

L3 Apprentissage

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LRI – LSV

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Overview

Introduction

Linear changes of representation

- Principal Component Analysis

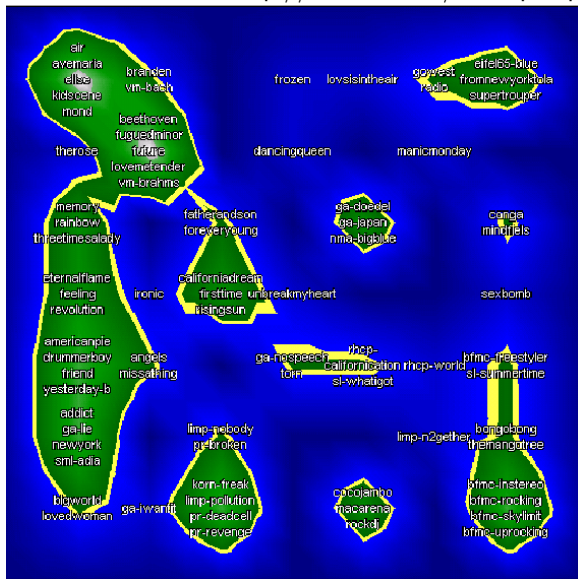
- Random projections

- Latent Semantic Analysis

Non linear changes of representation

Clustering

<http://www.ofai.at/~elias.pampalk/music/>



Unsupervised learning

$$\mathcal{E} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$$

Applications

- ▶ Documents, text mining
 - ▶ indexing, retrieval
- ▶ e-commerce, banks, insurance
 - ▶ user profiling, recommender systems

Jain, 2010

The representation of the data is closely tied with the purpose of the grouping. The representation must go hand in hand with the end goal of the user. We do not believe there can be a true clustering definable solely in terms of the data — truth is relative to the problem being solved.

Unsupervised learning – WHAT

$$\mathcal{E} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$$

Questions

- ▶ What is in the data
divide and conquer
- ▶ Abstraction: (lossy) compression
tradeoff precision/model size
- ▶ Regularities/ Anomaly detection
- ▶ What is the (generative) model ?
how to account for prior knowledge

Goals

Unsupervised learning – HOW

Position of the problem

- ▶ Stationary data
- ▶ Online data
Tradeoff precision / noise

Algorithmic issues

clustering; density estimation

Data streaming

- ▶ Non-stationary data:

change detection / noise

Real-time ? Limited resources ?

Validation

- ▶ exploratory data analysis (subjective)
- ▶ density estimation (likelihood)

Abstraction

Artifacts

- ▶ Represent a set of instances $\mathbf{x}_i \in X$ by an element $z \in X$
- ▶ Examples:
 - ▶ Mean state of a system
 - ▶ Sociological profile

Prototypes

- ▶ Find the most representatives instances among $\mathbf{x}_1, \dots, \mathbf{x}_n$
- ▶ How many prototypes ? (avoid overfitting)
- ▶ Examples:
 - ▶ Faces
 - ▶ Molecules



Generative models

Given

$$\mathcal{E} = \{x_i \in X\} \quad P(x)$$

Find $\hat{P}(x)$.

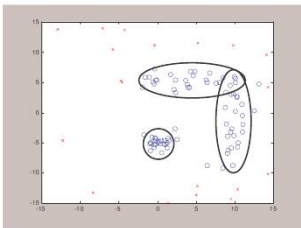
Issues

- ▶ A distribution (sums to 1)
- ▶ Parametric (e.g. mixture of Gaussians) vs non- ?
- ▶ Which criterion ? optimize (log) likelihood of data.

One-class SVM

Formulation

$$\left\{ \begin{array}{ll} \text{Min.} & \frac{1}{2} \|\mathbf{w}\|^2 - \rho + \mathbf{C} \sum_i \xi_i \\ \text{s.t.} & \forall i = 1 \dots n \\ & \langle \mathbf{w}, \mathbf{x}_i \rangle \geq \rho - \xi_i \end{array} \right.$$



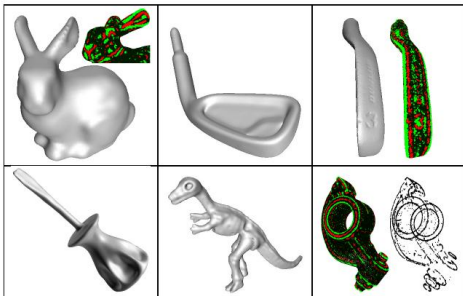
Dual problem

$$\left\{ \begin{array}{ll} \text{Min.} & \sum_{i,j} \alpha_i \alpha_j \langle \mathbf{x}_i, \mathbf{x}_j \rangle \\ \text{s.t.} & \forall i = 1 \dots n \quad 0 \leq \alpha_i \leq C \\ & \sum_i \alpha_i = 0 \end{array} \right.$$

Implicit surface modelling

Schoelkopf et al, 04 **Goal:** find the surface formed by the data points

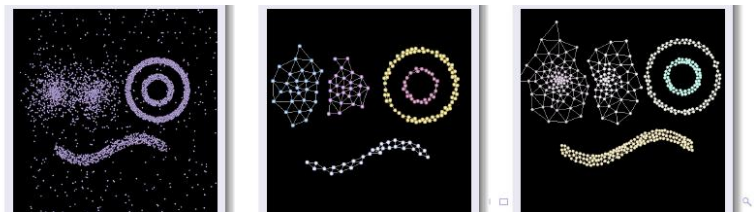
$$\langle \mathbf{w}, \mathbf{x}_i \rangle \geq \rho \text{ becomes } -\varepsilon \leq (\langle \mathbf{w}, \mathbf{x}_i \rangle - \rho) \leq \varepsilon$$



Working assumptions

Clustering assumption

Clusters are separated by a low-density region



Strange clusterings...

[http://en.wikipedia.org/wiki/Celestial_Emporium_of_Benevolent_Knowledge%](http://en.wikipedia.org/wiki/Celestial_Emporium_of_Benevolent_Knowledge%27)

... a certain Chinese encyclopedia called the Heavenly Emporium of Benevolent Knowledge.

In its distant pages it is written that animals are divided into (a) those that belong to the emperor; (b) embalmed ones; (c) those that are trained; (d) suckling pigs; (e) mermaids; (f) fabulous ones; (g) stray dogs; (h) those that are included in this classification; (i) those that tremble as if they were mad; (j) innumerable ones; (k) those drawn with a very fine camel's-hair brush; (l) etcetera; (m) those that have just broken the flower vase; (n) those that at a distance resemble flies.

Borges, 1942

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Non linear changes of representation

Dimensionality Reduction – Intuition

Degrees of freedom

- ▶ Image: 4096 pixels; but not independent
- ▶ Robotics: ($\#$ camera pixels + $\#$ infra-red) $\times$ time; but not independent

Goal

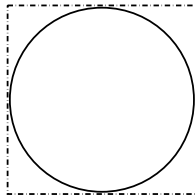
Find the (low-dimensional) structure of the data:

- ▶ Images
- ▶ Robotics
- ▶ Genes

Dimensionality Reduction

In high dimension

- ▶ Everybody lives in the corners of the space
Volume of Sphere $V_n = \frac{2\pi r^2}{n} V_{n-2}$
- ▶ All points are far from each other



Approaches

- ▶ Linear dimensionality reduction
 - ▶ Principal Component Analysis
 - ▶ Random Projection
- ▶ Non-linear dimensionality reduction

Criteria

- ▶ Complexity/Size
- ▶ Prior knowledge

e.g., relevant distance

Linear Dimensionality Reduction

Training set

unsupervised

$$\mathcal{E} = \{(\mathbf{x}_k), \mathbf{x}_k \in \mathbb{R}^D, k = 1 \dots N\}$$

Projection from $\mathbb{R}^D$ onto $\mathbb{R}^d$

$$\mathbf{x} \in \mathbb{R}^D \rightarrow \begin{aligned} h(\mathbf{x}) &\in \mathbb{R}^d, \quad d \ll D \\ h(\mathbf{x}) &= A\mathbf{x} \end{aligned}$$

$$\text{s.t. minimize} \quad \sum_{k=1}^N \|\mathbf{x}_k - h(\mathbf{x}_k)\|^2$$

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Principal Component Analysis

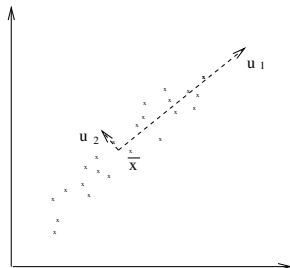
Covariance matrix S

Mean
$$\mu_i = \frac{1}{N} \sum_{k=1}^N att_i(\mathbf{x}_k)$$

$$S_{ij} = \frac{1}{N} \sum_{k=1}^N (att_i(\mathbf{x}_k) - \mu_i)(att_j(\mathbf{x}_k) - \mu_j)$$

symmetric $\Rightarrow$ can be diagonalized

$$S = U\Delta U' \quad \Delta = \text{Diag}(\lambda_1, \dots, \lambda_D)$$



Thm: Optimal projection in dimension d

projection on the first d eigenvectors of S

Let u_i the eigenvector associated to eigenvalue λ_i $\lambda_i > \lambda_{i+1}$

$$h : \mathbb{R}^D \mapsto \mathbb{R}^d, h(\mathbf{x}) = \langle \mathbf{x}, u_1 \rangle u_1 + \dots + \langle \mathbf{x}, u_d \rangle u_d$$

Sketch of the proof

1. Maximize the variance of $h(\mathbf{x}) = A\mathbf{x}$

$$\sum_k \|\mathbf{x}_k - h(\mathbf{x}_k)\|^2 = \sum_k \|\mathbf{x}_k\|^2 - \sum_k \|h(\mathbf{x}_k)\|^2$$

$$\text{Minimize } \sum_k \|\mathbf{x}_k - h(\mathbf{x}_k)\|^2 \Rightarrow \text{Maximize } \sum_k \|h(\mathbf{x}_k)\|^2$$

$$\text{Var}(h(\mathbf{x})) = \frac{1}{N} \left(\sum_k \|h(\mathbf{x}_k)\|^2 - \left\| \sum_k h(\mathbf{x}_k) \right\|^2 \right)$$

As

$$\left\| \sum_k h(\mathbf{x}_k) \right\|^2 = \left\| A \sum_k \mathbf{x}_k \right\|^2 = N^2 \|A\mu\|^2$$

where $\mu = (\mu_1, \dots, \mu_D)$.

Assuming that $\mathbf{x}_k$ are centered ($\mu_i = 0$) gives the result.

Sketch of the proof, 2

2. Projection on eigenvectors u_i of S

Assume $h(\mathbf{x}) = A\mathbf{x} = \sum_{i=1}^d \langle \mathbf{x}, v_i \rangle v_i$ and show $v_i = u_i$.

$$\text{Var}(AX) = (AX)(AX)' = A(XX')A' = ASA' = A(U\Delta U')A'$$

Consider $d = 1$, $v_1 = \sum w_i u_i$

$$\sum w_i^2 = 1$$

remind $\lambda_i > \lambda_{i+1}$

$$\text{Var}(AX) = \sum \lambda_i w_i^2$$

maximized for $w_1 = 1, w_2 = \dots = w_N = 0$

that is, $v_1 = u_1$.

Principal Component Analysis, Practicalities

Data preparation

- Mean centering the dataset

$$\begin{aligned}\mu_i &= \frac{1}{N} \sum_{k=1}^N att_i(\mathbf{x}_k) \\ \sigma_i &= \sqrt{\frac{1}{N} \sum_{k=1}^N att_i(\mathbf{x}_k)^2 - \mu_i^2} \\ z_k &= \left(\frac{1}{\sigma_i} (att_i(\mathbf{x}_k) - \mu_i) \right)_{i=1}^D\end{aligned}$$

Matrix operations

- Computing the covariance matrix

$$S_{ij} = \frac{1}{N} \sum_{k=1}^N att_i(z_k) att_j(z_k)$$

- Diagonalizing $S = U' \Delta U$
might be not affordable...

Complexity $\mathcal{O}(D^3)$

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Random projection

Random matrix

$$A : \mathbb{R}^D \mapsto \mathbb{R}^d \quad A[d, D] \quad A_{i,j} \sim \mathcal{N}(0, 1)$$

define

$$h(\mathbf{x}) = \frac{1}{\sqrt{d}} A \mathbf{x}$$

Property: h preserves the norm in expectation

$$E[\|h(\mathbf{x})\|^2] = \|\mathbf{x}\|^2$$

With high probability

$$1 - 2\exp\{-(\varepsilon^2 - \varepsilon^3)\frac{d}{4}\}$$

$$(1 - \varepsilon)\|\mathbf{x}\|^2 \leq \|h(\mathbf{x})\|^2 \leq (1 + \varepsilon)\|\mathbf{x}\|^2$$

Random projection

Proof

$$h(\mathbf{x}) = \frac{1}{\sqrt{d}} A \mathbf{x}$$

$$\begin{aligned} E(\|h(\mathbf{x})\|^2) &= \frac{1}{d} E \left[\sum_{i=1}^d \left(\sum_{j=1}^D A_{i,j} X_j(\mathbf{x}) \right)^2 \right] \\ &= \frac{1}{d} \sum_{i=1}^d E \left[\left(\sum_{j=1}^D A_{i,j} X_j(\mathbf{x}) \right)^2 \right] \\ &= \frac{1}{d} \sum_{i=1}^d \sum_{j=1}^D E[A_{i,j}^2] E[X_j(\mathbf{x})^2] \\ &= \frac{1}{d} \sum_{i=1}^d \sum_{j=1}^D \frac{\|\mathbf{x}\|^2}{D} \\ &= \|\mathbf{x}\|^2 \end{aligned}$$

Random projection, 2

Johnson Lindenstrauss Lemma

For $d > \frac{9 \ln N}{\varepsilon^2 - \varepsilon^3}$, with high probability

$$(1 - \varepsilon) \|\mathbf{x}_i - \mathbf{x}_j\|^2 \leq \|h(\mathbf{x}_i) - h(\mathbf{x}_j)\|^2 \leq (1 + \varepsilon) \|\mathbf{x}_i - \mathbf{x}_j\|^2$$

More:

<http://www.cs.yale.edu/clique/resources/RandomProjectionMethod.pdf>

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Non linear changes of representation

Example

- c1: Human machine interface for ABC computer applications
 - c2: A survey of user opinion of computer system response time
 - c3: The EPS user interface management system
 - c4: System and human system engineering testing of EPS
 - c5: Relation of user perceived response time to error measurement
-
- m1: The generation of random, binary, ordered trees
 - m2: The intersection graph of paths in trees
 - m3: Graph minors IV: Widths of trees and well-quasi-ordering
 - m4: Graph minors: A survey

Example, followed

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

LSA, 2

Motivations

- ▶ Context: bag of words
- ▶ Curse of dimensionality
- ▶ Synonymy / Polysemy

$\mathbb{R}^D$

Goals

- ▶ Dimensionality reduction
- ▶ Build a decent topology / metric

$\mathbb{R}^d$

Remark

- ▶ vanilla similarity: cosine
- ▶ (why not ?)

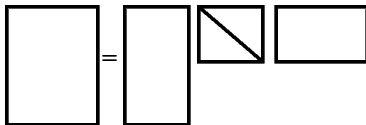
More

<http://lsa.colorado.edu>

LSA, 3

Input

Matrice $X = \text{mots} \times \text{documents}$



Principe

1. Changement de base des mots, documents aux concepts
2. Réduction de dimension

Différence Analyse en composantes principales

LSA $\equiv$ Singular Value Decomposition

Input

X matrice mots $\times$ documents

$m \times d$

$$X = U' S V$$

avec

- U : changement de base mots $m \times r$
- V : changement de base des documents $r \times d$
- S : matrice diagonale $r \times r$

Réduction de dimension

- S Ordonner par valeur propre décroissante
- $S' = S$ avec annulation de toutes les vp, sauf les (300) premières.

$$X' = U' S' V$$

Intuition

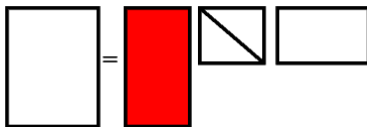
$$X = \begin{pmatrix} & m_1 & m_2 & m_3 & m_4 \\ d_1 & 0 & 1 & 1 & 1 \\ d_2 & 1 & 1 & 1 & 0 \end{pmatrix}$$

m_1 et m_4 ne sont pas “physiquement” ensemble dans les mêmes documents ; mais ils sont avec les mêmes mots ; “donc” ils sont un peu “voisins” ...

Après SVD + Réduction,

$$X = \begin{pmatrix} & m_1 & m_2 & m_3 & m_4 \\ d_1 & \epsilon & 1 & 1 & 1 \\ d_2 & 1 & 1 & 1 & \epsilon \end{pmatrix}$$

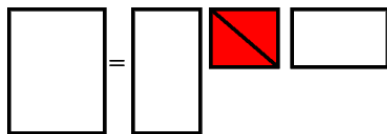
Algorithme



Singular value
Decomposition of the
words by contexts matrix

0.22	-0.11	0.29	-0.41	-0.11	-0.34	0.52	-0.06	-0.41
0.20	-0.07	0.14	-0.55	0.28	0.50	-0.07	-0.01	-0.11
0.24	0.04	-0.16	-0.59	-0.11	-0.25	-0.30	0.06	0.49
0.40	0.06	-0.34	0.10	0.33	0.38	0.00	0.00	0.01
0.64	-0.17	0.36	0.33	-0.16	-0.21	-0.17	0.03	0.27
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.30	-0.14	0.33	0.19	0.11	0.27	0.03	-0.02	-0.17
0.21	0.27	-0.18	-0.03	-0.54	0.08	-0.47	-0.04	-0.58
0.01	0.49	0.23	0.03	0.59	-0.39	-0.29	0.25	-0.23
0.04	0.62	0.22	0.00	-0.07	0.11	0.16	-0.68	0.23
0.03	0.45	0.14	-0.01	-0.30	0.28	0.34	0.68	0.18

Algorithme, 2



Singular value
Decomposition of the
words by contexts matrix

3.34

2.54

2.35

1.64

1.50

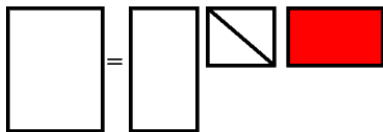
1.31

0.85

0.56

0.36

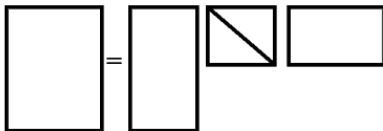
Algorithme. 3



Singular value
Decomposition of the
words by contexts matrix

0.20	0.61	0.46	0.54	0.28	0.00	0.01	0.02	0.08
-0.06	0.17	-0.13	-0.23	0.11	0.19	0.44	0.62	0.53
0.11	-0.50	0.21	0.57	-0.51	0.10	0.19	0.25	0.08
-0.95	-0.03	0.04	0.27	0.15	0.02	0.02	0.01	-0.03
0.05	-0.21	0.38	-0.21	0.33	0.39	0.35	0.15	-0.60
-0.08	-0.26	0.72	-0.37	0.03	-0.30	-0.21	0.00	0.36
0.18	-0.43	-0.24	0.26	0.67	-0.34	-0.15	0.25	0.04
-0.01	0.05	0.01	-0.02	-0.06	0.45	-0.76	0.45	-0.07
-0.06	0.24	0.02	-0.08	-0.26	-0.62	0.02	0.52	-0.45

Algorithme, 4

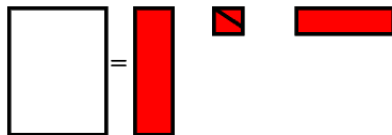


Singular value
Decomposition of the
words by contexts matrix

3.34

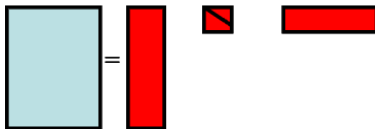
2.54

Algorithme, 5



Singular value
Decomposition of the
words by contexts matrix

Algorithm, 6



Singular value
Decomposition of the
words by contexts matrix

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	0.16	0.40	0.38	0.47	0.18	-0.05	-0.12	-0.16	-0.09
interface	0.14	0.37	0.33	0.40	0.16	-0.03	-0.07	-0.10	-0.04
computer	0.15	0.51	0.36	0.41	0.24	0.02	0.06	0.09	0.12
user	0.26	0.84	0.61	0.70	0.39	0.03	0.08	0.12	0.19
system	0.45	1.23	1.05	1.27	0.56	-0.07	-0.15	-0.21	-0.05
response	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
time	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
EPS	0.22	0.55	0.51	0.63	0.24	-0.07	-0.14	-0.20	-0.11
survey	0.10	0.53	0.23	0.21	0.27	0.14	0.31	0.44	0.42
trees	-0.06	0.23	-0.14	-0.27	0.14	0.24	0.55	0.77	0.66
graph	-0.06	0.34	-0.15	-0.30	0.20	0.31	0.69	0.98	0.85
minors	-0.04	0.25	-0.10	-0.21	0.15	0.22	0.50	0.71	0.62

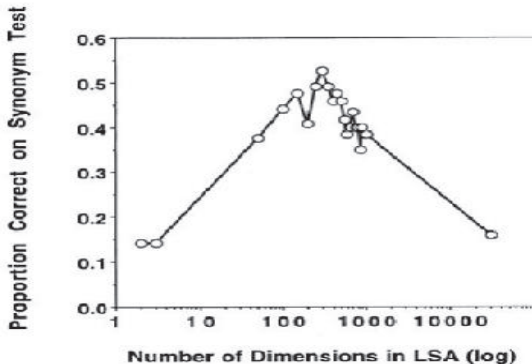
	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
Interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

Discussion

Une application

Test de synonymie

TOEFL



Déterminer le nb de dimensions/vp

Expérimentalement...

Quelques remarques

et la négation ?

battu par: nb de hits sur le Web

aucune importance (!)

P. Turney

Quelques applications

- ▶ Educational Text Selection
Permet de sélectionner automatiquement des textes permettant d'accroître les connaissances de l'utilisateur.
- ▶ Essay Scoring
Permet de noter la qualité d'une rédaction d'étudiant
- ▶ Summary Scoring & Revision
Apprendre à l'utilisateur à faire un résumé
- ▶ Cross Language Retrieval
permet de soumettre un texte dans une langue et d'obtenir un texte équivalent dans une autre langue

LSA – Analyse en composantes principales

Ressemblances

- ▶ Prendre une matrice
- ▶ La mettre sous forme diagonale
- ▶ Annuler toutes les valeurs propres sauf les plus grandes
- ▶ Projeter sur l'espace obtenu

Différences

	ACP	LSA
Matrice	covariance attributs	mots $\times$ documents
d	2-3	100-300

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Linear changes of representation

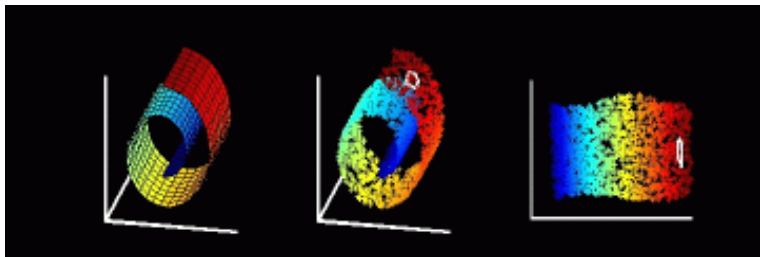
- Principal Component Analysis

- Random projections

- Latent Semantic Analysis

Non linear changes of representation

Non-Linear Dimensionality Reduction



Conjecture

Examples live in a manifold of dimension $d \ll D$

Goal: consistent projection of the dataset onto $\mathbb{R}^d$

Consistency:

- ▶ Preserve the structure of the data
- ▶ e.g. preserve the distances between points

Multi-Dimensional Scaling

Position of the problem

- ▶ Given $\{\mathbf{x}_1, \dots, \mathbf{x}_N, \mathbf{x}_i \in \mathbb{R}^D\}$
- ▶ Given $\text{sim}(\mathbf{x}_i, \mathbf{x}_j) \in \mathbb{R}^+$
- ▶ Find projection Φ onto $\mathbb{R}^d$

$$\begin{aligned}x \in \mathbb{R}^D &\rightarrow \Phi(x) \in \mathbb{R}^d \\ \text{sim}(\mathbf{x}_i, \mathbf{x}_j) &\sim \text{sim}(\Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j))\end{aligned}$$

Optimisation

Define X , $X_{i,j} = \text{sim}(\mathbf{x}_i, \mathbf{x}_j)$; X^Φ , $X_{i,j}^\Phi = \text{sim}(\Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j))$

Find Φ minimizing $\|X - X^\Phi\|$

Rq : Linear Φ = Principal Component Analysis

But linear MDS does not work: preserves all distances, while

only local distances are meaningful

Non-linear projections

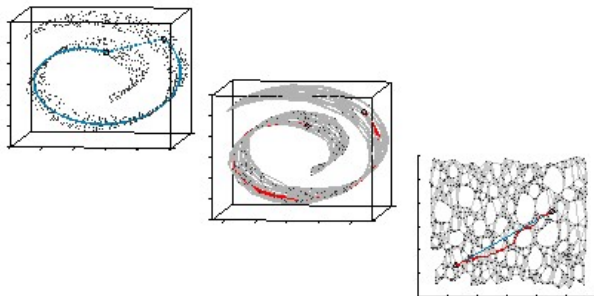
Approaches

- ▶ Reconstruct global structures from local ones and find global projection
- ▶ Only consider local structures

Isomap

LLE

Intuition: locally, points live in $\mathbb{R}^d$



Isomap

Tenenbaum, da Silva, Langford 2000

<http://isomap.stanford.edu>

Estimate $d(\mathbf{x}_i, \mathbf{x}_j)$

- ▶ Known if $\mathbf{x}_i$ and $\mathbf{x}_j$ are close
- ▶ Otherwise, compute the shortest path between $\mathbf{x}_i$ and $\mathbf{x}_j$
geodesic distance (dynamic programming)

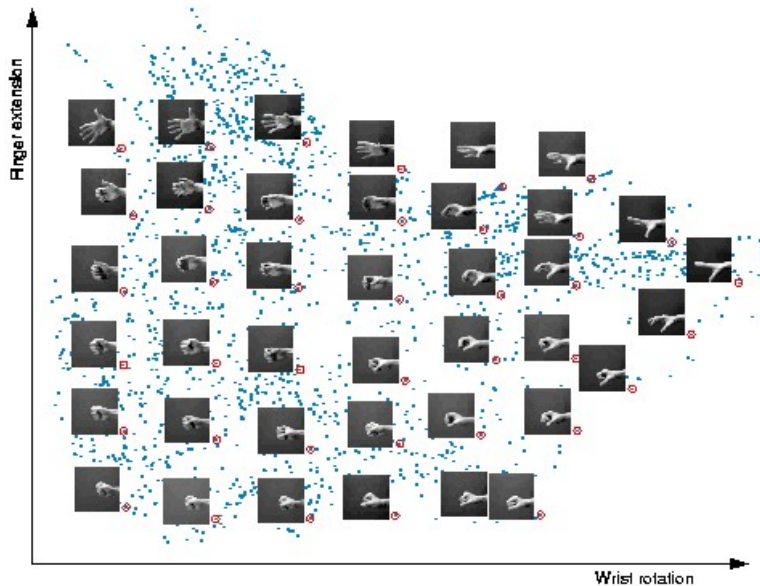
Requisite

If data points sampled in a convex subset of $\mathbb{R}^d$,
then geodesic distance $\sim$ Euclidean distance on $\mathbb{R}^d$.

General case

- ▶ Given $d(\mathbf{x}_i, \mathbf{x}_j)$, estimate $\langle \mathbf{x}_i, \mathbf{x}_j \rangle$
- ▶ Project points in $\mathbb{R}^d$

Isomap, 2



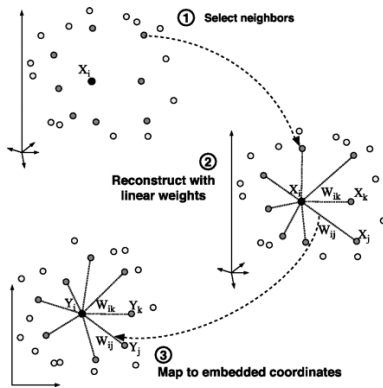
Locally Linear Embedding

Roweiss and Saul, 2000

<http://www.cs.toronto.edu/~roweis/lle/>

Principle

- Find local description for each point: depending on its neighbors



Local Linear Embedding, 2

Find neighbors

For each $\mathbf{x}_i$, find its nearest neighbors $\mathcal{N}(i)$

Parameter: number of neighbors

Change of representation

Goal Characterize $\mathbf{x}_i$ wrt its neighbors:

$$\mathbf{x}_i = \sum_{j \in \mathcal{N}(i)} w_{ij} \mathbf{x}_j \quad \text{with} \quad \sum_{j \in \mathcal{N}(i)} w_{ij} = 1$$

Property: invariance by translation, rotation, homothety

How Compute the local covariance matrix:

$$C_{j,k} = \langle \mathbf{x}_j - \mathbf{x}_i, \mathbf{x}_k - \mathbf{x}_i \rangle$$

Find vector w_i s.t. $Cw_i = 1$

Local Linear Embedding, 3

Algorithm

Local description: Matrix W such that

$$\sum_j w_{i,j} = 1$$

$$W = \operatorname{argmin} \left\{ \sum_{i=1}^N \left\| \mathbf{x}_i - \sum_j w_{i,j} \mathbf{x}_j \right\|^2 \right\}$$

Projection: Find $\{z_1, \dots, z_n\}$ in $\mathbb{R}^d$ minimizing

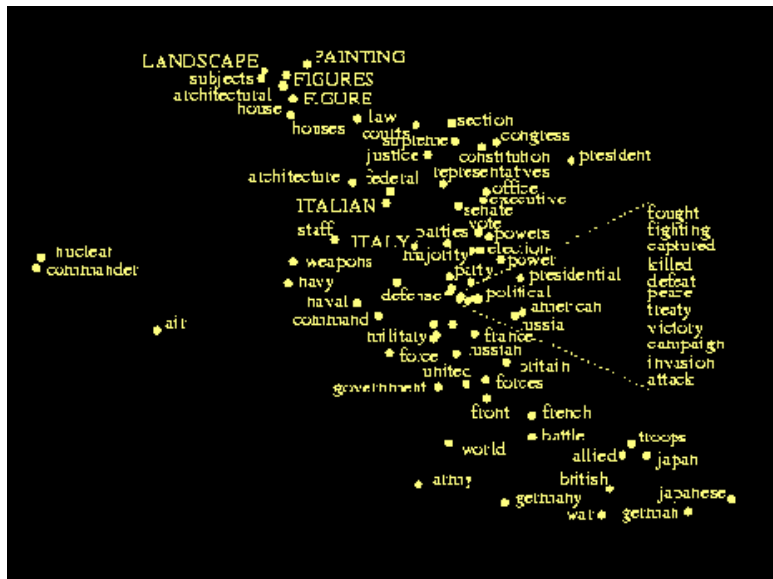
$$\sum_{i=1}^N \left\| z_i - \sum_j w_{i,j} z_j \right\|^2$$

$$\text{Minimize } ((I - W)Z)'((I - W)Z) = Z'(I - W)'(I - W)Z$$

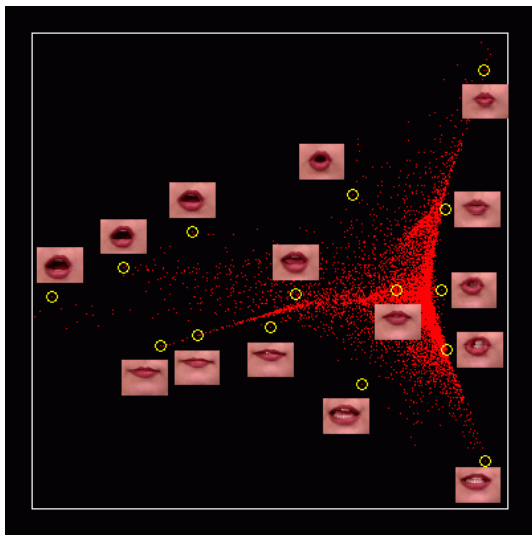
Solutions: vectors z_i are eigenvectors of $(I - W)'(I - W)$

- Keeping the d eigenvectors with lowest eigenvalues > 0

Example, Texts



Example, Images



LLE

Overview

Introduction

Linear changes of representation

- Principal Component Analysis

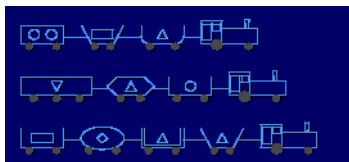
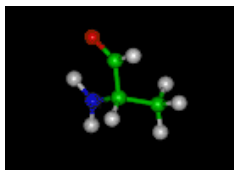
- Random projections

- Latent Semantic Analysis

Non linear changes of representation

Propositionalization

Relational domains



Relational learning

PROS

Use domain knowledge

CONS

Covering test $\equiv$ subgraph matching

Inductive Logic Programming

Data Mining
exponential complexity

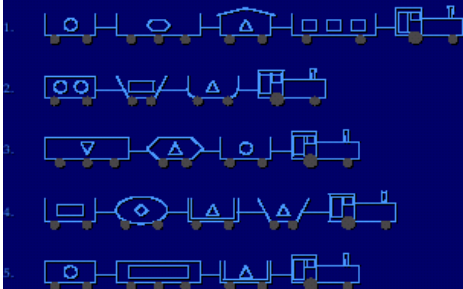
Getting back to propositional representation:

propositionalization

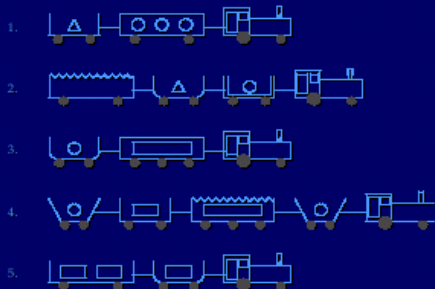
West - East trains

Michalski 1983

1. TRAINS GOING EAST



2. TRAINS GOING WEST



Propositionalization

Linus (ancestor)

Lavrac et al, 94

$West(a) \leftarrow Engine(a, b), first_wagon(a, c), roof(c), load(c, square, 3)...$
 $West(a') \leftarrow Engine(a', b'), first_wagon(a', c'), load(c', circle, 1)...$

West	Engine(X)	First Wagon(X,Y)	Roof(Y)	Load ₁ (Y)	Load ₂ (Y)
a	b	c	yes	square	3
a'	b'	c'	no	circle	1

Each column: a role predicate, where the predicate is determinate linked to former predicates (left columns) with a single instantiation in every example

Propositionalization

Stochastic propositionalization

Kramer, 98 Construct random formulas $\equiv$ boolean features

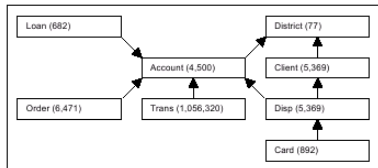
SINUS – RDS

<http://www.cs.bris.ac.uk/home/rawles/sinus>

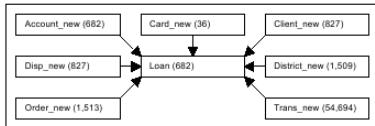
<http://labe.felk.cvut.cz/~zelezny/rsd>

- ▶ Use modes (user-declared) `modeb(2,hasCar(+train,-car))`
- ▶ Thresholds on number of variables, depth of predicates...
- ▶ Pre-processing (feature selection)

Propositionalization



DB Schema



Propositionalization

RELAGGS

Database aggregates

- ▶ average, min, max, of numerical attributes
- ▶ number of values of categorical attributes

Apprentissage par Renforcement Relationnel

Real Time Strategy Games



- Many objects of various types in complex interactions
- Good players can generalize across situations involving distinct object configurations

The Logistics Domain



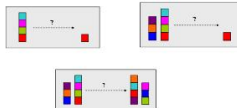
- Move many objects around with many other objects
- Identities and numbers of objects always changing

Robot Soccer



- Reasoning about relationship between objects (players and ball) key to good play

and of course Blocksworld



- Would like a policy that is independent of number of objects/blocks

Propositionalisation

Contexte variable

- ▶ Nombre de robots, position des robots
- ▶ Nombre de camions, lieu des secours

Besoin: Abstraire et Generaliser

Attributs

- ▶ Nombre d'amis/d'ennemis
- ▶ Distance du plus proche robot ami
- ▶ Distance du plus proche ennemi