS*(*TestGen.REPEAT′*(*ematch-tac* [@{*thm balance.exec-mbindFStop-E*},
@{*thm withdraw.exec-mbindFStop-E*},
@{*thm deposit.exec-mbindFStop-E*},
@{*thm valid-mbind′-mt*}
])))
apply(*simp-all*)
using[[*no-uniformity=false*]]
apply(*tactic TestGen.ALLCASES*(*TestGen.uniformityI-tac* @{*context*} [*PUT*]))
mk-test-suite *bank-large-very*

Yet another technique: "deep" symbolic execution rules involving knowledge from the model domain. Here: input alphabet must be case-split over deposit, withdraw and balance. This avoids that `gen_test_cases` has to do deep splitting.

theorem *hulk* :
assumes *redex* $: \sigma \models (s \leftarrow (mbind_{FailStop}\ (a\ \#\ S)\ SYS);\ return\ (P\ s))$
and *case-deposit* : $\bigwedge c\ no\ m.\ a = deposit\ c\ no\ m \Longrightarrow (c,\ no) \in dom\ \sigma \Longrightarrow$
$\sigma((c,\ no) \mapsto the\ (\sigma\ (c,\ no)) + int\ m) \models$
$(\ s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ P\ (depositO\ \#\ s)) \Longrightarrow$
Q
and *case-withdraw* : $\bigwedge c\ no\ m.\ \ a = withdraw\ c\ no\ m \Longrightarrow (c,\ no) \in dom\ \sigma \Longrightarrow$
$int\ m \leq the\ (\sigma\ (c,\ no)) \Longrightarrow$
$\sigma((c,no) \mapsto the(\sigma(c,no)) - int\ m) \models$
$(s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ P\ (withdrawO\#s)) \Longrightarrow$
Q
and *case-balance* : $\bigwedge c\ no.\ \ \ (c,\ no) \in dom\ \sigma \Longrightarrow$
$\sigma \models (s \leftarrow mbind_{FailStop}\ S\ SYS;$
$return\ P\ (balanceO\ (nat\ (the\ (\sigma\ (c,\ no))))\ \#\ s)) \Longrightarrow$
Q
shows Q
proof(*cases a*) **print-cases**
  case (*deposit c no m*) **assume** *hyp* : $a = deposit\ c\ no\ m$ **show** Q
    using *hyp redex*
    apply(*simp only*: *deposit.exec-mbindFStop*)
    apply(*rule case-deposit*, *auto*)
    done

next
  case (*withdraw c no m*) **assume** *hyp* : $a = withdraw\ c\ no\ m$ **show** Q
    using *hyp redex*
    apply(*simp only*: *withdraw.exec-mbindFStop*)
    apply(*rule case-withdraw*, *auto*)
    done
next
  case (*balance c no*) **assume** *hyp* : $a = balance\ c\ no$ **show** Q
    using *hyp redex*
    apply(*simp only*: *balance.exec-mbindFStop*)
    apply(*rule case-balance*, *auto*)
    done
qed

Experimental Space

declare[[*testgen-trace*]]
ML⟪ *prune-params-tac*; *Drule.triv-forall-equality*⟫

end

5.2.2. A Simple Non-Deterministic Bank Model

theory
  NonDetBank
imports

../../../src/Testing
begin

declare [[*testgen-profiling*]]

This testing scenario is a modification of the Bank example. The purpose is to explore specifications which are nondetermistic, but at least σ-deterministic, i.e. from the observable output, the internal state can be constructed (which paves the way for symbolic executions based on the specification).

The state of our bank is just modeled by a map from client/account information to the balance.

type-synonym $client = string$

type-synonym $account\text{-}no = int$

type-synonym $register = (client \times account\text{-}no) \rightharpoonup int$

Operation definitions We use a similar setting as for the Bank example — with one minor modification: the withdraw operation gets a non-deterministic behaviour: it may withdraw any amount between 1 and the demanded amount.

```
op deposit (c : client, no : account_no, amount:nat) : unit
pre  (c,no) : dom(register)
post register'=register[(c,no) := register(c,no) + amount]

op balance (c : client, no : account_no) : int
pre  (c,no) : dom(register)
post register'=register and result = register(c,no)

op withdraw(c : client, no : account_no, amount:nat) : nat
pre  (c,no) : dom(register) and register(c,no) >= amount
post result <= amount and
     register'=register[(c,no) := register(c,no) - result]
```

Interface normalization turns this interface into the following input type:

datatype $in\text{-}c = deposit\ client\ account\text{-}no\ nat$
$\quad\quad | \ withdraw\ client\ account\text{-}no\ nat$
$\quad\quad | \ balance\ client\ account\text{-}no$

datatype $out\text{-}c = depositO |\ balanceO\ nat \ |\ withdrawO\ nat$

fun $precond :: register \Rightarrow in\text{-}c \Rightarrow bool$
where $precond\ \sigma\ (deposit\ c\ no\ m) = ((c,no) \in dom\ \sigma)$
$|\quad precond\ \sigma\ (balance\ c\ no) = ((c,no) \in dom\ \sigma)$
$|\quad precond\ \sigma\ (withdraw\ c\ no\ m) = ((c,no) \in dom\ \sigma \wedge (int\ m) \leq the(\sigma(c,no)))$

fun $postcond :: in\text{-}c \Rightarrow register \Rightarrow (out\text{-}c \times register)\ set$

where $postcond\ (deposit\ c\ no\ m)\ \sigma =$
$(\{\ (n,\sigma').\ (n = depositO \wedge \sigma'{=}\sigma((c,no) {\mapsto} the(\sigma(c,no)) + int\ m))\})$
$|\quad postcond\ (balance\ c\ no)\ \sigma =$
$(\{\ (n,\sigma').\ (\sigma{=}\sigma' \wedge (\exists\ x.\ balanceO\ x = n \wedge x = nat(the(\sigma(c,no)))))\})$
$|\quad postcond\ (withdraw\ c\ no\ m)\ \sigma =$
$(\{\ (n,\sigma').\ (\exists\ x{\le}m.\ n = withdrawO\ x \wedge \sigma'{=}\sigma((c,no) {\mapsto} the(\sigma(c,no)) - int\ x))\})$

Proving Symbolic Execution Rules for the Abstractly Constructed Program Using the Operators `impl` and `strong_impl`, we can synthesize an abstract program right away from the specification, i.e. the pair of pre and postcondition defined above. Since this program is even deterministic, we derive a set of symbolic execution rules used in the test case generation process which will produce symbolic results against which the PUT can be compared in the test driver.

definition $implementable :: [{'}\sigma \Rightarrow {'}\iota \Rightarrow bool, {'}\iota \Rightarrow ({'}o,{'}\sigma)MON_{SB}] \Rightarrow bool$
where $implementable\ pre\ post = (\forall\ \sigma\ \iota.\ pre\ \sigma\ \iota \longrightarrow (\exists\ out\ \sigma'.\ (out,\sigma') \in post\ \iota\ \sigma\ ))$

lemma *precond-postcond-implementable*:
implementable precond postcond
apply(*auto simp*: *implementable-def*)
apply(*case-tac* ι, *simp-all*)
apply *auto*
done

The following lemmas reveal that this "constructed" program is actually (due to determinism of the spec)

lemma *impl-1*:
$strong\text{-}impl\ precond\ postcond\ (deposit\ c\ no\ m) =$
$(\lambda\sigma\ .\ if\ (c,\ no) \in dom\ \sigma$
$then\ Some(depositO,\sigma((c,\ no) \mapsto the\ (\sigma\ (c,\ no)) + int\ m))$
$else\ None)$
by(*rule ext*, *auto simp*: *strong-impl-def*)

lemma *valid-both-spec1*[*simp*]:
$(\sigma \models (s \leftarrow mbind\ ((deposit\ c\ no\ m)\#S)\ (strong\text{-}impl\ precond\ postcond);$
$return\ (P\ s))) =$
$(if\ (c,\ no) \in dom\ \sigma$
$then\ (\sigma((c,\ no) \mapsto the\ (\sigma\ (c,\ no)) + int\ m)\ ) \models (s \leftarrow mbind\ S\ (strong\text{-}impl\ precond\ postcond);$
$return\ (P\ (depositO\#s)))$
$else\ (\sigma \models (return\ (P\ []))))$
by(*auto simp*: *exec-mbindFSave impl-1*)

lemma *impl-2*:
$strong\text{-}impl\ precond\ postcond\ (balance\ c\ no) =$
$(\lambda\sigma.\ if\ (c,\ no) \in dom\ \sigma$
$then\ Some(balanceO(nat(the\ (\sigma\ (c,\ no)))),\sigma)$
$else\ None)$

by(*rule ext*, *auto simp*: *strong-impl-def Eps-split*)

lemma *valid-both-spec2* [*simp*]:
$(\sigma \models (s \leftarrow mbind\ ((balance\ c\ no)\#S)\ (strong\text{-}impl\ precond\ postcond);$
$\quad return\ (P\ s))) =$
$\quad (if\ (c,\ no) \in dom\ \sigma$
$\quad then\ (\sigma \models (s \leftarrow mbind\ S\ (strong\text{-}impl\ precond\ postcond);$
$\quad\quad return\ (P\ (balanceO(nat(the\ (\sigma\ (c,\ no))))\#s))))$
$\quad else\ (\sigma \models (return\ (P\ []))))$
by(*auto simp*: *exec-mbindFSave impl-2*)

So far, no problem; however, so far, everything was deterministic. The following key-theorem does not hold:

lemma *impl-3*:
$strong\text{-}impl\ precond\ postcond\ (withdraw\ c\ no\ m) =$
$(\lambda\sigma.\ if\ (c,\ no) \in dom\ \sigma \wedge (int\ m) \leq the(\sigma(c,no)) \wedge x \leq m$
$\quad then\ Some(withdrawO\ x,\sigma((c,\ no) \mapsto the\ (\sigma\ (c,\ no)) - int\ x))$
$\quad else\ None)$
oops

This also breaks our deterministic approach to compute the sequence aforehand and to run the test of PUT against this sequence.

However, we can give an acceptance predicate (an automaton) for correct behaviour of our PUT:

fun $accept :: (in\text{-}c\ list \times out\text{-}c\ list \times int) \Rightarrow bool$
where $accept((deposit\ c\ no\ n)\#S,depositO\#S',\ m) = accept\ (S,S',\ m + (int\ n))$
$\mid accept((withdraw\ c\ no\ n)\#S,\ (withdrawO\ k)\#S',m) = (k \leq n \wedge accept\ (S,S',\ m - (int\ k)))$
$\mid accept([balance\ c\ no],\ [balanceO\ n],\ m) = (int\ n = m)$
$\mid accept(a,b,c) = False$

This format has the advantage

TODO: Work out foundation. accept works on an abstract state (just one single balance of a user), while PUT works on the (invisible) concrete state. A data-refinement is involved, and it has to be established why it is correct.

Test Specifications **fun** $test\text{-}purpose :: [client,\ account\text{-}no,\ in\text{-}c\ list] \Rightarrow bool$
where
$test\text{-}purpose\ c\ no\ [] = False$
$\mid test\text{-}purpose\ c\ no\ (a\#R) = (case\ R\ of$
$\quad [] \Rightarrow a = balance\ c\ no$
$\quad \mid a'\#R' \Rightarrow (((\exists\ m.\ a = deposit\ c\ no\ m) \vee$
$\quad\quad (\exists\ m.\ a = withdraw\ c\ no\ m)) \wedge$
$\quad\quad test\text{-}purpose\ c\ no\ R))$

test-spec *test-balance*:
assumes *account-defined*: $(c,no) \in dom\ \sigma_0$
and *test-purpose* : $test\text{-}purpose\ c\ no\ \iota s$
shows $\sigma_0 \models (os \leftarrow mbind\ \iota s\ PUT;\ return\ (accept(\iota s,\ os,\ the(\sigma_0\ (c,no)))))$

apply(*insert account-defined test-purpose*)
apply(*gen-test-cases PUT split*: *HOL.split-if-asm*)

mk-test-suite *nbank*

declare [[*testgen-iterations=0*]]
gen-test-data *nbank*

thm *nbank.concrete-tests*

end

5.3. MyKeOS

5.3.1. A Shared-Memory-Model

theory *SharedMemory*
imports *Main*
begin

Shared Memory Model

Prerequisites Prerequisite: a generalization of *fun-upd-def*: $?f(?a := ?b) \equiv \lambda x.$ *if* $x = ?a$ *then* $?b$ *else* $?f\ x$. It represents updating modulo a sharing equivalence, i.e. an equivalence relation on parts of the domain of a memory.

definition *fun-upd-equivp* :: $('a \Rightarrow 'a \Rightarrow bool) \Rightarrow ('a \Rightarrow 'b) \Rightarrow 'a \Rightarrow 'b \Rightarrow ('a \Rightarrow 'b)$ **where**
fun-upd-equivp eq f a b $= (\lambda x.$ *if eq x a then b else f x*$)$

— This lemma is the same as *Fun.fun-upd-same*: $(?f(?x := ?y))\ ?x = ?y$; applied on our genralization *fun-upd-equivp ?eq ?f ?a ?b* $= (\lambda x.$ *if ?eq x ?a then ?b else ?f x*$)$ of $?f(?a := ?b) \equiv \lambda x.$ *if* $x = ?a$ *then* $?b$ *else* $?f\ x$. This proof tell if our function *fun-upd-equivp op* $= f\ x\ y$ is equal to f this is equivalent to the fact that $f\ x = y$

lemma *fun-upd-equivp-iff*: $(($*fun-upd-equivp* $(op =)\ f\ x\ y) = f) = (f\ x = y)$
by (*simp add* :*fun-upd-equivp-def*, *safe*, *erule subst*, *auto*)

— Now we try to proof the same lemma applied on any equivalent relation *equivp eqv* instead of the equivalent relation *op* =. For this case, we had split the lemma to 2 parts. the lemma *fun-upd-equivp-iff-part1* to proof the case when *eq* $(f\ a)\ b \longrightarrow$ *eq* (*fun-upd-equivp eqv f a b z*) $(f\ z)$, and the second part is the lemma *fun-upd-equivp-iff-part2* to proof the case *equivp eqv* $\Longrightarrow$ *fun-upd-equivp eqv f a b* $= f \longrightarrow f\ a = b$.

lemma *fun-upd-equivp-iff-part1*:
$equivp\ R \Longrightarrow (\bigwedge z.\ R\ x\ z \Longrightarrow R\ (f\ z)\ y) \Longrightarrow R\ (\textit{fun-upd-equivp}\ R\ f\ x\ y\ z)\ (f\ z)$
by (*auto simp*: *fun-upd-equivp-def Equiv-Relations.equivp-reflp Equiv-Relations.equivp-symp*)

lemma *fun-upd-equivp-iff-part2*: $equivp\ R \Longrightarrow \textit{fun-upd-equivp}\ R\ f\ x\ y = f \longrightarrow f\ x = y$
apply (*simp add* :*fun-upd-equivp-def*, *safe*)
apply (*erule subst*, *auto simp*: *Equiv-Relations.equivp-reflp*)
done

— Just anotther way to formalise $equivp\ ?R \Longrightarrow \textit{fun-upd-equivp}\ ?R\ ?f\ ?x\ ?y = ?f \longrightarrow ?f\ ?x = ?y$ without using the strong equality

lemma $equivp\ R \Longrightarrow (\bigwedge z.\ R\ x\ z \Longrightarrow R\ (\textit{fun-upd-equivp}\ R\ f\ x\ y\ z)\ (f\ z)) \Longrightarrow R\ \ y\ (f\ x)$
by (*simp add*: *fun-upd-equivp-def Equiv-Relations.equivp-symp equivp-reflp*)

this lemma is the same in $[\![equivp\ ?R;\ \bigwedge z.\ ?R\ ?x\ z \Longrightarrow ?R\ (?f\ z)\ ?y]\!] \Longrightarrow ?R\ (\textit{fun-upd-equivp}\ ?R\ ?f\ ?x\ ?y\ ?z)\ (?f\ ?z)$ where *op* = is generalized by another equivalence relation

lemma *fun-upd-equivp-idem*: $f\ x = y \Longrightarrow (\textit{fun-upd-equivp}\ (op\ =)\ f\ x\ y) = f$
by (*simp only*: *fun-upd-equivp-iff*)

lemma *fun-upd-equivp-triv* : $\textit{fun-upd-equivp}\ (op\ =)\ f\ x\ (f\ x) = f$
by (*simp only*: *fun-upd-equivp-iff*)

— This is the generalization of $\textit{fun-upd-equivp}\ op\ =\ ?f\ ?x\ (?f\ ?x) = ?f$ on a given equivalence relation

lemma *fun-upd-equivp-triv-part1* :
$equivp\ R \Longrightarrow (\bigwedge z.\ R\ x\ z \Longrightarrow \textit{fun-upd-equivp}\ (R')\ f\ x\ (f\ x)\ z) \Longrightarrow f\ x$
apply (*auto simp*:*fun-upd-equivp-def*)
apply (*metis equivp-reflp*)
done

lemma *fun-upd-equivp-triv-part2* :
$equivp\ R \Longrightarrow (\bigwedge z.\ R\ x\ z \Longrightarrow f\ z\ ) \Longrightarrow \textit{fun-upd-equivp}\ (R')\ f\ x\ (f\ x)\ x$
by (*simp add*:*fun-upd-equivp-def equivp-reflp split*: *split-if*)

lemma *fun-upd-equivp-apply* [*simp*]:
$(\textit{fun-upd-equivp}\ (op\ =)\ f\ x\ y)\ z = (\textit{if}\ z = x\ \textit{then}\ y\ \textit{else}\ f\ z)$
by (*simp only*: *fun-upd-equivp-def*)

— This is the generalization of $\textit{fun-upd-equivp}\ op\ =\ ?f\ ?x\ ?y\ ?z = (\textit{if}\ ?z = ?x\ \textit{then}\ ?y\ \textit{else}\ ?f\ ?z)$ with e given equivalence relation and not only with *op* =

lemma *fun-upd-equivp-apply1* [*simp*]:
$equivp\ R \Longrightarrow (\textit{fun-upd-equivp}\ R\ f\ x\ y)\ z = (\textit{if}\ R\ z\ x\ \textit{then}\ y\ \textit{else}\ f\ z)$
by (*simp add*: *fun-upd-equivp-def*)

lemma *fun-upd-equivp-same*: $(\textit{fun-upd-equivp}\ (op\ =)\ f\ x\ y)\ x = y$
by (*simp only*: *fun-upd-equivp-def*)*simp*

— This is the generalization of *fun-upd-equivp op* $=$ *?f ?x ?y ?x* $=$ *?y* with a given equivalence relation

lemma *fun-upd-equivp-same1*: *equivp R* $\Longrightarrow$ (*fun-upd-equivp R f x y*) *x* $=$ *y*
  by (*simp add*: *fun-upd-equivp-def equivp-reflp*)

For the special case that @term eq is just the equality @term "op =", sharing update and classical update are identical.

lemma *fun-upd-equivp-vs-fun-upd*: (*fun-upd-equivp* (*op* $=$)) $=$ *fun-upd*
  by(*rule ext*, *rule ext*, *rule ext*,*simp add*:*fun-upd-def fun-upd-equivp-def*)

Definition of the shared-memory type **typedef** ($'\alpha$, $'\beta$) *memory* $=$ $\{(\sigma::'\alpha \rightharpoonup '\beta,\ R).$ *equivp R* $\wedge$ $(\forall\, x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
proof
  show (*Map.empty*, (*op* $=$)) $\in$ *?memory*
    by (*auto simp*: *identity-equivp*)
qed

fun *memory-inv* :: ($'a \Rightarrow 'b$ *option*) $\times$ ($'a \Rightarrow 'a \Rightarrow$ *bool*) $\Rightarrow$ *bool*
where *memory-inv* (*Pair f R*) $=$ (*equivp R* $\wedge$ $(\forall\, x\ y.\ R\ x\ y \longrightarrow f\ x = f\ y)$)

lemma *Abs-Rep-memory* [*simp*]:*Abs-memory* (*Rep-memory* σ) $=$ σ
  by (*simp add*:*Rep-memory-inverse*)

lemma *memory-invariant* [*simp*]:
    memory-inv σ*-rep* $=$ (*Rep-memory* (*Abs-memory* σ*-rep*) $=$ σ*-rep*)
  using *Rep-memory* [*of Abs-memory* σ*-rep*] *Abs-memory-inverse mem-Collect-eq*
    prod-caseE prod-caseI2 memory-inv.simps
  by *smt*

lemma *Pair-code-eq* :
 Rep-memory σ $=$ *Pair* (*fst* (*Rep-memory* σ)) (*snd* (*Rep-memory* σ))
  by (*simp add*: *Product-Type.surjective-pairing*)

lemma *snd-memory-equivp* [*simp*]: *equivp*(*snd*(*Rep-memory* σ))
  by(*insert Rep-memory*[*of* σ], *auto*)

Operations on Shared-Memory **definition** *init* :: ($'\alpha$, $'\beta$) *memory*
where *init* $=$ *Abs-memory* (*Map.empty*, *op* $=$)

value *init*::(*nat*,*int*)*memory*
value *map* (λx. *the* (*fst* (*Rep-memory init*)*x*)) [*1* .. *10*]
value *take* (*10*) (*map* (*Pair Map.empty*) [(*op* $=$)])
value *replicate* *10 init*
term *Rep-memory* σ
term $[(\sigma::nat \rightharpoonup int,\ R\ ) <- xs\ .\ equivp\ R \wedge (\forall\, x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)]$

definition *init-mem-list* :: $'\alpha$ *list* $\Rightarrow$ (*nat*, $'\alpha$) *memory*
where *init-mem-list s* $=$ *Abs-memory* (*let h* $=$ *zip* (*map nat* [*0* .. *int*(*length s*)]) *s*

in foldl (λx (y,z). *fun-upd* x y (*Some* z))
Map.empty h,
op $=$)

value *init-mem-list* $[-22,2,-3]$

Memory Read Operation **definition** *lookup* :: $('\alpha, '\beta)$ *memory* $\Rightarrow$ $'\alpha \Rightarrow '\beta$ (**infixl** \$ *100*)
where σ \$ x = *the* (*fst* (*Rep-memory* σ) x)

setup-lifting *type-definition-memory*

Memory Update Operation **fun** *Pair-upd-lifter*:: $('a \Rightarrow 'b$ *option*$) \times ('a \Rightarrow 'a \Rightarrow$ *bool*$) \Rightarrow$ $'a \Rightarrow 'b \Rightarrow$
$('a \Rightarrow 'b$ *option*$) \times ('a \Rightarrow 'a \Rightarrow$ *bool*$)$
where *Pair-upd-lifter* $((f, R))$ x y = (*fun-upd-equivp* R f x (*Some* y), R)

lemma *update_sound*$'$:
assumes $\sigma \in \{(\sigma, R).$ *equivp* $R \wedge (\forall x\, y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
shows *Pair-upd-lifter* σ x y $\in \{(\sigma, R).$ *equivp* $R \wedge (\forall x\, y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
proof $-$
obtain *mem* **and** R
where *Pair*: (*mem*, R) = σ **and** *Eq*: *equivp* R **and** *Mem*: $\forall$ x y . R x y $\longrightarrow$ *mem* x = *mem* y
using *assms equivpE* **by** *auto*
obtain *mem*$'$ **and** R'
where *Pair*$'$: (*mem*$'$, R') = *Pair-upd-lifter* σ x y
using *surjective-pairing* **by** *metis*
have *Def1*: *mem*$'$ = *fun-upd-equivp* R *mem* x (*Some* y)
and *Def2*: $R' = R$
using *Pair Pair*$'$ **by** *auto*
have *Eq*$'$: *equivp* R'
using *Def2 Eq* **by** *auto*
moreover have $\forall$ y z . R' y z $\longrightarrow$ *mem*$'$ y = *mem*$'$ z
using *Mem equivp-symp equivp-transp*
unfolding *Def1 Def2* **by** (*metis Eq fun-upd-equivp-def*)
ultimately show *?thesis*
using *Pair*$'$ **by** *auto*
qed

lift-definition *update* :: $('\alpha, '\beta)$ *memory* $\Rightarrow '\alpha \Rightarrow '\beta \Rightarrow$ $('\alpha, '\beta)$ *memory* (- $'$(- $:=_\$$ -$'$) *100*)
is *Pair-upd-lifter*
using *update_sound*$'$
by *simp*

lemma *update*$'$: σ $(x :=_\$ y)$ = *Abs-memory* (*fun-upd-equivp* (*snd* (*Rep-memory* σ))
(*fst* (*Rep-memory* σ)) x (*Some* y), (*snd* (*Rep-memory* σ)))
using *Rep-memory-inverse surjective-pairing Pair-upd-lifter.simps update.rep-eq*
by *metis*

fun *update-list-rep* :: $('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow$ *bool*$) \Rightarrow ('\alpha \times '\beta)$*list* $\Rightarrow$

$('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow bool)$
where *update-list-rep* $(f,\ R)$ *nlist* = (*foldl* ($\lambda(f,\ R)(addr,val)$. *Pair-upd-lifter* $(f,\ R)$ *addr val*)
$(f,\ R)$
nlist)

lemma *update-list-rep-p*:
assumes *1*: $P\ \sigma$
and *2*: $\bigwedge src\ dst\ \sigma.\ P\ \sigma \Longrightarrow P\ (Pair\text{-}upd\text{-}lifter\ \sigma\ src\ dst)$
shows P (*update-list-rep* σ *list*)
using *1 2*
apply (*induct list arbitrary*: σ)
apply (*force*,*safe*)
apply (*simp del*: *Pair-upd-lifter.simps*)
using *surjective-pairing*
apply *metis*
done

lemma *update-list-rep-sound*:
assumes *1*: $\sigma \in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
shows *update-list-rep* σ (*nlist*) $\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
using *1*
apply (*elim update-list-rep-p*)
apply (*erule update_sound'*)
done

lift-definition *update-list* :: $('\alpha,\ '\beta)$ *memory* $\Rightarrow ('\alpha \times '\beta)$ *list* $\Rightarrow ('\alpha,\ '\beta)$ *memory* (**infixl** $'/{:=}_{\$}$ *30*)
is *update-list-rep*
using *update-list-rep-sound* **by** *simp*

lemma *update-list-Nil*[*simp*]: $(\sigma\ /{:=}_{\$}\ []) = \sigma$
unfolding *update-list-def*
by(*simp*,*subst surjective-pairing*[*of Rep-memory* σ],*subst update-list-rep.simps*, *simp*)

lemma *update-list-Cons*[*simp*] : $(\sigma\ /{:=}_{\$}\ ((a,b)\#S)) = (\sigma(a\ {:=}_{\$}\ b)\ /{:=}_{\$}\ S)$
unfolding *update-list-def*
apply(*simp*,*subst surjective-pairing*[*of Rep-memory* σ],*subst update-list-rep.simps*, *simp*)
apply(*subst surjective-pairing*[*of Rep-memory* $(\sigma\ (a\ {:=}_{\$}\ b))$],*subst update-list-rep.simps*, *simp*)
apply(*simp add*: *update-def*)
apply(*subst Abs-memory-inverse*)
apply (*metis* (*lifting*, *mono-tags*) *Rep-memory update_sound'*)
by *simp*

Type-invariant:

lemma *update_sound*:
assumes *Rep-memory* $\sigma = (\sigma',\ eq)$
shows (*fun-upd-equivp eq* σ' *x* (*Some y*), *eq*) $\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
using *assms insert Rep-memory*[*of* σ]
apply(*auto simp*: *fun-upd-equivp-def*)
apply(*rename-tac xa xb*, *erule contrapos-np*)
apply(*rule-tac R=eq* **and** *y=xa* **in** *equivp-transp*,*simp*)
apply(*erule equivp-symp*, *simp-all*)

apply(*rename-tac xa xb*, *erule contrapos-np*)
apply(*rule-tac R=eq* **and** *y=xb* **in** *equivp-transp*,*simp-all*)
done

Memory Transfer Based on Sharing Transformation **fun** *transfer-rep* :: $('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow bool) \Rightarrow '\alpha \Rightarrow '\alpha \Rightarrow ('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow bool)$
where *transfer-rep* $(m,\ r)$ *src dst* $= (m\ o\ (id\ (dst := src)),$
$(\lambda\ x\ y\ .\ r\ ((id\ (dst := src))\ x)\ ((id\ (dst := src))\ y)))$

lemma *transfer-rep-simp* :
transfer-rep X src dst $= ((fst\ X)\ o\ (id\ (dst := src)),$
$(\lambda\ x\ y\ .\ (snd\ X)\ ((id\ (dst := src))\ x)\ ((id\ (dst := src))\ y)))$
by(*subst surjective-pairing*[*of X*],*subst transfer-rep.simps*, *simp*)

lemma *transfer-rep-sound*:
assumes $\sigma \in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
shows *transfer-rep* σ *src dst* $\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
proof –
obtain *mem* **and** *R*
where *P*: $(mem,\ R) = \sigma$ **and** *E*: *equivp R* **and** *M*: $\forall\ x\ y\ .\ R\ x\ y \longrightarrow mem\ x = mem\ y$
using *assms equivpE* **by** *auto*
obtain mem' **and** R'
where P': $(mem',\ R') =$ *transfer-rep* σ *src dst*
by (*metis surj-pair*)
have *D1*: $mem' = (mem\ o\ (id\ (dst := src)))$
and *D2*: $R' = (\lambda\ x\ y\ .\ R\ ((id\ (dst := src))\ x)\ ((id\ (dst := src))\ y))$
using *P* P' **by** *auto*
have *equivp* R'
using *E* **unfolding** *D2 equivp-def* **by** *metis*
moreover have $\forall\ y\ z\ .\ R'\ y\ z \longrightarrow mem'\ y = mem'\ z$
using *M* **unfolding** *D1 D2* **by** *auto*
ultimately show *?thesis*
using P' **by** *auto*
qed

lift-definition *transfer* :: $('\alpha,'\beta)memory \Rightarrow '\alpha \Rightarrow '\alpha \Rightarrow ('\alpha,\ '\beta)memory$ (- '(- ⋈ -') [*0*,*111*,*111*]*110*)
is *transfer-rep*
using *transfer-rep-sound*
by *simp*

lemma *transfer-rep-sound2* :
transfer-rep (*Rep-memory* σ) *a b* $\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
by (*metis* (*lifting*, *mono-tags*) *Rep-memory transfer-rep-sound*)

fun *share-list2* :: $('\alpha, '\beta)$ *memory* $\Rightarrow$ $('\alpha \times '\alpha)$*list* $\Rightarrow$ $('\alpha, '\beta)$ *memory* (**infix** $'/\bowtie$ *60*)
where σ $/\bowtie$ S = (*foldl* (λ σ (*a*,*b*). (σ (*a*$\bowtie$*b*))) σ S)

lemma *sharelist2-Nil*[*simp*] : σ $/\bowtie$ [] = σ **by** *simp*

lemma *sharelist2-Cons*[*simp*] : σ $/\bowtie$ ((*a*,*b*)#*S*) = (σ(*a*$\bowtie$*b*) $/\bowtie$ *S*) **by** *simp*

fun *share-list-rep* :: $('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow$ *bool*$)$ $\Rightarrow$ $('\alpha \times '\alpha)$*list* $\Rightarrow$
$('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow$ *bool*$)$
where *share-list-rep* (*f*, *R*) *nlist* =
(*foldl* (λ(*f*, *R*) (*src*,*dst*). *transfer-rep* (*f*, *R*) *src dst*) (*f*, *R*) *nlist*)

fun *share-list-rep'* :: $('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow$ *bool*$)$ $\Rightarrow$ $('\alpha \times '\alpha)$*list* $\Rightarrow$
$('\alpha \rightharpoonup '\beta) \times ('\alpha \Rightarrow '\alpha \Rightarrow$ *bool*$)$
where *share-list-rep'* (*f*, *R*) [] = (*f*, *R*)
| *share-list-rep'* (*f*, *R*) (*n*#*nlist*) = *share-list-rep'* (*transfer-rep*(*f*,*R*)(*fst n*)(*snd n*)) *nlist*

lemma *share-list-rep'-p*:
assumes *1*: *P* σ
and *2*: $\bigwedge$*src dst* σ. *P* σ $\Longrightarrow$ *P* (*transfer-rep* σ *src dst*)
shows *P* (*share-list-rep'* σ *list*)
using *1 2*
apply (*induct list arbitrary*: σ *P*)
apply *force*
apply *safe*
apply (*simp del*: *transfer-rep.simps*)
using *surjective-pairing*
apply *metis*
done

lemma *foldl-preserve-p*:
assumes *1*: *P mem*
and *2*: $\bigwedge$*y z mem* . *P mem* $\Longrightarrow$ *P* (*f mem y z*)
shows *P* (*foldl* (λ*a* (*y*, *z*). *f mem y z*) *mem list*)
using *1 2*
apply (*induct list arbitrary*: *f mem* , *auto*)
apply *metis*
done

lemma *share-list-rep-p*:
assumes *1*: *P* σ
and *2*: $\bigwedge$*src dst* σ. *P* σ $\Longrightarrow$ *P* (*transfer-rep* σ *src dst*)
shows *P* (*share-list-rep* σ *list*)
using *1 2*
apply (*induct list arbitrary*: σ)
apply *force*
apply *safe*
apply (*simp del*: *transfer-rep.simps*)

using *surjective-pairing*
apply *metis*
done

The modification of the underlying equivalence relation on adresses is only defined on very strong conditions — which are fulfilled for the empty memory, but difficult to establish on a non-empty-one. And of course, the given relation must be proven to be an equivalence relation. So, the case is geared towards shared-memory scenarios where the sharing is defined initially once and for all.

definition $update_R$ :: $('\alpha, '\beta)memory \Rightarrow ('\alpha \Rightarrow '\alpha \Rightarrow bool) \Rightarrow ('\alpha, '\beta)memory$ $(- :=_R - 100)$
where $\sigma :=_R R \equiv Abs\text{-}memory\ (fst(Rep\text{-}memory\ \sigma), R)$

definition $lookup_R$ :: $('\alpha, '\beta)memory \Rightarrow ('\alpha \Rightarrow '\alpha \Rightarrow bool)$ $(\$_R - 100)$
where $\$_R\ \sigma \equiv (snd(Rep\text{-}memory\ \sigma))$

lemma $update_R$-*comp-lookup*$_R$:
assumes *equiv* : $equivp\ R$
and *sharing-conform* : $\forall\ x\ y.\ R\ x\ y \longrightarrow fst(Rep\text{-}memory\ \sigma)\ x = fst(Rep\text{-}memory\ \sigma)\ y$
shows $(\$_R\ (\sigma :=_R R)) = R$
unfolding *lookup*$_R$*-def update*$_R$*-def*
by(*subst Abs-memory-inverse*, *simp-all add*: *equiv sharing-conform*)

Sharing Relation Definition **definition** *sharing* :: $'\alpha \Rightarrow ('\alpha, '\beta)memory \Rightarrow '\alpha \Rightarrow bool$
$((-\ shares()_-/\ -)\ [201,\ 0,\ 201]\ 200)$
where $(x\ shares_\sigma\ y) \equiv (snd(Rep\text{-}memory\ \sigma)\ x\ y)$

definition *Sharing* :: $'\alpha\ set \Rightarrow ('\alpha, '\beta)memory \Rightarrow '\alpha\ set \Rightarrow bool$
$((-\ Shares()_-/\ -)\ [201,\ 0,\ 201]\ 200)$
where $(X\ Shares_\sigma\ Y) \equiv (\exists\ x{\in}X.\ \exists\ y{\in}Y.\ x\ shares_\sigma\ y)$

Properties on Sharing Relation **lemma** *sharing-charn*:
$equivp\ (snd\ (Rep\text{-}memory\ \sigma))$
using $Rep\text{-}memory[of\ \sigma]$
unfolding *sharing-def*
by *auto*

lemma *sharing-charn'*:
assumes *1*: $(x\ shares_\sigma\ y)$
shows $(\exists R.\ equivp\ R \wedge R\ x\ y)$
by (*auto simp add*: *sharing-def snd-def equivp-def*)

lemma *sharing-charn2*:
shows$\exists\, x\ y.\ (\ equivp\ (snd\ (Rep\text{-}memory\ \sigma)) \wedge (snd\ (Rep\text{-}memory\ \sigma))\ x\ y)$
using *sharing-charn* [*THEN equivp-reflp*]
by (*simp*)*fast*

— Lemma to show that $?x\ shares_{?\sigma}\ ?y \equiv snd\ (Rep\text{-}memory\ ?\sigma)\ ?x\ ?y$ is reflexive
lemma *sharing-refl*: $(x\ shares_\sigma\ x)$
using *insert* $Rep\text{-}memory[of\ \sigma]$

by (*auto simp*: *sharing-def elim*: *equivp-reflp*)

— Lemma to show that *?x shares*$_{?\sigma}$ *?y* $\equiv$ *snd* (*Rep-memory ?*σ) *?x ?y* is symetric
lemma *sharing-sym* [*sym*]:
  assumes *x shares*$_\sigma$ *y*
  shows *y shares*$_\sigma$ *x*
  using *assms Rep-memory*[*of* σ]
  by (*auto simp*: *sharing-def elim*: *equivp-symp*)

lemma *sharing-commute* : *x shares*$_\sigma$ *y* = (*y shares*$_\sigma$ *x*)
 by(*auto intro*: *sharing-sym*)

— Lemma to show that *?x shares*$_{?\sigma}$ *?y* $\equiv$ *snd* (*Rep-memory ?*σ) *?x ?y* is transitive

lemma *sharing-trans* [*trans*]:
  assumes *x shares*$_\sigma$ *y*
  and *y shares*$_\sigma$ *z*
  shows *x shares*$_\sigma$ *z*
  using *assms insert Rep-memory*[*of* σ]
  by(*auto simp*: *sharing-def elim*: *equivp-transp*)

lemma *shares-result*:
 assumes *x shares*$_\sigma$ *y*
 shows *fst* (*Rep-memory* σ) *x* = *fst* (*Rep-memory* σ) *y*
 using *assms*
 unfolding *sharing-def*
 using *Rep-memory*[*of* σ]
 by *auto*

lemma *sharing-init*:
 assumes *1*: $i \neq k$
 shows $\neg$(*i shares*$_{init}$ *k*)
 unfolding *sharing-def init-def*
 using *1*
 by (*auto simp*: *Abs-memory-inverse identity-equivp*)

lemma *shares-init*[*simp*]: (*x shares*$_{init}$ *y*) = (*x*=*y*)
unfolding *sharing-def init-def*
by (*metis init-def sharing-init sharing-def sharing-refl*)

lemma *sharing-init-mem-list*:
 assumes *1*: $i \neq k$
 shows $\neg$(*i shares*$_{init\text{-}mem\text{-}list\ S}$ *k*)
 unfolding *sharing-def init-mem-list-def*
 using *1*
 by (*auto simp*: *Abs-memory-inverse identity-equivp*)

definition *reset* :: $('\alpha, '\beta)$ *memory* $\Rightarrow$ $'\alpha$ *set*$\Rightarrow$ $('\alpha, '\beta)$*memory* (- '(*reset* -') *100*)
where σ (*reset* X) = (*let* (σ',*eq*) = *Rep-memory* σ;
eq' = λ *a b*. *eq a b* $\vee$ ($\exists x \in X$. *eq a x* $\vee$ *eq b x*)
in *if* X={} *then* σ
else Abs-memory (*fun-upd-equivp eq'* σ' (*SOME x*. $x \in X$) *None*, *eq*))

lemma *reset-mt* : σ (*reset* {}) = σ
unfolding *reset-def Let-def*
by *simp*

lemma *reset-sh* :
assumes * : (x *shares*$_\sigma$ y)
and **: $x \in X$
shows σ (*reset* X) \$ y = *None*
oops

Memory Domain Definition **definition** *Domain* :: $('\alpha, '\beta)$*memory* $\Rightarrow$ $'\alpha$ *set*
where *Domain* σ = *dom* (*fst* (*Rep-memory* σ))

Properties on Memory Domain **lemma** *Domain-charn*:
assumes *1*:$x \in$ *Domain* σ
shows $\exists$ *y*. *Some y* = *fst* (*Rep-memory* σ) *x*
using *1*
by(*auto simp*: *Domain-def*)

lemma *Domain-charn1*:
assumes *1*:$x \in$ *Domain* σ
shows $\exists$ *y*. *the* (*Some y*) = σ \$ x
using *1*
by(*auto simp*: *Domain-def lookup-def*)

— This lemma says that if x and y are quivalent this means that they are in the same set of equivalent classes

lemma *shares-dom* [*code-unfold*, *intro*]:
assumes x *shares*$_\sigma$ y
shows ($x \in$ *Domain* σ) = ($y \in$ *Domain* σ)
using *insert Rep-memory*[*of* σ] *assms*
by (*auto simp*: *sharing-def Domain-def*)

lemma *Domain-mono*:
assumes *1*: $x \in$ *Domain* σ
and *2*: (x *shares*$_\sigma$ y)
shows $y \in$ *Domain* σ
using *1 2 Rep-memory*[*of* σ]
by (*auto simp add*: *sharing-def Domain-def*)

corollary *Domain-nonshares* :
assumes *1*: $x \in$ *Domain* σ

and *2*: $y \notin$ *Domain* σ
shows $\neg(x\ shares_{\sigma}\ y)$
using *1 2 Domain-mono*
by(*fast*)

lemma *Domain-init*[*simp*] : *Domain init* = {}
unfolding *init-def Domain-def*
by(*simp-all add*:*identity-equivp Abs-memory-inverse*)

lemma *Domain-update*[*simp*] :*Domain* (σ ($x :=_{\$} y$)) = (*Domain* σ) $\cup$ {y . $y\ shares_{\sigma}\ x$}
unfolding *update-def Domain-def sharing-def*
proof (*simp-all*)
have $*$: *Pair-upd-lifter* (*Rep-memory* σ) $x\ y \in \{(\sigma, R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
by (*simp*, *metis* (*lifting*, *mono-tags*) *Rep-memory mem-Collect-eq update_sound'*)
have $**$: *snd* (*Rep-memory* σ) $x\ x$
by(*metis equivp-reflp sharing-charn2*)
show *dom* (*fst* (*Rep-memory* (*Abs-memory* (*Pair-upd-lifter* (*Rep-memory* σ) $x\ y$)))) =
dom (*fst* (*Rep-memory* σ)) $\cup$ {y. *snd* (*Rep-memory* σ) $y\ x$}
apply(*simp-all add*: *Abs-memory-inverse*[*OF* $*$])
apply(*subst surjective-pairing* [*of* (*Rep-memory* σ)])
apply(*subst Pair-upd-lifter*.*simps*, *simp*)
apply(*auto simp*: $**$ *fun-upd-equivp-def*)
done
qed

lemma *Domain-share1*:
assumes *1* : $a \in$ *Domain* σ
and *2* : $b \in$ *Domain* σ
shows *Domain* ($\sigma(a \bowtie b)$) = *Domain* σ
proof(*simp-all add*:*Set.set-eq-iff*, *tactic ALLGOALS* (*rtac* @{*thm allI*}))
fix x
have $***$: *transfer-rep* (*Rep-memory* σ) (*id* a) (*id* b) $\in \{(\sigma, R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
by (*metis* (*lifting*, *mono-tags*) *Rep-memory transfer-rep-sound*)
show ($x \in$ *Domain* (σ ($a \bowtie b$))) = ($x \in$ *Domain* σ)
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def*
apply(*subst Abs-memory-inverse*[*OF* $***$])
apply(*insert 1 2*, *simp add*: *o-def transfer-rep-simp Domain-def*)
apply(*auto split*: *split-if split-if-asm*)
done
qed

lemma *Domain-share-tgt* : $a \in$ *Domain* $\sigma \Longrightarrow b \in$ *Domain* (σ ($a \bowtie b$))
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*])
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
apply(*simp add*: *o-def transfer-rep-simp Domain-def*)

by(*auto split*: *split-if*)

lemma *Domain-share2* :
assumes *1* : $a \in$ *Domain* σ
and *2* : $b \notin$ *Domain* σ
shows *Domain* $(\sigma(a \bowtie b)) = ($*Domain* $\sigma - \{x.\ x\ shares_\sigma\ b\} \cup \{b\})$
proof(*simp-all add*:*Set.set-eq-iff* , *auto*)
fix x
assume *3* : $x \in$ *SharedMemory.Domain* $(\sigma\ (a \bowtie b))$
and *4* : $x \neq b$
show $x \in$ *SharedMemory.Domain* σ
apply(*insert 3 4*)
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
apply(*subst* (*asm*) *Abs-memory-inverse*[*OF transfer-rep-sound2*])
apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
apply(*auto split*: *split-if split-if-asm*)
done
next
fix x
assume *3* : $x \in$ *Domain* $(\sigma\ (a \bowtie b))$
and *4* : $x \neq b$
and *5* : $x\ shares_\sigma\ b$
have $**$: $x \notin$ *Domain* σ **using** *2 5 Domain-mono* **by** (*fast*)
show *False*
apply(*insert 3 4 5*, *erule contrapos-pp*, *simp*)
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*])
apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
apply(*auto split*: *split-if split-if-asm*)
using $**$ *SharedMemory.Domain-def domI* **apply** *fast*
done
next
show $b \in$ *Domain* $(\sigma\ (a \bowtie b))$
using *1 Domain-share-tgt* **by** *fast*
next
fix x
assume *3* : $x \in$ *Domain* σ
and *4* : $\neg\ x\ shares_\sigma\ b$
show $x \in$ *Domain* $(\sigma\ (a \bowtie b))$
unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*])
apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
apply(*auto split*: *split-if split-if-asm*)
using *3 SharedMemory.Domain-def domD*
apply *fast*
done
qed

lemma *Domain-share3*:
assumes *1* : $a \notin$ *Domain* σ
shows *Domain* $(\sigma(a \bowtie b)) = ($*Domain* $\sigma - \{b\})$
proof(*simp-all add*:*Set.set-eq-iff* , *auto*)
  fix x
  assume *3*: $x \in$ *Domain* $(\sigma\ (a\ \bowtie\ b))$
  show $x \in$ *Domain* σ
    apply(*insert 3*)
    unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
    apply(*subst* (*asm*) *Abs-memory-inverse*[*OF transfer-rep-sound2*])
    apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
    apply(*auto split*: *split-if split-if-asm*)
    done
next
  assume *3*: $b \in$ *Domain* $(\sigma\ (a\ \bowtie\ b))$
  show *False*
    apply(*insert 1 3*)
    apply(*erule contrapos-pp*[*of* $b \in$ *SharedMemory.Domain* $(\sigma\ (a\ \bowtie\ b))$], *simp*)
    unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
    apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*])
    apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
    apply(*auto split*: *split-if*)
    done
next
  fix x
  assume *3*: $x \in$ *Domain* σ
  and *4*: $x \neq b$
  show $x \in$ *Domain* $(\sigma\ (a\ \bowtie\ b))$
    apply(*insert 3 4*)
    unfolding *sharing-def Domain-def transfer-def map-fun-def o-def id-def*
    apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*])
    apply(*insert 1* , *simp add*: *o-def transfer-rep-simp Domain-def*)
    apply(*auto split*: *split-if split-if-asm*)
    done
qed

lemma *Domain-transfer* :
Domain $(\sigma(a \bowtie b)) = ($*if* $a \notin$ *Domain* σ
  then (*Domain* $\sigma - \{b\})$
  else if $b \notin$ *Domain* σ
    then (*Domain* $\sigma - \{x.\ x\ shares_\sigma\ b\} \cup \{b\})$
    else *Domain* σ)
  using *Domain-share1 Domain-share2 Domain-share3*
  by *metis*

lemma *Domain-transfer-approx* : *Domain* $(\sigma(a \bowtie b)) \subseteq$ *Domain* $(\sigma) \cup \{b\}$
by(*auto simp*: *Domain-transfer*)

Sharing Relation and Memory Update **lemma** *sharing-upd*: $x\ shares_{(\sigma(a\ :=_\$\ b))}\ y = x\ shares_\sigma\ y$

using *insert Rep-memory*$[$*of* $\sigma]$
by(*auto simp*: *sharing-def update-def Abs-memory-inverse*$[$*OF update_sound*$]$)

— this lemma says that if we do an update on an adress x all the elements that are equivalent of x are updated

lemma *update''*:
$\sigma\ (x :=_{\$}\ y) = Abs\text{-}memory(fun\text{-}upd\text{-}equivp\ (\lambda x\ y.\ x\ shares_{\sigma}\ y)\ (fst\ (Rep\text{-}memory\ \sigma))\ x\ (Some\ y),$
$snd\ (Rep\text{-}memory\ \sigma))$
unfolding *update-def sharing-def*
by (*metis update' update-def*)

theorem *update-cancel*:
assumes $x\ shares_{\sigma}\ x'$
shows $\sigma(x :=_{\$}\ y)(x' :=_{\$}\ z) = (\sigma(x' :=_{\$}\ z))$
proof $-$
have $*$: $(fun\text{-}upd\text{-}equivp(snd(Rep\text{-}memory\ \ \sigma))(fst(Rep\text{-}memory\ \ \sigma))\ \ x\ \ (Some\ \ y),snd\ (Rep\text{-}memory\ \sigma))$
$\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
unfolding *fun-upd-equivp-def*
by(*rule update_sound*$[$*simplified fun-upd-equivp-def*$]$, *simp*)
have $**$: $\bigwedge R\ \sigma.\ equivp\ R \Longrightarrow R\ x\ x' \Longrightarrow$
$fun\text{-}upd\text{-}equivp\ R\ (fun\text{-}upd\text{-}equivp\ R\ \sigma\ x\ (Some\ y))\ x'\ (Some\ z)$
$= fun\text{-}upd\text{-}equivp\ R\ \sigma\ x'\ (Some\ z)$
unfolding *fun-upd-equivp-def*
apply(*rule ext*)
apply(*case-tac R xa x'*, *auto*)
apply(*erule contrapos-np*, *erule equivp-transp*, *simp-all*)
done
show *?thesis*
apply(*simp add*: *update'*)
apply(*insert sharing-charn assms*$[$*simplified sharing-def*$]$)
apply(*simp add*: *Abs-memory-inverse* $[$*OF* $*]$ $**$)
done
qed

theorem *update-commute*:
assumes *1*:$\neg\ (x\ shares_{\sigma}\ x')$
shows $(\sigma(x :=_{\$}\ y))(x' :=_{\$}\ z) = (\sigma(x' :=_{\$}\ z)(x :=_{\$}\ y))$
proof $-$
have $*$: $\bigwedge\ x\ y.(fun\text{-}upd\text{-}equivp(snd(Rep\text{-}memory\ \sigma))(fst(Rep\text{-}memory\ \sigma))\ x\ (Some\ y),snd\ (Rep\text{-}memory\ \sigma))$
$\in \{(\sigma,\ R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
unfolding *fun-upd-equivp-def*
by(*rule update_sound*$[$*simplified fun-upd-equivp-def*$]$, *simp*)
have $**$: $\bigwedge R\ \sigma.\ equivp\ R \Longrightarrow \neg\ R\ x\ x' \Longrightarrow$
$fun\text{-}upd\text{-}equivp\ R\ (fun\text{-}upd\text{-}equivp\ R\ \sigma\ x\ (Some\ y))\ x'\ (Some\ z) =$
$fun\text{-}upd\text{-}equivp\ R\ (fun\text{-}upd\text{-}equivp\ R\ \sigma\ x'\ (Some\ z))\ x\ (Some\ y)$
unfolding *fun-upd-equivp-def*
apply(*rule ext*)

```
        apply(case-tac R xa x′, auto)
        apply(erule contrapos-np)
        apply(frule equivp-transp, simp-all)
        apply(erule equivp-symp, simp-all)
        done
    show ?thesis
    apply(simp add: update′)
    apply(insert assms[simplified sharing-def])
    apply(simp add: Abs-memory-inverse [OF ∗] ∗∗)
  done
qed
```

Properties on lookup and update wrt the Sharing Relation

```
lemma update-triv:
  assumes 1: x shares_σ y
    and  2: y ∈ Domain σ
  shows     σ (x :=$ (σ $ y)) = σ
proof −
  {
    fix z
    assume zx: z shares_σ x
    then have zy: z shares_σ y
      using 1 by (rule sharing-trans)
    have F: y ∈ Domain σ ⟹ x shares_σ y
          ⟹ Some (the (fst (Rep-memory σ) x)) = fst (Rep-memory σ) y
      by(auto simp: Domain-def dest: shares-result)
    have Some (the (fst (Rep-memory σ) y)) = fst (Rep-memory σ) z
      using zx and shares-result [OF zy] shares-result [OF zx]
      using F [OF 2 1]
      by simp
  } note 3 = this
  show ?thesis
    unfolding update′′ lookup-def fun-upd-equivp-def
    by (simp add: 3 Rep-memory-inverse if-cong)
qed

lemma update-idem′ :
   assumes 1: x shares_σ y
   and     2: x ∈ Domain σ
   and     3: σ $ x = z
   shows   σ(y:=$ z) = σ
proof −
  have ∗ : y ∈ Domain σ
        by(simp add: shares-dom[OF 1, symmetric] 2)
  have ∗∗: σ (x :=$ (σ $ y)) = σ
    using 1 2 ∗
    by (simp add: update-triv)
  also have (σ $ y) = σ $ x
    by (simp only: lookup-def shares-result [OF 1])
  finally show ?thesis
    using 1 2 3 sharing-sym update-triv
    by fast
```

qed

lemma *update-idem* :
  assumes *2*: $x \in$ *Domain* σ
  and *3*: σ \$ $x = z$
  shows $\sigma(x{:=}_{\$}\ z) = \sigma$
proof –
 show *?thesis*
 using *2 3 sharing-refl update-triv*
 by *fast*
qed

lemma *update-apply*: $(\sigma(x :=_{\$} y))$ \$ $z = ($*if* z *shares*$_\sigma$ x *then* y *else* σ \$ $z)$
proof –
  have $*$: (λz. *if* z *shares*$_\sigma$ x *then Some* y *else fst* (*Rep-memory* σ) z, *snd* (*Rep-memory* σ))
      $\in \{(\sigma, R).$ *equivp* $R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
      unfolding *sharing-def*
      by(*rule update_sound*[*simplified fun-upd-equivp-def*], *simp*)
  show *?thesis*
    proof (*cases* z *shares*$_\sigma$ x)
      case *True*
          assume *A*: z *shares*$_\sigma$ x
          show $\sigma\ (x :=_{\$} y)$ \$ $z = ($*if* z *shares*$_\sigma$ x *then* y *else* σ \$ $z)$
                unfolding *update'' lookup-def fun-upd-equivp-def*
                by(*simp add*: *Abs-memory-inverse* [*OF* $*$])
    next
      case *False*
          assume *A*: $\neg$ z *shares*$_\sigma$ x
          show $\sigma\ (x :=_{\$} y)$ \$ $z = ($*if* z *shares*$_\sigma$ x *then* y *else* σ \$ $z)$
                unfolding *update'' lookup-def fun-upd-equivp-def*
                by(*simp add*: *Abs-memory-inverse* [*OF* $*$])
    qed
qed

lemma *update-share*:
  assumes z *shares*$_\sigma$ x
  shows $\sigma(x :=_{\$} a)$ \$ $z = a$
  using *assms*
  by (*simp only*: *update-apply if-True*)

lemma *update-other*:
  assumes $\neg$(z *shares*$_\sigma$ x)
  shows $\sigma(x :=_{\$} a)$ \$ $z = \sigma$ \$ z
  using *assms*
  by (*simp only*: *update-apply if-False*)

lemma *lookup-update-rep*:
 assumes *1*: (*snd* (*Rep-memory* σ')) x y
 shows (*fst* (*Pair-upd-lifter* (*Rep-memory* σ') *src dst*)) x =
       (*fst* (*Pair-upd-lifter* (*Rep-memory* σ') *src dst*)) y
 using *1 shares-result sharing-def sharing-upd update.rep-eq*
 by (*metis* (*hide-lams*, *no-types*))

lemma *lookup-update-rep''*:
  assumes *1*: $x\ shares_\sigma\ y$
  shows $(\sigma\ (src :=_\$ dst))\ \$\ x = (\sigma\ (src :=_\$ dst))\ \$\ y$
  using *1 lookup-def lookup-update-rep sharing-def update.rep-eq*
  by *metis*

theorem *memory-ext* :
 assumes $*$: $\bigwedge x\ y.\ (x\ shares_\sigma\ y) = (x\ shares_{\sigma'}\ y)$
  and $**$: $Domain\ \sigma = Domain\ \sigma'$
  and $***$: $\bigwedge x.\ \sigma\ \$\ x = \sigma'\ \$\ x$
 shows $\sigma = \sigma'$
apply(*subst Rep-memory-inverse*[*symmetric*])
apply(*subst* (*3*) *Rep-memory-inverse*[*symmetric*])
apply(*rule arg-cong*[*of - - Abs-memory*])
apply(*auto simp*:*Product-Type.prod-eq-iff*)
proof −
  show *fst* (*Rep-memory* σ) = *fst* (*Rep-memory* σ')
      apply(*rule ext*, *insert* $**$ $***$, *simp add*: *SharedMemory.lookup-def Domain-def*)
      apply (*metis* (*lifting*, *no-types*) *domD domIff the.simps*)
      done
next
  show *snd* (*Rep-memory* σ) = *snd* (*Rep-memory* σ')
      by(*rule ext*, *rule ext*, *insert* $*$, *simp add*: *sharing-def*)
qed

Nice connection between sharing relation, domain of the memory and content equaltiy on the one hand and equality on the other; this proves that our memory model is fully abstract in these three operations.

corollary *memory-ext2*: $(\sigma = \sigma') = ((\forall\ x\ y.\ (x\ shares_\sigma\ y) = (x\ shares_{\sigma'}\ y))$
$\wedge\ Domain\ \sigma = Domain\ \sigma'$
$\wedge\ (\forall\ x.\ \sigma\ \$\ x = \sigma'\ \$\ x))$
by(*auto intro*: *memory-ext*)

Rules On Sharing and Memory Transfer **lemma** *transfer-rep-inv-E*:
  assumes *1* : $\sigma \in \{(\sigma, R).\ equivp\ R \wedge (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
  and *2* : *memory-inv* (*transfer-rep* σ *src dst*) $\Longrightarrow Q$
  shows Q
  using *assms transfer-rep-sound*[*of* σ]
  by (*auto simp*: *Abs-memory-inverse*)

lemma *transfer-rep-fst1*:
  assumes *1*: σ = *fst*(*transfer-rep* (*Rep-memory* σ') *src dst*)
  shows $\bigwedge x.\ x = dst \Longrightarrow \sigma\ x$ = (*fst* (*Rep-memory* σ')) *src*
  using *1* **unfolding** *transfer-rep-simp*
  by *simp*

lemma *transfer-rep-fst2*:

assumes *1*: $\sigma = fst(transfer\text{-}rep\ \ (Rep\text{-}memory\ \ \sigma')\ src\ dst)$
shows $\bigwedge x.\ x \neq\ \ dst \Longrightarrow \sigma\ x = (fst\ (Rep\text{-}memory\ \ \sigma'))\ (id\ x)$
using *1* **unfolding** *transfer-rep-simp*
by *simp*

lemma *lookup-transfer-rep'*:
$(fst\ (transfer\text{-}rep\ (Rep\text{-}memory\ \ \sigma')\ src\ dst))\ src =$
$(fst\ (transfer\text{-}rep\ (Rep\text{-}memory\ \ \sigma')\ src\ dst))\ dst$
using *Rep-memory* [*of* σ']
apply (*erule-tac src= src* **and** *dst = dst* **in** *transfer-rep-inv-E*)
apply (*rotate-tac 1*)
apply (*subst* (*asm*) *surjective-pairing*[*of* (*transfer-rep* (*Rep-memory* σ') *src dst*)])
unfolding *memory-inv.simps*
apply (*erule conjE*)
apply (*erule allE*)+
apply (*erule impE*)
unfolding *transfer-rep-simp*
apply *auto*
using *equivp-reflp snd-memory-equivp*
apply *metis*
done

theorem *share-transfer*:
$$x\ shares_{\sigma(a\ \bowtie\ b)}\ y = (\ (y = b \wedge (x = b \vee (x \neq b \wedge\ \ x\ shares_{\sigma}\ a))) \vee (y \neq b \wedge ((x = b \wedge a\ shares_{\sigma}\ y) \vee (x \neq b \wedge x\ shares_{\sigma}\ y))))$$
unfolding *sharing-def transfer-def*
unfolding *transfer-def map-fun-def o-def id-def*
apply(*subst Abs-memory-inverse*[*OF transfer-rep-sound2*], *simp add*: *transfer-rep-simp*)
by (*metis equivp-reflp sharing-charn2*)

lemma *transfer-share*: $a\ shares_{\sigma(a\ \bowtie\ b)}\ b$ **by**(*simp add*: *share-transfer sharing-refl*)

lemma *transfer-share-sym*: $a\ shares_{\sigma\ (b\ \bowtie\ a)}\ b$ **by**(*simp add*: *share-transfer sharing-refl*)

lemma *transfer-share-mono*: $x\ shares_{\sigma}\ y \Longrightarrow \neg(x\ shares_{\sigma}\ b) \Longrightarrow (x\ shares_{\sigma\ (a\ \bowtie\ b)}\ y)$
by(*auto simp*: *share-transfer sharing-refl*)

lemma *transfer-share-charn*:
$\neg(x\ shares_{\sigma}\ b) \Longrightarrow \neg(y\ shares_{\sigma}\ b) \Longrightarrow x\ shares_{\sigma(a\ \bowtie\ b)}\ y = x\ shares_{\sigma}\ y$
by(*auto simp*: *share-transfer sharing-refl*)

lemma *transfer-share-trans*: $(a\ shares_{\sigma}\ x) \Longrightarrow (x\ shares_{\sigma(a\ \bowtie\ b)}\ b)$
by(*auto simp*: *share-transfer sharing-refl sharing-sym*)

lemma *transfer-share-trans-sym*:$(a\ shares_{\sigma}\ y) \Longrightarrow (b\ shares_{(\sigma(a\ \bowtie\ b))}\ y)$
using *transfer-share-trans sharing-sym* **by** *fast*

lemma *transfer-share-trans'*: $(a\ shares_{(\sigma(a\ \bowtie\ b))}\ z) \Longrightarrow (b\ shares_{(\sigma(a\ \bowtie\ b))}\ z)$
using *transfer-share sharing-sym sharing-trans* **by** *fast*

lemma *transfer-tri* : $x\ shares_{\sigma\ (a\ \bowtie\ b)}\ y \Longrightarrow x\ shares_{\sigma}\ b \lor b\ shares_{\sigma}\ y \lor x\ shares_{\sigma}\ y$
by (*metis sharing-sym transfer-share-charn*)

lemma *transfer-tri'* : $\neg\ x\ shares_{\sigma\ (a\ \bowtie\ b)}\ y \Longrightarrow\ y\ shares_{\sigma}\ b \lor \neg\ x\ shares_{\sigma}\ y$
by (*metis sharing-sym sharing-trans transfer-share-mono*)

lemma *transfer-dest'* :
assumes *1* : $a\ shares_{\sigma\ (a\ \bowtie\ b)}\ y$
and *2* : $b \neq y$
shows $a\ shares_{\sigma}\ y$
using *assms* **by**(*auto simp*: *share-transfer sharing-refl sharing-sym*)

lemma *transfer-dest* :
assumes *1* : $\neg(x\ shares_{\sigma}\ a)$
and *2* : $x \neq b$
and *3* : $x\ shares_{\sigma}\ b$
shows $\neg(x\ shares_{\sigma\ (a\ \bowtie\ b)}\ b)$
using *assms* **by**(*auto simp*: *share-transfer sharing-refl sharing-sym*)

lemma *transfer-dest''*:$x = b \Longrightarrow y\ shares_{\sigma}\ a \Longrightarrow x\ shares_{\sigma(a\ \bowtie\ b)}\ y$
by (*metis sharing-sym transfer-share-trans-sym*)

thm *share-transfer*
transfer-share
transfer-share-sym
sharing-sym [*THEN transfer-share-trans*]

sharing-sym [*THEN transfer-share-trans-sym*]

transfer-share-trans'
transfer-dest''
transfer-dest'
transfer-tri'
transfer-share-mono
transfer-tri
transfer-share-charn

transfer-dest

Properties on Memory Transfer and Lookup **lemma** *transfer-share-lookup1*: $(\sigma(x \bowtie y))\ \$\ x = \sigma\ \$\ x$
using *lookup-transfer-rep′ transfer-rep-fst1*
unfolding *lookup-def transfer.rep-eq*
by *metis*

lemma *transfer-share-lookup2*:
$(\sigma(x \bowtie y))\ \$\ y = \sigma\ \$\ x$
using *transfer-rep-fst1*
unfolding *transfer.rep-eq lookup-def*
by *metis*

lemma add_e*-not-share-lookup*:
assumes *1*: $\neg(x\ shares_\sigma\ z)$
and *2*: $\neg(y\ shares_\sigma\ z)$
shows $\sigma\ (x \bowtie y)\ \$\ z = \sigma\ \$\ z$
using *assms*
unfolding *sharing-def lookup-def transfer.rep-eq*
using *id-def sharing-def sharing-refl transfer-rep-fst2*
by *metis*

lemma *transfer-share-dom*:
assumes *1*: $z \in Domain\ \sigma$
and *2*: $\neg(y\ shares_\sigma\ z)$
shows $(\sigma(x \bowtie y))\ \$\ z = \sigma\ \$\ z$
using *assms*
unfolding *Domain-def sharing-def lookup-def*
using *2 transfer.rep-eq id-apply sharing-refl transfer-rep-fst2*
by *metis*

lemma *shares-result′*:
assumes *1*: $(x\ shares_\sigma\ y)$
shows $\sigma\ \$\ x = \sigma\ \$\ y$
using *assms lookup-def shares-result*
by *metis*

lemma *transfer-share-cancel1*:
assumes *1*: $(x\ shares_\sigma\ z)$
shows $(\sigma(x \bowtie y))\ \$\ z = \sigma\ \$\ x$
using *1 transfer.rep-eq transfer-share-trans lookup-def*
transfer-rep-fst1 shares-result
by (*metis*)

lemmas *sharing-refl-smt* = *sharing-refl*

An Intrastructure for Global Memory Spaces Memory spaces are common concepts in Operating System (OS) design since it is a major objective of OS kernels to separate logical, linear memory spaces belonging to different processes (or in other terminologies such as PiKeOS: tasks) from each other. We achieve this goal by modeling the adresses of memory spaces as a *pair* of a subject (e.g. process or task, denominated by a process-id or task-id) and a location (a conventional adress).

Our model is still generic - we do not impose a particular type for subjects or locations (which could be modeled in a concrete context by an enumeration type as well as integers of bitvector representations); for the latter, however, we require that they are instances of the type class $'\alpha$ assuring that there is a minimum of infrastructure for address calculation: there must exist a $0::'a$-element, a distinct $1::'a$-element and an addition operation with the usual properties.

fun $init_{globalmem}$ :: $(('sub \times 'loc::comm\text{-}semiring\text{-}1),\ '\beta)$ *memory*
$\Rightarrow$ $('sub \times 'loc) \Rightarrow '\beta$ *list*
$\Rightarrow$ $(('sub \times 'loc),\ '\beta)$ *memory* (- |> - <| - [60,60,60] 70)
where σ |> *start* <| [] = σ
| σ |> (*sub*,*loc*) <| (a # S) = ((σ((*sub*,*loc*):=$_\$$ a)) |> (*sub*, *loc*+1)<| S)

lemma *Domain-mem-init-Nil* : *Domain*(σ |> *start* <| []) = *Domain* σ
by *simp*

Example **type-synonym** *task-id* = *int*
type-synonym *loc* = *int*

type-synonym *global-mem* = ((*task-id*$\times$*loc*), *int*)*memory*

definition σ_0 :: *global-mem*
where σ_0 $\equiv$ *init* |> (0,0) <| [0,0,0,0]
|> (2,0) <| [0,0]
|> (4,0) <| [2,0]

lemma σ_0-*Domain*: *Domain* σ_0 = {(4, 1), (4, 0), (2, 1), (2, 0), (0, 3), (0, 2), (0, 1), (0, 0)}
unfolding σ_0-*def*
by(*simp add*: *sharing-upd*)

Memory Transfer Based on Sharing Closure (Experimental) One might have a foundamentally different understanding on memory transfer — at least as far as the sharing relation is concerned. The prior definition of sharing is based on the idea that the overridden part is "carved out" of the prior equivalence. Instead of transforming the equivance relation, one might think of transfer as an operation where the to be shared memory is synchronized and then the equivalence relation closed via reflexive-transitive closure.

definition *transfer'* :: $('a,'b)$*memory* $\Rightarrow 'a \Rightarrow 'a \Rightarrow ('a,\ 'b)$*memory* (- '(- |⋈| -') [0,111,111]110)
where σ(i |⋈| k) =
(σ(i :=$_\$$ (σ \$ k)) :=$_R$ (*rtranclp*($\lambda x\ y.$ (\$$_R$ σ) $x\ y \vee (x{=}i \wedge y = k) \vee (x{=}k \wedge y = i)$)))

lemma *transfer'-rep-sound*:

$(fst(Rep\text{-}memory\ (\sigma(i{:=}_{\$}(\sigma\ \$\ k)))),(\lambda xa\ ya.\ (\$_R\ \sigma)\ xa\ ya \lor xa = x \land ya = y \lor xa = y \land ya = x)^{**})$
$\in$
$\{(\sigma, R).\ equivp\ R \land (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$
unfolding *update-def*
proof(*auto*)
let $?R' = ((\lambda xa\ ya.\ (\$_R\ \sigma)\ xa\ ya \lor xa = x \land ya = y \lor xa = y \land ya = x)^{**})$
have *E* : *equivp* $(\$_R\ \sigma)$ **unfolding** $lookup_R$*-def* **by** (*metis snd-memory-equivp*)
have *fact1* : *symp* $?R'$
unfolding *symp-def*
apply (*auto*)
apply (*erule Transitive-Closure.rtranclp-induct*,*auto*)
apply (*drule E*[*THEN equivp-symp*])
by (*metis* (*lifting*, *full-types*) *converse-rtranclp-into-rtranclp*)+
have *fact2* : *transp* $?R'$
unfolding *transp-def*
by (*metis* (*lifting*, *no-types*) *rtranclp-trans*)
have *fact3* : *reflp* $?R'$
unfolding *reflp-def*
by (*metis* (*lifting*) *rtranclp.rtrancl-refl*)
show *equivp* $(\lambda xa\ ya.\ (\$_R\ \sigma)\ xa\ ya \lor xa = x \land ya = y \lor xa = y \land ya = x)^{**}$
using *fact1 fact2 fact3 equivpI* **by** *auto*
next
fix *xa ya*
assume *H* : $(\lambda xa\ ya.\ (\$_R\ \sigma)\ xa\ ya \lor xa = x \land ya = y \lor xa = y \land ya = x)^{**}\ xa\ ya$
have $*$: (*fun-upd-equivp* (*snd* (*Rep-memory* σ)) (*fst* (*Rep-memory* σ)) *i* (*Some* $(\sigma\ \$\ k)$),
snd (*Rep-memory* σ))
$\in \{(\sigma, R).\ equivp\ R \land (\forall x\ y.\ R\ x\ y \longrightarrow \sigma\ x = \sigma\ y)\}$ **sorry**
show *fst* (*Rep-memory* (*Abs-memory* (*Pair-upd-lifter* (*Rep-memory* σ) *i* $(\sigma\ \$\ k)$))) *xa* =
fst (*Rep-memory* (*Abs-memory* (*Pair-upd-lifter* (*Rep-memory* σ) *i* $(\sigma\ \$\ k)$))) *ya*
apply(*subst surjective-pairing*[*of* (*Rep-memory* σ)])
apply(*subst Pair-upd-lifter.simps*)
apply(*subst* (*4*)*surjective-pairing*[*of* (*Rep-memory* σ)])
apply(*subst Pair-upd-lifter.simps*)
apply(*auto simp*: *Abs-memory-inverse*[*OF* $*$])
apply(*simp add*: *SharedMemory.lookup-def*)
apply(*insert H*, *simp add*: *SharedMemory.lookup*$_R$*-def*)
oops

Framing Conditions on Shared Memories (Experimental) The Frame of an action — or a monadic operation — is the smallest possible subset of the domain of a memory, in which the action has effect, i.e. it modifies only locations in this set.

definition *Frame* :: $(('\alpha, '\beta)memory \Rightarrow ('\alpha, '\beta)memory) \Rightarrow '\alpha\ set$
where $Frame\ A \equiv Least(\lambda X.\ \forall\ \sigma.\ (\sigma(reset\ X)) = ((A\ \sigma)(reset\ X)))$

lemma *Frame-update* : $Frame\ (\lambda\sigma.\ \sigma(x :=_{\$} y)) = \{x\}$
oops

lemma *Frame-compose* : $Frame\ (A\ o\ B) \subseteq Frame\ A \cup Frame\ B$

oops

notation *transfer* (add_e)
lemmas add_e*-def* = *transfer-def*
lemmas add_e*-rep-eq* = *transfer.rep-eq*
lemmas *transfer-share-old-new-trans* = *transfer-share-trans-sym*
lemmas *sharing-commute-smt* = *sharing-commute*
lemmas *update-apply-smt* = *update-apply*
lemmas *transfer-share-lookup2-smt* = *transfer-share-lookup2*
lemmas *transfer-share-lookup1-smt* = *transfer-share-lookup1*
lemmas *transfer-share-smt* = *SharedMemory.transfer-share*

end

theory *SharedMemory-test*
imports *../../../src/Testing*
SharedMemory
begin

Our Local Instance of a Global memory Model **type-synonym** *task-id* = *int*
type-synonym *loc* = *int*

type-synonym *global-mem* = ((*task-id*×*loc*), *int*)*memory*

definition σ_0 :: *global-mem*
where $\sigma_0 \equiv init\ |\!>\ (0,0)\ <\!|\ [0,0,0,0]$
$|\!>\ (2,0)\ <\!|\ [0,0]$
$|\!>\ (4,0)\ <\!|\ [2,0]$

find-theorems *sharing*

lemma σ_0*-Domain*: *Domain* $\sigma_0 = \{(4,\ 1), (4,\ 0), (2,\ 1), (2,\ 0), (0,\ 3), (0,\ 2), (0,\ 1), (0,\ 0)\}$
unfolding σ_0*-def*
by(*simp add*: σ_0*-Domain sharing-upd*)

datatype *in-c* = *load* *task-id loc*
| *store* *task-id loc int*
| *share* *task-id loc task-id loc*

thm *in-c.split*

datatype *out* = *load-ok* *int*
| *store-ok*
| *share-ok*

fun *precond* :: *global-mem* ⇒ *in-c* ⇒ *bool*
where *precond* σ (*load tid addr*) = ((*tid*,*addr*) ∈ *Domain* σ)
| *precond* σ (*store tid addr n*) = *True*
| *precond* σ (*share tid addr tid′ addr′*) = ((*tid*,*addr*) ∈ *Domain* σ ∧ (*tid′*,*addr′*) ∈ *Domain* σ)

term *load-ok* (σ_0 \$ (*tid*,*addr*))

fun *postcond* :: *in-c* ⇒ *global-mem* ⇒ (*out* × *global-mem*) *set*
where *postcond* (*load tid addr*) σ = {(*n*,σ'). (*n* = *load-ok* (σ \$ (*tid*,*addr*))) ∧ σ'=σ}
| *postcond* (*store tid addr m*) σ = {(*n*,σ'). (*n* = *store-ok* ∧ σ'= σ((*tid*, *addr*):=$_\$$ *m*))}
| *postcond* (*share tid addr tid′ addr′*) σ = { (*n*,σ'). *n* = *share-ok* ∧ σ'=σ((*tid*,*addr*) ⋈ (*tid′*,*addr′*))}

definition *SYS* = (*strong-impl precond postcond*)

lemma *SYS-is-strong-impl* : *is-strong-impl precond postcond SYS*
by(*simp add*: *SYS-def is-strong-impl*)

lemma *precond-postcond-implementable*:
implementable precond postcond
apply(*auto simp*: *implementable-def*)
apply(*case-tac* ι, *simp-all*)
done

thm *SYS-is-strong-impl*[*simplified is-strong-impl-def*,*THEN spec*,*of* (*alloc c no m*), *simplified*, *standard*]

lemma *Eps-split-eq′* : (*SOME* (x', y'). x'= x ∧ y'= y) = (*SOME* (x', y'). x = x' ∧ y = y')
by(*rule arg-cong*[*of - - Eps*], *auto*)

consts *PUT* :: *in-c* ⇒ ′σ ⇒ (*out* × ′σ) *option*

interpretation *load* : *efsm-det*
precond postcond SYS (*load tid addr*)
$\lambda\sigma$. *load-ok* (σ \$ (*tid*,*addr*))
λ σ. σ
λ σ. (*tid*,*addr*) ∈ *Domain* σ
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq′*)

interpretation *store* : *efsm-det*
precond postcond SYS (*store tid addr m*)
λ-. *store-ok*
λ σ. σ((*tid*, *addr*):=$_\$$ *m*)
λ σ.(*True*)
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq′*)

interpretation *share* : *efsm-det*
precond postcond SYS (*share tid addr tid′ addr′*)

λ-. *share-ok*
$\lambda\ \sigma.\ \sigma((tid,addr) \bowtie (tid',addr'))$
$\lambda\ \sigma.\ ((tid,addr) \in Domain\ \sigma \wedge (tid',addr') \in Domain\ \sigma)$
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq′*)

The TestGen Setup **set-pre-safe-tac**⟪
(*fn ctxt => TestGen.ALLCASES*(
TestGen.CLOSURE (
TestGen.case-tac-typ ctxt [*SharedMemory-test.in-c*])))
⟫

declare *Monads.mbind′-bind*[*simp del*]

lemmas *update-simps* = *update-share sharing-upd update-apply update-other*
update-cancel update-triv update-commute

shares-dom Domain-transfer Domain-update

shares-init sharing-refl sharing-upd transfer-share share-transfer
sharing-commute

thm *update-simps*

An Abstract Test-Case Generation Scenario

Scenario with tests starting on an fixed initialized memory. This corresponds roughly to checking that all inductively defined shared memories build over store, load and transfer reveal the behaviour rescribed by the model.

test-spec *test-status*:
assumes *sym-exec-spec*:
$init \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $init \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$
apply(*insert assms*)
apply(*tactic TestGen.mp-fy 1*,*rule-tac x*=*x* **in** *spec*[*OF allI*])
apply(*tactic asm-full-simp-tac* @{*context*} *1*)
using [[*no-uniformity*]]
apply(*gen-test-cases 5 1 PUT*)
apply(*tactic ALLGOALS*(*TestGen.REPEAT′*(*ematch-tac* [@{*thm load.exec-mbindFStop-E*},
@{*thm store.exec-mbindFStop-E*},
@{*thm share.exec-mbindFStop-E*},
@{*thm valid-mbind′-mtE*}
])))

Normalization

apply(*simp-all add*: *update-simps*)
apply(*tactic TestGen.ALLCASES*(*TestGen.TRY′*(*fn n => REPEAT-DETERM1* ((*safe-steps-tac* @{*context*} *n*)))))
apply(*simp-all add*: *update-simps*)

Closing : Extracting PO's

using [[*no-uniformity=false*]]

apply(*tactic TestGen.ALLCASES*(*TestGen.uniformityI-tac* @{*context*} [*PUT*]))
mk-test-suite *SharedMemoryNB*

Concrete Test Data Selection

declare [[*testgen-iterations=0*]]
declare [[*testgen-SMT*]]

gen-test-data *SharedMemoryNB*
thm *SharedMemoryNB.concrete-tests*
thm *SharedMemoryNB.test-inst-thm*

end

5.3.2. The MyKeOS Case Study

theory *MyKeOS*
imports

../../../src/Testing

begin

This example is drawn from the operating system testing domain; it is a rough abstraction of PiKeOS and explains the underlying techniques of this particular case study on a small example. The full paper can be found under [5].

This is a fun-operating system — closely following the Bank example — intended to explain the principles of symbolic execution used in our PikeOS study.
Moreover, in this scenario, we assume that the system under test is deterministic.

The state of a thread (belonging to a task, i. e. a Unix/PosiX like "process" just modeled by a map from task-id/thread-id information to the number of a resource (a communication channel descriptor, for example) that was allocated to a thread.

type-synonym *task-id* = *int*

type-synonym *thread-id* = *int*

type-synonym *thread-local-var-tab* = (*task-id* $\times$ *thread-id*) $\rightharpoonup$ *int*

Operation definitions A standard, JML or OCL or VCC like interface specification might look like:

```
Init:  forall (c,no) : dom(data_base::thread_local_var_tab). data_base(c,no)>=0

op alloc (c : task_id, no : thread_id, amount:nat) : unit
```

```
pre  (c,no) : dom(data_base)
post data_base'=data_base[(c,no) := data_base(c,no) + amount]

op release(c : task_id, no : thread_id, amount:nat) : unit
pre  (c,no) : dom(data_base) and data_base(c,no) >= amount
post data_base'=data_base[(c,no) := data_base(c,no) - amount]

op status (c : task_id, no : thread_id) : int
pre  (c,no) : dom(data_base)
post data_base'=data_base and result = data_base(c,no)
```

Interface normalization turns this interface into the following input type:

datatype $\textit{in-c} = \textit{alloc} \quad \textit{task-id thread-id nat}$
$\quad | \ \textit{release task-id thread-id nat}$
$\quad | \ \textit{status} \quad \textit{task-id thread-id}$

typ $\textit{MyKeOS.in-c}$

datatype $\textit{out-c} = \textit{alloc-ok} \mid \textit{release-ok} \mid \textit{status-ok nat}$

fun $\quad \textit{precond} :: \textit{thread-local-var-tab} \Rightarrow \textit{in-c} \Rightarrow \textit{bool}$
where $\textit{precond}\ \sigma\ (\textit{alloc c no m}) = ((c,no) \in \textit{dom}\ \sigma)$
$\quad | \ \textit{precond}\ \sigma\ (\textit{release c no m}) = ((c,no) \in \textit{dom}\ \sigma \wedge (\textit{int}\ m) \leq \textit{the}(\sigma(c,no)))$
$\quad | \ \textit{precond}\ \sigma\ (\textit{status c no}) = ((c,no) \in \textit{dom}\ \sigma)$

fun $\quad \textit{postcond} :: \textit{in-c} \Rightarrow \textit{thread-local-var-tab} \Rightarrow (\textit{out-c} \times \textit{thread-local-var-tab})\ \textit{set}$
where $\textit{postcond}\ (\textit{alloc c no m})\ \sigma =$
$\quad\quad \{\ (n,\sigma').\ (n = \textit{alloc-ok} \wedge \sigma'{=}\sigma((c,no) \mapsto \textit{the}(\sigma(c,no)) + \textit{int}\ m))\}$
$\quad | \ \textit{postcond}\ (\textit{release c no m})\ \sigma =$
$\quad\quad \{\ (n,\sigma').\ (n = \textit{release-ok} \wedge \sigma'{=}\sigma((c,no) \mapsto \textit{the}(\sigma(c,no)) - \textit{int}\ m))\}$
$\quad | \ \textit{postcond}\ (\textit{status c no})\ \sigma =$
$\quad\quad \{\ (n,\sigma').\ (\sigma{=}\sigma' \wedge (\exists\ x.\ \textit{status-ok}\ x = n \wedge x = \textit{nat}(\textit{the}(\sigma(c,no))))))\}$

Constructing an Abstract Program Using the Operators `impl` and `strong_impl`, we can synthesize an abstract program right away from the specification, i.e. the pair of pre- and postcondition defined above. Since this program is even deterministic, we derive a set of symbolic execution rules used in the test case generation process which will produce symbolic results against which the PUT can be compared in the test driver.

lemma *precond-postcond-implementable*:
  implementable precond postcond
apply(*auto simp*: *implementable-def*)
apply(*case-tac* ι, *simp-all*)
done

Based on this machinery, it is now possible to construct the system model as the canonical completion of the (functional) specification consisting of pre- and post-conditions

definition *SYS* = (*strong-impl precond postcond*)

lemma *SYS-is-strong-impl* : *is-strong-impl precond postcond SYS*
by(*simp add*: *SYS-def is-strong-impl*)

thm *SYS-is-strong-impl*[*simplified is-strong-impl-def*, *THEN spec*,*of* (*alloc c no m*), *simplified*, *standard*]

Proving Symbolic Execution Rules for the Abstractly Program The following lemmas reveal that this "constructed" program is actually (due to determinism of the spec):

lemma *Eps-split-eq'* : $(SOME\ (x', y').\ x'= x\ \wedge\ y'= y) = (SOME\ (x', y').\ x = x' \wedge y = y')$
by(*rule arg-cong*[*of - - Eps*], *auto*)

interpretation *alloc* : *efsm-det*
precond postcond SYS (*alloc c no m*) λ-. *alloc-ok*
$\lambda\ \sigma.\ \sigma((c,\ no) \mapsto\ (the(\sigma(c,\ no)) +\ int\ m))\ \lambda\ \sigma.\ ((c,\ no) \in\ dom\ \sigma)$
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

interpretation *release* : *efsm-det*
precond postcond SYS (*release c no m*) λ-. *release-ok*
$\lambda\ \sigma.\ \sigma((c,\ no) \mapsto\ (the(\sigma(c,\ no)){-}int\ m))$
$\lambda\ \sigma.((c,\ no){\in}dom\ \sigma) \wedge (int\ m){\le}the(\sigma(c,no))$
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

interpretation *status* : *efsm-det*
precond postcond SYS (*status c no*)
$\lambda\sigma.\ (status\text{-}ok\ (nat(the(\sigma(c,\ no)))))$
$\lambda\ \sigma.\ \sigma\ \lambda\ \sigma.\ ((c,\ no) \in\ dom\ \sigma)$
by *unfold-locales* (*auto simp*: *SYS-def Eps-split-eq'*)

Setup Now we close the theory of symbolic execution by *exluding* elementary rewrite steps on $mbind_{FailSave}$, i. e. the rules $mbind_{FailSave}$ [] *?iostep ?σ = Some* ([], *?σ*) $mbind_{FailSave}$ (*?a # ?S*) *?iostep ?σ* = (*case ?iostep ?a ?σ of None* ⇒ *Some* ([], *?σ*) | *Some* (*out*, *σ'*) ⇒ *case* $mbind_{FailSave}$ *?S ?iostep σ' of None* ⇒ *Some* ([*out*], *σ'*) | *Some* (*outs*, *σ''*) ⇒ *Some* (*out # outs*, *σ''*))

declare *mbind.simps*(*1*) [*simp del*]
mbind.simps(*2*) [*simp del*]

Here comes an interesting detail revealing the power of the approach: The generated sequences still respect the preconditions imposed by the specification - in this case, where we are talking about a `task_id` for which a defined account exists and for which we will never produce traces in which we release more money than available on it.

Restricting the Test-Space by Test Purposes We introduce a constraint on the input sequence, in order to limit the test-space a little and eliminate logically possible, but irrelevant test-sequences for a specific test-purpose. In this case, we narrow down on test-sequences concerning a specific `task_id` *c* with a specific bank-account number *no*.

We make the (in this case implicit, but as constraint explicitly stated) test hypothesis, that the SUT is correct if it behaves correct for a single `task_id`. This boils down to the assumption that they are implemented as atomic transactions and interleaved processing does not interfere with a single thread.

fun *test-purpose* :: $[(\textit{task-id} \times \textit{thread-id})\ \textit{list},\ \textit{in-c list}] \Rightarrow \textit{bool}$
where
  $\textit{test-purpose}\ ((c,no)\#R)\ [\textit{status}\ c'\ no'] = ((c=c' \wedge no=no') \vee \textit{test-purpose}\ R\ [\textit{status}\ c'\ no'])$
$|\ \textit{test-purpose}\ ((c,no)\#R)\ ((\textit{alloc}\ c'\ no'\ m)\#S) = ((c=c' \wedge no=no' \wedge \textit{test-purpose}\ ((c,no)\#R)\ S)$
$\vee\ \textit{test-purpose}\ R\ ((\textit{alloc}\ c'\ no'\ m)\#S))$
$|\ \textit{test-purpose}\ ((c,no)\#R)\ ((\textit{release}\ c'\ no'\ m)\#S) = ((c=c' \wedge no=no' \wedge \textit{test-purpose}\ ((c,no)\#R)$
$S)$
$\vee\ \textit{test-purpose}\ R\ ((\textit{release}\ c'\ no'\ m)\#S))$
$|\ \textit{test-purpose}$ - - = *False*

lemma [*simp*] : *test-purpose* [] *a* = *False* **by** *simp*
lemma [*simp*] : *test-purpose* *r* [] = *False* **by** *simp*

lemma [*simp*] : $\textit{test-purpose}\ ((c,no)\#R)\ [a] = ((a = \textit{status}\ c\ no) \vee \textit{test-purpose}\ R\ [a])$
proof (*induct R*)
  case *Nil* **show** *?case* **by**(*cases a*, *auto*)
next
  case (*Cons* a' R') **then show** *?case*
    apply(*cases a*, *simp-all*)
    apply(*cases* a', *simp*)
    apply(*cases* a', *simp*)
    apply(*cases* a', *simp*)
    apply(*rename-tac int1 int2 aa b*)
    apply(*case-tac* $c = \textit{int1} \wedge no = \textit{int2}$, *auto*)
    done
qed

find-theorems *name*:*in-c name* :*split*
lemma [*simp*] : $R \neq [] \Longrightarrow \textit{test-purpose}\ [(c,no),(c',no')]\ (a\#R) =$
$(((\exists\, m.\ a = (\textit{alloc}\ c\ no\ m)) \vee$
$(\exists\, m.\ a = (\textit{release}\ c\ no\ m)) \vee$
$(\exists\, m.\ a = (\textit{alloc}\ c'\ no'\ m)) \vee$
$(\exists\, m.\ a = (\textit{release}\ c'\ no'\ m)))$
$\wedge\ \textit{test-purpose}\ [(c,no),(c',no')]\ R)$
apply(*simp add*: *List.neq-Nil-conv*, *elim exE*,*simp*)
apply(*auto split*: *in-c.split in-c.split-asm*)
apply(*cases a*, *auto*)
sorry

consts *PUT* :: $\textit{in-c} \Rightarrow {}'\sigma \Rightarrow (\textit{out-c} \times {}'\sigma)\ \textit{option}$

end

5.3.3. The MyKeOS "Classic" Data-sequence enumeration approach

theory *MyKeOS-test*
imports *MyKeOS*
../../../src/codegen-fsharp/Code-Integer-Fsharp
begin

The purpose of these test-scenarios is to apply the bute-force data-exploration approach to a little operation system example. It is conceptually very close the the Bank-example, essentially a renaming. However, the present "data-exploration" based approach is an interesting intermediate step to the subsequently shown scenarios based on:

1. exploration if the interleaving space
2. optimized exploration if the interleaving space, including theory for partial-order reduction.

declare [[*testgen-profiling*]]

The TestGen Setup The default configuration of `gen_test_cases` does *not* descend into sub-type expressions of type constructors (since this is not always desirable, the choice for the default had been for "non-descent"). This case is relevant here since *in-c list* has just this structure but we need ways to explore the input sequence type further. Thus, we need configure, for all test cases, and derivation descendants of the relusting clauses during splitting, again splitting for all parameters of input type *in-c*:

set-pre-safe-tac⟪
(*fn ctxt* => *TestGen.ALLCASES*(
TestGen.CLOSURE (
TestGen.case-tac-typ ctxt [*MyKeOS.in-c*])))
⟫

The Scenario We construct test-sequences for a concrete `task_id` (implicitely assuming that interleaving actions with other `task_id`'s will not influence the system behaviour. In order to prevent HOL-TestGen to perform case-splits over names — i.e. list of characters — we define it as constant.

definition tid_0 :: *task-id* **where** $tid_0 = 0$
definition tid_1 :: *task-id* **where** $tid_1 = 1$

definition $thid_0$ :: *thread-id* **where** $thid_0 = 4$
definition $thid_1$ :: *thread-id* **where** $thid_1 = 6$

declare[[*goals-limit* = *500*]]

Making my own test-data generation — temporarily **lemma** *HH* : $(A \wedge (A \longrightarrow B)) = (A \wedge B)$ **by** *auto*

Some Experiments with nitpick as Testdata Selection Machine. Exists in two formats : General Fail-Safe Tests (which allows for scenarios with normal *and* exceptional behaviour; and Fail-Stop Tests, which generates Tests only for normal behaviour and correspond to inclusion test refinement.

lemma H: $((((X586X11506, X587X11507) \in dom\ X588X11508 \longrightarrow$
$[status\text{-}ok\ (nat\ (the\ (X588X11508\ (X586X11506, X587X11507))))] = X590X11510 \wedge$
$X588X11508\ (X586X11506, X587X11507) = Some\ X589X11509) \wedge$
$((X586X11506, X587X11507) \notin dom\ X588X11508 \longrightarrow$
$[] = X590X11510 \wedge X588X11508\ (X586X11506, X587X11507) = Some\ X589X11509)))$
nitpick[*satisfy*,*debug*]
oops

lemma H: $(((X586X11506, X587X11507) \in dom$
$([(X586X11506, X587X11507) \mapsto X589X11509]) \longrightarrow$
$[status\text{-}ok\ (nat\ (the\ ($
$([(X586X11506, X587X11507) \mapsto X589X11509])\ (X586X11506, X587X11507))))] =$
$X590X11510 \wedge$
$([(X586X11506, X587X11507) \mapsto X589X11509])\ (X586X11506, X587X11507) = Some$
$X589X11509) \wedge$
$((X586X11506, X587X11507) \notin dom$
$([(X586X11506, X587X11507) \mapsto X589X11509]) \longrightarrow$
$[] = X590X11510 \wedge ([(X586X11506, X587X11507) \mapsto X589X11509])\ (X586X11506,$
$X587X11507) = Some\ X589X11509))$
nitpick[*satisfy*,*debug*,*timeout=500*]
oops

In the following, we discuss a test-scenario with error-abort semantics; i. e. in each test-case, a sequence may be chosen (by the test data selection) where the `task_id` has several accounts. . .

test-spec *test-status*:
assumes *account-def* : $(c_0,no) \in dom\ \sigma_0 \wedge (c_0,no') \in dom\ \sigma_0$
and *test-purpose* : *test-purpose* $[(c_0,no),(c_0,no')]\ S$
and *sym-exec-spec* : $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*rule rev-mp*[*OF sym-exec-spec*])
apply(*rule rev-mp*[*OF account-def*])
apply(*rule rev-mp*[*OF test-purpose*])
apply(*rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process.

apply(*gen-test-cases 4 1 PUT*)

apply(*simp-all add*: *HH split*: *HOL.split-if-asm*)
mk-test-suite *mykeos-simpleSNXB*

And now the Fail-Stop scenario — this corresponds exactly to inclusion test.

declare *Monads.mbind′-bind* [*simp del*]

test-spec *test-status2*:
assumes *system-def* : $(c_0, no) \in dom\ \sigma_0 \wedge (c_0, no') \in dom\ \sigma_0$
and *test-purpose* : *test-purpose* $[(c_0, no), (c_0, no')]\ S$
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*rule rev-mp*[*OF sym-exec-spec*])
apply(*rule rev-mp*[*OF system-def*])
apply(*rule rev-mp*[*OF test-purpose*])
apply(*rule-tac* $x{=}x$ **in** *spec*[*OF allI*])

Starting the test generation process.

using[[*no-uniformity*]]
apply(*gen-test-cases 3 1 PUT*)

So lets go for a more non-destructive approach:

apply *simp-all*

using[[*no-uniformity=false*]]
apply(*tactic TestGen.ALLCASES*(*TestGen.uniformityI-tac* @{*context*} [*PUT*]))

mk-test-suite *mykeos-simpleNB*

Test-Data Generation **thm** *mykeos-simpleSNXB.test-thm*

declare [[*testgen-iterations=0*]]
declare [[*testgen-SMT*]]

declare tid_0*-def* [*testgen-smt-facts*]
declare tid_1*-def* [*testgen-smt-facts*]
declare $thid_0$*-def* [*testgen-smt-facts*]
declare $thid_1$*-def* [*testgen-smt-facts*]

declare *mem-Collect-eq* [*testgen-smt-facts*]
declare *Collect-mem-eq* [*testgen-smt-facts*]
declare *dom-def* [*testgen-smt-facts*]
declare *the.simps* [*testgen-smt-facts*]

gen-test-data *mykeos-simpleSNXB*
thm *mykeos-simpleSNXB.concrete-tests*

gen-test-data *mykeos-simpleNB*
thm *mykeos-simpleNB.concrete-tests*

Generating the Test-Driver for an SML and C implementation The generation of the test-driver is non-trivial in this exercise since it is essentially two-staged: Firstly, we chose to generate an SML test-driver, which is then secondly, compiled to a C program that is linked to the actual program under test. Recall that a test-driver consists of four components:

- `../../../../../harness/sml/main.sml` the global controller (a fixed element in the library),
- `../../../../../harness/sml/main.sml` a statistic evaluation library (a fixed element in the library),
- `bank_simple_test_script.sml` the test-script that corresponds merely one-to-one to the generated test-data (generated)
- `bank_adapter.sml` a hand-written program; in our scenario, it replaces the usual (black-box) program under test by SML code, that calls the external C-functions via a foreign-language interface.

On all three levels, the HOL-level, the SML-level, and the C-level, there are different representations of basic data-types possible; the translation process of data to and from the C-code under test has therefore to be carefully designed (and the sheer space of options is sometimes a pain in the neck). Integers, for example, are represented in two ways inside Isabelle/HOL; there is the mathematical quotient construction and a "numerals" representation providing 'bit-string-representation-behind- the-scene" enabling relatively efficient symbolic computation. Both representations can be compiled "natively" to data types in the SML level. By an appropriate configuration, the code-generator can map "int" of HOL to three different implementations: the SML standard library `Int.int`, the native-C interfaced by `Int32.int`, and the `IntInf.int` from the multi-precision library `gmp` underneath the polyml-compiler.

We do a three-step compilation of data-reresentations model-to-model, model-to-SML, SML-to-C.

A basic preparatory step for the initializing the test-environment to enable code-generation is:

generate-test-script *mykeos-simpleNB*
thm *mykeos-simpleNB.test-script*
generate-test-script *mykeos-simpleSNXB*

In the following, we describe the interface of the SML-program under test, which is in our scenario an *adapter* to the C code under test. This is the heart of the model-to-SML translation. The the SML-level stubs for the program under test are declared as follows:

consts *status-stub* :: *task-id* $\Rightarrow$ *thread-id* $\Rightarrow$ $(int, {}'\sigma)MON_{SE}$
code-const *status-stub* $(SML\ MyKeOSAdapter.status)$
consts *alloc-stub* :: *task-id* $\Rightarrow$ *thread-id* $\Rightarrow$ *int* $\Rightarrow$ $(unit, {}'\sigma)MON_{SE}$
code-const *alloc-stub* $(SML\ MyKeOSAdapter.alloc)$
consts *release-stub*:: *task-id* $\Rightarrow$ *thread-id* $\Rightarrow$ *int* $\Rightarrow$ $(unit, {}'\sigma)MON_{SE}$
code-const *release-stub* $(SML\ MyKeOSAdapter.release)$

Note that this translation step prepares already the data-adaption; the type `nat` is seen as an predicative constraint on integer (which is actually not tested). On the model-to-model level, we provide a global step function that distributes to individual interface functions via stubs (mapped via the code generation to SML ...). This translation also represents uniformly nat by int's.

fun *my-nat-conv* :: *int* $\Rightarrow$ *nat*
where *my-nat-conv x* =(*if x* <= *0 then 0 else Suc* (*my-nat-conv*(*x* − *1*)))

fun *stepAdapter* :: (*in-c* $\Rightarrow$(*out-c*, $'\sigma$)MON_{SE})
where *stepAdapter*(*status tid thid*) =
        (*x* $\leftarrow$ *status-stub tid thid*; *return*(*status-ok* (*my-nat-conv x*)))
  | *stepAdapter*(*alloc tid thid amount*) =
        (- $\leftarrow$ *alloc-stub thid thid* (*int amount*); *return*(*alloc-ok*))
  | *stepAdapter*(*release tid thid amount*)=
        (- $\leftarrow$ *release-stub tid thid* (*int amount*); *return*(*release-ok*))

The *stepAdapter* function links the HOL-world and establishes the logical link to HOL stubs which were mapped by the code-generator to adapter functions in SML (which call internally to C-code inside `bank_adapter.sml` via a foreign language interface) ... We configure the code-generator to identify the `PUT` with the generated SML code implicitely defined by the above *stepAdapter* definition.

code-const *PUT* (*SML* *stepAdapter*)

And there we go and generate the `mykeos_simpleNB_test_script.sml` as well as the `mykeos_simpleSNXB_test_script.sml`:

export-code *stepAdapter mykeos-simpleSNXB.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/mykeos-simpleSNXB-test-script.sml*
export-code *stepAdapter mykeos-simpleNB.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c/mykeos-simpleNB-test-script.sml*

More advanced Test-Case Generation Scenarios Exploring a bit the limits ...

Rewriting based approach of symbolic execution ... FailSave Scenario

test-spec *test-status*:
assumes *account-def* : (c_0,*no*) $\in$ *dom* σ_0 $\wedge$ (c_0,*no'*) $\in$ *dom* σ_0
and *test-purpose* : *test-purpose* [(c_0,*no*),(c_0,*no'*)] *S*
and *sym-exec-spec* :
  $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailSave}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*insert account-def test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1*,*rule-tac x*=*x* **in** *spec*[*OF allI*])

Starting the test generation process.

apply(*gen-test-cases 3 1 PUT*)

Symbolic Execution:

apply(*simp-all add*: *HH split*: *HOL.split-if-asm*)

mk-test-suite *mykeos-large*

gen-test-data *mykeos-large*
thm *mykeos-large.concrete-tests*

Rewriting based approach of symbolic execution ... FailSave Scenario

test-spec *test-status*:
assumes *account-def* : $(c_0,no) \in dom\ \sigma_0 \wedge (c_0,no') \in dom\ \sigma_0$
and *test-purpose* : *test-purpose* $[(c_0,no),(c_0,no')]\ S$
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$

Prelude: Massage of the test-theorem — representing the assumptions of the test explicitely in HOL and blocking x from beeing case-splitted (which complicates the process).

apply(*insert account-def test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1,rule-tac x=x* **in** *spec*[*OF allI*])

Starting the test generation process.

using [[*no-uniformity*]]
apply(*gen-test-cases 3 1 PUT*)

Symbolic Execution:

apply(*simp-all add*: *HH split*: *HOL.split-if-asm*)

apply(*auto*)
mk-test-suite *mykeos-large2*

gen-test-data *mykeos-large2*
thm *mykeos-large2.concrete-tests*

And now, to compare, elimination based procedures ...

declare *alloc.exec-mbindFSave-If* [*simp del*]
status.exec-mbindFSave-If [*simp del*]
release.exec-mbindFSave-If [*simp del*]
alloc.exec-mbindFStop [*simp del*]
status.exec-mbindFStop [*simp del*]
release.exec-mbindFStop [*simp del*]

thm *alloc.exec-mbindFSave-E release.exec-mbindFSave-E status.exec-mbindFSave-E*

ML⟪ *open Tactical*; ⟫
test-spec *test-status*:
assumes *account-defined*: $(c_0,no) \in dom\ \sigma_0 \wedge (c_0,no') \in dom\ \sigma_0$
and *test-purpose* : *test-purpose* $[(c_0,no),(c_0,no')]\ S$
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (s = x))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$
apply(*insert account-defined test-purpose sym-exec-spec*)
apply(*tactic TestGen.mp-fy 1,rule-tac x=x* **in** *spec*[*OF allI*])
apply(*tactic asm-full-simp-tac* @{*context*} *1*)

using [[*no-uniformity*]]
apply(*gen-test-cases 3 1 PUT*)

apply(*tactic ALLGOALS*(*TestGen.REPEAT'*(*ematch-tac* [@{*thm status.exec-mbindFStop-E*},
@{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*},
@{*thm valid-mbind'-mtE*}
])))
apply(*simp-all*)
mk-test-suite *mykeos-very-large*

end

5.3.4. Implementation of integer numbers by target-language integers

theory *Code-Target-Int*
imports *Main*
begin

code-datatype *int-of-integer*

lemma [*code*, *code del*]:
integer-of-int = *integer-of-int* **..**

lemma [*code*]:
integer-of-int (*int-of-integer k*) = *k*
by *transfer rule*

lemma [*code*]:
Int.Pos = *int-of-integer* ∘ *integer-of-num*
by *transfer* (*simp add*: *fun-eq-iff*)

lemma [*code*]:
Int.Neg = *int-of-integer* ∘ *uminus* ∘ *integer-of-num*
by *transfer* (*simp add*: *fun-eq-iff*)

lemma [*code-abbrev*]:
int-of-integer (*numeral k*) = *Int.Pos k*
by *transfer simp*

lemma [*code-abbrev*]:
int-of-integer (*neg-numeral k*) = *Int.Neg k*
by *transfer simp*

lemma [*code*, *symmetric*, *code-post*]:
0 = *int-of-integer 0*
by *transfer simp*

lemma [*code*, *symmetric*, *code-post*]:
1 = *int-of-integer 1*

by *transfer simp*

lemma [*code*]:
$k + l = \textit{int-of-integer}\ (\textit{of-int}\ k + \textit{of-int}\ l)$
by *transfer simp*

lemma [*code*]:
$-\ k = \textit{int-of-integer}\ (-\ \textit{of-int}\ k)$
by *transfer simp*

lemma [*code*]:
$k - l = \textit{int-of-integer}\ (\textit{of-int}\ k - \textit{of-int}\ l)$
by *transfer simp*

lemma [*code*]:
$\textit{Int.dup}\ k = \textit{int-of-integer}\ (\textit{Code-Numeral.dup}\ (\textit{of-int}\ k))$
by *transfer simp*

lemma [*code*, *code del*]:
$\textit{Int.sub} = \textit{Int.sub}$ **..**

lemma [*code*]:
$k * l = \textit{int-of-integer}\ (\textit{of-int}\ k * \textit{of-int}\ l)$
by *simp*

lemma [*code*]:
$\textit{Divides.divmod-abs}\ k\ l = \textit{map-pair int-of-integer int-of-integer}$
$(\textit{Code-Numeral.divmod-abs}\ (\textit{of-int}\ k)\ (\textit{of-int}\ l))$
by (*simp add*: *prod-eq-iff*)

lemma [*code*]:
$k\ \textit{div}\ l = \textit{int-of-integer}\ (\textit{of-int}\ k\ \textit{div}\ \textit{of-int}\ l)$
by *simp*

lemma [*code*]:
$k\ \textit{mod}\ l = \textit{int-of-integer}\ (\textit{of-int}\ k\ \textit{mod}\ \textit{of-int}\ l)$
by *simp*

lemma [*code*]:
$\textit{HOL.equal}\ k\ l = \textit{HOL.equal}\ (\textit{of-int}\ k :: \textit{integer})\ (\textit{of-int}\ l)$
by *transfer* (*simp add*: *equal*)

lemma [*code*]:
$k \le l \longleftrightarrow (\textit{of-int}\ k :: \textit{integer}) \le \textit{of-int}\ l$
by *transfer rule*

lemma [*code*]:
$k < l \longleftrightarrow (\textit{of-int}\ k :: \textit{integer}) < \textit{of-int}\ l$
by *transfer rule*

lemma (**in** *ring-1*) *of-int-code*:
$\textit{of-int}\ k = (\textit{if}\ k = 0\ \textit{then}\ 0$

```
    else if k < 0 then − of-int (− k)
    else let
      (l, j) = divmod-int k 2;
      l′ = 2 ∗ of-int l
    in if j = 0 then l′ else l′ + 1)
proof −
  from mod-div-equality have ∗: of-int k = of-int (k div 2 ∗ 2 + k mod 2) by simp
  show ?thesis
    by (simp add: Let-def divmod-int-mod-div mod-2-not-eq-zero-eq-one-int
      of-int-add [symmetric]) (simp add: ∗ mult-commute)
qed

declare of-int-code [code]

lemma [code]:
  nat = nat-of-integer ∘ of-int
  by transfer (simp add: fun-eq-iff)

code-identifier
  code-module Code-Target-Int ⇀
    (SML) Arith and (OCaml) Arith and (Haskell) Arith

end
```

5.3.5. Avoidance of pattern matching on natural numbers

theory *Code-Abstract-Nat*
imports *Main*
begin

When natural numbers are implemented in another than the conventional inductive *0* / *Suc* representation, it is necessary to avoid all pattern matching on natural numbers altogether. This is accomplished by this theory (up to a certain extent).

Case analysis Case analysis on natural numbers is rephrased using a conditional expression:

lemma [*code*, *code-unfold*]:
  nat-case = (λ*f g n*. *if n* = *0 then f else g* (*n* − *1*))
  by (*auto simp add*: *fun-eq-iff dest*!: *gr0-implies-Suc*)

Preprocessors The term *Suc n* is no longer a valid pattern. Therefore, all occurrences of this term in a position where a pattern is expected (i.e. on the left-hand side of a code equation) must be eliminated. This can be accomplished – as far as possible – by applying the following transformation rule:

lemma *Suc-if-eq*: (⋀*n*. *f* (*Suc n*) ≡ *h n*) ⟹ *f 0* ≡ *g* ⟹
  f n ≡ *if n* = *0 then g else h* (*n* − *1*)
  by (*rule eq-reflection*) (*cases n*, *simp-all*)

The rule above is built into a preprocessor that is plugged into the code generator.

```
setup ⟪
let

fun remove-suc thy thms =
  let
    val vname = singleton (Name.variant-list (map fst
      (fold (Term.add-var-names o Thm.full-prop-of) thms []))) n;
    val cv = cterm-of thy (Var ((vname, 0), HOLogic.natT));
    fun lhs-of th = snd (Thm.dest-comb
      (fst (Thm.dest-comb (cprop-of th))));
    fun rhs-of th = snd (Thm.dest-comb (cprop-of th));
    fun find-vars ct = (case term-of ct of
        (Const (@{const-name Suc}, -) $ Var -) => [(cv, snd (Thm.dest-comb ct))]
      | - $ - =>
        let val (ct1, ct2) = Thm.dest-comb ct
        in
          map (apfst (fn ct => Thm.apply ct ct2)) (find-vars ct1) @
          map (apfst (Thm.apply ct1)) (find-vars ct2)
        end
      | - => []);
    val eqs = maps
      (fn th => map (pair th) (find-vars (lhs-of th))) thms;
    fun mk-thms (th, (ct, cv')) =
      let
        val th' =
          Thm.implies-elim
           (Conv.fconv-rule (Thm.beta-conversion true)
             (Drule.instantiate'
               [SOME (ctyp-of-term ct)] [SOME (Thm.lambda cv ct),
                 SOME (Thm.lambda cv' (rhs-of th)), NONE, SOME cv']
               @{thm Suc-if-eq})) (Thm.forall-intr cv' th)
      in
        case map-filter (fn th'' =>
            SOME (th'', singleton
              (Variable.trade (K (fn [th'''] => [th''' RS th']))
                (Variable.global-thm-context th'')) th'')
          handle THM - => NONE) thms of
            [] => NONE
          | thps =>
              let val (ths1, ths2) = split-list thps
              in SOME (subtract Thm.eq-thm (th :: ths1) thms @ ths2) end
      end
  in get-first mk-thms eqs end;

fun eqn-suc-base-preproc thy thms =
  let
    val dest = fst o Logic.dest-equals o prop-of;
    val contains-suc = exists-Const (fn (c, -) => c = @{const-name Suc});
  in
    if forall (can dest) thms andalso exists (contains-suc o dest) thms
```

then thms |> *perhaps-loop* (*remove-suc thy*) |> (*Option.map o map*) *Drule.zero-var-indexes*
else NONE
end;

val eqn-suc-preproc = *Code-Preproc.simple-functrans eqn-suc-base-preproc*;

in

Code-Preproc.add-functrans (*eqn-Suc*, *eqn-suc-preproc*)

end;
⟫

end

5.3.6. Implementation of natural numbers by target-language integers

theory *Code-Target-Nat*
imports *Code-Abstract-Nat*
begin

Implementation for *nat* **lift-definition** *Nat* :: *integer* ⇒ *nat*
is *nat*
.

lemma [*code-post*]:
Nat 0 = *0*
Nat 1 = *1*
Nat (*numeral k*) = *numeral k*
by (*transfer*, *simp*)+

lemma [*code-abbrev*]:
integer-of-nat = *of-nat*
by *transfer rule*

lemma [*code-unfold*]:
Int.nat (*int-of-integer k*) = *nat-of-integer k*
by *transfer rule*

lemma [*code abstype*]:
Code-Target-Nat.Nat (*integer-of-nat n*) = *n*
by *transfer simp*

lemma [*code abstract*]:
integer-of-nat (*nat-of-integer k*) = *max 0 k*
by *transfer auto*

lemma [*code-abbrev*]:
nat-of-integer (*numeral k*) = *nat-of-num k*
by *transfer* (*simp add*: *nat-of-num-numeral*)

lemma [*code abstract*]:
integer-of-nat (*nat-of-num n*) = *integer-of-num n*
by *transfer* (*simp add*: *nat-of-num-numeral*)

lemma [*code abstract*]:
integer-of-nat 0 = *0*
by *transfer simp*

lemma [*code abstract*]:
integer-of-nat 1 = *1*
by *transfer simp*

lemma [*code*]:
Suc n = *n* + *1*
by *simp*

lemma [*code abstract*]:
integer-of-nat (*m* + *n*) = *of-nat m* + *of-nat n*
by *transfer simp*

lemma [*code abstract*]:
integer-of-nat (*m* − *n*) = *max 0* (*of-nat m* − *of-nat n*)
by *transfer simp*

lemma [*code abstract*]:
integer-of-nat (*m* * *n*) = *of-nat m* * *of-nat n*
by *transfer* (*simp add*: *of-nat-mult*)

lemma [*code abstract*]:
integer-of-nat (*m div n*) = *of-nat m div of-nat n*
by *transfer* (*simp add*: *zdiv-int*)

lemma [*code abstract*]:
integer-of-nat (*m mod n*) = *of-nat m mod of-nat n*
by *transfer* (*simp add*: *zmod-int*)

lemma [*code*]:
Divides.divmod-nat m n = (*m div n*, *m mod n*)
by (*simp add*: *prod-eq-iff*)

lemma [*code*]:
HOL.equal m n = *HOL.equal* (*of-nat m* :: *integer*) (*of-nat n*)
by *transfer* (*simp add*: *equal*)

lemma [*code*]:
m ≤ *n* ⟷ (*of-nat m* :: *integer*) ≤ *of-nat n*
by *simp*

lemma [*code*]:
m < *n* ⟷ (*of-nat m* :: *integer*) < *of-nat n*
by *simp*

lemma *num-of-nat-code* [*code*]:
num-of-nat = *num-of-integer* ∘ *of-nat*
by *transfer* (*simp add*: *fun-eq-iff*)

lemma (**in** *semiring-1*) *of-nat-code*:
of-nat n = (*if n* = *0 then 0*
else let
(*m*, *q*) = *divmod-nat n 2*;
m′ = *2* ∗ *of-nat m*
in if q = *0 then m′ else m′* + *1*)
proof −
from *mod-div-equality* **have** ∗: *of-nat n* = *of-nat* (*n div 2* ∗ *2* + *n mod 2*) **by** *simp*
show *?thesis*
by (*simp add*: *Let-def divmod-nat-div-mod mod-2-not-eq-zero-eq-one-nat*
of-nat-add [*symmetric*])
(*simp add*: ∗ *mult-commute of-nat-mult add-commute*)
qed

declare *of-nat-code* [*code*]

definition *int-of-nat* :: *nat* ⇒ *int* **where**
[*code-abbrev*]: *int-of-nat* = *of-nat*

lemma [*code*]:
int-of-nat n = *int-of-integer* (*of-nat n*)
by (*simp add*: *int-of-nat-def*)

lemma [*code abstract*]:
integer-of-nat (*nat k*) = *max 0* (*integer-of-int k*)
by *transfer auto*

lemma *term-of-nat-code* [*code*]:
— Use *nat-of-integer* in term reconstruction instead of *Code-Target-Nat.Nat* such that reconstructed terms can be fed back to the code generator
term-of-class.term-of n =
Code-Evaluation.App
(*Code-Evaluation.Const* (*STR ″Code-Numeral.nat-of-integer″*)
(*typerep.Typerep* (*STR ″fun″*)
[*typerep.Typerep* (*STR ″Code-Numeral.integer″*) [],
typerep.Typerep (*STR ″Nat.nat″*) []]))
(*term-of-class.term-of* (*integer-of-nat n*))
by(*simp add*: *term-of-anything*)

lemma *nat-of-integer-code-post* [*code-post*]:
nat-of-integer 0 = *0*
nat-of-integer 1 = *1*
nat-of-integer (*numeral k*) = *numeral k*
by(*transfer*, *simp*)+

code-identifier
code-module *Code-Target-Nat* ⇀
(*SML*) *Arith* **and** (*OCaml*) *Arith* **and** (*Haskell*) *Arith*

end

5.3.7. Implementation of natural and integer numbers by target-language integers

theory *Code-Target-Numeral*
imports *Code-Target-Int Code-Target-Nat*
begin

end

theory *Code-gdb-script*
imports *Main ../TestLib*

begin

datatype *gdb-comand* =
  break string gdb-comand
 | *commands gdb-comand*
 | *silent gdb-comand*
 | *continue gdb-comand*
 | *thread gdb-comand*
 | *end gdb-comand*
 | *sharp string*
 |*start*

datatype *gdb-option* =
 logging gdb-option
 |*on*
 |*off*
 |*pagination gdb-option*
 |*file string*
 |*print gdb-option*

writing on file using Isabelle/ML **ML**⟪
```
  val file-path-try = ../../add−ons/OS−IFP−test/OS-kernel-model/IPC/example-gdb-impl/c/yakoub.gdb
                      |> Path.explode
                      |> Path.append (Thy-Load.master-directory @{theory });
  val file-check =  file-path-try |> File.exists;
  (*val file-write = File.write file-path-office #yakoub;*)
```
⟫

ML⟪
```
  fun writeFiles  - - [] = []
 |  writeFiles  filePath fileExtension (gdb-script :: gdb-script-list) =
```

```
    ([filePath] @ [(gdb-script :: gdb-script-list) |> length |> Int.toString] @
     [fileExtension] |> String.concat |> Path.explode |> File.write-list) gdb-script::
    writeFiles filePath fileExtension gdb-script-list;
  ⟫
```

ML ```
⟪Thy-Load.master-directory @{theory};
  fun masterPath-add theory Path = Path
                          |> Path.explode
                          |> Path.append (Thy-Load.master-directory theory)
                          |> Path.implode;
  ⟫
```

**Printing a list of terms in column using Pretty** **ML**
```
⟪
   fun pretty-terms' context terms = terms |> (Syntax.pretty-term context
                                          |> List.map)
                                       |> Pretty.chunks;

    Pretty.writeln (pretty-terms' @{context} [@{term 2::int}, @{term 2::int}]);
  ⟫
```

**Going from a list of terms to ASCII string** **ML**
```
⟪(*fun render-thm ctxt thm =
    Print-Mode.setmp [xsymbols]
   (fn - => Display.pretty-thm ctxt thm
          |> Pretty.str-of
          |> YXML.parse-body
          |> XML.content-of) ();
   render-thm @{context} @{thm conjI};*)
   fun render-term ctxt term =
    Print-Mode.setmp [xsymbols]
   (fn - => Syntax.pretty-term ctxt term
          |> Pretty.str-of
          |> YXML.parse-body
          |> XML.content-of) ();

  render-term @{context} @{term 1::int};

  fun render-term-list ctxt term =
    Print-Mode.setmp [xsymbols]
   (fn - => pretty-terms' ctxt term
          |> Pretty.str-of
          |> YXML.parse-body
          |> XML.content-of) ();
  render-term-list @{context} [@{term 1::int}, @{term 1::int}];
⟫
```

**GDB terms script to control scheduler** **ML**
```
⟪val gdb-header =
        @{term ''#setting gdb options''} $ @{term ''{''} $
        @{term set} $ @{term logging (file ''Example-sequential.log'')} $ @{term ''{''} $
        @{term set} $ @{term logging on} $ @{term ''{''} $
        @{term set} $ @{term pagination off} $ @{term ''{''} $
```
```

```
    @{term set ''target−async''} $ @{term  on} $ @{term ''{''} $
    @{term set ''non−stop''} $ @{term  on} $ @{term ''{''} $
    @{term set ''print thread−events off''} $ @{term ''{''} $ @{term ''{''}
    ;

  fun gdb-break-point-entry fun-nam-term thread-id-term =
    @{term ''#setting thread entry''} $ @{term ''{''} $
    @{term break}  $ fun-nam-term $ @{term ''{''} $
    @{term commands} $ @{term ''{''} $
    @{term silent} $ @{term ''{''} $
    @{term thread} $ thread-id-term $ @{term ''{''} $
    @{term continue} $ @{term ''{''} $
    @{term end} $ @{term ''{''} $ @{term ''{''};


  fun gdb-break-point-exist line-number-term thread-id-term =
    @{term ''#setting thread exit''} $ @{term ''{''} $
    @{term break}  $ line-number-term $ @{term ''{''} $
    @{term commands} $ @{term ''{''} $
    @{term silent} $ @{term ''{''} $
    @{term thread} $ thread-id-term $ @{term ''{''} $
    @{term continue} $ @{term ''{''} $
    @{term end} $ @{term ''{''} $ @{term ''{''};

  fun gdb-break-main-entry fun-nam-term  =
    @{term ''#setting main thread entry''} $ @{term ''{''} $
    @{term break}  $ fun-nam-term $ @{term ''{''} $
    @{term commands} $ @{term ''{''} $
    @{term silent} $ @{term ''{''} $
    @{term set} $ @{term ''scheduler−locking''} $ @{term  on} $ @{term ''{''} $
    @{term continue} $ @{term ''{''} $
    @{term end} $ @{term ''{''} $ @{term ''{''};

   fun gdb-break-main-exit line-number-term thread-id-term =
    @{term ''#wait for thread creation''} $ @{term ''{''} $
    @{term break}  $ line-number-term $ @{term ''{''} $
    @{term commands} $ @{term ''{''} $
    @{term silent} $ @{term ''{''} $
    @{term thread} $ thread-id-term $ @{term ''{''} $
    @{term continue} $ @{term ''{''} $
    @{term end} $ @{term ''{''} $ @{term ''{''};

  val gdb-start-term = @{term start} $ @{term ''{''};

  val gdb-endFile =  @{term ''#endFile''}

⟫
```

ML ⟪ *gdb-header*⟫

removing quotes and parentheses from ASCII string ML ⟪ *fun remove-char nil =* []

```
  | remove-char (x::xs) = (if ((x = #( orelse x = #)) orelse x = #')
                          then remove-char xs
                          else x::remove-char xs);
⟫
```

Jump to the next line ML ⟪ *fun next-line nil =* []

```
  | next-line (x::xs) = (if x = #{
                         then next-line (#\n::xs)
                         else x::next-line xs);
⟫
```

Going from a simple list to a list of terms ML ⟪*render-term-list* @{*context*} [@{*term* ″{″}]⟫

Terms constructors and scheme destructors ML⟪

```
fun thm-to-term thm = thm
                    |> concl-of |> HOLogic.dest-Trueprop;
fun thms-to-terms thms = thms
                       |> (thm-to-term |> map);

fun dest-valid-SE-term terms = terms |> ((fn term => case term of
                                   ((Const(@{const-name valid-SE},-) $ -)
                                    $(Const(@{const-name bind-SE},-) $ T $ -)) => T
                                          | - => term)
                                 |> map);

fun dest-mbind-term terms = terms |> ((fn term => case term of
                                      Const (@{const-name mbind}, -)
                                      $ LIST $ - => LIST
                                       |- => term )
                              |> map);

fun dest-mbind-term' terms = terms |> ((fn term => case term of
                                      Const (@{const-name mbind'}, -)
                                      $ LIST $ - => LIST
                                       |- => term )
                              |> map);

fun dest-List-term terms = terms |> ((fn term => HOLogic.dest-list term) |> map);

⟫
```

From a test thm to terms of input sequences ML ⟪*fun thm-to-inputSeqTerms test-facts* =

```
        test-facts
        |> thms-to-terms |> dest-valid-SE-term
        |> dest-mbind-term |> dest-List-term;

  fun thm-to-inputSeqTerms' test-facts =
```

```
    test-facts
    |> thms-to-terms |> dest-valid-SE-term
    |> dest-mbind-term' |> dest-List-term;
 ⟫
```

from input seuquences to strings **ML** ⟪ *fun inputSeq-to-gdbStrings actTerm-to-gdbTerm inputSeqTerms* =

```
    inputSeqTerms
    |> ((fn terms => [gdb-header]
               @(terms |> (actTerm-to-gdbTerm |> map))
               @[gdb-start-term]
               |> (render-term @{context} |> map))
                  |> map);

 fun
 breakpoint-setup (term::terms) =
 ((term::terms) |> length) :: (terms |> breakpoint-setup)   ;

 ⟫
```

ML ⟪*open List*⟫
ML ⟪*open HOLogic*;⟫

from sequeces of strings to a gdb script **ML** ⟪ *fun gdbStrings-to-gdbScripts gdbStrings* =

```
    gdbStrings
    |> ((fn strings => strings
                |> (String.implode o next-line o
                    remove-char o String.explode |> map))
        |> map);
 ⟫
```

concat terms **ML** ⟪

```
fun add-entry-exist-terms [] [] = []
  |  add-entry-exist-terms terms [] = terms
  |  add-entry-exist-terms [] terms = terms
  |  add-entry-exist-terms (term :: terms) (term'::terms') =
     term $ term':: add-entry-exist-terms terms terms';

  fun add-entry-exist-termsS [] [] = []
  |  add-entry-exist-termsS termsS [] = termsS
  |  add-entry-exist-termsS [] termsS = termsS
  |  add-entry-exist-termsS (terms :: termsS) (terms'::termsS') =
     add-entry-exist-terms terms terms'::add-entry-exist-termsS termsS termsS';

  fun add-entry-exist-termsS' [] [] = []
  |  add-entry-exist-termsS' termsS [] = termsS
  |  add-entry-exist-termsS' [] termsS = termsS
  |  add-entry-exist-termsS' (terms :: termsS) (terms'::termsS') =
     (terms @ terms')::add-entry-exist-termsS' termsS termsS';

⟫
```

from thms to gdb scripts **ML** ⟪

```
fun thms-to-gdbScripts inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos thms =
  thms
  |> thm-to-inputSeqTerms
  |> ((fn terms => inputSeq-to-gdbMain infos terms) |> map)
  |> add-entry-exist-termsS'
    (thms |> thm-to-inputSeqTerms |> ((fn terms => inputSeq-to-gdbEx infos terms)|> map))
  |> add-entry-exist-termsS
    (thms |> thm-to-inputSeqTerms |> ((fn terms => inputSeq-to-gdbEn infos terms)|> map))
  |> inputSeq-to-gdbStrings (fn term => term)
            |> gdbStrings-to-gdbScripts;

fun thms-to-gdbScripts' inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos thms =
  thms
  |> thm-to-inputSeqTerms'
  |> ((fn terms => inputSeq-to-gdbMain infos terms) |> map)
  |> add-entry-exist-termsS'
    (thms |> thm-to-inputSeqTerms' |> ((fn terms => inputSeq-to-gdbEx infos terms)|> map))
  |> add-entry-exist-termsS
    (thms |> thm-to-inputSeqTerms' |> ((fn terms => inputSeq-to-gdbEn infos terms)|> map))
  |> inputSeq-to-gdbStrings (fn term => term)
            |> gdbStrings-to-gdbScripts;
```

⟫

isa markup **ML** ⟪

```
  fun gen-gdb-scripts
    inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos theory path thms =
    thms
    |> thms-to-gdbScripts inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos
    |> writeFiles (path |> masterPath-add theory) .gdb;


  (*For mbind'*)
  fun gen-gdb-scripts'
    inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos theory path thms =
    thms
    |> thms-to-gdbScripts' inputSeq-to-gdbEn inputSeq-to-gdbEx inputSeq-to-gdbMain infos
    |> writeFiles (path |> masterPath-add theory) .gdb;


 (* val - = Outer-Syntax.command
        @{command-spec gen-gdb-script}
        store test state (theorem)
        ;*)

 (*For mbind*)

 (*val gen-gdb-script = @{thms mykeos-simple.test-data}
             |> thm-to-inputSeqTerms
```

|> *inputSeq-to-gdbStrings actTerm-to-gdbTerm*
|> *gdbStrings-to-gdbScripts*∗)

⟫

end

theory *MyKeOS-test-conc*
imports *MyKeOS*

~~/*src*/*HOL*/*Library*/*Code-Target-Numeral*
../../../*src*/*codegen-gdb*/*Code-gdb-script*
begin

declare [[*testgen-profiling*]]

Interleaving

The purpose of this example is to model system calls that consists of a number of (internal) atomic actions; the global behavior is presented by the interleaving of the actions actions

definition *syscall tid thid m m′* = [*alloc tid thid m*, *release tid thid m′*, *status tid thid*]

value *interleave* (*syscall 5 0 m m′*) (*syscall 5 1 m m′*)

In the following, we do a predicate abstraction on the interleace language, leading to an automaton represented as a set of rewrites ...

fun *Interleave* :: *in-c list* ⇒ *nat* × *nat* ⇒ *int* ⇒ *nat* ⇒ *nat* ⇒ *nat* ⇒ *nat* ⇒ *bool* (**infixl** ⋈*100*)
where *S* ⋈ (*a*, *b*) = (λ *tid m m′ m″ m‴*. (*S* ∈ *interleave* (*drop a* (*syscall tid 0 m m′*))
(*drop b* (*syscall tid 1 m″ m‴*))))
lemma *init-Interleave* : (*S* ⋈ (*0*, *0*)) *tid m m′ m″ m‴* =
(*S* ∈ *interleave* (*syscall tid 0 m m′*)
(*syscall tid 1 m″ m‴*))
by *simp*

value *interleave* (*syscall tid 0 m m′*) (*syscall tid 1 m″ m‴*)

find-theorems *name*:*Interleave*

lemma *ref-mt* [*simp*]: ¬([] ⋈ (*0*, *0*)) *tid m m′ m″ m‴*
by (*simp add*: *syscall-def*)
lemma *ref-0-0* [*simp*]: ¬(((*status a b*) # *R*) ⋈ (*0*, *0*)) *tid m m′ m″ m‴*
by (*simp add*: *syscall-def*)
lemma *ref-1-0* [*simp*]: ¬(((*status a b*) # *R*) ⋈ (*1*, *0*)) *tid m m′ m″ m‴*
by (*simp add*: *syscall-def*)
lemma *ref-0-1* [*simp*]: ¬(((*status a b*) # *R*) ⋈ (*0*, *1*)) *tid m m′ m″ m‴*
by (*simp add*: *syscall-def*)
lemma *ref-1-1* [*simp*]: ¬(((*status a b*) # *R*) ⋈ (*1*, *1*)) *tid m m′ m″ m‴*
by (*simp add*: *syscall-def*)
lemma *ref-3-1* [*simp*]: ¬(((*status a b*) # *R*) ⋈ (*3*, *1*)) *tid m m′ m″ m‴*

by (*simp add*: *syscall-def*)
lemma *ref-1-3* [*simp*]: $\neg(((status\ a\ b)\ \#\ R) \bowtie (1,\ 3))\ tid\ m\ m'\ m''\ m'''$
by (*simp add*: *syscall-def*)

lemma *trans-0-0* [*simp*]: $(((a\ \#\ R) \bowtie (0,\ 0))\ tid\ m\ m'\ m''\ m''') =$
$((a = alloc\ tid\ 0\ m \wedge (R \bowtie (1,\ 0))\ tid\ m\ m'\ m''\ m''') \vee$
$(a = alloc\ tid\ 1\ m'' \wedge (R \bowtie (0,\ 1))\ tid\ m\ m'\ m''\ m'''))$
by (*simp add*: *syscall-def*, *rule iffI*, *metis*, *metis*)
lemma *trans-1-0* [*simp*]: $(((a\ \#\ R) \bowtie (1,\ 0))\ tid\ m\ m'\ m''\ m''') =$
$((a = release\ tid\ 0\ m' \wedge (R \bowtie (2,\ 0))\ tid\ m\ m'\ m''\ m''') \vee$
$(a = alloc\ tid\ 1\ m'' \wedge (R \bowtie (1,\ 1))\ tid\ m\ m'\ m''\ m'''))$
by (*simp add*: *syscall-def*, *rule iffI*, *metis*, *metis*)
lemma *trans-2-0* [*simp*]: $(((a\ \#\ R) \bowtie (2,\ 0))\ tid\ m\ m'\ m''\ m''') =$
$(\ (a = status\ tid\ 0 \wedge R = [alloc\ tid\ 1\ m'',\ release\ tid\ 1\ m''',\ status\ tid\ 1]) \vee$
$(a = alloc\ tid\ 1\ m'' \wedge (R \bowtie (2,\ 1))\ tid\ m\ m'\ m''\ m'''))$
by (*simp add*: *syscall-def*, *rule iffI*, *metis*, *metis*)

lemma *trans-2-1* [*simp*]: $(((a\ \#\ R) \bowtie (2,\ 1))\ tid\ m\ m'\ m''\ m''') =$
$(\ (a = status\ tid\ 0 \wedge R = [release\ tid\ 1\ m''',\ status\ tid\ 1]) \vee$
$(a = release\ tid\ 1\ m''' \wedge (R \bowtie (2,\ 2))\ tid\ m\ m'\ m''\ m'''))$
by (*simp add*: *syscall-def*, *rule iffI*, *metis*, *metis*)

lemma *trans-2-2* [*simp*]: $(((a\ \#\ R) \bowtie (2,\ 2))\ tid\ m\ m'\ m''\ m''') =$
$(\ (a = status\ tid\ 0 \wedge R = [status\ tid\ 1]) \vee$
$(a = status\ tid\ 1 \wedge R = [status\ tid\ 0]))$
by (*simp add*: *syscall-def*, *rule iffI*, *metis*, *metis*)

value *interleave* (*drop 0* (*syscall tid 0 m m'*))(*drop 0* (*syscall tid 1 m'' m'''*))

TestData Hack:

lemma *PO-norm0* [*simp*]: *PO True* **by**(*simp add*: *PO-def*)

The following scenario is meant to describe the symbolic execution step by step.

declare *Monads.mbind'-bind* [*simp del*]

find-theorems $mbind_{FailStop}$ []

lemma *example-symbolic-execution-simulation* :
  assumes *H*: $S = [alloc\ tid\ 1\ m'',\ release\ tid\ 0\ m',\ release\ tid\ 1\ m''',\ status\ tid\ 1]$
  assumes *SE*: $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (x = s))$
  shows *P*
apply(*insert SE H*)
apply(*hypsubst*)
apply(*tactic ematch-tac* [@{*thm status.exec-mbindFStop-E*}, @{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*}] *1*)
apply(*tactic ematch-tac* [@{*thm status.exec-mbindFStop-E*}, @{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*}] *1*)
apply(*tactic ematch-tac* [@{*thm status.exec-mbindFStop-E*}, @{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*}] *1*)
apply(*tactic ematch-tac* [@{*thm status.exec-mbindFStop-E*}, @{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*}] *1*)

apply(*tactic ematch-tac* [@{*thm status.exec-mbindFStop-E*}, @{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*}, @{*thm valid-mbind′-mtE*}] *1*)
apply *simp*
oops

lemma
assumes *valid*: $(\sigma \models (\ s \leftarrow mbind_{FailSave}\ (alloc\ c\ no\ m\ \#\ S)\ SYS;\ unit_{SE}\ (P\ s)))$
and *case1*:
$(c,\ no) \in dom\ \sigma \Longrightarrow$
$\sigma((c,\ no) \mapsto the\ (\sigma\ (c,\ no)) + int\ m) \models$
$(s \leftarrow mbind_{FailSave}\ S\ SYS;\ unit_{SE}\ (P\ (alloc\text{-}ok\ \#\ s))) \Longrightarrow Q$
and *case2*: $(c,\ no) \notin dom\ \sigma \Longrightarrow \sigma \models unit_{SE}\ (P\ []) \Longrightarrow Q$
shows Q
apply (*insert assms*)
apply (*erule MyKeOS.alloc.exec-mbindFSave-E*)
apply *metis*
by *metis*

thm *MyKeOS.alloc.exec-mbindFSave-E*

ML⟨⟨*hyp-subst-tac* ⟩⟩
test-spec *test-status*:
assumes *account-defined*: $(tid,0) \in dom\ \sigma_0 \wedge (tid,1) \in dom\ \sigma_0$
and *test-purpose* : $S \in interleave\ (syscall\ tid\ 0\ m\ m')\ (syscall\ tid\ 1\ m''\ m''')$
and *sym-exec-spec* :
$\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ SYS;\ return\ (x = s))$
shows $\sigma_0 \models (s \leftarrow mbind_{FailStop}\ S\ PUT;\ return\ (s = x))$
apply(*insert account-defined test-purpose sym-exec-spec*)
apply(*frule length-interleave*)

apply(*simp add*: *syscall-def*)
apply(*tactic TestGen.mp-fy 1*,*rule-tac x=x* **in** *spec*[*OF allI*])

just case elaboration of test-cases

apply (*clarify*, *elim disjE*)
apply (*tactic ALLGOALS*(*hyp-subst-tac* @{*context*}))

symbolic execution

apply(*tactic ALLGOALS*(*TestGen.REPEAT′*(*ematch-tac* [@{*thm status.exec-mbindFStop-E*},
@{*thm release.exec-mbindFStop-E*},
@{*thm alloc.exec-mbindFStop-E*},
@{*thm valid-mbind′-mtE*}
])))

elimination of infeasible executions

apply(*simp-all*)
apply(*tactic ALLGOALS*(*hyp-subst-tac* @{*context*}))
apply(*tactic ALLGOALS*(*TestGen.uniformityI-tac* @{*context*} [*PUT*]))
mk-test-suite *mykeos-interleave*

declare [[*testgen-iterations=0*]]

declare [[*testgen-SMT*]]
gen-test-data *mykeos-interleave*
thm *mykeos-interleave.concrete-tests*

generate-test-script *mykeos-interleave*
thm *mykeos-interleave.test-script*

Generation of a gdb file **ML** ⟨⟨ (*building the gdb term*)

```
    fun actTerm-to-gdbTerm (Const(@{const-name alloc}, typ) $ - $ B $ -)=
      gdb-break-point-entry (Const(@{const-name alloc}, typ)) B $
      gdb-break-point-exist @{term 0} B
    | actTerm-to-gdbTerm (Const(@{const-name release}, typ) $ - $ B $ -)=
      gdb-break-point-entry (Const(@{const-name release}, typ)) B $
      gdb-break-point-exist @{term 0} B
    | actTerm-to-gdbTerm (Const(@{const-name status}, typ) $ - $ B)=
      gdb-break-point-entry (Const(@{const-name status}, typ)) B $
      gdb-break-point-exist @{term 0} B
    | actTerm-to-gdbTerm (Const(@{const-name end}, -) $ -)=
      gdb-start-term $ gdb-endFile;
⟩⟩
```

ML⟨⟨

```
 type info-threads = {task-id: int,
                      th-id: int,
                      order: int,
                      break-alloc: int * int,
                      break-release: int * int,
                      break-status: int * int,
                      break-main: int * int};
⟩⟩
```

ML⟨⟨

```
 type info-threads-configure = {input-type : typ,
                                get-task-id : term -> int,
                                       (* precond: term must be of type typ*)
                                get-thread-id : term -> int,
                                       (* precond: term must be of type typ*)
                                config-atomic-actions : (string -> int * int) -> unit,
                                set-break-main : (int * int) -> unit
                               };

(* So, the package gdb-script-generator can provide a function: *)

fun generate-gdb-script-config (X: info-threads-configure)
                               (testenv : string) (* in this case: mykeos-interleave *)
   = []: (string * int * int) list

⟩⟩
```

ML ⟨⟨ @{*thms mykeos-interleave.concrete-tests*}⟩⟩

ML⟪

```
val thread-info1 = {task-id = 5 , th-id = 1 , order = 2,
                  break-alloc = (50, 59), break-release= (59, 61), break-status = (61, 63),
                  break-main = (123, 137)};

val thread-info12 = {task-id = 5 , th-id = 0 , order = 3,
                   break-alloc = (68, 77), break-release= (77, 79), break-status = (79, 81),
                   break-main = (123, 137)};

val thread-info2 = {task-id = 3 , th-id = 1 , order = 2,
                  break-alloc = (94 , 96), break-release= (96, 98), break-status = (98, 100),
                  break-main = (123, 137)};

val thread-info31 = {task-id = 3 , th-id = 0 , order = 3,
                   break-alloc = (112, 114), break-release= (114, 116), break-status = (116, 118),
                   break-main = (123, 137)};


val needed-informations = [thread-info1, thread-info12, thread-info2, thread-info31,
                          thread-info2, thread-info31]
```

⟫

ML ⟪

```
 fun check-identity (info:info-threads) task-id th-id =
     ((info |> #task-id) = (task-id |> HOLogic.dest-number|> snd) andalso
     (info |> #th-id) = (th-id |> HOLogic.dest-number|> snd));

 fun get-successor-order [] - = 99 |> mk-number @{typ int}

 |  get-successor-order ((info:info-threads)::infos)
                      (Const(@{const-name alloc}, typ) $ task-id $ th-id $ value) =
      if check-identity info task-id th-id
      then info |> #order |> mk-number @{typ int}
      else get-successor-order infos (Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)
 |  get-successor-order ((info:info-threads)::infos)
                      (Const(@{const-name release}, typ) $ task-id $ th-id $ value) =
      if check-identity info task-id th-id
      then info |> #order |> mk-number @{typ int}
      else get-successor-order infos (Const(@{const-name release}, typ) $ task-id $ th-id $ value)
  | get-successor-order ((info:info-threads)::infos)
                      (Const(@{const-name status}, typ) $ task-id $ th-id) =
      if check-identity info task-id th-id
      then info |> #order |> mk-number @{typ int}
      else get-successor-order infos (Const(@{const-name status}, typ) $ task-id $ th-id)
  | get-successor-order - (- $ - $ - $ - $ - $ - $ - $ - $ - $ term11 $ - $ - $ - $ - $ - $ -)
                      = term11;

 fun inputSeq-to-gdbEn [] [] = []
```

```
|   inputSeq-to-gdbEn - [] = []
|   inputSeq-to-gdbEn [] terms = terms

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                 ([(Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)]) =
    if check-identity info task-id th-id
    then gdb-break-point-entry (info |> #break-alloc |> fst |> mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int}) :: []
    else inputSeq-to-gdbEn infos [(Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)]

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                 ([(Const(@{const-name release}, typ) $ task-id $ th-id $ value)]) =
    if check-identity info task-id th-id
    then gdb-break-point-entry (info |> #break-release |> fst |>mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int}):: []
    else inputSeq-to-gdbEn infos [(Const(@{const-name release}, typ) $ task-id $ th-id $ value)]

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                 ([(Const(@{const-name status}, typ) $ task-id $ th-id)]) =
    if check-identity info task-id th-id
    then gdb-break-point-entry (info |> #break-status |> fst |> mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int}) :: []
    else inputSeq-to-gdbEn infos [(Const(@{const-name status}, typ) $ task-id $ th-id)]

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                  ((Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)::terms) =
    if check-identity info task-id th-id
    then(gdb-break-point-entry (info |> #break-alloc |> fst |> mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int}))::
            inputSeq-to-gdbEn (info::infos) terms
    else inputSeq-to-gdbEn infos
        ((Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)::
         inputSeq-to-gdbEn (info::infos) terms)

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                 ((Const(@{const-name release}, typ) $ task-id $ th-id $ value)::terms) =
    if check-identity info task-id th-id
    then (gdb-break-point-entry (info |> #break-release |> fst |> mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int}))::
            inputSeq-to-gdbEn (info::infos) terms
    else inputSeq-to-gdbEn infos
        ((Const(@{const-name release}, typ) $ task-id $ th-id $ value)::
         inputSeq-to-gdbEn (info::infos) terms)

|   inputSeq-to-gdbEn ((info:info-threads)::infos)
                 ((Const(@{const-name status}, typ) $ task-id $ th-id)::terms) =
    if check-identity info task-id th-id
    then (gdb-break-point-entry (info |> #break-status |> fst |> mk-number @{typ int})
                              (info |> #order |> mk-number @{typ int})) ::
            inputSeq-to-gdbEn (info::infos) terms
    else inputSeq-to-gdbEn infos
        ((Const(@{const-name status}, typ) $ task-id $ th-id)::
```

```
      inputSeq-to-gdbEn (info::infos) terms)
  |inputSeq-to-gdbEn infos (term1 $ term2 $ term3 $ term4 $ term5 $ term6 $ ter7 $ term8 $
                      term9 $ term10 $ term11 $ term12 $ term13 $ term14 $ term15 $ term16 $
                        term17::terms) =
    term1 $ term2 $ term3 $ term4 $ term5 $ term6 $ ter7 $ term8 $
    term9 $ term10 $ term11 $ term12 $ term13 $ term14 $ term15 $ term16 $
                          term17 :: inputSeq-to-gdbEn infos terms ;

  (*|  inputSeq-to-gdbEn infos (term::terms) =
     term :: inputSeq-to-gdbEn infos terms*)


fun inputSeq-to-gdbEx [] [] = []
|   inputSeq-to-gdbEx - [] = []
|   inputSeq-to-gdbEx [] terms = terms

|   inputSeq-to-gdbEx ((info:info-threads)::infos)
                   ([(Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)]) =
    if check-identity info task-id th-id
    then gdb-break-point-exist (info |> #break-alloc |> snd |> mk-number @{typ int})
         (info |> #order |> mk-number @{typ int}):: []

    else inputSeq-to-gdbEx infos [(Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)]

|   inputSeq-to-gdbEx ((info:info-threads)::infos)
                   ([(Const(@{const-name release}, typ) $ task-id $ th-id $ value)]) =
    if check-identity info task-id th-id
    then gdb-break-point-exist (info |>  #break-release |> snd |> mk-number @{typ int})
         (info |> #order |> mk-number @{typ int}):: []
    else inputSeq-to-gdbEx infos [(Const(@{const-name release}, typ) $ task-id $ th-id $ value)]

|   inputSeq-to-gdbEx ((info:info-threads)::infos)
                   ([(Const(@{const-name status}, typ) $ task-id $ th-id)]) =
    if check-identity info task-id th-id
    then gdb-break-point-exist (info |> #break-status |> snd |> mk-number @{typ int})
         (info |> #order |> mk-number @{typ int}):: []
    else inputSeq-to-gdbEx infos [(Const(@{const-name status}, typ) $ task-id $ th-id )]

|   inputSeq-to-gdbEx ((info:info-threads)::infos)
                    ((Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)::terms) =
    if check-identity info task-id th-id
    then gdb-break-point-exist (info |> #break-alloc |> snd |> mk-number @{typ int})
                            (if terms = []
                             then (info |> #order |> mk-number @{typ int})
                             else get-successor-order (info::infos)
                                                     (hd terms))::
               inputSeq-to-gdbEx (info::infos) terms
    else inputSeq-to-gdbEx infos
       ((Const(@{const-name alloc}, typ) $ task-id $ th-id $ value)::
        terms)

 |  inputSeq-to-gdbEx ((info:info-threads)::infos)
```

```
                 ((Const(@{const-name release}, typ) $ task-id $ th-id $ value)::terms) =
   if check-identity info task-id th-id
   then gdb-break-point-exist (info |> #break-release |> snd |> mk-number @{typ int})
                              (if terms = []
                               then (info |> #order |> mk-number @{typ int})
                               else get-successor-order (info::infos)
                                                        (hd terms)) ::
              inputSeq-to-gdbEx (info::infos) terms
   else inputSeq-to-gdbEx infos
       ((Const(@{const-name release}, typ) $ task-id $ th-id $ value)::
         terms)

 |  inputSeq-to-gdbEx ((info:info-threads)::infos)
                 ((Const(@{const-name status}, typ) $ task-id $ th-id)::terms) =
   if check-identity info task-id th-id
   then gdb-break-point-exist (info |> #break-status |> snd |> mk-number @{typ int})
                              (if terms = []
                               then (info |> #order |> mk-number @{typ int})
                               else get-successor-order (info::infos)
                                                        (hd terms)) ::
              inputSeq-to-gdbEx (info::infos) terms
   else inputSeq-to-gdbEx infos
       ((Const(@{const-name status}, typ) $ task-id $ th-id)::
         terms)

 |inputSeq-to-gdbEx infos (term1 $ term2 $ term3 $ term4 $ term5 $ term6 $ ter7 $ term8 $
                         term9 $ term10 $ term11 $ term12 $ term13 $ term14 $ term15 $ term16 $
                           term17::terms) =
   term1 $ term2 $ term3 $ term4 $ term5 $ term6 $ ter7 $ term8 $
   term9 $ term10 $ term11 $ term12 $ term13 $ term14 $ term15 $ term16 $
                             term17  :: inputSeq-to-gdbEx infos terms
  (*|  inputSeq-to-gdbEx infos (term::terms) =
    term :: inputSeq-to-gdbEn infos terms*);
gdb-break-main-entry (main |> HOLogic.mk-string);

fun
 add-gdb-main [] terms = terms
|   add-gdb-main ((info:info-threads)::infos)
               (Const(@{const-name alloc}, typ) $ task-id $ th-id $ value::terms) =
    if check-identity info task-id th-id
    then
        [gdb-break-main-entry (info |> #break-main |> fst |> mk-number @{typ int})$
         gdb-break-main-exit (info |> #break-main |> snd |> mk-number @{typ int})
                             (info |> #order |> mk-number @{typ int})]
    else add-gdb-main infos (Const(@{const-name alloc}, typ) $ task-id $ th-id $ value::terms)

|   add-gdb-main ((info:info-threads)::infos)
               (Const(@{const-name release}, typ) $ task-id $ th-id $ value::terms) =
    if check-identity info task-id th-id
    then
        [gdb-break-main-entry (info |> #break-main |> fst |> mk-number @{typ int})$
         gdb-break-main-exit (info |> #break-main |> snd |> mk-number @{typ int})
```

```
                    (info |> #order |> mk-number @{typ int})]
    else add-gdb-main infos (Const(@{const-name release}, typ) $ task-id $ th-id $ value::terms)

  |  add-gdb-main ((info:info-threads)::infos)
            (Const(@{const-name status}, typ) $ task-id $ th-id::terms) =
    if check-identity info task-id th-id
    then
      [gdb-break-main-entry (info |> #break-main |> fst |> mk-number @{typ int})$
       gdb-break-main-exit (info |> #break-main |> snd |> mk-number @{typ int})
                    (info |> #order |> mk-number @{typ int})]
    else add-gdb-main infos (Const(@{const-name status}, typ) $ task-id $ th-id::terms)

  |  add-gdb-main - - = [];

»
```

ML «

```
gen-gdb-scripts'
     inputSeq-to-gdbEn inputSeq-to-gdbEx add-gdb-main needed-informations
    @{theory} impl/c-conc/MyKeOS (@{thms mykeos-interleave.concrete-tests});

  »
```

Experimental Space

declare[[*testgen-trace*]]

Code Generation Setup For Concurrent Scenario

Generation of an SML file to put datatypes **definition** *program-dum-conc*
::(*int* × *int* ⇒ *int option*) ⇒ *in-c* ⇒ *out-c* ⇒ (*int* × *int* ⇒ *int option*)
where *program-dum-conc* σ *a outs* = [(*0*, *0*)↦ *0*]

export-code *program-dum-conc* **in** *SML*
module-name *Datatypes* **file** *impl/c-conc/datatypes.sml*

Code Setup for Datatypes **code-printing**
type-constructor *in-c* => (*SML*) *Datatypes.in'-c*
|**constant** *alloc* => (*SML*) !(*Datatypes.Alloc* (- , - , -))
|**constant** *release* => (*SML*) !(*Datatypes.Release* (- , - , -))
|**constant** *status* => (*SML*) !(*Datatypes.Status* (- , -))

code-printing
type-constructor *out-c* => (*SML*) *Datatypes.out'-c*
|**constant** *alloc-ok* => (*SML*) *Datatypes.Alloc'-ok*
|**constant** *release-ok* => (*SML*) *Datatypes.Release'-ok*
|**constant** *status-ok* => (*SML*) !(*Datatypes.Status'-ok* (-))

code-printing
type-constructor *int* =>

(*SML*) *Datatypes.int*
| **constant** *int-of-integer* =>
(*SML*) !(*Datatypes.Int'-of'-integer* (-))

code-printing
type-constructor *nat* =>
(*SML*) *Datatypes.nat*
| **constant** *Nat* => (*SML*) !(*Datatypes.Nat* (-))

HOL to SML adapter

Constant definitions: stubs **consts** *MyKeOS-conc1* :: *int* ⇒ *int* ⇒ *int* ⇒ (*int*, $'\sigma$)MON_{SE}

Conversion: Integer to Action Output **fun** *my-nat-conv* :: *int* ⇒ *nat*
where *my-nat-conv x* =(*if x* <= *0 then 0 else Suc* (*my-nat-conv*(*x* − *1*)))

fun *stubs-to-out-conc*::*in-c* ⇒ (*int* × *int* ⇒ *int option*) ⇒ *int* ⇒ *out-c*
where
stubs-to-out-conc (*alloc task-id th-id res*) σ σ-*impl* =
(*if* (σ-*impl* = (*plus* ((*the o* σ)(*task-id*,*th-id*)) (*int res*)))
then alloc-ok
else release-ok)
| *stubs-to-out-conc* (*release task-id th-id res*) σ σ-*impl* =
(*if* (σ-*impl* = (*minus* ((*the o* σ)(*task-id*,*th-id*)) (*int res*)))
then release-ok
else alloc-ok)
| *stubs-to-out-conc* (*status task-id th-id*) σ σ-*impl* =
(*if* (σ-*impl* = ((*the o* σ)(*task-id*,*th-id*)))
then status-ok (*my-nat-conv* σ-*impl*)
else release-ok)

fun *mykeAdapter-con*::*in-c* ⇒ (*int* × *int* ⇒ *int option*) ⇒ (*out-c* × (*int* × *int* ⇒ *int option*)) *option*
where
mykeAdapter-con (*alloc task-id th-id res*) σ =
(*out* ← *MyKeOS-conc1 task-id th-id res*;
return(*stubs-to-out-conc* (*alloc task-id th-id res*) σ
((*fst o the*) (*MyKeOS-conc1 task-id th-id res* σ)))) σ
| *mykeAdapter-con* (*release task-id th-id res*) σ =
(*out* ← *MyKeOS-conc1 task-id th-id res*;
return(*stubs-to-out-conc* (*alloc task-id th-id res*) σ
((*fst o the*) (*MyKeOS-conc1 task-id th-id res* σ)))) σ
| *mykeAdapter-con* (*status task-id th-id*) σ =
(*out* ← *MyKeOS-conc1 task-id th-id* (*the* (σ (*task-id*, *th-id*)));
return(*status-ok* ((*my-nat-conv o the*) (σ (*task-id*, *th-id*))))) σ

Serialisation: semantics of conc stubs **code-printing**
constant *MyKeOS-conc1* => (*SML*) !(*MyKeOSAdapter.get′-state* (-) (-) (-) (-))

export-code *mykeAdapter-con* **in** *SML*
module-name *MykeAdapter* **file** *impl/c-conc/mykeAdapter.sml*

Serialisation: semantics of SUT **code-printing**
constant *PUT* => (*SML*) !(*MykeAdapter.mykeAdapter′-con* (-) (-))

export-code *mykeos-interleave.test-script* **in** *SML*
module-name *TestScript* **file** *impl/c-conc/mykeos-test-script.sml*

end

A. Glossary

Abstract test data : In contrast to pure ground terms over constants (like integers $1, 2, 3$, or lists over them, or strings ...) abstract test data contain arbitrary predicate symbols (like *triangle 3 4 5*).

Regression testing: Repeating of tests after addition/bug fixes have been introduced into the code and checking that behavior of unchanged portions has not changed.

Stub: Stubs are "simulated" implementations of functions, they are used to simulate functionality that does not yet exist ore cannot be run in the test environment.

Test case: An abstract test stimuli that tests some aspects of the implementation and validates the result.

Test case generation: For each operation the pre/postcondition relation is divided into sub-relations. It assumes that all members of a sub-relation lead to a similar behavior of the implementation.

Test data: One or more representative for a given test case.

Test data generation (Test data selection): For each test case (at least) one representative is chosen so that coverage of all test cases is achieved. From the resulting test data, test input data processable by the implementation is extracted.

Test execution: The implementation is run with the selected test input data in order to determine the test output data.

Test executable: An executable program that consists of a test harness, the test script and the program under test. The Test executable executes the test and writes a test trace documenting the events and the outcome of the test.

Test harness: When doing unit testing the program under test is not a runnable program in itself. The *test harness* or *test driver* is a main program that initiates test calls (controlled by the test script), i. e. drives the method under test and constitutes a test executable together with the test script and the program under test.

Test hypothesis : The hypothesis underlying a test that makes a successful test equivalent to the validity of the tested property, the test specification. The current implementation of HOL-TestGen only supports uniformity and regularity hypotheses, which are generated "on-the-fly" according to certain parameters given by the user like *depth* and *breadth*.

Test specification : The property the program under test is required to have.

Test result verification: The pair of input/output data is checked against the specification of the test case.

Test script: The test program containing the control logic that drives the test using the test harness. HOL-TestGen can automatically generate the test script for you based on the generated test data.

Test theorem: The test data together with the test hypothesis will imply the test specification. HOL-TestGen conservatively computes a theorem of this form that relates testing explicitly with verification.

Test trace: Output made by a test executable.

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