

A Generic Method for Statistical Testing

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Abstract

This paper addresses the problem of selecting finite test sets and automating this selection. Among these methods, some are deterministic and some are statistical. The kind of statistical testing we consider has been inspired by the work of Thevenod-Fosse and Waeselynck. There, the choice of the distribution on the input domain is guided by the structure of the program or the form of its specification.

In the present paper, we describe a new generic method for performing statistical testing according to any given graphical description of the behavior of the system under test. This method can be fully automated. Its main originality is that it exploits recent results and tools in combinatorics, precisely in the area of random generation of combinatorial structures.

Uniform random generation routines are used for drawing paths from the set of execution paths or traces of the system under test. Then a constraint resolution step is performed, aiming to design a set of test data that activate the generated paths. This approach applies to a number of classical coverage criteria. Moreover, we show how linear programming techniques may help to improve the quality of test, i.e. the probabilities for the elements to be covered by the test process.

The paper presents the method in its generality. Then, in the last section, experimental results on applying it to structural statistical software testing are reported.

1. Introduction

In the area of software testing, numerous methods have been proposed and used for selecting finite test sets and automating this selection. Among these methods some are deterministic and some are probabilistic. Depending on the authors, methods of this last class are called random testing or statistical testing.

Random testing as in [11, 12, 28], consists in selecting test data uniformly at random from the input domain of the

program. When the random selection is based on some operational profile, it is sometimes called statistical or operational testing and can be used to make reliability estimates [26]. In this article, the kind of statistical testing we consider has been inspired by the work of Thévenod-Fosse and Waeselynck [31]. There, the choice of the distribution on the input domain is guided by some coverage criteria of either the program (structural statistical testing) or some specification (functional statistical testing).

In the recent years, the general problem of studying and simulating random processes has particularly benefitted from progresses in the area of random generation of combinatorial structures. The seminal works of Wilf and Nijenhuis in the late 70's [36, 27] have led to efficient algorithms for generating uniformly at random a variety of combinatorial structures. In 1994, Flajolet, Zimmermann and Van Cutsem [13] have widely generalized and systematized the approach. Briefly, their approach is based on a non-ambiguous recursive decomposition of the combinatorial structures to be generated. Their work constitutes the basis of powerful tools for uniform random generation of complex entities, as graphs, trees, words, paths, etc. In the present paper, we explore the idea of using such concepts and tools for random software testing.

Actually, there are several ways to use uniform generation in the area of testing.

As mentioned above, a natural idea is to uniformly draw data from the input domain. This approach of random testing, was studied some time ago for numerical data [11, 12], and turned out to have an uneven detection power when applied to realistic complex programs [4, 31]. This is confirmed by recent comparisons and evaluations [28].

In this paper, we follow another idea: We describe a generic method for using these tools as soon as there is a graphical description of the behavior of the system under test. It may be the control graph of the program, or some specification, either directly graphical (State-charts, Petri nets) or indirectly via some semantics in terms of transition systems, automaton, state machines, Kripke structures, etc. Such behavioral graphs can be described as combina-

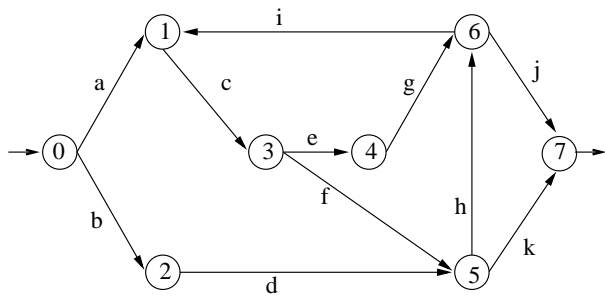


Figure 1. A graph with starting and ending vertices

torial structures. Therefore, uniform generation can be used for drawing paths from the set of execution paths or traces of the system under test, or, more efficiently, among some subsets satisfying some coverage conditions.

Our approach was sketched in a previous paper [16], where we presented a first example for structural statistical testing. This paper presents the method in its generality, and avoids the heuristic used in [16] for the implementation of the all-statements and all-branches criteria.

The paper is organized as follows: Section 2 presents our general method to count and uniformly draw paths in a graph, based on some translation into a combinatorial structure specification; Section 3 recalls some basic notions of statistical testing, coverage criteria and test quality; In Section 4 we present a path generation scheme guided by test quality, and in Section 5 we discuss the issue of deriving test inputs, once a set of paths has been generated. In the two last sections, we give some experimental results on applying our method to structural statistical testing and we sketch some perspectives.

2. Combinatorial preliminaries

We present here some combinatorial concepts and methods which will be used in the sequel of the paper. Let us consider a connected directed graph G where vertices, as well as edges, are labeled in such a way that any two distinct vertices (resp edges) have distinct labels. Furthermore, there exist two vertices v_s (starting vertex) and v_e (ending vertex) such that, for any vertex v , there exists a path from v_s to v and a path from v to v_e in G . Figure 1 presents such a graph, where vertices are labeled with numbers from 0 to 7 and edges are labeled with letters from 'a' to 'k'; vertices 0 and 7 are the starting and ending vertices respectively. If n is a positive integer, \mathcal{P}_n (resp. $\mathcal{P}_{\leq n}$) denotes the set of paths of length n (resp. whose length is $\leq n$) in G from v_s to v_e , and $\mathcal{P}_{\leq \infty}$ denotes the whole (possibly infinite) set of paths from v_s to v_e .

2.1. Uniform random generation of paths in a graph

Our aim is, given an integer n , to generate uniformly at random (u.a.r.) one or several paths of length $\leq n$ from v_s to v_e . Uniformly means that all paths in $\mathcal{P}_{\leq n}$ have the same probability to be generated. At first, let us focus on a slightly different problem: the generation of paths of length n exactly. We will see further that a small change in the graph allows to generate paths of length $\leq n$. Remark that generally the number of paths of length n grows exponentially with n .

The principle of the generation process is simple: Starting from vertex v_s , draw a path step by step. At each step, the process consists in choosing a successor of the current vertex and going to it. The problem is to proceed in such a way that only (and all) paths of length n can be generated, and that they are equiprobably distributed. This is done by choosing successors with suitable probabilities. Given any vertex v , let $f_v(m)$ denote the number of paths of length m which connect v to the end vertex v_e . Suppose that, at any step of the generation, we are on vertex v which has k successors denoted v_1, v_2, \dots, v_k . In addition, suppose that $m > 0$ edges remain to be crossed in order to get a path of length n to v_e . Then the condition for uniformity is that the probability of choosing vertex v_i ($1 \leq i \leq k$) equals $f_{v_i}(m-1)/f_v(m)$. In other words, the probability to go to any successor of v must be proportional to the number of paths of suitable length from this successor to v_e .

So we need to compute the numbers $f_v(i)$ for any $0 \leq i \leq n$ and any vertex v of the graph. This can be done by using the following recurrence rules:

$$\begin{aligned} f_v(0) &= 1 && \text{if } v = v_e \\ &= 0 && \text{otherwise} \\ f_v(i) &= \sum_{v \rightarrow v'} f_{v'}(i-1) && \text{for } i > 0 \end{aligned}$$

where $v \rightarrow v'$ means that there exists an edge from v to v' (note that v' may be equal to v if loops are allowed in the graph). Table 1 presents the recurrence rules which correspond to the graph of Figure 1.

Now the generation scheme is as follows:

- Preprocessing stage: Compute a table of the $f_v(i)$'s for all $0 \leq i \leq n$ and all vertices.
- Generation stage: Draw the path according to the scheme seen above.

Note that the preprocessing stage must be done only once, whatever the number of paths to be generated. Easy computations show that the memory space requirement is $n \times |G|$ integer numbers, where $|G|$ stands for the number of vertices in the graph. The number of arithmetic operations needed for the preprocessing stage is in the worst case in $O(n|G|^2)$ and the generation stage is $O(n \log n)$. When

$$\begin{aligned}
f_0(0) &= f_1(0) = f_2(0) = 0 \\
f_3(0) &= f_4(0) = f_5(0) = f_6(0) = 0 \\
f_7(0) &= 1 \\
f_0(k) &= f_1(k-1) + f_2(k-1) & (k > 0) \\
f_1(k) &= f_3(k-1) & (k > 0) \\
f_2(k) &= f_5(k-1) & (k > 0) \\
f_3(k) &= f_4(k-1) + f_5(k-1) & (k > 0) \\
f_4(k) &= f_6(k-1) & (k > 0) \\
f_5(k) &= f_6(k-1) + f_7(k-1) & (k > 0) \\
f_6(k) &= f_1(k-1) + f_7(k-1) & (k > 0) \\
f_7(k) &= 0 & (k > 0)
\end{aligned}$$

Table 1. Recurrences for the $f_i(k)$.

the degree of the graph is bounded by a constant, the complexity reduces to $O(n|G|)$ for the preprocessing stage and $O(n)$ for the generation stage. This ensures a very efficient generation process [13].

Now we address the problem of generating paths of length $\leq n$ instead of exactly n . The only change is the following: Add to the graph a new vertex v'_s which becomes the new start vertex, with an edge from v'_s to v_s and a loop edge from v'_s to itself. Each path of length $n+1$ from v'_s to v_e in this new graph crosses k times the loop edge for some k such that $0 \leq k \leq n$ and once the one from v'_s to v_s . With this path we obviously associate a path of length $n-k$ in the previous graph. It is straightforward to verify that any path of length $\leq n$ can be generated in such a way, and the generation is uniform.

Note that the above developments are a special case of a general method of generation of combinatorial structures, which has been first addressed by Wilf [36] and then generalized and systematized by Flajolet, Zimmermann and Van Cutsem [13]. More precisely, the problem of generating paths of a given length in G is equivalent to the one of uniform random generation of words of so-called *regular languages*, which has first been discussed in [17]. Indeed, a regular language is defined by a particular labeled graph called *finite state automaton*, and any word of the language corresponds to a path in the automaton. We show in Table 2 the set of words which correspond to the paths of length ≤ 10 of the graph of Figure 1.

In our implementation, the generation of paths is programmed in MuPAD, using the CS package. MuPAD [29] is a formal and algebraic calculus tool, developed at the University of Paderborn. CS [8, 9], is a package devoted to counting and randomly generating combinatorial structures, based on the general notion of “decomposable structures” defined in [13]. CS is now part of the MuPAD-Combinat package [18] which is freely available at the following address: <http://mupad-combinat.sourceforge.net>

length	words
3	bdk
4	acfk, bdhj
5	acegj, acfhj
7	bdhicfk
8	acegicfk, acfhicfk, bdhicegj, bdhicfhj
9	acegicegj, acegicfhj, acfhicegj, acfhicfhj

Table 2. The 14 paths of length ≤ 10 from vertex $v_s = 0$ to vertex $v_e = 7$.

2.2. Constraints on paths and graphs transformations

As we will see in Section 4, our method of statistical testing involves counting and random generation of paths subject to additional constraints. In this subsection, we show how to change the graph in order to take into account such constraints.

Let us focus first on a rather simple constraint: we aim to construct, given a labeled connected graph G and an edge label ℓ , a graph H whose set of paths is equal to the set of paths of G which cross edge labeled ℓ . This can be done by using the following procedure:

1. Create a copy G' of graph G , in which the edges are labeled exactly as the edges of G , and in which any vertex label v in G becomes v' in G' .
2. Suppose that the edge labeled ℓ joins vertex u to vertex v in G . Then delete this edge and replace it with a new edge labeled ℓ between vertex u (in G) and vertex v' (in G').
3. set v'_e as the ending vertex, instead of v_e (but v_s remains the start vertex.)
4. Delete all the vertices (and their adjacent edges) to which no path from v_s exists.
5. Delete all the vertices (and their adjacent edges) from which no path to v'_e exists.

This concludes the construction of H . Figure 2 shows the result of the procedure, given the graph of Figure 1 and the edge labeled ‘e’. This transformation can be done in linear time and linear memory requirement with respect to the size of the graph. Note that steps 4 and 5 are not mandatory: they are used only to “clean” the final graph by deleting useless elements.

Like in the previous subsection, this process can be stated in terms of operations on regular languages: the graph H may be seen as a finite automaton of the regular language which is the intersection of the language of G and the (regular) language of those words which contain at least once the

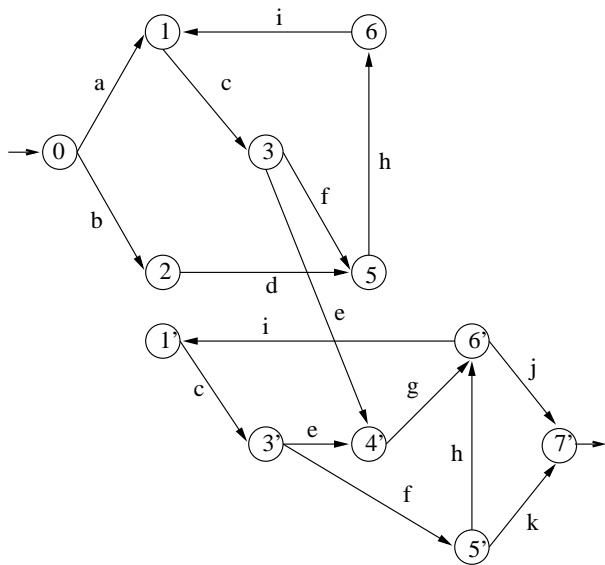


Figure 2. Graph which contains only the paths of the graph of Figure 1 which cross edge labeled 'e'.

letter ℓ . This approach can be generalized in order to perform more complex transformations of graph G as for example, constructing a graph which contains the paths which cross exactly k times a given edge, or which cross two or more given edges, or which take k times a given cycle in the graph. Roughly, it suffices to be able to express the desired constraint in terms of a regular language, i.e. to design a regular expression or a finite automaton which recognizes the whole set of words which satisfy the constraint. Then a standard algorithm for intersecting regular languages (see e.g. [20]) gives H . In our special case, this general method consists exactly in the procedure described above. Very similar procedures apply if we are given vertices instead of edges.

3. Coverage criteria and statistical testing

The idea of combining coverage criteria and random testing aims at overcoming some drawbacks of both approaches.

Applying coverage criteria corresponds to a decomposition of the input domain into some (very often non disjoint) sub-domains: Each element to be covered defines a sub-domain that is the subset of the inputs that cause its execution. The main drawback here is that these sub-domains are generally not homogeneous, i.e. some of their inputs may result in a failure, and some others may yield correct results.

Random testing lessens this drawback since it allows intensive test campaigns where the same element of the pro-

gram may be executed several times with different data. However, in its pure uniform version it induces a bad coverage of cases corresponding to small input sub-domains.

In [31, 32, 33, 34], Thévenod-Fosse and Waeselynck developed a statistical testing method where the input distribution takes into account some coverage criteria in order to avoid the existence of low probability cases. They have reported several experiments, which led to the conclusion that their approach has a better fault detection power than uniform random testing and deterministic testing based on classical coverage criteria. However, the construction of the input distribution is difficult since it requires the resolution of as many equations as paths in the program (or the specification). For large programs, or in presence of loops, the construction is empirical, based on preliminary observations of the behavior of the program [34].

Here, we avoid the construction of the distribution by using the tools presented in Section 2 for generating paths. Before presenting our approach in detail, we must state precisely what it means for a statistical method to take into account a coverage criteria.

A notion of test quality for statistical testing methods has been defined first in [30]. We slightly reformulate it for our context.

Let D be some description of a system under test. D may be a specification or a program, depending on the kind of test we are interested in (functional or structural). We assume that D is based on a graph (or, more generally, on some kind of combinatorial structure). On the basis of this graph, it is possible to define coverage criteria: all-vertices, all-edges, all-paths-of a certain-kind, etc. More precisely, a coverage criterion C characterizes for a given description D a set of elements $E_C(D)$ of the underlying graph (noted E in the sequel when C and D are obvious). In the case of deterministic testing, the criterion is satisfied if every element of the set is exercised by at least one test.

In the case of statistical testing, the satisfaction of a coverage criteria C by a testing method for a description D is characterized by the minimal probability $q_{C,N}(D)$ of covering any element of $E_C(D)$ when drawing N tests. In [30], $q_{C,N}(D)$ is called the test quality of the method with respect to C .

The test quality $q_{C,N}(D)$ can be easily stated if $q_{C,1}(D)$ is known. Indeed, one gets $q_{C,N}(D) = 1 - (1 - q_{C,1}(D))^N$, since when drawing N tests, the probability of reaching an element is one minus the probability of not reaching it N times.

Let us come back to the example of Section 2, where the set of all paths of Figure 1 has been expressed as a specification of some combinatorial structure, and the CS system is used for uniformly drawing among paths of length $\leq n$. Let us note $\mathcal{P}_{\leq n}$ the set of such paths, as in Section 2. Considering the coverage criterion “all paths of length $\leq n$ ”,

noted below $AP_{\leq n}$, we get the following test quality:

$$q_{AP_{\leq n}, N} = 1 - \left(1 - \frac{1}{|\mathcal{P}_{\leq n}|}\right)^N$$

In the example, choosing $n = 10$ allows the coverage of all elementary paths. Since there are 14 paths of length less or equal to 10 (see Table 2) we have:

$$q_{AP_{\leq 10}, N} = 1 - \left(1 - \frac{1}{14}\right)^N$$

Table 3 gives the number of tests required for four values of test quality, for the criterion “all paths of length ≤ 10 ”.

q	0.9	0.99	0.999	0.9999
N	32	63	94	125

Table 3. Number of tests N required for a test quality q

The assessment of test quality is more complicated in general. Let us consider more practicable coverage criteria, such as “all-vertices” or “all-edges”, and some given statistical testing method. The elements to be covered generally have different probabilities to be reached by a test. Some of them are covered by all the tests, for instance the initial and terminal vertices v_s and v_e mentioned in Section 2. Some of them may have a very weak probability, due to the structure of the behavioral graph or to some specificity of the testing method. For instance, in our example edges b and d appear in 5 paths of length ≤ 10 only. Edges a and c appear in 9 such paths. It means that drawing uniformly from $\mathcal{P}_{\leq 10}$ leads to a probability of $\frac{5}{14}$ to reach edge b , and $\frac{9}{14}$ to reach edge a .

Let $E_C(D) = \{e_1, e_2, \dots, e_m\}$ and for any $i \in (1..m)$, p_i the probability for the element e_i to be exercised during the execution of a test generated by the considered statistical testing method. Then

$$q_{C, N}(D) = 1 - (1 - p_{min})^N \quad (1)$$

where $p_{min} = \min\{p_i | i \in (1..m)\}$. Consequently, the number N of tests required to reach a given quality $q_C(D)$ is

$$N \geq \frac{\log(1 - q_C(D))}{\log(1 - p_{min})}$$

By definition of the test quality, p_{min} is just $q_{C,1}(D)$. Thus, from the formula above one immediately deduces that for any given D , for any given N , maximizing the quality of a statistical testing method with respect to a coverage criteria C reduces to maximizing $q_{C,1}(D)$, i. e. p_{min} .

4. Generation of paths guided by the QoT

4.1. General scheme.

Now let us consider a given coverage criterion C . As a preliminary remark, note that the set of elements $E_C(D)$ must be finite, otherwise the quality of test would be zero. This implies, in particular, that the coverage criterion “all paths” is irrelevant as soon as there is a cycle in the description, like in our example (figure 1). Thus, this criterion has to be bounded by additional conditions, for example “all paths of length $\leq n$ ”, “all paths of length between given n_1 and n_2 ”, or “all paths which take at most m times each cycle in the graph”. For the sake of simplicity, we consider in the following that paths are generated within $\mathcal{P}_{\leq n}$, the set of paths of length $\leq n$ that go from v_s to v_e .

We consider two cases, according to the nature of the elements of $E_C(D)$. If $E_C(D)$ denotes a set of paths in the graph, we immediately state that the quality of test is optimal if the paths of $E_C(D)$ are generated uniformly, i.e. any path has the same probability $1/|E_C(D)|$ to be generated. Indeed, if the probability of one or several paths was greater than $1/|E_C(D)|$, then there would exist at least one path with probability less than $1/|E_C(D)|$, therefore the quality of test would be lower. We saw in Section 2.1 how to generate uniformly random paths of given length n in a graph, and how to modify the graph in order to fit with the criterion “all paths of length $\leq n$ ”. The method easily applies to other criteria that involve paths, as those given above, by ways similar to the ones seen in Section 2.2.

Now, we consider the case where the elements of $E_C(D)$ are not paths, but are constitutive elements of the graph as, for example, vertices, edges, or cycles. Clearly, uniform generation of paths does not ensure optimal quality of test in this case. Ideally, the distribution on paths should ensure that the minimal probability to reach any element of $E_C(D)$ is maximal. Unfortunately, computing this distribution would require the resolution as many equations as paths. This is generally impracticable. Thus we propose to generate a path in two steps:

1. pick at random one element e of $E_C(D)$, according to a suitable probability distribution (which will be discussed in Section 4.2);
2. generate uniformly at random one path of length $\leq n$ that goes through e . (This ensures a balanced coverage of the set of paths which cross e .)

Algorithms for achieving the second step are detailed in Section 2. The next subsection deals with the first step.

4.2. Probability distribution for an optimal quality of test.

The problem consists in choosing the suitable probability distribution over $E_C(D)$ in order to maximize the quality of test. Given $E_C(D) = \{e_1, e_2, \dots, e_m\}$, with $m > 0$, the probability p_i for the element e_i (for any i in $(1..m)$) to be reached by a path is

$$p_i = \pi_i + \sum_{j \in (1..m) - \{i\}} \pi_j \frac{\alpha_{i,j}}{\alpha_j},$$

where

- α_i is the number of paths of $\mathcal{P}_{\leq n}$ which takes element e_i ;
- $\alpha_{i,j}$ is the number of paths which take both elements e_i and e_j ; (note that $\alpha_{i,i} = \alpha_i$ and $\alpha_{i,j} = \alpha_{j,i}$);
- π_i is the probability of choosing element e_i during step 1 of the above process.

Indeed, the probability of choosing element e_i in step 1 is π_i ; and the probability of reaching e_i by drawing a random path which goes through another element e_j is $\frac{\alpha_{i,j}}{\alpha_j}$. The above equation simplifies to

$$p_i = \sum_{j=1}^m \pi_j \frac{\alpha_{i,j}}{\alpha_j} \quad (2)$$

since $\alpha_{i,i} = \alpha_i$. Note that coefficients α_j and $\alpha_{i,j}$ are easily computed by ways given in Section 2.

Now we have to determine probabilities $\{\pi_1, \pi_2, \dots, \pi_m\}$ with $\sum \pi_i = 1$, which maximize $p_{min} = \min\{p_i, i \in [1..m]\}$. This can be stated as a linear programming problem:

$$\begin{aligned} &\text{Maximize } p_{min} \text{ under the constraints:} \\ &\left\{ \begin{array}{l} \forall i \leq m, \quad p_{min} \leq p_i ; \\ \pi_1 + \pi_2 + \dots + \pi_m = 1 ; \end{array} \right. \end{aligned}$$

where the p_i 's are computed as in Equation (2). Standard methods lead to a solution in time polynomial according to m .

Let us illustrate this with our example. Given the coverage criterion "all the edges" and given $n = 10$, Table 4 presents the coefficients $\alpha_{i,j}$, where i and j denote letters from 'a' to 'k'. For example, the value '9' in row 'f' and column 'c' means that $\alpha_{c,f} = 9$, i.e. there are exactly 9 paths of length lower or equal to 10 from v_s to v_e which cross both edges c and f in the graph of Figure 1.

The corresponding linear program is shown in Table 5. Each line, but the last one, is an inequation which corresponds to a row in Table 4. The first term of the inequation is p_{min} , the value to be maximized. The second term is one of the p_i 's, computed according to Formula 2. For example, the first line means that p_{min} must be lower or

	a	b	c	d	e	f	g	h	i	j	k
a	9	0	9	0	5	7	5	5	6	6	3
b	0	5	3	5	1	2	1	4	3	3	2
c	9	3	12	3	6	9	6	8	9	8	4
d	0	5	3	5	1	2	1	4	3	3	2
e	5	1	6	1	6	3	6	3	5	5	1
f	7	2	9	2	3	9	3	7	7	5	4
g	5	1	6	1	6	3	6	3	5	5	1
h	5	4	8	4	3	7	3	9	7	7	2
i	6	3	9	3	5	7	5	7	9	6	3
j	6	3	8	3	5	5	5	7	6	9	0
k	3	2	4	2	1	4	1	2	3	0	5

Table 4. Table of the α_{ij} .

equal to p_a , the probability of reaching edge 'a' with a random path. By maximizing p_{min} , one maximizes the lowest p_i , so that the quality of test is optimal. The last line ensures that the probabilities π_i that we are searching for sum to 1.

Solving this linear program leads to $\pi_a = \pi_c = \pi_d = \pi_f = \pi_g = \pi_h = \pi_i = \pi_j = 0$, while $\pi_b = \pi_k = \frac{5}{16}$ and $\pi_e = \frac{6}{16}$. This gives $p_{min} = \frac{1}{2}$, therefore the optimal quality of test equals $1 - \frac{1}{2^N}$, according to Formula 1.

5. From paths to input data

So far we have presented a generic method for generating execution paths in a way that maximizes test quality. This method relies on existing algorithms and tools and can be fully automated. A last step is to generate, for every path, input values that will cause its execution.

5.1. The trivial case of finite models

First let us consider the case where the graphical description corresponds to a finite model of the system under test (Finite State Automaton, Finite State Machine, etc.)[7]. Every edge of the description is labeled by some symbol of a finite alphabet that represents some input or event. This symbol may be coupled with some other symbol indicating some expected reaction (output, action). Coverage criteria typically used in such cases are variants of transition coverage, the main problem being state identification before and after a tested transition. The method described in Section 4 can be used to draw paths for such coverage criteria. Then, given a path, the test data for executing it is just the sequence of inputs labeling its edges, possibly followed by some additional inputs in order to observe that the system under test is in the expected state [21, 6].

$p_{min} \triangleq$	π_a		$+\frac{3}{4}\pi_c$		$+\frac{5}{6}\pi_e$	$+\frac{7}{9}\pi_f$	$+\frac{5}{6}\pi_g$	$+\frac{5}{9}\pi_h$	$+\frac{2}{3}\pi_i$	$+\frac{2}{9}\pi_j$	$+\frac{3}{9}\pi_k$
$p_{min} \triangleq$		π_b	$+\frac{3}{4}\pi_c$	$+\pi_d$	$+\frac{1}{6}\pi_e$	$+\frac{2}{9}\pi_f$	$+\frac{1}{6}\pi_g$	$+\pi_h$	$+\frac{1}{3}\pi_i$	$+\frac{1}{9}\pi_j$	$+\pi_k$
$p_{min} \triangleq$	π_a	$+\frac{3}{9}\pi_b$	$+\pi_c$	$+\frac{3}{9}\pi_d$	$+\pi_e$	$+\pi_f$	$+\pi_g$	$+\pi_h$	$+\pi_i$	$+\pi_j$	$+\pi_k$
$p_{min} \triangleq$		π_b	$+\frac{1}{4}\pi_c$	$+\pi_d$	$+\frac{1}{6}\pi_e$	$+\frac{2}{9}\pi_f$	$+\frac{1}{6}\pi_g$	$+\pi_h$	$+\frac{1}{3}\pi_i$	$+\pi_j$	$+\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{3}\pi_c$	$+\frac{1}{9}\pi_d$	$+\pi_e$	$+\frac{1}{3}\pi_f$	$+\pi_g$	$+\pi_h$	$+\frac{1}{3}\pi_i$	$+\pi_j$	$+\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{4}\pi_c$	$+\pi_d$	$+\frac{1}{2}\pi_e$	$+\pi_f$	$+\frac{1}{2}\pi_g$	$+\pi_h$	$+\frac{1}{3}\pi_i$	$+\pi_j$	$+\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{3}\pi_c$	$+\pi_d$	$+\pi_e$	$+\frac{1}{2}\pi_f$	$+\pi_g$	$+\frac{1}{3}\pi_h$	$+\frac{1}{3}\pi_i$	$+\pi_j$	$+\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{3}\pi_c$	$+\pi_d$	$+\frac{1}{2}\pi_e$	$+\frac{2}{9}\pi_f$	$+\frac{1}{2}\pi_g$	$+\pi_h$	$+\frac{1}{9}\pi_i$	$+\frac{2}{9}\pi_j$	$+\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{3}\pi_c$	$+\pi_d$	$+\frac{5}{6}\pi_e$	$+\frac{2}{9}\pi_f$	$+\frac{5}{6}\pi_g$	$+\frac{7}{9}\pi_h$	$+\pi_i$	$+\frac{2}{3}\pi_j$	$+\frac{5}{9}\pi_k$
$p_{min} \triangleq$	$\frac{5}{9}\pi_a$	$+\frac{1}{9}\pi_b$	$+\frac{1}{3}\pi_c$	$+\pi_d$	$+\frac{5}{6}\pi_e$	$+\frac{2}{9}\pi_f$	$+\frac{5}{6}\pi_g$	$+\frac{7}{9}\pi_h$	$+\frac{2}{3}\pi_i$	$+\pi_j$	
$1 =$	π_a	$+\pi_b$	$+\pi_c$	$+\pi_d$	$+\pi_e$	$+\pi_f$	$+\pi_g$	$+\pi_h$	$+\pi_i$	$+\pi_j$	$+\pi_k$

Table 5. The linear program.

5.2. The general case of infinite models

The problem is more difficult as soon as the model underlying the description is not finite [21]. It is the case for various sorts of Extended Finite State Machines [19], State-charts [5], or the control graph of pieces of code [16], namely any description including non-trivial data types and guards.

An example is given in Figure 3, using a notation close to UML state charts. It is presented in details in [22]. In this example, the M variable is of a given type *Message*; every message has a priority. The Q variable is of type *PriorityQueue*. The *get* operation returns the oldest message of the queue with the best priority. The boxes labeled by *Buffer(Q)* and *ClientReady(Q)* denote infinite classes of states, (as many states as possible values for the Q variable). A possible trace (or more exactly class of traces) is: $_ / Q.init(); ?M/Q.add(M); ?ready[\neg Q.isEmpty()]/_ ; !Q.get()/Q.remove()$

Given some path or some trace made of conditioned statements or guarded commands, how to find some inputs triggering its execution? It is a classical issue in structural testing, or in functional testing based on specifications with data types. Constructing, via symbolic evaluation techniques, the predicate characterizing the input domain of the path, can solve it: This predicate is the conjunction of the guards (or conditions) encountered on the path, adequately updated in function of the variables assignments (see for instance [16]). Then the problem reduces to a constraint-solving problem. Any data satisfying the above predicate is an input executing the path. At this stage, the tool to be used is highly dependent on the kind of guards and data types allowed in the description: There exist a lot of specialized constraint solvers for

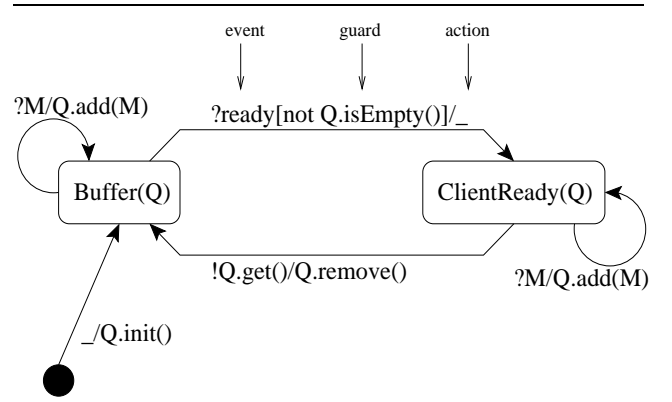


Figure 3. A state-chart specification of a buffer with priorities

various types of variables and constraints, which ensure termination and completeness. However, in full generality the problem is only semi-decidable and a general-purpose solver may not terminate when searching for a solution. Lately, significant advances have been achieved with the introduction of powerful heuristics and randomization techniques, such as those used in the LOFT and GATEL tools [23, 24] or the BZ-tools in [2]. Other uses of constraint solvers for test generation are reported in [3, 14, 25]. A classical difficulty at this stage is that unfeasible paths may arise. For instance, in the example of Figure 3, all paths beginning by $_ / Q.init(); ?ready[\neg Q.isEmpty()]/_ ; \dots$ are unfeasible since the *init* method assigns an empty state to the Queue. In the next section we show how we cope with this problem in the prototype that we have developed for structural statistical testing.

6. The AuGuSTe prototype

In order to validate the applicability of our approach, Sandrine Gouraud has developed a tool for statistical structural testing [15]. The programs under test are written in a small programming language inspired from C. The data types are booleans, integers, and arrays. The constraint solver is the one of GATEL [24], extended to arrays.

When an unfeasible path is detected (or suspected) by the constraint solver, it is rejected and another path is drawn. This strategy does not affect the uniform distribution on paths: any feasible path is still drawn with uniform probability. This ensures that, if the coverage criterion involves paths only (like e.g. "all paths of length $\leq n$ "), the quality of test stays optimal. However, in other cases, it may decrease with regard to its theoretical value, depending on the distribution of unfeasible paths in the graph.

Actually, our first experiments with AuGuSTe show that the difference may be significant in presence of big numbers of unfeasible paths. We are currently investigating methods for improving this experimental quality of test. For example, in some cases a number of unfeasible paths can be detected by static analysis of the description of the system. Then the combinatorial specification of the graph can be modified in order to avoid these paths.

	# lines	# paths	coverage criterion C	cardinality of C
FCT1	30	17	all paths	17
FCT2	43	9	all paths	9
FCT3	135	33	all paths	33
FCT4	77	∞	all branches	41

Table 6. The four tested programs

Our tool has been used for testing the same set of four industrial C programs as in [33] (see also table 6), where Pascale Thévenod-Fosse, H el ene Waeselynck and Yves Crouzet presented the first experimental evaluation of the detection power of statistical structural testing. Thanks to them, it was possible to reuse the same sets of mutants and to replay almost the same set of experiments. In [33], the statistical method is different from here, since it is based on the explicit construction of a distribution on the input domain, either analytically, or empirically (when there is some loop). In our case, we draw paths and then use constraint-solving tools to produce inputs. Of course, this induces a distribution on the input domain. As this distribution is highly dependent on the implementation of the constraint solver, it remains implicit. The only way to compare the detection power of the two methods is by experiments. Despite the difference between the two approaches, the results

of the experiments are quite similar (see Table 8), with the advantage that our new approach is fully automated.

More than 10000 runs were performed on 2914 mutants (see [33, 35] for details on the mutation operators) for performing 12 experiments (see Table 7). Test data were generated in order to obtain a quality of test of $q_N = 0.9999$. The test generation time for the programs FCT1, FCT2 and FCT3 was few minutes. The differences between the mutation scores obtained for FCT3 (see Table 8) are mainly due to a problem of non independence of the test experiments because some global variables are not initialized in this program. When these variables are initialized, the mutation score is the same.

	#tests N	#mutants
FCT1	170	279
FCT2	80	563
FCT3	5×405	1467
FCT4	5×850	605

Table 7. Number of mutants and tests for each program

	Mutation scores	
	[35]	AuGuSTe
FCT1	1	1
FCT2	1	1
FCT3	min=1 ave=1 max=1	min=0.9951 ave=0.9989 max=1
FCT4(1)	min=0.9898 ave=0.9901 max=0.9915	min=0.9854 ave=0.9854 max=0.9854
FCT4(2)	min=0.9898 ave=0.9901 max=0.9915	min=0.9634 ave=0.9762 max=0.9854

Table 8. Mutation scores

The fourth program, FCT4, was the most difficult and the most interesting. It contains both a loop and a huge number of unfeasible paths. The coverage criterion was "all the edges with maximal path length of 234", in number of edges of the control graph (thus much more in number of statements). The length 234 was chosen according to the characteristics of the loop. Consequently, the predicates to be solved were rather long too: At least one out of two vertices

on a path corresponds to a decision point, thus to a condition to be added to the predicate.

In order to reduce the number of unfeasible paths, we have adapted the combinatorial structure according to the characteristics of the program [15]. Such modification can be done systematically by some classical analysis, without altering the program. This manipulation reduces the test generation time (drawing paths and solving predicates) from a week to two hours. The preprocessing stage (construction of the combinatorial structure and counting the number of paths) lasted two days: it means constructing the α_{ij} table for 41 edges. This preprocessing stage could be improved by avoiding unnecessary intersections, using dominator tree [1] for determining equivalences between α_{ij} , and optimizing combinatorial structures. We are currently working in this direction.

The obtained mutation scores are presented in Table 8, line FCT4(1). The line FCT4(2) correspond to mutation scores obtained by using for the π_i a distribution based on the heuristic presented in [16]. The results are slightly lower but the preprocessing stage lasts one hour only.

These first experiments let think that the method scales up well.

7. Conclusion and perspectives

We have shown how uniform generation of combinatorial structures can be used for statistical testing as soon as some graphical description of the program under test is available. If the description is at the program level (control flow graph), our method applies to structural statistical testing. If the description is at the specification level, it applies to functional statistical testing.

This paper brings two main novelties with respect to our previous paper [16], where we presented a first application of combinatorial methods to the main criteria of structural testing: First, we give a generalization of the method to any graphical description and to any coverage criteria ; Moreover, in Section 4, we show how to build a probability distribution on the elements to be covered for any given criteria. This distribution ensures an optimal quality of test when associated with the methods of Section 2.2 for randomly generating paths constrained to cross these elements. This replaces the heuristic used in [16] for the implementation of the all-statements and all-branches criteria.

As reported in Section 6, this approach has been validated on realistic examples.

It exhibits a similar detection power as Thévenod-Fosse and Waeselynck's method, which is better than pure random testing or deterministic testing.

More generally, we think that this approach provides a basis for a new class of tools in the domain of testing, com-

bining random generation of combinatorial structures, linear programming techniques, and constraint solvers.

Some interesting perspectives are still open. The CS tool can deal with languages more complex than regular languages (for instance, with cardinality constraints such as paths with the same number of iterations in loop 1 and loop 2). Practically, it means that it could be possible to compile behavioral graphs into more elaborated combinatorial structures, taking into account some knowledge on the system under test, or some results of static analysis. This could improve significantly the efficiency of the tools, by eliminating some major sources of unfeasible paths.

Another possibility worth to explore is the use of the new approach proposed recently by Flajolet & al. [10] for random generation of combinatorial structures : It is based on Boltzmann models and could avoid the introduction of a bound on the length of the considered paths.

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