A Two-level Auction for Resource Allocation in Multi-tenant C-RAN

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Abstract

We consider in this paper a dynamic resource allocation scheme between several Mobile Virtual Network Operators (MVNOs), sharing common radio resources at a Cloud-based Radio Access Network (C-RAN) run by a central operator. We specifically propose a two-level coupled auction so as to enhance resource utilization and maximize the revenues both for the central operator and the MVNOs: at the lower level, end users belonging to a given MVNO bid for resources and, at the higher-level, MVNOs compete for resources at the central operator based on the output of the lower-level auction. We show fundamental economic properties of our proposal: truthfulness and individual rationality, and propose a greedy algorithm to enhance its computational efficiency. We prove the existence of Nash equilibrium for the global auction and its uniqueness in a typical duopoly scenario. Further numerical results illustrate the performance of our proposal in various network settings.

Keywords: ***Index terms—Resource Allocation, C-RAN, MVNO, Auction, Nash Equilibrium.***

1. Introduction

Next generation (5G) mobile networks are targeting twenty five-fold data rates provided by the current generation of mobile networks, with higher efficiency, enhanced mobility support and seamless management of connected devices. In order to provide such features, at reduced Capital Expenditure (CAPEX) and Operational Expenditure (OPEX) [1], the Cloud-RAN, also termed Virtual-RAN (V-RAN), paradigm has been recently proposed [2],[3].

The C-RAN architecture is based on two key features: (1) *Centralization*, wherein computational resources of base stations, namely Base Band Units (BBUs), are pooled together in a central Cloud, and (2) *Virtualization*, with the possibility that several Mobile Virtual Network Operators (MVNOs) share the radio resources and the BBUs in order to reduce physical resources' costs and maintenance [4][5]. This evolution poses however multiple challenges, especially in terms of dynamic resource allocation between the users as well as between the MVNOs.

In this paper, we consider a multi-tenant C-RAN, where MVNOs compete dynamically for the shared resources with the aim of serving their customers demands while maximizing their revenues. To this end, we propose a two-level auction, coupled in a hierarchical way: a higher-level auction between the MVNOs and the central C-RAN operator, and a lower-level one between end users of a single MVNO.

Our motivation for using auction-based resource allocation is both technical and economical: technically, auctions enable to increase the efficiency of resource utilization and economically, they are well adapted to maximize the revenues of the sellers (or auctioneers) [6].

At the higher-level, we make use of weighted proportional fairness [7] in the sharing of resources between the competing MVNOs. Each MVNO receives a share of resources proportional to its bid versus the bids of the other MVNOs, where the bid consists of the price the MVNO is willing to pay in order to purchase a given quantity of resources.

At the lower level, the individual users submit their bids consisting of the required resources and the price the user is willing to pay. The effective price (what the user should effectively pay) is determined according to the Vickrey-Clarke-Groves (VCG) auction [8] based on the *virtual valuation* concept introduced by Myerson in [9], which is a second price sealed bid auction known to enforce truthfulness of the bidding. The rationale behind VCG auction is that each bidder will have an incentive to declare its true valuation for the commodity it desires so as to maximize its individual payoff [10]. This property guarantees, in turn, a high revenue for the MVNO.

We formulate the revenue maximization problem of each MVNO using Integer Linear Programming (ILP), and we determine the optimal set of users who win the bidding process as well as the price each one has to pay to obtain the requested resources.

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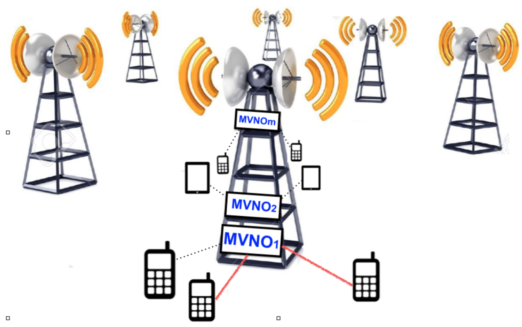


Figure 1: System model with different base stations, owned by a single physical operator, m mobile virtual network operators (MVNOs) and N end users

In summary, our main contributions in this paper are the following:

- We propose a coupled, two-level auction especially tailored for the multi-tenant C-RAN setting;
- For the lower-level auction, as VCG alone does not guarantee revenue maximization for the MVNO [11], we make use of the *virtual valuation* concept introduced by Myerson in [9] so as to obtain an optimal VCG auction enforcing truthfulness and maximizing revenues. This solution can be applied in Bayesian settings, where the MVNOs know (or can estimate) the probability distribution of their users' valuations, which is a reasonable assumption commonly used in the literature.
- We propose a sub-optimal greedy algorithm so as to solve the ILP-based winner and price determination problems in polynomial time, contrary to the optimal solution which is known to be NP-hard and is hence solved in an exponential time.
- We study the Nash equilibrium for the global auction, consisting of the two levels, and show its existence and uniqueness under some conditions.
- We perform an extensive numerical analysis, implementing our proposed approach in several, typical mobile network scenarios, and illustrate the effectiveness of our scheme in terms of efficient resource allocation as well as revenue maximization.

To our knowledge, our work is the first that tackles in a systematic manner, by means of a hierarchical two-coupled auction, the resource allocation problem in the context of C-RAN.

The remainder of this paper is organized as follows. Section 2 gives an overview of related works. In Section 3, we describe the system model, formulate our proposed two-level auction, derive the optimal solution and show its main properties. In Section 4, we prove the existence and uniqueness of the Nash equilibrium of the global auction. In Section 5, we evaluate numerically the performance of our proposed algorithms and compare them to other allocation strategies. Finally, Section 6 concludes the paper.

2. Related works

Dynamic resource allocation is especially tailored for Cloud computing and shared data centers, where virtual technologies are adopted to optimize resource usage [12, 13, 14, 15]. Dynamic spectrum sharing is a key problem in cognitive radio networks, especially due to the time-varying nature of shared resources, in particular the spectrum [16, 17, 18]. The work in [5] shows the importance of dynamic resource allocation in increasing the multiplexing gain brought by the new C-RAN architecture. Furthermore, the application of Software Defined Networking (SDN) and virtualization concepts to the C-RAN paradigm is proposed in [3, 19] to converge towards the future mobile generation network with the capability of handling heterogeneous types of applications and technologies. In particular, [19] addresses the issue of competitive spectrum sharing among the tenant operators and the possibility of adopting auction-based approaches.

Auction theory has been extensively applied to address the problem of dynamic resource allocation in Cloud computing [15], communication systems [20, 21] and cognitive radio networks [22, 23], in order to optimize resource utilization and social welfare. In [24], the authors introduce a low-complexity periodic auction on radio resources between the spectrum provider (government) and operators, in order to improve efficiency in terms of spectrum usage.

Recent works considered the use of auctions in the context of C-RAN, but only few tackled the case of a two-level hierarchical one, as is the case of our setting. The work in [25] proposes different auction approaches that can be applied to radio resource allocation, such as spectrum and power allocation. The authors in [26] introduce an auction design between an MVNO and a radio service provider with the aim of maximizing the social welfare through an efficient and fair allocation. This work also suggests a greedy algorithm to reduce the time complexity of the proposed solution. In [27], a two-level hierarchical combinatorial auction has been considered for 5G networks between the infrastructure provider, MVNO and user equipment, and two models have been proposed: a single seller/multiple buyer model and a multiple buyer/multiple seller one. A backward induction method has been proposed to solve the winner and price determination problems. Myerson's virtual valuation concept was adopted in [28] to design a truthful dynamic spectrum access allocation between competing base stations with approximate expected revenue or social welfare. The authors in [17] propose an online auction to allocate spectrum between primary and secondary users. Myerson's scheme was used to design a strategy-proof revenue maximization mechanism. We observe that the Nash equilibrium concept was not tackled by [17, 26, 27, 28].

In a different context, the authors in [29] investigate an auction-based allocation scheme for network resources between multi-tenant software defined networks, and they show the existence and uniqueness of the Nash equilibrium. They also introduce a learning algorithm to facilitate convergence to the unique Nash equilibrium.

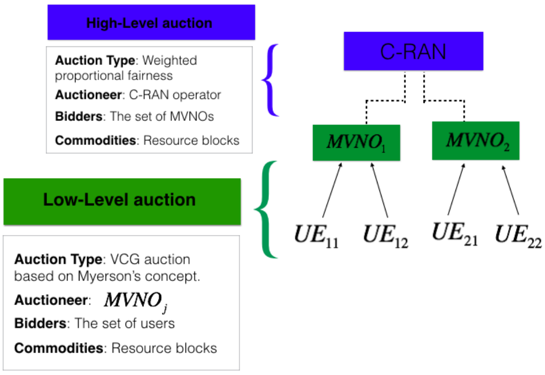


Figure 2: Hierarchical/two-level auction game

With respect to the previous discussed related works, and to the best of our knowledge, our work is the first to adopt a two-level allocation scheme that guarantees fairness and revenue maximization at the same time. Furthermore, we are the first to consider the allocation mechanism, in the context of C-RAN, as a non cooperative game and to prove the existence and uniqueness of the Nash equilibrium.

3. System model and two-level auction

3.1. System model

We consider a system with multiple base stations, equipped with Remote Radio Heads (RRHs), and a centralized pool of Base Band Units (BBUs), owned by a single central operator. We assume that this C-RAN consists of a certain number of resource blocks, denoted by Q, which the central operator dynamically allocates to m MVNOs (indexed from 1 to m), with $MVNO_j$ serving N users, as shown in Figure 1.

3.2. The two-level auction proposal

We propose a two-level auction-based resource allocation scheme, as shown in Figure 2: the higher-level auction runs between the C-RAN operator and the MVNOs; the lower-level auction between each MVNO and the end users it serves. Recall that the C-RAN operator owns the physical resources, notably the base stations which are to be shared by the MVNOs. The latter have to serve their end users, as illustrated in Figure 2.

Our two-level auction mechanism is executed periodically so as to tailor to system dynamics. The allocation is thus fixed for the entire duration between two executions of the auction. However, the time range depends on the system setting and parameters, and can be gradually adapted and adjusted during the execution. Typical time range varies from 30 minutes to several hours or half day.

Specifically, the agents of the auction are:

- **The C-RAN operator:** It is the auctioneer at the higher-level auction; it initiates the auction over Q resource blocks.

Table 1: Higher-level auction glossary

Notation		Interpretation
Auctioneer	C-RAN	C-RAN operator spectrum owner
Bidders	$MVNO$	Set of m $\{MVNO_j\}$; $1 \leq j \leq m$
Commodity	RB	Q resource blocks
Bids	S_j	$MVNO_j$'s bid vector $S_j = P_j$
Bid price	P_j	The price $MVNO_j$ pays to get R_j
Allocation	R_j	$MVNO_j$ allocation in terms of resource blocks
Reserved bid	S_0	Bid set by the C-RAN

- **MVNOs:** Each $MVNO_j$ ($1 \leq j \leq m$) has two roles: a bidder in the higher-level auction and an auctioneer in the lower-level auction.
- **End users:** End users are bidders in the lower-level auction. They bid for resource blocks to satisfy their service needs. Users served by $MVNO_j$ participate in the lower-level auction of this MVNO.

The commodities of the auctions are the resource blocks. The bids are signals that inform the auctioneer about the bidders demands in terms of resource blocks and the offered price they are willing to pay in order to purchase the commodities.

Our two-level auction proceeds as follows (please refer to Tables 1 and 2 for notations):

1. In the first step, each user associated to $MVNO_j$, denoted as $UE_{i,j}$, submits a bid vector $b_{i,j} = (d_{i,j}, w_{i,j})$ to $MVNO_j$, where $d_{i,j}$ is an integer indicating the number of resource blocks required by $UE_{i,j}$ and $w_{i,j}$ is the price that user $UE_{i,j}$ is willing to pay to purchase $d_{i,j}$. $w_{i,j}$ is always less than or equal to $v_{i,j}$, the true valuation of $UE_{i,j}$ for receiving $d_{i,j}$. $v_{i,j}$ is a private information, only known by the user itself.

   We define $\boldsymbol{b_j} = \left\langle b_{1,j}, ...b_{N,j} \right\rangle$ as $MVNO_j$ users bids set.

2. In the second step, each $MVNO_j$ submits a bid vector S_j based on the bids received in the lower-level auction, i.e., $\boldsymbol{b_j}$. We define the m-dimension bid vector $\boldsymbol{S} = \langle S_1, ...S_m \rangle$.

   Based on $\boldsymbol{S}$, the C-RAN operator allocates to $MVNO_j$ R_j resource blocks and charges it $P_j = S_j$. We denote by $\boldsymbol{R} = \langle R_1, ...R_m \rangle$ the allocation set to the m MVNOs.

3. In the third step, each $MVNO_j$ determines the winner vector $\boldsymbol{x_j} = \left\langle x_{1,j}, .x_{i,j}..x_{N,j} \right\rangle$ according to R_j, where

$$x_{i,j} = \begin{cases} 1 & \text{if } UE_{i,j} \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

The satisfaction of the agents participating in the auction can be expressed in terms of their utilities as follows:

Table 2: Lower-level auction glossary

Notation		Interpretation
Auctioneer	$MVNO_j$	j-th MVNO
Bidders	$UE_{i,j}$	Set of $MVNO_j$'s users $\{UE_{i,j}\}_{1\leq i\leq N}$
Commodity	RB	R_j Resource blocks
Bids	$b_{i,j}$	$UE_{i,j}$'s bid vector $b_{i,j} = (d_{i,j}, w_{i,j})$
Demands	$d_{i,j}$	Number of resource blocks requested by $UE_{i,j}$
Bid price	$w_{i,j}$	$UE_{i,j}$'s declared valuation for $d_{i,j}$ resource blocks
Valuation	$v_{i,j}$	$UE_{i,j}$'s true valuation for $d_{i,j}$ resource blocks
Decision vector	$\boldsymbol{x_j}$	$\boldsymbol{x_j} = \{x_{i,j}\}$, $1\leq i\leq N$; $x_{i,j} = 1$ if $UE_{i,j}$ wins and 0 otherwise
Price vector	$\boldsymbol{p_j}$	$\boldsymbol{p_j} = \{p_{i,j}\}$, $1\leq i\leq N$; $p_{i,j} = p^{VCG}_{i,j}$ if $UE_{i,j}$ wins and 0 otherwise

- $UE_{i,j}$'s utility depends on the true valuation $v_{i,j}$ of the user for receiving the requested resource blocks and the price $p_{i,j}$ it is going to pay:

$$u_{i,j} = \begin{cases} v_{i,j} - p_{i,j} & \text{if } UE_{i,j} \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

- $MVNO_j$'s utility depends on the profit it is going to make from reselling the R_j resource blocks and the price P_j it is going to pay:

$$U_j = \sum_{i=1}^{N} p_{i,j} - P_j \tag{2}$$

We next illustrate how resources are allocated in both levels of the auction, as well as the winners' determination and the corresponding price each player has to pay.

3.3. Resource allocation at the higher-level auction

In the higher-level auction, the C-RAN operator allocates R_j resource blocks to $MVNO_j$ as follows:

$$R_j = \frac{S_j Q}{\sum_{i=1}^{m} S_i + S_0}, \tag{3}$$

where S_0 is a *reserved bid* set by the C-RAN operator to avoid selling the spectrum at a low price.

The rationale of (3) stems from the weighted proportional fairness which gives to each MVNO a share of resources proportional to its bid as well as to the bids of its competitors. The weighted proportional fair allocation has many advantages other than guaranteeing fairness: authors in [7] show that this mechanism is efficient in terms of signaling complexity.

The utility of $MVNO_j$ is:

$$U_j = \sum_{i=1}^{N} p_{i,j} - S_j. \tag{4}$$

Each $MVNO_j$ calculates the optimal S_j^* maximizing its profit U_j.

3.4. Solving the lower-level auction

In the lower-level auction, we first derive the optimal VCG solution for each MVNO which consists of implementing an Integer Linear Program (ILP) to solve the winner and price determination problem by maximizing the revenue of the MVNO, as shown hereafter.

3.4.1. VCG auction

The VCG auction is an auction where the bidder either wins all it asked for ($d_{i,j}$), or nothing, and then pays the *harm* it causes to the other players ($p^{VCG}_{i,j}$) [30]. The winners are determined based on a social welfare maximization problem where the number of winners is limited to the number of resource blocks owned by $MVNO_j$ from the higher-level auction (R_j), given that a resource block cannot be assigned to more than one user.

The following optimization model is to solve the winners and price determination problem for the given $MVNO_j$:

$$\max \sum_{i=1}^{N} w_{i,j} x_{i,j} \tag{5}$$

$$s.t. \quad \sum_{k=1}^{K_j=\lfloor R_j \rfloor} r^k_{i,j} = d_{i,j} x_{i,j}, \forall i \in \{1, \ldots, N\} \tag{6}$$

$$\sum_{i=1}^{N} r^k_{i,j} \leq 1 \tag{7}$$

where

$$x_{i,j} = \begin{cases} 1 & \text{if } UE_{i,j} \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

$$p_{i,j} = \begin{cases} p^{VCG}_{i,j} & \text{if } UE_{i,j} \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

We denote by $r^k_{i,j}$ the k-th resource block $MVNO_j$ owns and allocates to $UE_{i,j}$, such that $1 \leq k \leq K_j$, where we consider $K_j = \lfloor R_j \rfloor$.

The objective function in (5) aims at maximizing the social welfare of $MVNO_j$. Constraint (6) ensures that $UE_{i,j}$ wins $d_{i,j}$ or nothing and constraint (7) ensures that the k-th resource block cannot be allocated to more than one user.

We denote by $p^{VCG}_{i,j}$ the VCG price that $UE_{i,j}$ pays; $p^{VCG}_{i,j}$ is calculated as follows:

$$p^{VCG}_{i,j} = \max_{b_{-i,j}} \sum_{k \neq i}^{N} w_{k,j} x_{k,j} - \max_{b_j} \sum_{k \neq i}^{N} w_{k,j} x_{k,j} \tag{8}$$

where b_j denotes the set of all $MVNO_j$'s users bid vectors, while $b_{-i,j}$ indicates the set of all $MVNO_j$'s users bid vectors except for $UE_{i,j}$'s bid: $b_j = (b_{i,j}, b_{-i,j})$.

3.4.2. *Truthful solution maximizing revenue*

Given that the VCG auction may fail to ensure high revenue [11], we use Myerson virtual valuation concept [9] which ensures truthfulness and guarantees maximum revenues.

The virtual valuation concept was introduced by Myerson in [9]. It states that the true revelation of the user valuation is the best strategy for him to maximize its own profit, which means truthfulness. Moreover, according to the revenue equivalence theorem in [9], the expected revenue P_E of any truthful mechanism under the Bayesian setting is equal to its expected virtual surplus, $\sum_{i,j} x_i \phi(w_{i,j})$ where x_i is equal to 1 when user i wins the bid and 0 otherwise and $\phi_{i,j}$ is the virtual valuation. $\phi_{i,j}$ is expressed in terms of the probability distribution $F_{i,j}(w_{i,j})$ of the user valuation $w_{i,j}$ and the probability density $f_{i,j}(w_{i,j})$ as follows:

$$\phi_{i,j}(w_{i,j}) = w_{i,j} - \frac{1 - F_{i,j}(w_{i,j})}{f_{i,j}(w_{i,j})} \tag{9}$$

where

$$f_{i,j} = \frac{\partial F_{i,j}(z)}{\partial z} \tag{10}$$

Hence, $MVNO_j$ can maximize its expected revenue:

$$\max P_E = \max \sum_{i=1}^{N} \phi_{i,j}(w_{i,j}) x'_{i,j}$$

instead of maximizing expression (5).

According to the conventional Bayesian approach, we consider that the valuation $w_{i,j}$ of the buyers follows a distribution $F_{i,j}$ known to the seller. We assume $F_{i,j}$ to be monotone increasing and $\frac{f_{i,j}}{1-F_{i,j}}$ to be monotone non-decreasing, therefore the virtual valuation becomes monotone non-decreasing [31].

To determine the winners, $MVNO_j$ solves the following optimization problem:

$$\max \sum_{i=1}^{N} \phi_{i,j}(w_{i,j}) x'_{i,j} \tag{11}$$

$$s.t. \quad \sum_{k=1}^{K_j = \lfloor R_j \rfloor} r^k_{i,j} = d_{i,j} x'_{i,j}, \forall i \in \{1, \ldots, N\} \tag{12}$$

$$\sum_{i=1}^{N} r^k_{i,j} \leq 1 \tag{13}$$

where

$$x'_{i,j} = \begin{cases} 1 & \text{if } UE_{i,j} \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

We define the virtual price $p'_{i,j}$ as follows:

$$p'_{i,j} = \max_{b_{-i,j}} \sum_{k \neq i}^{N} \phi_{k,j}(w_{k,j}) x_{k,j} - \max_{b_j} \sum_{k \neq i}^{N} \phi_{k,j}(w_{k,j}) x_{k,j} \tag{14}$$

The final allocation $x_{i,j}$ is set to $x'_{i,j}$ and the final price $p_{i,j}$ is set to

$$p_{i,j} = \phi^{-1}_{i,j}(p'_{i,j}) \tag{15}$$

Algorithm 1 Winner determination

Input R_j, b_j.
Output x_j, the winner vector
set R $\leftarrow R_j$, *set the number of resource blocks owned by $MVNO_j$ to its allocation from the higher-level auction R_j*
For $i = 0$ **to** N
    $l_{i,j} \leftarrow \frac{\phi_{i,j}(w_{i,j})}{d_{i,j}}$
end
$[I, B]$=sort $L/\ l_{i,j}$, *where I is the sorted list and B is the list of the weights' indexes in I*
For $n = 1$ **to** N
    $k = I(n)$
        if $\phi_{kj}(w_{kj}) > 0$ & $d_{kj} \leq R$
            $x_{kj} \leftarrow 1$
            $R \leftarrow R - R_j$
        else
            $x_{kj} \leftarrow 0$
        end
end

3.4.3. *Truthful sub-optimal solution*

Given the complexity of the integer linear optimization in the winner and price determination problem, we propose hereafter a greedy algorithm executed by each MVNO to determine the winners and the corresponding prices to be paid.

The greedy algorithm proceeds in two phases as follows.

- *Winner determination (Algorithm 1)*: we sort, in decreasing order, a list L of N weights $l_{i,j}$, where $l_{i,j} = \frac{\phi_{i,j}(w_{i,j})}{d_{i,j}}$. We start by allocating resources to the users according to the order of the corresponding $l_{i,j}$ value in the sorted list L. In other words, with respect to the order, $UE_{i,j}$ is a winner if $d_{i,j} \leq R_j$, i.e., $x_{i,j}$ is set to 1 and R_j is updated. Otherwise, $x_{i,j}$ is set to 0.

- *Price setting*: $UE_{i,j}$ pays $p_{i,j} = l_{k,j} d_{i,j}$ such that $l_{k,j}$ is the critical weight as described in [31]; if $l_{i,j} \geq l_{k,j}$ $UE_{i,j}$ wins, otherwise loses. According to the sub-optimal method in [1] we compute the critical price $c_{k,j}(i)$ (Algorithm 2) such that if bidder $UE_{i,j}$ bids $l_{i,j} \geq l_{k,j}(c_{k,j}(i))$ he wins, and loses if $l_{i,j} < l_{k,j}(c_{k,j}(i))$.

The algorithm we propose has polynomial complexity, thus inducing only limited delay between collecting users requests in the lower level auction and the resource allocation phase.

3.5. *Analysis of the VCG auction properties*

It is widely known that it is desirable for auctions to meet the following economic properties: truthfulness, rationality and computational efficiency [10, 26].

1. **Individual rationality:** Individual rationality ensures that none of the bidders utilities should be negative, i.e., any of the users should pay more than its declared valuation: $p^{VCG}_{i,j} \leq w_{i,j}$.

Algorithm 2 Critical price determination

Input $R_j, b_j, x_j, L, I.$
Output p_j, the price vector
set R $\leftarrow R_j$
For $n = 1$ **to** N
  $k = I(n)$
  if $x_{kj} = 1$
   for l=1:N & $l \neq k$
    $R \leftarrow R - d_{lj}$
    if $d_{kj} \geq R$
     $c_{kj} = \Phi_{lj}(w_{lj})$
    end
   end
  else
   $c_{kj} = 0$
  end
end

2. **Truthfulness:** Truthfulness is a very desirable property in auction theory which guarantees that the users don't lie about their true valuations. A truthful auction is an auction where the bidders cannot do better by bidding other than their true valuation.
3. **Computational efficiency:** Computational efficiency can be satisfied when the computation time for the winner and price determination is polynomial.

While it is proven that the VCG auction is individually rational (theorem 3 in [26]) and truthful (theorem 1 in [18] and theorem 2 in [26]), VCG can possibly generate low revenues [10]. We addressed this issue in Section 3.4.2, and suggested an approach that guarantees both truthfulness and a high revenue. Moreover, a major concern of the VCG auction is the exponential time complexity due to the ILP optimization formulation. We addressed this limitation by proposing a greedy algorithm in Section 3.4.3 that reduces the time complexity to a polynomial. Hence, all desired properties are satisfied in our auction algorithm.

4. Nash equilibrium analysis of the global auction

We now analyze the existence and uniqueness of the Nash Equilibrium (NE) of the global auction consisting of the two levels: high and low.

Since the utility function in (2), $U_j = \sum_{i=1}^{N} p_{i,j} - P_j$, is not explicit in terms of the agents' strategies, i.e., MVNOs bids S_j ($1 \leq j \leq m$), we start by approximating it to an explicit form in terms of these latter terms. We consider the case where the distribution of the valuation $F_{i,j}(v_{i,j})$ is uniform in $[0, 1]$ $\forall i$, i.e., $F_{i,j}(v_{i,j}) = v_{i,j}$ and we conduct the analysis per unit of resource block[1]. The methodology we employed can be extended straightforwardly to the general case with generic distribution $F(v_{i,j})$, (please refer to the details in Appendix A.1). It follows from our analysis in the previous section that users bid their true valuations; this is enforced by the lower-level auction. We consider the asymptotic case with a large number of users participating in the lower-level auction.

[1] We assume that users are homogeneous and bid for 1 resource block from MVNO_j in the lower-level auction.

4.1. Utility function approximation

Given that $F_{i,j}(v_{i,j}) = v_{i,j}$, Equation (9) becomes:

$$\phi_{i,j}(v_{i,j}) = \phi(v_{i,j}) = v_{i,j} - \frac{1 - v_{i,j}}{1} = 2v_{i,j} - 1. \tag{16}$$

Lemma 4.1. *The utility function of $MVNO_j$, given in (2), per unit of resource block can be written as:*

$$U_j(S_j, S_{-j}) \hat{=} \sum_{i=1}^{N} \left([F_{i,j}]^N - [F_{i,j}]^{N - \frac{S_j Q}{S_j + S_{-j} + S_0}} [1 - F_{i,j}]^{\frac{S_j Q}{S_j + S_{-j} + S_0}} \right) - S_j \tag{17}$$

Proof 4.1. *Under the above assumptions, it is easy to see that in total we have $n = R_j$ winners. More precisely, the first $n = \lfloor R_j \rfloor$ users with the highest valuation will win the auction. Recall that $\lfloor R_j \rfloor$ is the total number of resource blocks allocated to $MVNO_j$ by the Cloud-RAN operator in the higher-level auction.*

Hence, the problem is assimilated to sorting a list in a decreasing order in terms of valuation and assigning the $n = \lfloor R_j \rfloor$ resources to the first n users in the sorted list. Knowing that the distributions of valuations are known, we can derive the probability of user $UE_{i,j}$ being a winner as follows:

$$\begin{aligned} p(x_{i,j} = 1) = &\prod_{(k \neq i)}^{(N-1)} p(v_{k,j} \leq v_{i,j}) + \\ &\prod_{(k \neq i)}^{(N-2)} p(v_{k,j} \leq v_{i,j})(1 - p(v_{k,j} \leq v_{i,j})) + \ldots + \\ &\prod_{(k \neq i)}^{(N-R_j)} p(v_{kj} \leq v_{i,j}) \prod_{k \neq i}^{R_j - 1} (1 - p(v_{k,j} \leq v_{i,j})). \end{aligned} \tag{18}$$

The first term on the right hand side of Equation (18) equals the probability that $UE_{i,j}$ has the highest valuation, $v_{i,j}$, among the N users participating in the auction. The last term equals the probability that $v_{i,j}$ is the $N - R_j + 1$ highest valuation, i.e., there are $R_j - 1$ valuations larger than $UE_{i,j}$'s valuation $v_{i,j}$.

Knowing that $p(v_{kj} \leq (v_{i,j})) = F_{kj}(v_{i,j})$ and that $F_{k,j}(v_{i,j}) = F(v_{i,j}), \forall k \in \{1, \ldots, N\}$, the probability that $UE_{i,j}$ wins in the lower-level auction becomes:

$$\begin{aligned} p(x_{i,j} = 1) =& [F(v_{i,j})]^{N-1} + [F(v_{i,j})]^{N-2}(1 - F(v_{i,j})) \\ &+ \ldots + [F(v_{i,j})]^{N-R_j}(1 - F(v_{i,j}))^{R_j - 1} \\ =& \frac{[F(v_{i,j})]^N - F(v_{i,j})^{N-R_j}(1 - F(v_{i,j}))^{R_j}}{2F(v_{i,j}) - 1}. \end{aligned} \tag{19}$$

Replacing Equations (16) and (19) in Equation (2) gives Equation (17) (complete proof in Appendix A).

4.2. Nash equilibrium: existence

In this section, we prove the existence of a NE point according to the following theorem.

Theorem 4.2. *An equilibrium point exists for every concave n-person game [32].*

In other words, a NE point S_j^* exists if the utility function $U_j(S_j, S_{-j})$ is continuous in S_j and its second derivative with respect to S_j is negative.

Lemma 4.3. *Our m-person game has at least one Nash equilibrium when $F_{i,j} > 0.5, \forall i \in \{1, \dots, N\}$.*

Proof 4.3. *$U_j(S_j, S_{-j})$ is continuous in S_j. Its second derivative is $\frac{\partial^2 U_j(S)}{\partial^2 S_j} = C_j^2 \sum_{i=1}^{N} -A_{i,j}B_{i,j}E_{i,j}$, which is less than or equal to zero due to the positivity of expressions $A_{i,j}$, $B_{i,j}$, C_j and $E_{i,j}$ under the condition that $F_{i,j} > 0.5, \forall i \in \{1, \dots, N\}$ (please refer to the proof in Appendix B).*

4.3. Nash equilibrium: uniqueness

In this subsection, we prove the uniqueness of the NE in the duopoly case with 2 $MVNOs$ participating in the auction: $MVNO_1$ and $MVNO_2$ having S_1 and S_2 as bid strategies. We base our proof of NE point uniqueness on the following theorems 4, 5 and 6 in [32].

Let us first introduce some definitions:

- $g(S,r)$: the pseudo-gradient of the weighted sum $\sigma(S,r) = \sum_{j=1}^{m} r_j \Delta(U_j)$, where $r = \{r_j\}$ with $r_j \geq 0 \ \forall \ j$. $g(S,r)$ is such as:

$$g(S,r) = \begin{pmatrix} r_1\Delta(U_1) \\ r_2\Delta(U_2) \\ . \\ . \\ r_m\Delta(U_m) \end{pmatrix}$$

- $G(S,r)$: the Jacobian of $g(S,r)$, with respect to r, is an $m * m$ matrix where the element a_{kl} of $G(S,r)$ is such that $a_{kl} = \frac{\partial^2 U_k(S_k, S_{-k})}{\partial S_k \partial S_l}$, $\{k, l\} \in \{1, 2\}$
- $G^t(S,r)$: the transpose matrix of $G(S,r)$

Theorem 4.4. *(theorem 4 [32]) If $\sigma(S,r)$ satisfies the diagonal strict concavity property, the Nash equilibrium is unique when it exists.*

$\sigma(S,r)$ is diagonally strictly concave if for a given $r > 0$ and for every S_1, S_2 we have [33]:

$$(S_1 - S_0)(g(S_1, r) - g(S_0, r)) < 0$$

Theorem 4.5. *(theorem 6 [32]) $\sigma(S,r)$ is diagonally strictly concave if for a given $r > 0$ the matrix $[G(S,r) + G(S,r)^t]$ is definite negative, i.e., if the eigenvalues of $[G(S,r) + G(S,r)^t]$ are negative.*

Table 3: Expressions table

Notation	Interpretation
$A_{i,j}=$	$F_{ij}^{\frac{S_j Q}{S_j + S_{-j}}}$
$B_{i,j}=$	$(1 - F_{ij})^{\frac{S_j Q}{S_j + S_{-j}}}$
$C_{i,j}=$	$\frac{S_{-j}}{(S_j + S_{-j})^2}$
$E_{i,j}=$	$(\log(\frac{F_{ij}}{1-F_{ij}}))^2 + \frac{2}{Q}(1 + \frac{S_j}{S_j + S_{-j}}) \log(\frac{F_{ij}}{1-F_{ij}})$
$K_j=$	$-Q^2 \sum_{i=1}^{N} A_{i,j}B_{i,j}E_{i,j}$
$L_j=$	$Q^2 \sum_{i=1}^{N} A_{i,j}B_{i,j}H_{i,j} = -K_j$
$\beta_j=$	$\frac{(S_j + S_0)^2}{(S_j + S_{-j} + S_0)^4}$
$\gamma=$	$\frac{S_j(S_{-j} + S_0)}{(S_j + S_{-j})^4}$

Lemma 4.6. *For the duopoly scenario, $[G(S,r) + G(S,r)^t]$ is definite negative and, therefore, the Nash equilibrium $S_j{}^*$ for the two-level auction is unique under the condition $F_{i,j} > 0.5, \forall i \in \{1, \dots, N\}, j \in \{1, 2\}$.*

Proof 4.6. *The Jacobian matrix with 2 MVNOs is the following:*

$$G(S,r) = \begin{pmatrix} r_1\beta_2 K_1 & -r_1\gamma K_1 \\ -r_2\gamma K_2 & r_2\beta_1 K_2 \end{pmatrix}$$

(The detailed expressions of the matrix elements are given in Table 3.)

and

$$[G(S,r) + G(S,r)^t] = \begin{pmatrix} 2r_1\beta_2 K_1 & -\gamma(K_1 + K_2)(r_1 - r_2) \\ -\gamma(K_2 + K_1)(r_2 - r_1) & 2r_2\beta_1 K_2 \end{pmatrix} \tag{20}$$

In Appendix C we prove that when $F_{i,j} = 0.5, \forall i \in \{1, \dots, N\}, j \in \{1, 2\}$, the eigenvalues of the matrix $[G(S,r) + G(S,r)^t]$ are negative, and it then follows that $[G(S,r) + G(S,r)^t]$ is definite negative and that the Nash equilibrium is unique.

5. Numerical results

In this section, we evaluate numerically the two-level auction approach we propose in this paper. We first simulate the lower-level auction and then the global one consisting of the two coupled auctions. We implemented our proposed optimization models in MATLAB on a server equipped with an Intel CPU at 2.60 GHz and 64 GByte of RAM. All numerical results are obtained by averaging 50 random extractions to achieve very narrow 95% confidence intervals (not shown in the figures for the sake of clarity).

Table 4: Lower-level auction scenarios

Scenario	Number of users	Number of resource blocks (R_1)	Requested number of resource blocks (d_{i1})
1(a)	$N \in [0, 200]$	$R_1 = 50$	$d_{i1} = 1$
1(b)	$N \in [0, 200]$	$R_1 = 50$	$d_{i1} \in [1, 5]$
1(c)	$N = 100$	$R_1 \in [0, 120]$	$d_{i1} = 1$
1(d)	$N = 100$	$R_1 \in [0, 400]$	$d_{i1} \in [1, 5]$

5.1. *Lower-level auction*

We simulate the lower-level auction run by $MVNO_1$, with N users. We assume that the auction is run over R_1 (resource blocks). Each user UE_{i1} submits a bid vector $b_{i1} = [d_{i1}, w_{i1}]$. After collecting the bids from its users, $MVNO_1$ determines the winners $\boldsymbol{x_1} =< x_{i1}, ..., x_{N1} >$ and the corresponding price vector $\boldsymbol{p_1} =< p_{i1}, ..., p_{N1} >$, and calculates the revenue $R_{rev1} = \sum_{i=1}^{N} p_{i1}$, where p_{i1} is the price that user i should pay to $MVNO_1$.

The winner and price vectors $\boldsymbol{x_1} =< x_{i1}, \ldots, x_{N1} >$ and $\boldsymbol{p_1} =< p_{i1}, \ldots, p_{N1} >$ are determined using the following algorithms, which are compared to each other:

1. *VCG-Integer Linear Program (V-ILP):* The winner determination is realized by solving the ILP verifying Equations (5), (6) and (7), described in Section 3.4.1. The price vector is constituted by VCG prices determined by Equation (8).

2. *Optimal ILP (O-ILP):* This algorithm is based on Myerson's concept of virtual valuation ($\phi_{i1}(v_{i1}) = v_{i1} - \frac{1-F_{i1}(v_{i1})}{f_{i1}(v_{i1})}$). The winners are determined by the ILP that aims at maximizing the expected revenue, as in Equations (11), (12) and (13). The virtual prices vector is determined by Equations (14) and (15).

3. *Greedy algorithm (GA):* The winner and price determination are given by the greedy algorithms (Algorithm 1 and Algorithm 2) described in Section 3.4.3.

4. *Random allocation scheme (RAS):* For comparison reasons, we further implement a baseline allocation approach where the winning users are selected randomly from those whose valuations are higher than the price set by the MVNO, p_0 ($v_{i,j} > p_0$). Each user will pay a fixed price p_0 per resource block; the price here is the average of users' valuations.

We evaluate and compare the above algorithms in the scenarios described in Table 4. These scenarios are repeated for the two cases where the users true valuations $v_{i,1}$ are drawn from a *uniform* distribution as well as an *exponential* distribution; we consider in all scenarios that the users bid truthfully, i.e., $w_{i,1} = v_{i,1}$.

Table 5: Global auction scenarios

Scenario	Number of MVNOs	Number of users	Number of resource blocks	Req. number of resource blocks
2(a)	$m = 2$	$N = \{50, 100, 150\}$	$R_1 = 100$	$d_{i1} = 1$
2(b)	$m = 2$	$N = \{50, 100, 150\}$	$R_1 = 100$	$d_{i1} \in [1, 5]$
2(c)	$m = 4$	$N = \{50, 100, 150\}$	$R_1 = 100$	$d_{i1} = 1$
2(d)	$m = 4$	$N = \{50, 100, 150\}$	$R_1 = 100$	$d_{i1} \in [1, 5]$

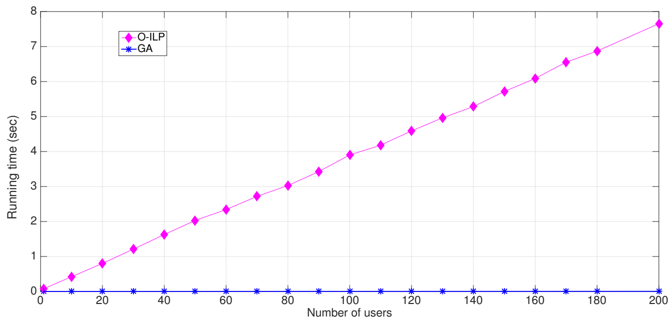


Figure 3: Running time for scenario 1(a)

5.1.1. *Discussion*

We observe, in Figure 4 and for all scenarios, that the Greedy Algorithm (GA) is very similar to Optimal ILP (O-ILP) in terms of revenue when varying the number of users as well as the number of resource blocks, at the remarkable advantage of reducing the running time: as shown in Figure (3), for scenario 1(a), the Greedy Algorithm (GA) takes less than 1% of the computing time of the Optimal ILP-based one (O-ILP).

A known problem of VCG is that it can generate very low revenues when the number of users is small or when the number of commodities is redundant [11]. This is the explanation of the low revenue generated by the VCG auction (used in V-ILP) in Figures 4a, 4b, 5a and 5b, when the number of users is small, and in Figures 4c, 4d, 5c and 5d, when the number of resource blocks is redundant. However, the mechanism based on the concept of Myerson's virtual valuation (O-ILP) performs better than the VCG auction, and as well as the random allocation, in terms of revenue. Consequently, we observe from Figures 4 and 5 the importance of Myerson's virtual valuation concept in guaranteeing a positive payoff due to the reservation effect, while V-ILP can lead to a zero-payoff when users' demands are higher than, or equal to, the number of resource blocks R_1 owned by $MVNO_1$. The advantage of Myerson's concept, which discriminates against negative/low valuations, is to set an incentive on the users to give their true valuation, and by that guarantee an optimal revenue. This can be assimilated to a market where the price of a commodity increases when the demands increase, and decreases otherwise. The number of demands has the same effect as the market on the VCG auction; however by adopting Myerson's concept, $MVNO_1$ is able to maintain a high revenue.

As of the auction approach versus the RAS scheme, it can be observed from the different scenarios that when the number of users increases, the auction-based approaches outperform the Random Allocation Scheme (RAS) in terms of revenue, especially for a medium-to-high number of users competing in the lower-level auction. In fact, in our proposed approach, the resource blocks are allocated to the users that value the resource blocks the most and, by that, the revenue of the MVNO is maximized.

Finally, the case where users valuations are extracted according to an exponential distribution yields similar results with respect to the uniform one, as shown in Figure 5. This behavior emphasizes the independence of the allocation scheme from the distribution type.

5.2. Global auction

We now evaluate the global auction (i.e., the two-level auction approach). To this end, we consider in total m $MVNO$s, with the same settings, participating in this global auction with a central C-RAN operator. The latter runs the higher-level auction over $Q = 100$ resource blocks. Each $MVNO_j$, which plays the role of a seller in the lower-level auction and bidder in the higher-level one, submits S_j as its bid, obtains $R_j = \frac{S_j Q}{S_j + S_{-j} + S_0}$ and pays S_j. Regarding the lower-level auction, we consider the same settings described in subsection 5.1.

5.2.1. Discussion

In scenarios (2a) and (2b), we set $m = 2$ and evaluate the payoff of a given MVNO ($MVNO_1$ in this case) by varying $MVNO_1$'s bid S_1 and $MVNO_2$'s bid S_2 in the range $[0, 20]$, and by considering the settings described in Table 5. The results are shown in Figures 6 and 7 for scenario (2a) and scenario (2b), respectively. We notice that the payoff U_1 increases with S_1, reaches its maximum and then decreases when S_1 continues to increase. More specifically, this maximum increases with the number of users N and is substantially lower than this latter since $\max \sum_i x_{i1} p_{i1} = N$. Moreover, U_1 also depends on S_2 and, as expected, decreases when S_2 increases. It is worth noting that, since we consider the same settings for both $MVNOs$, the results are the same for $MVNO_2$ when varying $MVNO_1$'s bid S_1 and $MVNO_2$'s bid S_2 in the same range $[0, 20]$.

In scenarios (2c) and (2d), we set $m = 4$ and evaluate the payoff of $MVNO_1$ by varying $MVNO_1$'s bid S_1 and the sum of all the other $MVNO$s' bids $S_{-1} = \sum_{j=2:4} S_j$, in the range $[0, 60]$, and by considering the parameters setting described in Table 5. The results are shown in Figures 8 and 9 for scenarios (2c) and (2d), respectively. We notice that when the number of $MVNOs$ participating in the higher level auction increases, i.e., when the competition increases, and for a fixed value of S_1, the maximum values that $MVNO_1$'s payoff can take are lower compared with the one obtained with $m = 2$.

We observe from the concavity of the curves, in Figures 6, 7, 8 and 9, that the global auction is a n-concave game, which guarantees that the Nash equilibrium indeed exists, even for the general case where $m > 2$.

We underline that in both scenarios 2(b) and 2(d) we evaluate the performance of our proposed architecture with an increasing number of resource blocks demanded (i.e., from 1 up to 5). Figures 7, and 9 show that when users bid for up to 5 resource blocks, the revenue generated by $MVNO_1$ is higher than the one generated in scenarios 2(a) and 2(c), where the users can bid for only one resource block. However, the concavity of the payoff function is very similar to the one obtained in scenarios 2(a) and 2(c), and so the existence of the Nash equilibrium is guaranteed for the general case where users can bid for more than 1 resource block.

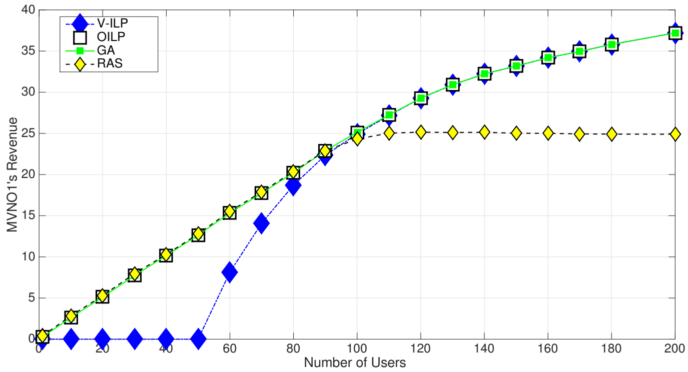
Please note that our proposed auction scheme achieves the maximum revenue for the operators among all simulated schemes, which does not imply its optimality. In fact, it is widely known that achieving truthfulness and maximizing revenue simultaneously is impossible [34]. In this regard, the main contribution with our auction scheme is to ensure *truthfulness*, a fundamental and challenging economic property of any auction scheme, and we achieve this without sacrificing drastically the revenue for the MVNO, as illustrated above. As for the end-users, our auction scheme naturally ensures that those users with high valuation are served before those with low valuation, which is a reasonable property.

6. Conclusion

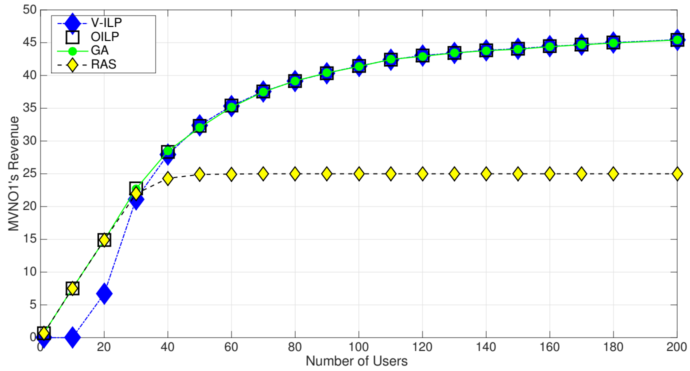
We considered in this paper a multi-tenant C-RAN where several MVNOs, each having its own end users, compete for shared resources. We proposed a two-level auction to enable both the C-RAN operator and the MVNOs allocating the radio resource to the end users efficiently and truthfully. Technically, we adopted Myerson's virtual valuation concept in order to guarantee revenue maximization, and showed that our proposal is truthful and individually rational. We further implemented a greedy algorithm to ensure the computational efficiency of our auction framework. We showed the existence of Nash equilibrium for the global auction and its uniqueness in a typical duopoly scenario. Finally, we evaluated numerically our proposal in typical network scenarios, and demonstrated the efficiency of our proposal in terms of revenue maximization for the MVNOs.

The focus of the present paper was on maximizing the C-RAN operator profit, and so we maximized its resource utilization. Energy is yet another important parameter to consider in this setting as well; its minimization could constitute an interesting future perspective. One way to do so would be by incorporating it as a second term in the global utility function, in addition to the resource utilization term. In this case, each end user will not only be represented by a request for resources but also by a certain energy consumption figure which depends on the required resources, its radio conditions, modulation and coding scheme, etc.

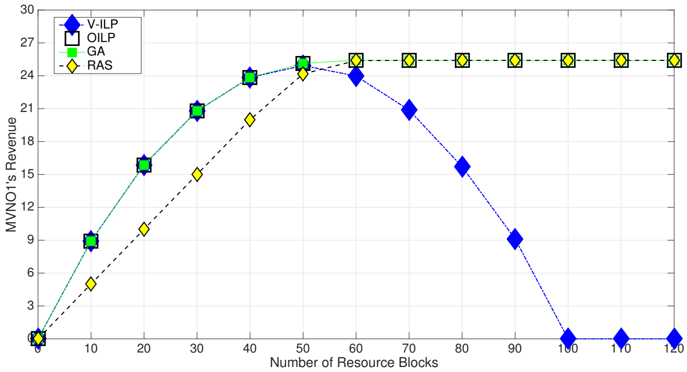
Another perspective would be to test our proposal on a platform implementing multi-tenant C-RAN and compare our scheme with existing ones.



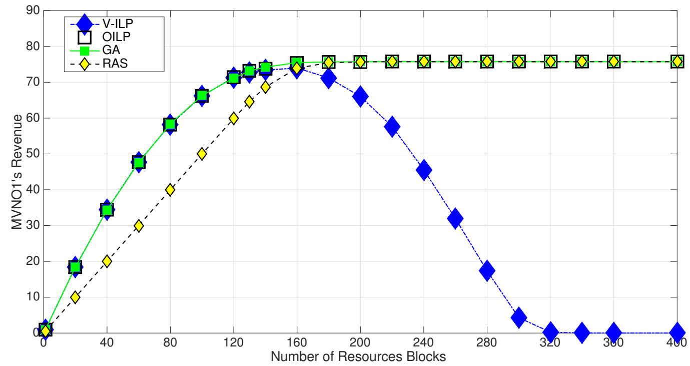
(a) scenario (1a)



(b) scenario (1b)

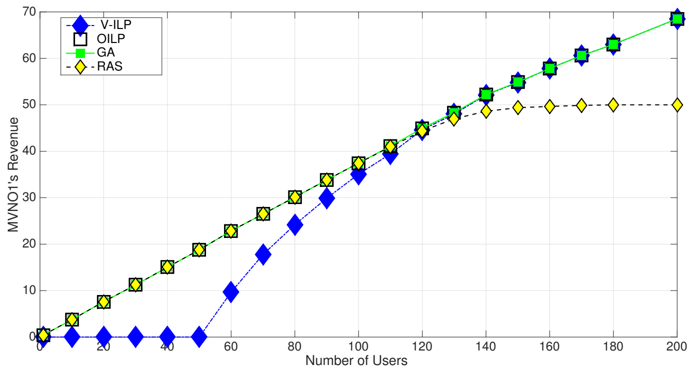


(c) scenario (1c)

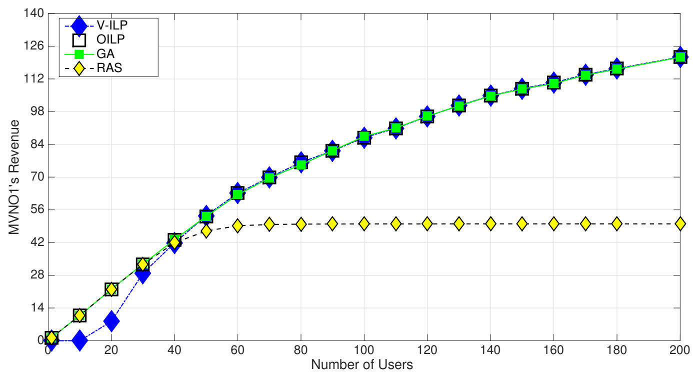


(d) scenario (1d)

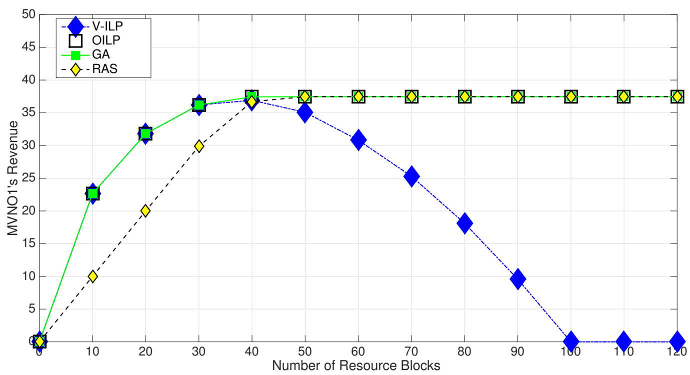
Figure 4: MVNO_1 revenue using V-ILP, OILP, GA and RAS algorithms in the lower-level auction - Uniform distribution



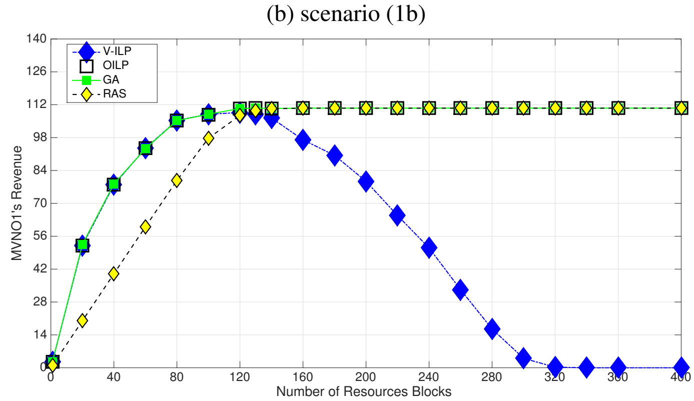
(a) scenario (1a)



(b) scenario (1b)

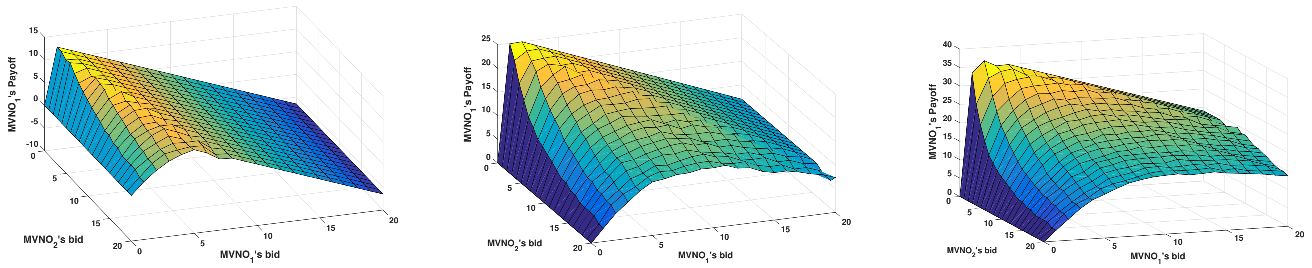


(c) scenario (1c)



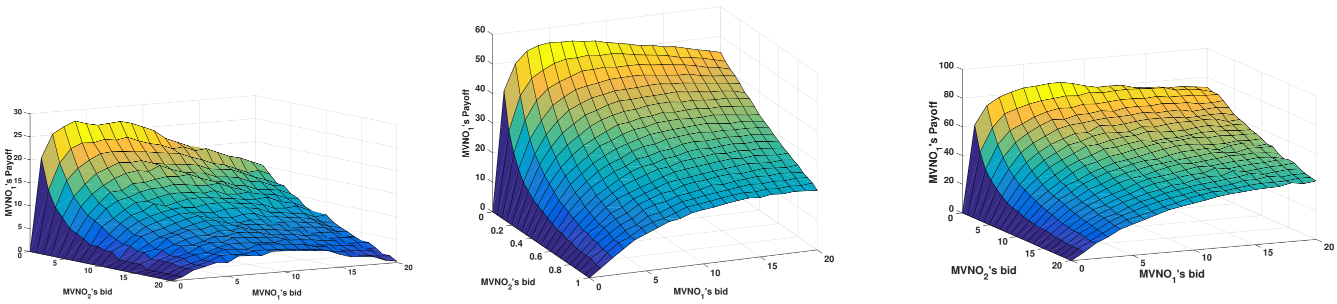
(d) scenario (1d)

Figure 5: MVNO_1 revenue using V-ILP, OILP, GA and RAS algorithms in the lower-level auction - Exponential distribution



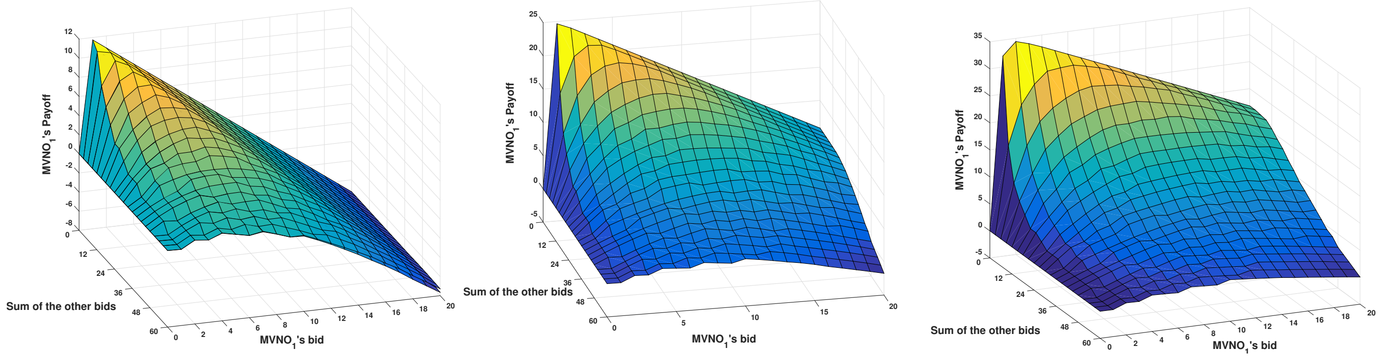
(a) Global Auction : N=50 (b) Global Auction : N=100 (c) Global Auction : N=150

Figure 6: MVNO$_1$'s payoff from the global auction - Scenario 2(a)



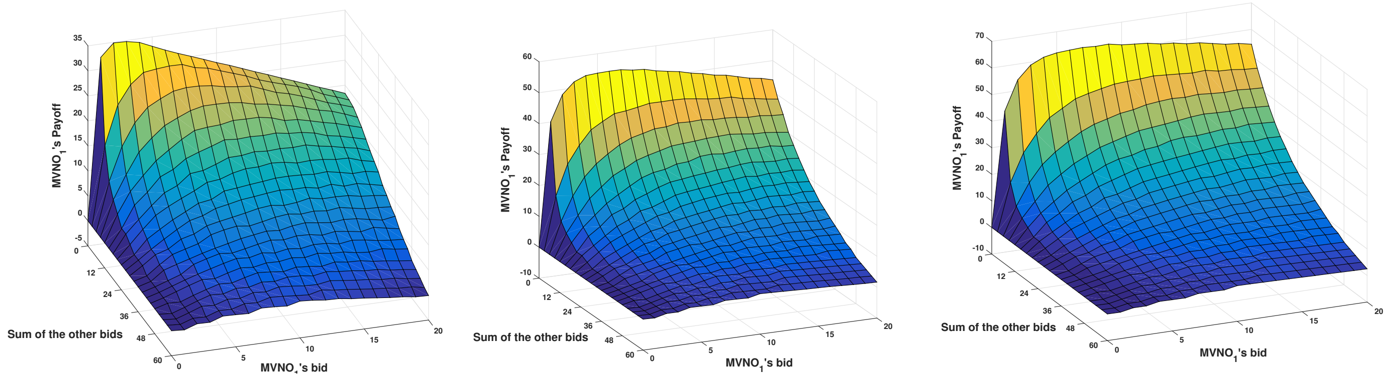
(a) N=50 (b) N=100 (c) N=150

Figure 7: MVNO$_1$'s payoff from the global auction - Scenario 2(b)



(a) N=50 (b) N=100 (c) N=150

Figure 8: MVNO$_1$'s payoff from the global auction - Scenario 2(c)



(a) N=50 (b) N=100 (c) N=150

Figure 9: MVNO$_1$'s payoff from the global auction - Scenario 2(d)

Acknowledgments

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7. References

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Appendix A.

Lemma Appendix A.1. *The utility function of a given MVNO can be expressed as:*

$$U_j(S_j, S_{-j}, N) \triangleq \sum_{i=1}^{N} \Big([F_{i,j}]^N - [F_{i,j}]^{N - \frac{S_j Q}{S_j + S_{-j} + S_0}} [1 - F_{i,j}]^{\frac{S_j Q}{S_j + S_{-j} + S_0}} \Big) - S_j \tag{A.1}$$

Proof Appendix A.1. *Given that $d_{i,j} = 1$ $\forall$ the users and given that the distribution $F_{i,j}(v_{i,j}) = v_{i,j}$ is uniform for all users, we have:*

$$\phi_{i,j}(v_{i,j}) = \phi(v_{i,j}) = v_{i,j} - \frac{1 - v_{i,j}}{1} = 2v_{i,j} - 1 \tag{A.2}$$

$$\begin{aligned} p(x_{i,j} = 1) = &\prod_{(k\neq i)}^{(N-1)} p(v_{k,j} \leq v_{i,j}) + \\ &\prod_{(k\neq i)}^{(N-2)} p(v_{k,j} \leq v_{i,j})(1 - p(v_{k,j} \leq v_{i,j})) + \ldots + \\ &\prod_{(k\neq i)}^{(N-R_j)} p(v_{k,j} \leq v_{i,j}) \prod_{k\neq i}^{R_j - 1} (1 - p(v_{k,j} \leq v_{i,j})) \end{aligned} \tag{A.3}$$

Knowing that $p(v_{k,j} \leq (v_{i,j})) = F_{k,j}(v_{i,j})$ Equation (A.3) becomes:

$$\begin{aligned} p(x_{i,j} = 1) = &\prod_{(k\neq i)}^{(N-1)} F_{kj}(v_{i,j}) + \prod_{(k\neq i)}^{(N-2)} F_{k,j}(v_{i,j})(1 - F_{k,j}(v_{i,j})) \\ &+ \ldots + \prod_{(k\neq i)}^{(N-R_j)} F_{kj}(v_{i,j}) \prod_{(k\neq i)}^{(R_j - 1)} (1 - F_{kj}(v_{i,j})) \end{aligned}$$

And so, knowing that $F_{k,j}(v_{i,j}) = F(v_{i,j})$ $\forall$ $k \in \{1, N\}$:

$$\begin{aligned} p(x_{i,j} = 1) = &[F(v_{i,j})]^{N-1} + [F(v_{i,j})]^{N-2}(1 - F(v_{i,j})) \\ &+ \ldots + [F(v_{i,j})]^{N-R_j}(1 - F(v_{i,j}))^{R_j - 1} \end{aligned}$$

$$\begin{aligned} p(x_{i,j} = 1) = &[(1 - F(v_{i,j}))]^{R_j - 1} F(v_{i,j})^{N - R_j} \\ &[\frac{F(v_{i,j})^{Rj-1}}{(1 - F(v_{i,j}))^{Rj-1}} + \frac{F(v_{i,j})^{Rj-2}}{(1 - F(v_{i,j}))^{Rj-2}} \\ &+ \ldots \frac{F(v_{i,j})}{1 - F(v_{i,j})} + 1] \end{aligned} \tag{A.4}$$

The terms between the brackets in Equation (A.4) follow a geometric series that can be written as:

$$\frac{q^{R_j} - 1}{q - 1} \tag{A.5}$$

where $q = \frac{F(v_{i,j})}{1 - F(v_{i,j})}$

By replacing Equation (A.5) in Equation (A.4), we obtain

$$p(x_{i,j} = 1) = [1 - F(v_{i,j})]^{R_j - 1} F(v_{i,j})^{N - R_j} \Big(\frac{q^{R_j} - 1}{q - 1} \Big)$$

$$p(x_{i,j} = 1) = \frac{[F(v_{i,j})]^N - F(v_{i,j})^{N - R_j}(1 - F(v_{i,j}))^{R_j}}{2F(v_{i,j}) - 1} \tag{A.6}$$

By replacing $x_{i,j}$ by $p(x_{i,j} = 1)$ and Equations (A.2) and (A.6) in $\sum_{i=1}^{N} p_{i,j} = \sum_{i=1}^{N} \phi_{i,j}(w_{i,j}) * p(x_{i,j} = 1)$, we obtain:

$$\sum_{i=1}^{N} p_{i,j} \triangleq \sum_{i=1}^{N} (2v_{i,j} - 1) \frac{[F(v_{i,j})]^N - F(v_{i,j})^{N - R_j}(1 - F(v_{i,j}))^{R_j}}{2F(v_{i,j}) - 1} \tag{A.7}$$

Since $F(v_{i,j}) = F_{i,j}$ (for a simpler notation)

$$\sum_{i=1}^{N} p_{i,j} \triangleq \sum_{i=1}^{N} [F_{i,j}]^N - F_{i,j}^{N - R_j}(1 - F_{i,j})^{R_j} \tag{A.8}$$

By replacing in Equation (A.1) we obtain :

$$U_j(S_j, S_{-j}, N) \triangleq \sum_{i=1}^{N} \Big([F_{i,j}]^N - [F_{i,j}]^{N - \frac{S_j Q}{S_j + S_{-j} + S_0}} [1 - F_{i,j}]^{\frac{S_j Q}{S_j + S_{-j} + S_0}} \Big) - S_j \tag{A.9}$$

Appendix A.1.

Given any distribution F other that the uniform, we can apply the same method as presented in this paper, although the particular results may differ, depending on the specific properties of F_{ij}. More concretely, the method consists of:

1. *Establishing (eventually approximately) the utility function by injecting F into equations (A.2) as well as (A.6) by following a procedure similar to the one described in Section Appendix A. In particular, Equation A.9 will have the general form:*

$$U_j(S_j, S_{-j}, N) \hat{=}$$
$$\sum_{i=1}^{N} \Phi_{ij}(v_{ij})\Big(\frac{[F_{i,j}]^N - [F_{i,j}]^{N-\frac{S_jQ}{S_j+S_{-j}+S_0}}[1-F_{i,j}]^{\frac{S_jQ}{S_j+S_{-j}+S_0}}}{2F_{ij}-1}\Big) - S_j$$

2. *Studying the existence of Nash equilibrium by investigating the concavity of the utility function derived in step 1, as in Section 4.2;*

3. *Studying the uniqueness of the Nash equilibrium by checking the diagonal strict concavity property of $\delta(S,r)$, as in Section 4.3.*

Appendix B.

Lemma Appendix B.1. *The second derivative of a given MVNO utility, $\frac{\partial^2 U_j(S_j,S_{-j})}{\partial S_j^2}$ is negative.*

Proof Appendix B.1. *First derivative of Equation (A.1) gives:*

$$\frac{\partial U_j(S_j, S_{-j})}{\partial S_j} = C_j \sum_{i=1}^{N}\Big(A_{i,j}B_{i,j}\log(\frac{F_{i,j}}{1-F_{i,j}})\Big) - 1 \tag{B.1}$$

Second derivative of (A.1) gives :

$$\frac{\partial^2 U_j(S_j, S_{-j})}{\partial S_j^2} = C_j{}^2 \sum_{i=1}^{N} -A_{i,j}B_{i,j}E_{i,j} \tag{B.2}$$

We study the sign of $\frac{\partial^2 U_j(S)}{\partial^2 S_j} = C_j{}^2 \sum_{i=1}^{N} -A_{i,j}B_{i,j}E_{i,j}$. Since $A_{i,j}$, $B_{i,j}$ and C_j are positive, we study the sign of $E_{i,j}(F_{i,j}) = I_{i,j}(F_{i,j}) + J_{i,j}(F_{i,j})$. To achieve this, we consider the function $f(x) = I(x) + J(x) = (\log(\frac{x}{1-x}))^2 + \alpha\log(\frac{x}{1-x})$ where $x = F_{i,j}$ and $\alpha = \frac{2}{Q}(1 + \frac{S_j}{S_{-j}+S_0})$. We start by calculating the derivative with respect to x. We have: $f(x) > 0$ for $x > 0.5$ leading to $\frac{\partial^2 U_j(S)}{\partial^2 S_j} = C_j{}^2 \sum_{i=1}^{N} -A_{i,j}B_{i,j}E_{i,j} < 0$ for $F_{i,j} > 0.5$ $\{1, N\}$.

Appendix C.

Lemma Appendix C.1. *The symmetric matrix $[G(S,r)+G(S,r)^t]$ is negative definite due to the negativity of it eigenvalues.*

Proof Appendix C.1. *We start by calculating the Jacobian G(S,r) elements:*

$$a_{11} = -\frac{(S_2+S_0)^2Q^2}{(S_1+S_2+S_0))^4}\sum_{i=1}^{N} A_{i1}B_{i1}E_{i1}$$

$$a_{12} = \frac{S_1(S_2+S_0)Q^2}{(S_1+S_2+S_0)^4}\sum_{i=1}^{N} A_{i1}B_{i1}H_{i1}$$

$$a_{21} = \frac{(S_1+S_0)S_2Q^2}{(S_1+S_2+S_0)^4}\sum_{i=1}^{N} A_{i2}B_{i2}H_{i2}$$

$$a_{22} = -\frac{(S_1+S_0)^2Q^2}{(S_1+S_2+S_0)^4}\sum_{i=1}^{N} A_{i2}B_{i1}E_{i2}$$

Furthermore, we denote by

$$K_j = -Q^2\sum_{i=1}^{N} A_{i,j}B_{i,j}E_{i,j}$$

$$L_j = Q^2\sum_{i=1}^{N} A_{i,j}B_{i,j}H_{i,j} = -K_j$$

$$\beta_j = \frac{(S_j+S_0)^2}{(S_j+S_{-j}+S_0)^4}$$

$$\gamma = \frac{S_j(S_{-j}+S_0)}{(S_j+S_{-j})^4}$$

We suppose that for $F_{i,j} > 0\ \forall\ i\in\{1, N\}$, $I_{i,j} >> J_{i,j}$ and so $E_{i,j} = I_{i,j}$ leading to $H_{i,j} = E_{i,j}$

G(S,r) becomes

$$\begin{pmatrix} r_1\beta_2K_1 & -r_1\gamma K_1 \\ -r_2\gamma K_2 & r_2\beta_1K_2 \end{pmatrix} \tag{C.1}$$

Now we calculate $[G(S,r) + G(S,r)^t]$:

$$[G(S,r) + G(S,r)^t] = \begin{pmatrix} 2r_1\beta_2K_1 & -\gamma(K_1+K_2)(r_1-r_2) \\ -\gamma(K_2+K_1)(r_2-r_1) & 2r_2\beta_1K_2 \end{pmatrix} \tag{C.2}$$

We have to prove that the matrix above is negative definite. This is the case if all the eigenvalues are negative:

$$[G(S,r) + G(S,r)^t] = \begin{pmatrix} 2r_1\beta_2K_1 - \lambda & -\gamma(K_1+K_2)(r_1-r_2) \\ -\gamma(K_2+K_1)(r_2-r_1) & 2r_2\beta_1K_2 - \lambda \end{pmatrix} \tag{C.3}$$

For $r_1 = r_2$, we have $\lambda_0 = r_1\alpha_2K_1 < 0$ and $\lambda_1 = r_2\beta_1K_2 < 0$ since we have

- *r_1 and $r_2 > 0$ (positive weights)*
- *β_1 and $\beta_2 > 0$*
- *K_1 and $K_2 < 0$ ($K_j = -Q^2\sum_{i=1}^{N} A_{i,j}B_{i,j}E_{i,j} < 0$)*

which justify the uniqueness of the Nash equilibrium. Now for $r_1 \neq r_2$, we have the following quadratic equation in terms of λ

$$\begin{aligned}\lambda^2 - 2(r_1\beta_2 K_1 + r_2\beta_1 K_2)\lambda + \\ \gamma^2[(K_1 + K_2)^2(r_1 - r_2)^2] + 4K_1K_2r_1r_2\beta_1\beta_2\end{aligned} \tag{C.4}$$

Knowing that the solutions of a quadratic equation satisfy $x^2 - Sx + P = 0$, where S is the sum of the solution and P is their product. Comparing with Equation (C.4) we have

$$S = 2(r_1\beta_2 K_1 + r_2\beta_1 K_2) < 0$$

$$P = \gamma^2[(K_1 + K_2)^2(r_1 - r_2)^2] + 4K_1K_2r_1r_2\beta_1\beta_2 > 0$$

leading to negative eigenvalues.

Appendix D.

Table D.6: Expression table

Notation	Interpretation
$C-RAN$	C-RAN operator spectrum owner
$MVNO_j$	j-th MVNO; $1 \leq j \leq m$
$UE_{i,j}$	i-th user of $MVNO_j$; $1 \leq i \leq N$
S_j	$MVNO_j$'s bid vector $S_j = P_j$
P_j	The price $MVNO_j$ pays to get R_j
R_j	$MVNO_j$ allocation in terms of resource blocks
S_0	Bid set by the C-RAN
$b_{i,j}$	$UE_{i,j}$'s bid vector $b_{i,j} = (d_{i,j}, w_{i,j})$
$d_{i,j}$	Number of resource blocks requested by $UE_{i,j}$
$w_{i,j}$	$UE_{i,j}$'s declared valuation for $d_{i,j}$ resource blocks
$v_{i,j}$	$UE_{i,j}$'s true valuation for $d_{i,j}$ resource blocks
$\boldsymbol{x_j}$	**Decision vector** $\boldsymbol{x_j} = \{x_{i,j}\}$, $1 \leq i \leq N$; $x_{i,j} = 1$ if $UE_{i,j}$ wins, 0 otherwise
$\boldsymbol{p_j}$	**Price vector** $\boldsymbol{p_j} = \{p_{i,j}\}, 1 \leq i \leq N$; $p_{i,j} = p_{i,j}^{VCG}$ if $UE_{i,j}$ wins, 0 otherwise
$A_{i,j}=$	$F_{ij}^{\frac{S_jQ}{S_j+S_{-j}}}$
$B_{i,j}=$	$(1 - F_{ij})^{\frac{S_jQ}{S_j+S_{-j}}}$
$C_{i,j}=$	$\frac{S_{-j}}{(S_j+S_{-j})^2}$
$E_{i,j}=$	$(\log(\frac{F_{ij}}{1-F_{ij}}))^2 + \frac{2}{Q}(1 + \frac{S_j}{S_j+S_{-j}})\log(\frac{F_{ij}}{1-F_{ij}})$
$K_j=$	$-Q^2 \sum_{i=1}^{N} A_{i,j}B_{i,j}E_{i,j}$
$L_j=$	$Q^2 \sum_{i=1}^{N} A_{i,j}B_{i,j}H_{i,j} = -K_j$
$\beta_j=$	$\frac{(S_j+S_0)^2}{(S_j+S_{-j}+S_0)^4}$
$\gamma=$	$\frac{S_j(S_{-j}+S_0)}{(S_j+S_{-j})^4}$

***Authors Bios**

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