Backtracking Iterators

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ML Iterators

abstract datatype `t` for a **collection** of elements of type `elt`

it is usual to provide **iterators** such as

`fold : (elt` $\rightarrow \alpha \rightarrow \alpha$`)` $\rightarrow$ `t` $\rightarrow \alpha \rightarrow \alpha$

and

`iter : (elt` $\rightarrow$ `unit)` $\rightarrow$ `t` $\rightarrow$ `unit`

massively used: one occurrence each 40 lines of code

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Drawbacks

in some situations, the higher-order iterators are not convenient

1. premature **interruption**
   e.g. is there an element in the collection satisfying `p:elt→bool`?

2. **simultaneous** traversal of several collections
   e.g. the same-fringe problem (do two binary trees have the same leaves when read from left to right?)

3. **backtracking**
   e.g. greedy graph-coloring algorithm based on a given iterator to visit the graph nodes (DFS, BFS, etc.)

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Cursors in Object-Oriented Programming

an iterator is an object of a class such as

```
class Iterator {
  boolean hasNext();
  Object next();
}
```

and is used with the following pattern

```
for (Iterator i = t.iterator(); i.hasNext(); )
  ...  visit i.next() ...
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nicely addresses drawbacks 1 (interruption) and 2 (simultaneous traversals) but inelegant (hidden side effect) and does not allow backtracking

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Persistent Iterators

a different paradigm of iteration: a **persistent iterator**

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type enum
val start : t → enum
val step  : enum → elt × enum
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nice solution to our three issues

already exists in the ML folklore

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Binary Tree Comparison

total ordering of binary search trees in the Ocaml or SML standard libraries

```
type t = E | N of t × int × t

type enum = End | More of int × t × enum

let rec left t e = match t with
  | E → e
  | N (l, x, r) → left l (More (x, r, e))

let start t = left t End

let step = function
  | End → raise Exit
  | More (x, r, e) → x, left r e
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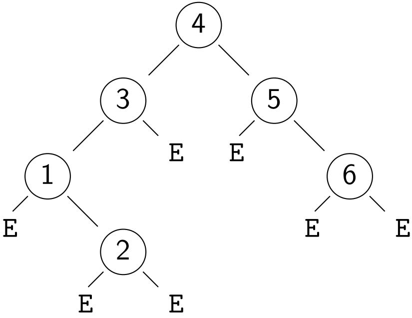

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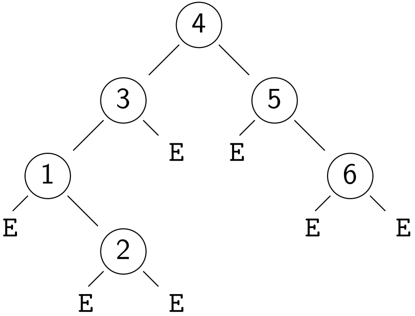
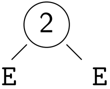
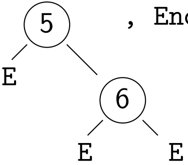


More(1,  , More(3, E, More(4,  , End)))

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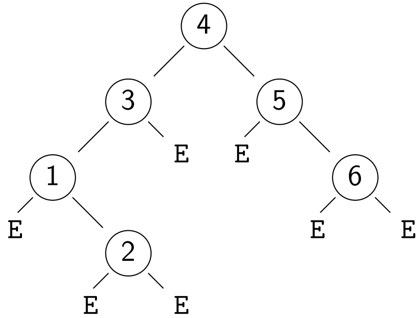
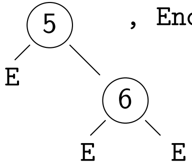


1 More(2, E, More(3, E, More(4, 

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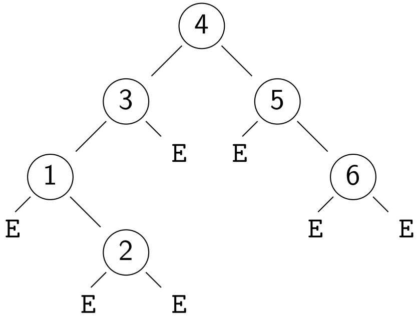
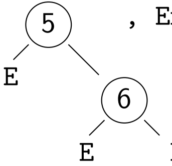


1 2 `More(3, E, More(4,`  `, End))`

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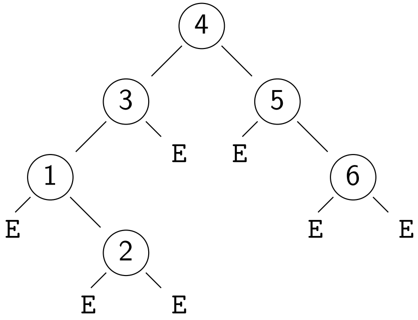
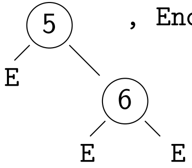


1 2 3 More(4, 

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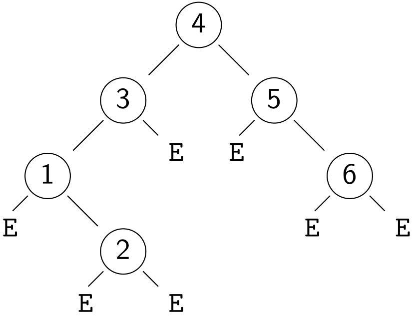
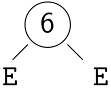
Example



1 2 3 4 More(5, 

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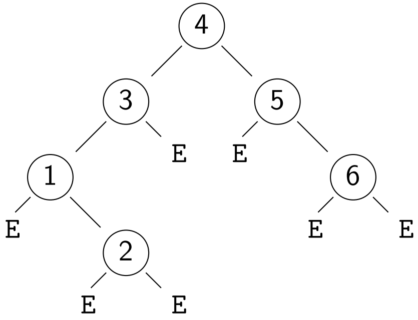
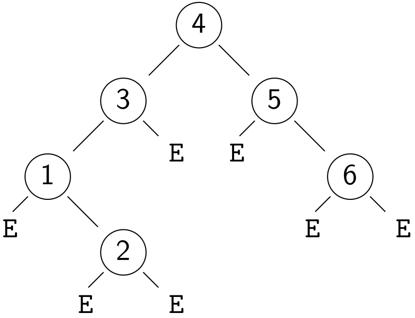


1 2 3 4 5 More(6, E, End)

Example



1 2 3 4 5 6 End

A Systematic Way?

we can build similar **ad-hoc** persistent iterators for preorder, postorder or breadth-first traversals (cf the proceedings)

but is there a **systematic** way to build persistent iterators?

at least one: **lazy lists**

a lazy list is a **function** returning its first element and the lazy list of the remaining elements, i.e.

```
type enum = unit → int × enum
```

hence `step` is immediate

```
let step k = k ()
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Lazy List Example: Inorder Traversal

it is convenient to switch to Continuation Passing Style

```
let start t = run t (fun () → raise Exit)
```

```
let rec run t k = match t with
  | E → k
  | N (l,x,r) → run l (fun () → (x, run r k))
```

if efficiency really matters, we can even **defunctionalize** the lazy list

then we get solutions roughly similar to the ad-hoc data structures

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Another Approach

Gérard Huet. *The Zipper*. Journal of Functional Programming, 1997

the **zipper** is to the **applicative** structure
what the **pointer** is to the **imperative** structure

it is a mean to move within an applicative structure
and to make local **modifications**

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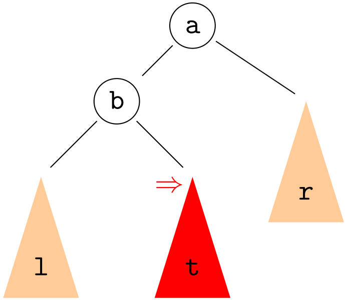
Zipper for Binary Trees

```
type path = Top | Left of path × int × t | Right of t × int × path
type location = t × path
```



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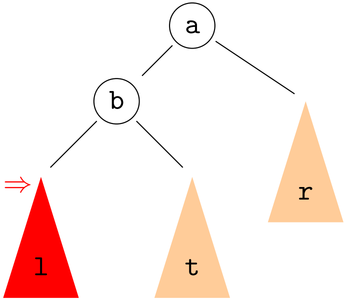

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Zipper for Binary Trees

```
let create t = (t, Top)

let go_down_left = function
  | E, _ → invalid_arg "go_down_left"
  | N (l, x, r), p → l, Left (p, x, r)

let go_left = function
  | _, Top | _, Left _ → invalid_arg "go_left"
  | r, Right (l, x, p) → l, Left (p, x, r)

let change (_, p) t = (t, p)

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Inorder Traversal using the Zipper

persistent iterator = zipper

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let start t = (t, Top)
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  | E, Top → raise Exit
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constructor Right not even used ⇒

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type path = Top | Left of path × int × t
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type **isomorphic** to

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constructor `Right` not even used ⇒

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Similarly...

for preorder and postorder traversals, we retrieve solutions isomorphic to the ad-hoc ones using the Zipper (cd the proceedings)

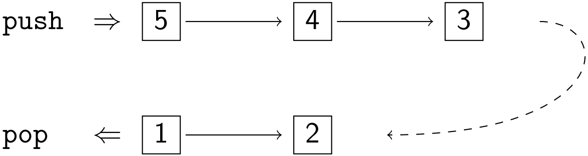
Breadth-First Traversal

usual iterator implemented using a **queue**

```
let bfs f t =
  let q = Queue.create () in
  Queue.push t q;
  while not (Queue.is_empty q) do match Queue.pop q with
    | E → ()
    | N (l, x, r) → f x; Queue.push l q; Queue.push r q
  done
```

Persistent Queues

```
module Q = struct
  type α t = α list × α list
```




```
  let push = ...
  let pop = ...
  ...
end
```

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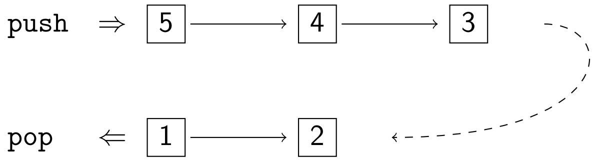


```
  let push = ...
  let pop = ...
  ...
end
```

Persistent Iterator using a Persistent Queue

```
type enum = t Q.t

let start t = Q.push t Q.empty

let rec step e =
  try match Q.pop e with
    | E, e → step e
    | N (l, x, r), e → x, Q.push r (Q.push l e)
  with Q.Empty →
    raise Exit
```

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What about the Zipper?

we need a Zipper generalized to **forests**

```
type path = Top | Node of t list × path × t list
type location = t × path
```

navigation

```
let go_left = function
  | t, Node (l :: ll, p, r) → l, Node (ll, p, t :: r)
  | _ → invalid_arg "go_left"

let go_right = function
  | t, Node (l, p, r :: rr) → r, Node (t :: l, p, rr)
  | _ → invalid_arg "go_right"
```

What about the Zipper?

we need a Zipper generalized to **forests**

```
type path = Top | Node of t list × path × t list
type location = t × path
```

navigation

```
let go_left = function
  | t, Node (l :: ll, p, r) → l, Node (ll, p, t :: r)
  | _ → invalid_arg "go_left"

let go_right = function
  | t, Node (l, p, r :: rr) → r, Node (t :: l, p, rr)
  | _ → invalid_arg "go_right"
```

Persistent Iterator using the Zipper

```
type enum = location
let start t = t, Node ([], Top, [])

let rec step = function
  | E, Node ([], _, []) →
      raise Exit
  | N (l, x, r), Node (ll, p, rr) →
      x, (E, Node (r :: l :: ll, p, rr))
  | E, Node (ll, p, r :: rr) →
      step (r, Node (ll, p, rr))
  | E, Node (ll, p, []) →
      step (E, Node ([], p, List.rev ll))
```

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Simplification

the Zipper is actually always of the shape `Node(_,Top,_)`

we simplify:

```
type enum = t list × t list
let start t = [], [t]
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  | [], [] → raise Exit
  | ll, [] → step ([], List.rev ll)
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this is **exactly** the solution using persistent queues (which are here *inlined*)

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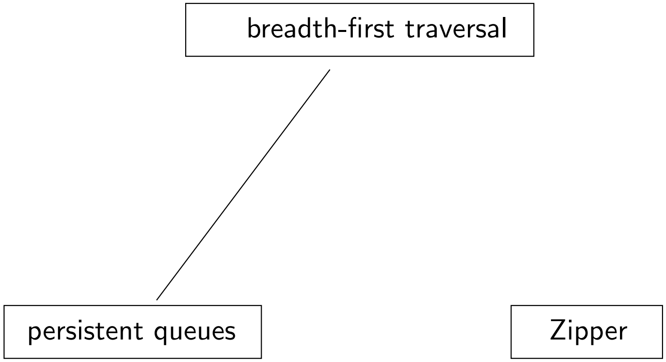
Connections



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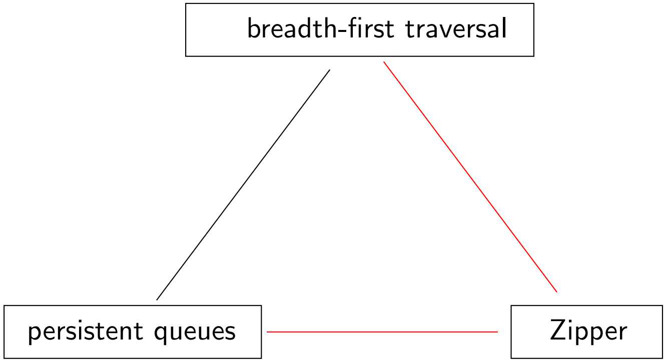

Performances

traversal	code	random	left	right	full
inorder	ad-hoc	1.07	0.86	0.11	0.25
	zipper	1.14	1.12	0.11	0.27
	lazy list	1.28	1.38	0.12	0.29
preorder	ad-hoc	1.01	0.76	0.10	0.26
	zipper	0.99	0.99	0.11	0.27
	lazy list	1.51	0.14	0.16	0.41
postorder	ad-hoc	1.26	0.86	1.04	0.28
	zipper	1.42	1.08	1.06	0.35
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bfs	manual	14.57	0.44	0.47	4.62
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Back on Graph Coloring

in Ocamlgraph, persistent iterators are provided for depth-first and breadth-first traversals ⇒ direct application to graph coloring

```
let rec iterate iter =
  let v = Bfs.get iter in
  let m = G.Mark.get v in
  if m > 0 then
    iterate (Bfs.step iter)
  else begin
    for i = 1 to k do
      try try_color v i; iterate (Bfs.step iter)
      with NoColoring → ()
    done;
    uncolor v; raise NoColoring
  end
in
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solving the **Sudoku** as a 9-coloring of a graph

tests show that

- a breadth-first traversal is really more efficient than a depth-first traversal
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- persistent iterators should be part of standard libraries
- the Zipper is a useful tool to build such iterators
- as a bonus, we rediscovered the two-lists implementation of persistent queues

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