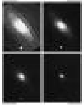
Master Recherche IAC
TC2: Apprentissage Statistique & Optimisation

Alexandre Allauzen – Anne Auger – Michèle Sebag
LIMSI – LRI

Sept. 22nd, 2014

Where we are

Ast. series 

Pierre de Rosette 

World

Natural phenomenons

Human–related phenomenons

Data / Principles

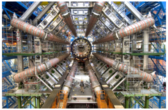
Maths. Modelling

Common Sense

You are here

Labex DigiCosme

Where we are

Sc. data 

World

Natural phenomenons

Human–related phenomenons

Data / Principles

Maths. Modelling

Common Sense

You are here

Harnessing Big Data

Watson (IBM) defeats human champions at the quiz game Jeopardy (Feb. 11)

i	1	2	3	4	5	6	7	8	
1000^i	kilo	mega	giga	tera	peta	exa	zetta	yotta	bytes

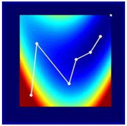
- Google: 24 petabytes/day
- Facebook: 10 terabytes/day; Twitter: 7 terabytes/day
- Large Hadron Collider: 40 terabytes/seconds

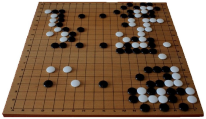
Machine Learning and Optimization

Machine Learning

World $\rightarrow$ instance $\mathbf{x}_i$ $\rightarrow$ Oracle $\downarrow$ y_i

Optimization





ML and Optimization

- ML is an optimization problem: find the best model
- Smart optimization requires learning about the optimization landscape

Types of Machine Learning problems

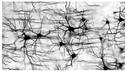
WORLD – DATA – USER

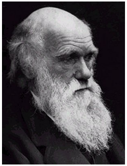
Observations	**+ Target**	+ Rewards
Understand **Code**	**Predict** **Classification/Regression**	Decide Action Policy/Strategy
Unsupervised **LEARNING**	**Supervised** **LEARNING**	Reinforcement LEARNING

The module

1. Introduction. Decision trees. Validation.
2. Support Vector Machines
3. Learning from sequences
4. Unsupervised learning
5. Representation changes
6. Bayesian learning
7. Optimisation

Rev. T. Bayes





Pointers

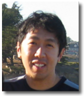
- Slides of this module:
  http://tao.lri.fr/tiki-index.php?page=Courses
  http://www.limsi.fr/Individu/allauzen/wiki/index.php/
- Andrew Ng courses
  http://ai.stanford.edu/~ang/courses.html
- PASCAL videos
  http://videolectures.net/pascal/
- Tutorials NIPS Neuro Information Processing Systems
  http://nips.cc/Conferences/2006/Media/
- About ML/DM
  http://hunch.net/

Today

1. Part 1. Generalities
2. Part 2. Decision trees
3. Part 3. Validation

Examples

- Vision
- Control
- Netflix
- Spam
- Playing Go
- Google



http://ai.stanford.edu/~ang/courses.html

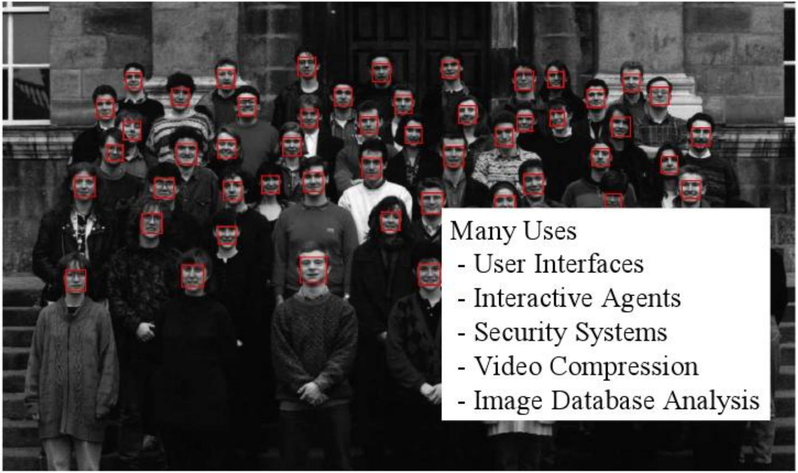
Reading cheques

LeCun et al. 1990

MNIST: The drosophila of MI

Fig. 4. Size-normalized examples from the MNIST database.

Detecting faces

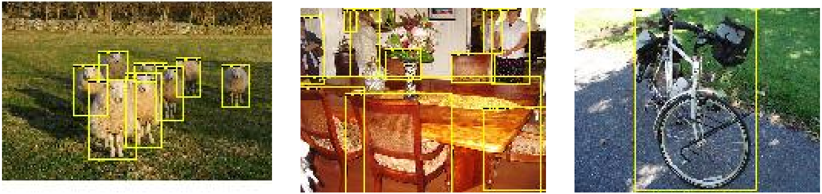


2

Viola and Jones, Robust object detection using a boosted cascade of simple features, CVPR 2001

The 2005-2012 Visual Object Chall

A. Zisserman, C. Williams, M. Everingham, L. v.d. Gool



The supervised learning setting

Input: set of $(\mathbf{x}, y)$

- An instance $\mathbf{x}$ e.g. set of pixels, $\mathbf{x} \in \mathbb{R}^D$
- A label y in $\{1, -1\}$ or $\{1, \ldots, K\}$ or $\mathbb{R}$

The supervised learning setting

Input: set of $(\mathbf{x}, y)$

- An instance $\mathbf{x}$ e.g. set of pixels, $\mathbf{x} \in \mathbb{R}^D$
- A label y in $\{1, -1\}$ or $\{1, \ldots, K\}$ or $\mathbb{R}$

Pattern recognition

- Classification **Does the image contain the target concept ?**

$$h : \{\text{ Images}\} \mapsto \{1, -1\}$$

- Detection **Does the pixel belong to the img of target concept?**

$$h : \{\text{ Pixels in an image}\} \mapsto \{1, -1\}$$

- Segmentation

  Find contours of all instances of target concept in image

The 2005 Darpa Challenge

Thrun, Burgard and Fox 2005

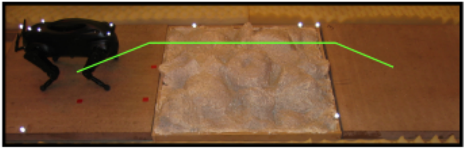
Autonomous vehicle Stanley – Terrains

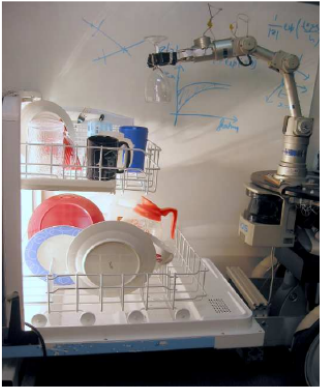
The Darpa challenge and the AI ag

What remains to be done	Thrun 2005
▸ Reasoning	10%
▸ Dialogue	60%
▸ Perception	90%

Robots

Ng, Russell, Veloso, Abbeel, Peters, Schaal, ...

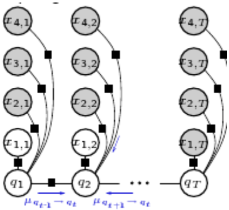




Reinforcement learning **Classification**

Robots, 2

Toussaint et al. 2010



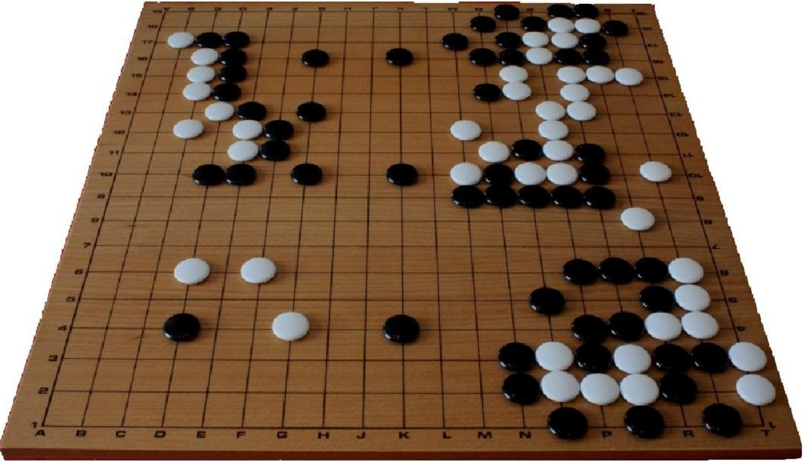
(a) Factor graph modelling the variable interactions

(b) Behaviour of the 39-DOF Humanoid:
Reaching goal under Balance and Collision constraints

Bayesian Inference for Motion Control and Planning

Go as AI Challenge

Gelly Wang 07; Teytaud et al. 2008-2011



Reinforcement Learning, Monte-Carlo Tree Search

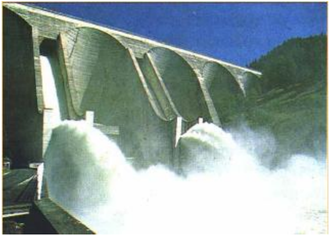
Energy policy

Claim

Many problems can be phrased as optimization in front of the uncertainty.

Adversarial setting	2 two-player game
uniform setting	a single player game

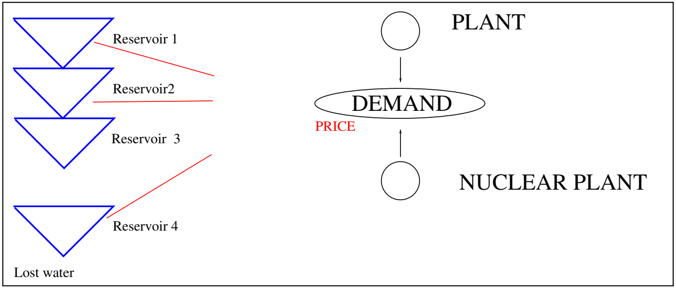
Management of energy stocks under uncertainty



States and Decisions

States

- Amount of stock (60 nuclear, 20 hydro.)
- Varying: price, weather   alea or archive
- Decision: release water from one reservoir to another
- Assessment: meet the demand, otherwise buy energy



Netflix Challenge 2007-2008

Collaborative Filtering

Collaborative filtering

Input

- A set of users n_u, ca 500,000
- A set of movies n_m, ca 18,000
- A $n_m \times n_u$ matrix: person, movie, rating
  Very sparse matrix: less than 1% filled...

Output

- Filling the matrix !

Collaborative filtering

Input

- A set of users n_u, ca 500,000
- A set of movies n_m, ca 18,000
- A $n_m \times n_u$ matrix: person, movie, rating
  Very sparse matrix: less than 1% filled...

Output

- Filling the matrix !

Criterion

- (relative) mean square error
- ranking error

Spam – Phishing – Scam

Classification, Outlier detection

The power of big data

- Now-casting outbreak of flu
- Public relations >> Advertizing

Mc Luhan and Google

We shape our tools and afterwards our tools shape us

Marshall McLuhan, 1964

First time ever a tool is observed to modify human cognition that fast.

Sparrow et al., Science 2011

Types of application

Domain	But : Modelling
Physical phenomenons	**analysis & control**
manufacturing, experimental sciences, numerical engineering	
Vision, speech, robotics..	
Social phenomenons	**+ privacy**
Health, Insurance, Banks ...	
Individual phenomenons	**+ dynamics**
Consumer Relationship Management, User Modelling	
Social networks, games...	

PASCAL : http://pascallin2.ecs.soton.ac.uk/

Banks, Telecom, CRN

Ex: KDD 2009 – Orange

1. Churn
2. Appetency
3. Up-selling

Objectives

1. Ads. efficiency
2. Less fraud

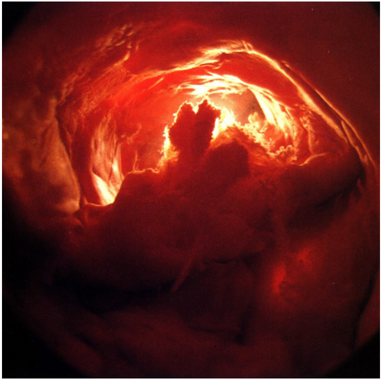
Health, bio-informatics

Ex: Risk factors

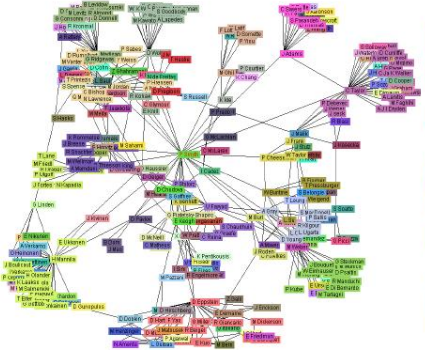
1. Cardio-vascular diseases
2. Carcinogenic Molecules
3. Obesity genes ...

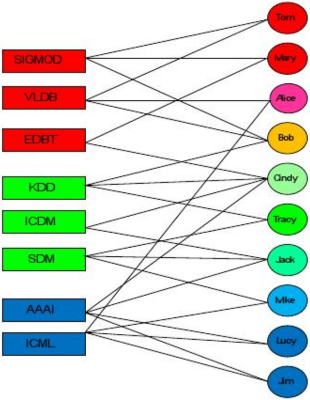
Objectives

1. Diagnostic
2. Personalized care
3. Identification



Scientific Social Network





Questions

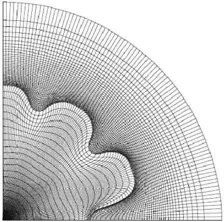
1. Who does what ?
2. Good conferences ?
3. Hot/emerging topics ?
4. Is Mr Q. Lee same as Mr Quoc N. Lee ?

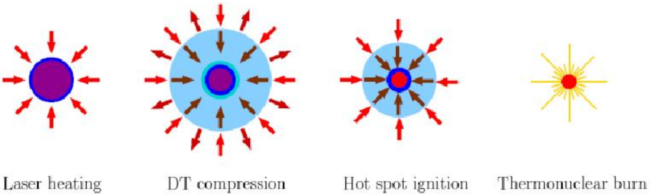
[tr Jiawei Han 2010]

e-Science, Design

Numerical Engineering

- Codes
- Computationally heavy
- Expertise demanding





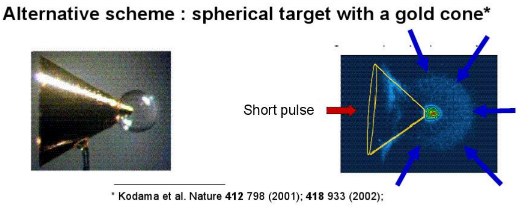
Fusion based on inertial confinement, ICF

e-Science, Design (2)

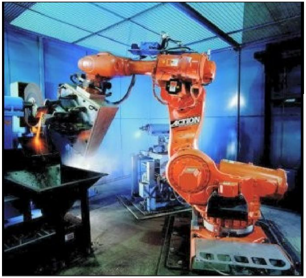
Objectives

- Approximate answer
- .. in tenth of seconds
- Speed up the design cycle
- Optimal design

More is Different

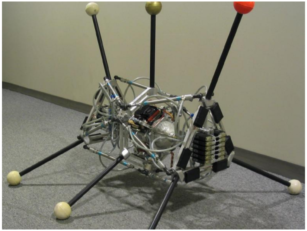


Autonomous robotics



Complexe, monde fermé — simple, random

Design



[tr. Hod Lipson, 2010]

Autonomous robotics, 2

Reality Gap

- Design in silico (simulator)
- Run the controller on the robot (in vivo)

Autonomous robotics, 2

Reality Gap

- Design in silico (simulator)
- Run the controller on the robot (in vivo)
- Does not work !

Closing the reality Gap

1. Simulator-based design
2. On-board trials safe environnement
3. Log the data, update the simulator
4. Goto 1

Active learning Co-evolution

[tr. Hod Lipson, 2010]

Overview

Types of Machine Learning probl

WORLD – DATA – USER

Observations	**+ Target**	+ Rewards
Understand **Code**	**Predict** **Classification/Regression**	Decide Policy
Unsupervised **LEARNING**	**Supervised** **LEARNING**	Reinforcement LEARNING

Data

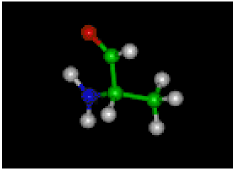
Example

- row : example/ case
- column : feature/ variable/ attribute
- attribute : class/ label

age	employme	education	edun	marital	...	job	relation	race	gender	hour	country	wealth
					...							
39	State_gov	Bachelors	13	Never_mar	...	Adm_cleri	Not_in_fam	White	Male	40	United_Sta	poor
51	Self_emp_	Bachelors	13	Married	...	Exec_man	Husband	White	Male	13	United_Sta	poor
39	Private	HS_grad	9	Divorced	...	Handlers_	Not_in_fam	White	Male	40	United_Sta	poor
54	Private	11th	7	Married	...	Handlers_	Husband	Black	Male	40	United_Sta	poor
28	Private	Bachelors	13	Married	...	Prof_speci	Wife	Black	Female	40	Cuba	poor
38	Private	Masters	14	Married	...	Exec_man	Wife	White	Female	40	United_Sta	poor
50	Private	9th	5	Married_sp	...	Other_serv	Not_in_fam	Black	Female	16	Jamaica	poor
52	Self_emp_	HS_grad	9	Married	...	Exec_man	Husband	White	Male	45	United_Sta	rich
31	Private	Masters	14	Never_mar	...	Prof_speci	Not_in_fam	White	Female	50	United_Sta	rich
42	Private	Bachelors	13	Married	...	Exec_man	Husband	White	Male	40	United_Sta	rich
37	Private	Some_coll	10	Married	...	Exec_man	Husband	Black	Male	80	United_Sta	rich
30	State_gov	Bachelors	13	Married	...	Prof_speci	Husband	Asian	Male	40	India	rich
24	Private	Bachelors	13	Never_mar	...	Adm_cleri	Own_child	White	Female	30	United_Sta	poor
33	Private	Assoc_acc	12	Never_mar	...	Sales	Not_in_fam	Black	Male	50	United_Sta	poor
41	Private	Assoc_voc	11	Married	...	Craft_repai	Husband	Asian	Male	40	*MissingV	rich
34	Private	7th_8th	4	Married	...	Transport_	Husband	Amer_India	Male	45	Mexico	poor
26	Self_emp_	HS_grad	9	Never_mar	...	Farming_fi	Own_child	White	Male	35	United_Sta	poor
33	Private	HS_grad	9	Never_mar	...	Machine_c	Unmarried	White	Male	40	United_Sta	poor
38	Private	11th	7	Married	...	Sales	Husband	White	Male	50	United_Sta	poor
44	Self_emp_	Masters	14	Divorced	...	Exec_man	Unmarried	White	Female	45	United_Sta	rich
41	Private	Doctorate	16	Married	...	Prof_speci	Husband	White	Male	60	United_Sta	rich
:	:	:	:	:	:	:	:	:	:	:	:	:

Instance space $\mathcal{X}$

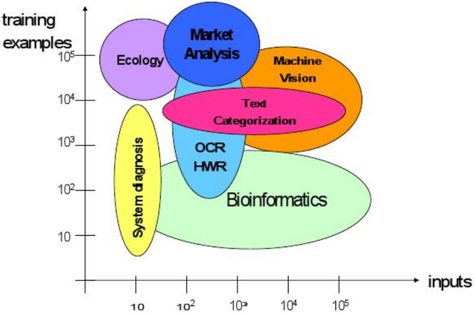
- Propositionnal : $\mathcal{X} \equiv \mathbb{R}^d$
- Structured : sequential, spatio-temporal, relational.



aminoacid

Data / Applications

▶ Propositionnal data	80% des applis.
▶ Spatio-temporal data	alarms, mines, accidents
▶ Relationnal data	chemistry, biology
▶ Semi-structured data	text, Web
▶ Multi-media	images, music, movies,..



Difficulty factors

Quality of data / of representation

– Noise; missing data
+ Relevant attributes **Feature extraction**
– Structured data: spatio-temporal, relational, text, videos,..

Data distribution

+ Independants, identically distributed examples
– Other: robotics; data streams; heterogeneous data

Prior knowledge

+ Goals, interestingness criteria
+ Constraints on target hypotheses

Difficulty factors, 2

Learning criterion

- $+$ Convex optimization problem
- $\searrow$ Complexity : n, $n\log n$, n^2 **Scalability**
- $-$ Combinatorial optimization

H. Simon, 1958:
In complex real-world situations, optimization becomes approximate optimization since the description of the real-world is radically simplified until reduced to a degree of complication that the decision maker can handle.
Satisficing seeks simplification in a somewhat different direction, retaining more of the detail of the real-world situation, but settling for a satisfactory, rather than approximate-best, decision.

Learning criteria, 2

The user's criteria

- Relevance, causality,
- INTELLIGIBILITY
- Simplicity
- Stability
- Interactive processing, visualisation
- ... Preference learning

Difficulty factors, 3

Crossing the chasm

- No *killer algorithm*
- Little expertise about algorithm selection

How to assess an algorithm

- Consistency

When number n of examples goes to infinity
and target concept h^* is in $\mathcal{H}$
h^* is found:

$$lim_{n \rightarrow \infty} h_n = h^*$$

- Speed of convergence

$$||h^* - h_n|| = \mathcal{O}(1/n), \mathcal{O}(1/\sqrt{n}), \mathcal{O}(1/\ln n)$$

Context

Disciplines et critères

- Data bases, Data Mining

  Scalability

- Statistics, data analysis

  Predefined models

- Machine learning

  Prior knowledge; complex data/hypotheses

- Optimisation

  well / ill posed problems

- Computer Human Interaction

  No final solution: a process

- High performance computing

  Distributed processing; safety

Supervised Learning, notations

Context

$$\text{World} \rightarrow \text{Instance } \mathbf{x}_i \rightarrow \begin{array}{c}\text{Oracle}\\ \downarrow \\ y_i\end{array}$$

INPUT $\sim P(\mathbf{x}, y)$

$$\mathcal{E} = \{(\mathbf{x}_i, y_i), x_i \in \mathcal{X}, y_i \in \mathcal{Y}, i = 1 \ldots n\}$$

HYPOTHESIS SPACE

$$\mathcal{H} \quad h : \mathcal{X} \mapsto \mathcal{Y}$$

LOSS FUNCTION

$$\ell : \mathcal{Y} \times \mathcal{Y} \mapsto \mathbb{R}$$

OUTPUT

$$h^* = arg\ max\{score(h), h \in \mathcal{H}\}$$

Classification and criteria

Supervised learning

- $\mathcal{Y}$ = True/False — classification
- $\mathcal{Y} = \{1, \ldots k\}$ — multi-class discrimination
- $\mathcal{Y} = \mathbb{R}$ — regression

Generalization Error

$$Err(h) = E[\ell(y, h(\mathbf{x}))] = \int \ell(y, h(\mathbf{x}))dP(x, y)$$

Empirical Error

$$Err_e(h) = \frac{1}{n}\sum_{i=1}^{n} \ell(y_i, h(\mathbf{x}_i))$$

Bound

structural risk

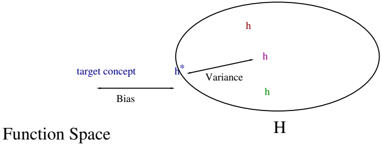
$$Err(h) < Err_e(h) + \mathcal{F}(n, d(\mathcal{H}))$$

$d(\mathcal{H})$ = Vapnik Cervonenkis dimension of $\mathcal{H}$, see later

The Bias-Variance Trade-off

Biais Bias $(\mathcal{H})$: error of the best hypothesis h^* de $\mathcal{H}$

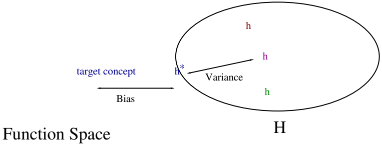
Variance Variance of h_n as a function of $\mathcal{E}$



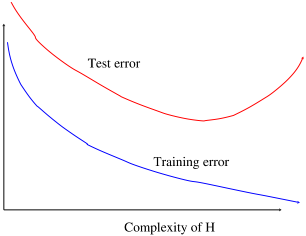
The Bias-Variance Trade-off

Biais Bias ($\mathcal{H}$): error of the best hypothesis h^* de $\mathcal{H}$

Variance Variance of h_n as a function of $\mathcal{E}$



Overfitting



Key notions

- The main issue regarding supervised learning is overfitting.

- How to tackle overfitting:
  - Before learning: use a sound criterion regularization
  - After learning: cross-validation **Case studies**

Summary

- Learning is a search problem
- What is the space ? What are the navigation operators ?

Hypothesis Spaces

Logical Spaces

$$\text{Concept} \leftarrow \bigvee \bigwedge \text{Literal,Condition}$$

- Conditions = [color = blue]; [age < 18]
- Condition $f : X \mapsto \{True, False\}$
- Find: disjunction of conjunctions of conditions

- Ex: (unions of) rectangles of the 2D-plane$X$.

Hypothesis Spaces

Numerical Spaces

$$\text{Concept} \;= (h() > 0)$$

- $h(x)$ = polynomial, neural network, ...
- $h : X \mapsto \mathbb{R}$
- Find: (structure and) parameters of h

Hypothesis Space $\mathcal{H}$

Logical Space

- h covers one example x iff $h(x) = True$.
- $\mathcal{H}$ is structured by a partial order relation

$$h \prec h' \text{ iff } \forall x, h(x) \rightarrow h'(x)$$

Numerical Space $\mathcal{H}$

- $h(x)$ is a real value (more or less far from 0)
- we can define $\ell(h(x), y)$
- $\mathcal{H}$ is structured by a partial order relation

$$h \prec h' \text{ iff } E[\ell(h(x), y)] < E[\ell(h'(x), y)]$$

Hypothesis Space $\mathcal{H}$ / Navigati

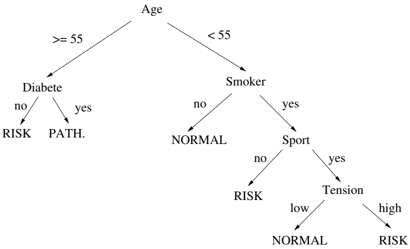
	$\mathcal{H}$	navigation operators
Version Space	Logical	spec / gen
Decision Trees	Logical	specialisation
Neural Networks	Numerical	gradient
Support Vector Machines	Numerical	quadratic opt.
Ensemble Methods	–	adaptation $\mathcal{E}$

Overview

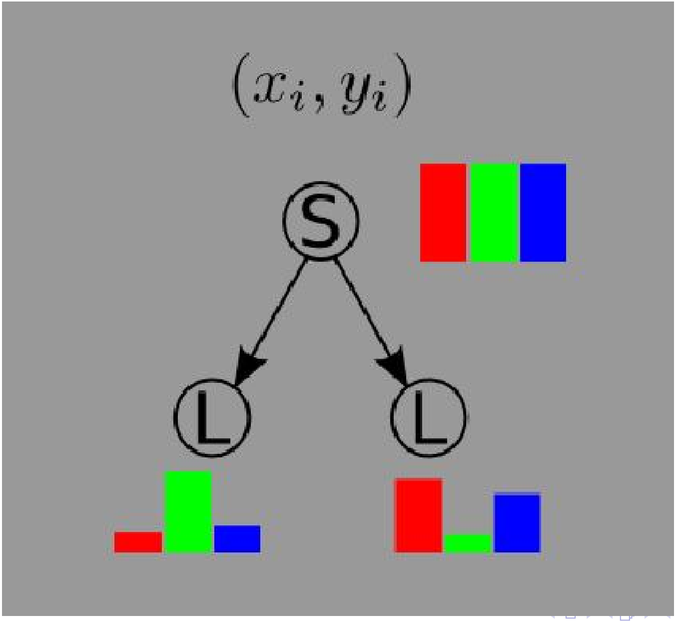
Decision Trees

C4.5 (Quinlan 86)

- Among the most widely used algorithms
- Easy
  - to understand
  - to implelement
  - to use
  - and cheap in CPU time
- J48, Weka, SciKit



Decision Trees



Decision Trees (2)

Procedure DecisionTree($\mathcal{E}$)

1. Assume $\mathcal{E} = \{(x_i, y_i)_{i=1}^n, \; x_i \in \mathbb{R}^D, \; y_i \in \{0,1\}\}$
   - If $\mathcal{E}$ single-class (i.e., $\forall i, j \in [1, n]; y_i = y_j$), return
   - If n too small (i.e., $<$ threshold), return
   - Else, find the most informative attribute *att*
2. Forall value *val* of *att*
   - Set $\mathcal{E}_{val} = \mathcal{E} \cap [att = val]$.
   - Call DecisionTree($\mathcal{E}_{val}$)

Criterion: information gain

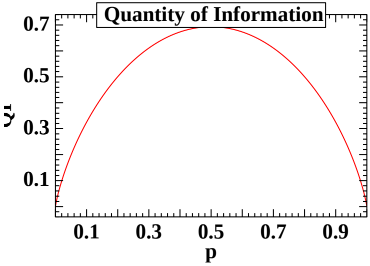
$$\begin{aligned}
p &= Pr(Class = 1 | att = val) \\
I([att = val]) &= -p \log p - (1-p) \log (1-p) \\
I(att) &= \sum_i Pr(att = val_i).I([att = val_i])
\end{aligned}$$

Decision Trees (3)

Contingency Table

wealth values:		poor	rich
agegroup	10s	2507	3
	20s	11262	743
	30s	9468	3461
	40s	6738	3986
	50s	4110	2509
	60s	2245	809
	70s	668	147
	80s	115	16
	90s	42	13

Quantity of Information (QI)



Computation

value	p(value)	p(poor | value)	QI (value)	p(value) * QI (value)
[0,10[	0.051	0.999	0.00924	0.000474
[10,20[	0.25	0.938	0.232	0.0570323
[20,30[	0.26	0.732	0.581	0.153715

Decision Trees (4)

Limitations

- XOR-like attributes
- Attributes with many values
- Numerical attributes
- Overfitting

Limitations

Numerical Attributes

- Order the values $val_1 < \ldots < val_t$
- Compute $\text{QI}([att < val_i])$
- $\text{QI}(\text{att}) = \max_i \text{QI}([att < val_i])$

The XOR case

Bias the distribution of the examples

Complexity

Quantity of information of an attribute

$$n \ln n$$

Adding a node

$$D \times n \ln n$$

Tackling Overfitting

Penalize the selection of an already used variable

- Limits the tree depth.

Do not split subsets below a given minimal size

- Limits the tree depth.

Pruning

- Each leaf, one conjunction;
- Generalization by pruning litterals;
- Greedy optimization, QI criterion.

Decision Trees, Summary

Still around after all these years

- Robust against noise and irrelevant attributes
- Good results, both in quality and complexity

Random Forests Breiman 00

Overview

Validation issues

1. What is the result ?
2. My results look good. Are they ?
3. Does my system outperform yours ?
4. How to set up my system ?

Validation: Three questions

Define a good indicator of quality

- Misclassification cost
- Area under the ROC curve

Computing an estimate thereof

- Validation set
- Cross-Validation
- Leave one out
- Bootstrap

Compare estimates: Tests and confidence levels

Which indicator, which estimate: de

Settings

- Large/few data

Data distribution

- Dependent/independent examples
- balanced/imbalanced classes

Overview

Performance indicators

Binary class

- h^* the truth
- $\hat{h}$ the learned hypothesis

Confusion matrix

$\hat{h}$ / h^*	1	0	
1	a	b	a+b
0	c	d	c+d
	a+c	b+d	a + b + c + d

Performance indicators, 2

$\hat{h}$ / h^*	1	0	
1	a	b	a+b
0	c	d	c+d
	a+c	b+d	a + b + c + d

- Misclassification rate $\frac{b+c}{a+b+c+d}$
- Sensitivity (recall), True positive rate (TP) $\frac{a}{a+c}$
- Specificity, False negative rate (FN) $\frac{b}{b+d}$
- Precision $\frac{a}{a+b}$

Note: always compare to random guessing / baseline alg.

Performance indicators, 3

The Area under the ROC curve

- ROC: Receiver Operating Characteristics
- Origin: Signal Processing, Medicine

Principle

$$h : X \mapsto \mathbb{R} \qquad h(x) \text{ measures the risk of patient } x$$

h leads to order the examples:

$$+ + + - + - + + + + - - - + - - - + - - - - - - - - - - - -$$

Performance indicators, 3

The Area under the ROC curve

- ROC: Receiver Operating Characteristics
- Origin: Signal Processing, Medicine

Principle

$$h : X \mapsto \mathbb{R} \qquad h(x) \text{ measures the risk of patient } x$$

h leads to order the examples:

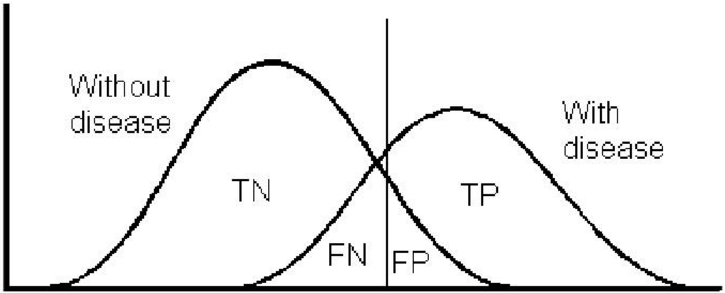
$$+ + + - + - + + + + - - - + - - - + - - - - - - - - - - - - -$$

Given a threshold θ, h yields a classifier: Yes iff $h(x) > \theta$.

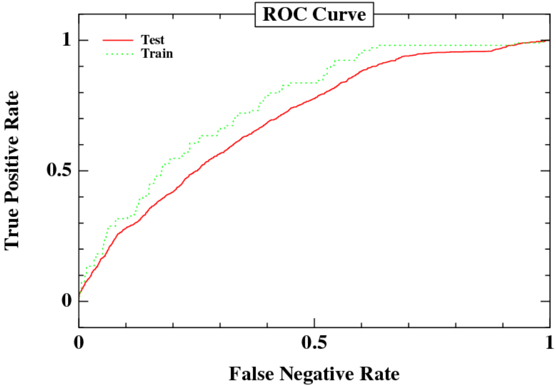
$$+ + + - + - + + + + + \quad | - - - + - - - + - - - - - - - - - - - - -$$

Here, TP (θ)= .8; FN (θ) = .1

ROC



The ROC curve



Ideal classifier: (0 False negative,1 True positive)
Diagonal (True Positive = False negative) $\equiv$ nothing learned.

ROC Curve, Properties

Properties

ROC depicts the trade-off True Positive / False Negative.

Standard: misclassification cost (Domingos, KDD 99)

$$\text{Error} = \#\text{ false positive} + c \times \ \#\text{ false negative}$$

In a multi-objective perspective, ROC $=$ Pareto front.

Best solution: intersection of Pareto front with $\Delta(-c, -1)$

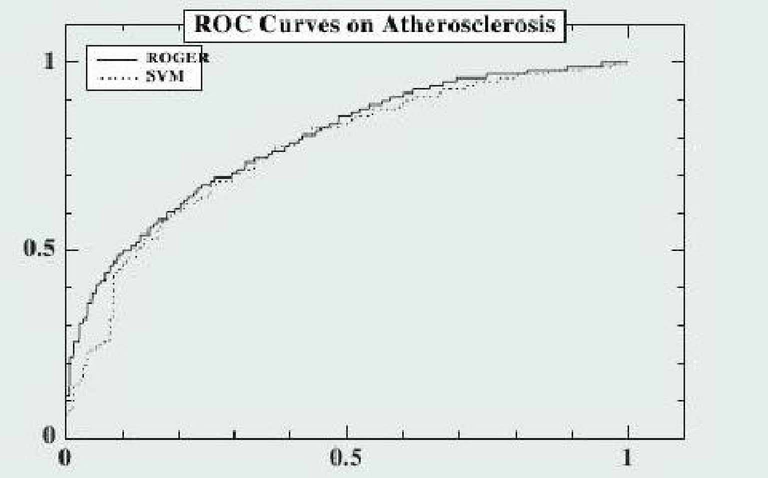
ROC Curve, Properties, foll'd

Used to compare learners Bradley 97

multi-objective-like

insensitive to imbalanced distributions

shows sensitivity to error cost.



Area Under the ROC Curve

Often used to select a learner

Don't ever do this ! Hand, 09

Sometimes used as learning criterion Mann Whitney Wilcoxon

$$AUC = Pr(h(x) > h(x')|y > y')$$

WHY Rosset, 04

- More stable $\mathcal{O}(n^2)$ vs $\mathcal{O}(n)$
- With a probabilistic interpretation Clemençon et al. 08

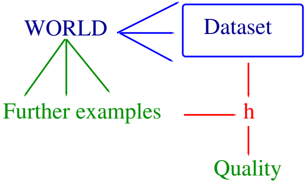
HOW

- SVM-Ranking Joachims 05; Usunier et al. 08, 09
- Stochastic optimization

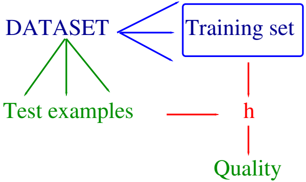
Overview

Validation, principle

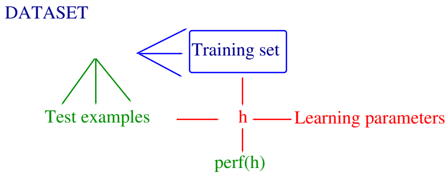
Desired: performance on further instances



Assumption: Dataset is to World, like Training set is to Dataset.

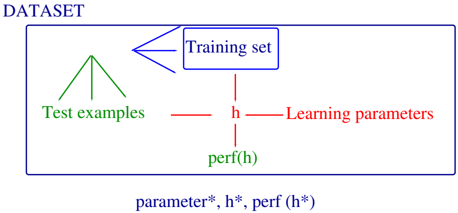


Validation, 2



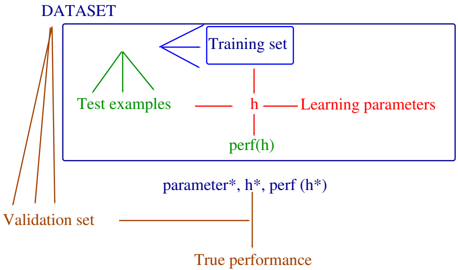
Unbiased Assessment of Learning Algorithms
T. Scheffer and R. Herbrich, 97

Validation, 2



Unbiased Assessment of Learning Algorithms
T. Scheffer and R. Herbrich, 97

Validation, 2



Unbiased Assessment of Learning Algorithms
T. Scheffer and R. Herbrich, 97

Overview

Confidence intervals

Definition

Given a random variable X on $\mathbb{R}$, a p%-confidence interval is $I \subset \mathbb{R}$ such that

$$Pr(X \in I) > p$$

Binary variable with probability ϵ

Probability of r events out of n trials:

$$P_n(r) = \frac{n!}{r!(n-r)!}\epsilon^r(1-\epsilon)^{n-r}$$

- Mean: $n\epsilon$
- Variance: $\sigma^2 = n\epsilon(1-\epsilon)$

Gaussian approximation

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}}exp^{-\frac{1}{2}\frac{x-\mu}{\sigma}^2}$$

Confidence intervals

Bounds on (true value, empirical value) for n trials, $n > 30$

$$Pr(|\hat{x}_n - x^*| > \underset{z}{1.96} \sqrt{\frac{\hat{x}_n.(1-\hat{x}_n)}{n}}) < \underset{\varepsilon}{.05}$$

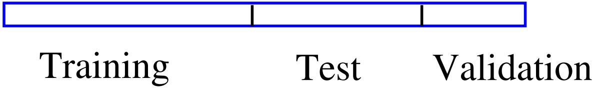
Table

z	.67	1.	1.28	1.64	1.96	2.33	2.58
ε	50	32	20	10	5	2	1

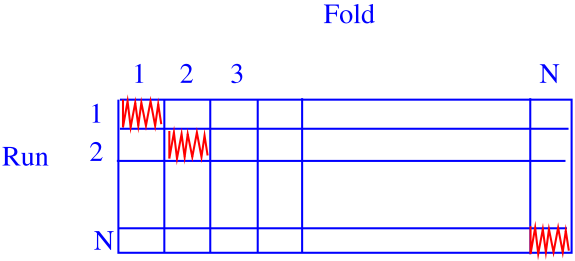
Empirical estimates

When data abound

(MNIST)



Cross validation



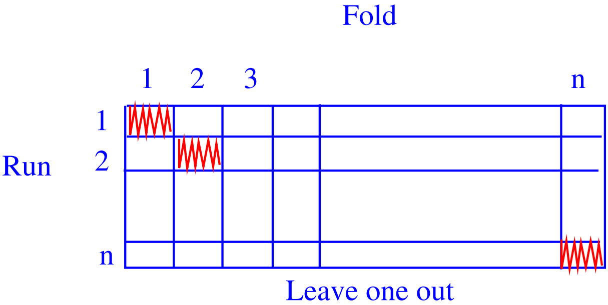
N–fold Cross Validation

Error = Average (error on of h

learned from)

Empirical estimates, foll'd

Cross validation → Leave one out



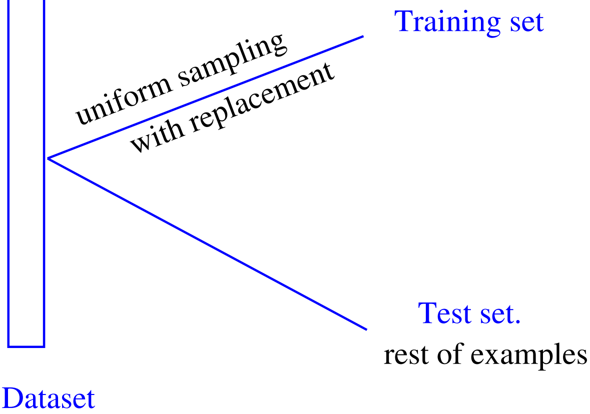
Same as N-fold CV, with N = number of examples.

Properties

Low bias; high variance; underestimate error if data not independent

Empirical estimates, foll'd

Bootstrap



Average indicator over all (Training set, Test set) samplings.

Beware

Multiple hypothesis testing

- If you test many hypotheses on the same dataset
- one of them will appear confidently true...

More

- Tutorial slides:
  http://www.lri.fr/ sebag/Slides/Validation_Tutorial_11.pdf
- Video and slides (soon): ICML 2012, Videolectures, Tutorial Japkowicz & Shah
  http://www.mohakshah.com/tutorials/icml2012/

Validation, summary

What is the performance criterion

- Cost function
- Account for class imbalance
- Account for data correlations

Assessing a result

- Compute confidence intervals
- Consider baselines
- Use a validation set

If the result looks too good, don't believe it