

24MES JOURNÉES D'ARITHMÉTIQUES FAIBLES  
JOURNÉES  
COMPLEXITÉ, MODÈLES FINIS ET BASES DE DONNÉES  
2005  
[HTTP://WWW.UNIV-PARIS12.FR/LACL/DURAND/  
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## Présentation

Les 24mes journées d'arithmétique faible et les journées du groupe de travail "Complexité, modèles finis et bases de données" du GDR ALP se sont tenues en même temps à l'IUT de Fontainebleau (université Paris 12), les jeudi 26, vendredi 27 et samedi 28 mai 2005.

Les rencontres du groupe de travail "Complexité, modèles finis et bases de données" ont pour but de permettre tout chercheur qui le souhaite de présenter ses travaux en complexité, en théorie des modèles finis ou dans un de leur nombreux champs d'application comme : les bases de données, la vérification de programmes, la satisfaction de contraintes, les jeux formels, etc... Les éditions précédentes ont eu lieu en 1995 Paris XI (organisateur : M. Santha), en 1996 à l'IUT de Fontainebleau (P. Cegielski), en 1997 Paris II (M. de Rougemont), en 1998 à l'ENS de Lyon (P. Koïran), en 1999 Paris VI (I. Guessarian), en 2000 Paris XII (A. Durand), en 2001 Paris II (M. Hermann), en 2002 Arcachon (D. Janin), en 2003 Paris-Dauphine (C. Bazgan et A. Durand) et en 2004 Lausanne (J. Duparc).

Les arithmétiques faibles jouent un rôle fondamental dans plusieurs domaines des mathématiques, de l'informatique et de la philosophie en étudiant les propriétés des nombres entiers d'un point de vue logique. Ces journées sont l'occasion pour les chercheurs du domaine qui étudient ou appliquent les arithmétiques faibles de se rencontrer et d'échanger des idées. Les JAF ont eu lieu précédemment : Lyon (1990), Paris (1990, 1991, 1996), Clermont (1991, 1992, 1993, 1994, 1995, 1999, 2000), Fontainebleau (1994, 2001), Metz (1996), St-Petersburg (1997, 2002), Mons (1997), Warsaw (1998), New-York (1999), Naples (2003), et Yerevan (2004).

Une grande partie des sessions des deux événements sont susceptibles d'intéresser les deux communautés et ont donc été communes. Les organisateurs de cette année sont Patrick Cegielski, Arnaud Durand, Michel de Rougemont.

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## Abstracts

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### Thomas Brihaye

Université de Mons-Hainaut

*Model-Checking, bisimulation and word combinatorics.*

**Abstract.** In this talk we explain briefly what model-checking is. Then we recall the notion of bisimulation and explain why the study of bisimulations is relevant in the context of model-checking. In particular we focus on bisimulations of dynamical systems and we explain how the dynamics of these systems can be recovered through some words encoding. This technique leads to nice finiteness results for o-minimal dynamical systems.

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**Andres Cordón-Franco**

University of Seville (Spain)

*On axiom schemes for  $D(n+1)(T)$ -formulas (joint work with A. Fernández-Margarit and F.F. Lara-Martín)*

**Abstract.** Motivated by J. Paris' problem on the equivalence between the schemes of induction and minimization for  $\Delta(n+1)$ -formulas, in [3] and [6] we introduced theories  $\mathbf{I}\Delta(n+1)(\mathbf{T})$ ,  $\mathbf{L}\Delta(n+1)(\mathbf{T})$  and  $\mathbf{B}*\Delta(n+1)(\mathbf{T})$  (in what follows referred to as *relativized fragments*), where  $\mathbf{T}$  is an arbitrary theory in the language of first-order Arithmetic extending  $\mathbf{I}\Delta 0$ . These fragments are obtained by restricting the schemes of induction, minimization and (a version of) collection to  $\Delta(n+1)(\mathbf{T})$ -formulas (that is,  $\Sigma(n+1)$ -formulas which are equivalent in  $\mathbf{T}$  to some  $\Pi(n+1)$ -formula).

The work developed in [3]–[6] provides evidence that relativized fragments are interesting subsystems of Peano Arithmetic (=  $\mathbf{PA}$ ). Firstly, relativized fragments are closely related to fragments of Arithmetic described in terms of inference rules (see [1], [2]). In fact,  $\mathbf{T} + \mathbf{I}\Delta(n+1)(\mathbf{T})$  and  $\mathbf{T} + \mathbf{B}*\Delta(n+1)(\mathbf{T})$  are deductively equivalent to the closure of  $\mathbf{T}$  by unnested applications of  $\Delta(n+1)$ -induction rule and  $\Sigma(n+1)$ -collection rule, respectively. Thus, the study of relativized fragments in [3], [6] can be considered to be a model-theoretic analysis of inference rules. Secondly, fragments for  $\Delta 1(\mathbf{T})$ -formulas are relevant to some fundamental problems on Complexity Theory. As an example, in [4] it is shown that the provably total recursive functions of the parameter free  $\Delta 1(\mathbf{T})$ -induction scheme are, precisely, the primitive recursive functions which are provably total in  $\mathbf{T}$ ; and sufficient conditions for separating the Polynomial Time Hierarchy and PSPACE are established. Finally, relativized fragments provide a general framework for the analysis of extensions of bounded quantifier complexity of *classical* fragments of Arithmetic. The results in [5] show that fragments for parameter free  $\Sigma(n+1)$  and  $\Pi(n+1)$  formulas have extensions of *small* quantifier complexity (that is, of quantifier complexity less than that of their axiomatizations) while their parameter counterparts do not. This divergent behaviour can be explained from the study of fragments for  $\Delta(n+1)(N)$ -formulas (that is,  $\Sigma(n+1)$ -formulas equivalent in the standard model of Arithmetic,  $N$ , to some  $\Pi(n+1)$ -formula).

In this work we present a new method for the study of relativized fragments. The crucial property is that, under certain assumptions on the theory  $\mathbf{T}$ , these fragments give axiomatizations of the classes of structures which are either models of the  $\Pi(n+2)$ -consequences of  $\mathbf{T}$  or models of a certain classical fragment of Arithmetic. More concretely, it holds that (let us denote by  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  the  $\Pi(n+2)$ -consequences of  $\mathbf{T}$ ):

**Theorem 1.** (*Transfer Theorem*)

1. If  $M$  is a model of  $\mathbf{I}\Delta(n+1)(T)$ , either  $M$  is a model of  $\mathbf{Th}(\Pi(n+2))(T)$  or  $M$  is a model of  $\mathbf{I}\Sigma(n+1)$ .
2. If  $M$  is a model of  $\mathbf{B}*\Delta(n+1)(T)$ , either  $M$  is a model of  $\mathbf{Th}(\Pi(n+2))(T)$  or  $M$  is a model of  $\mathbf{B}\Sigma(n+1)$ .

In addition, we establish versions of the theorem above for the parameter free counterparts of the relativized fragments:  $\mathbf{I}\Delta(n+1)(\mathbf{T})-$ ,  $\mathbf{L}\Delta(n+1)(\mathbf{T})-$  and  $\mathbf{B}*\Delta(n+1)(\mathbf{T})-$ .

By applying these Transfer Theorems, we develop a systematical analysis of the relative strength, finite axiomatizability and quantifier complexity properties of relativized fragments (for both their parametric and parameter free versions). In fact, Transfer Theorems allow one to reduce the study of these properties to the analysis of the existence of extension of bounded quantifier complexity of classical fragments of Arithmetic.

(A) On relative strength.

From Theorem 1 and the fact that  $\mathbf{B}\Sigma(n+1) + exp$  does not have consistent  $\Sigma(n+3)$ -extensions (Proposition 4.3 in [5]), we can deduce the following result:

**Theorem 2.**

1. Assume that  $\mathbf{T1}$ ,  $\mathbf{T2}$  are closed under  $\Delta(n+1)$ -induction rule. Then  $\mathbf{I}\Delta(n+1)(\mathbf{T1}) \equiv \mathbf{I}\Delta(n+1)(\mathbf{T2})$  if, and only if,  $\mathbf{Th}(\Pi(n+2))(\mathbf{T1}) \equiv \mathbf{Th}(\Pi(n+2))(\mathbf{T2})$ .

2. Assume that  $\mathbf{T1}$ ,  $\mathbf{T2}$  prove  $exp$  and are closed under  $\Sigma(n+1)$ -collection rule. Then

$\mathbf{B}*\Delta(n+1)(\mathbf{T1}) \equiv \mathbf{B}*\Delta(n+1)(\mathbf{T2})$  if, and only if,  $\mathbf{Th}(\Pi(n+2))(\mathbf{T1}) \equiv \mathbf{Th}(\Pi(n+2))(\mathbf{T2})$ .

As a consequence, we deduce Hierarchy Theorems for  $\Delta(n+1)(\mathbf{T})$ -induction and  $\Delta(n+1)(\mathbf{T})$ -collection.

**Theorem 3.** (*Hierarchy Theorem*)

1.  $\mathbf{I}\Delta(n+1)(\mathbf{I}\Sigma(n)) \subset \mathbf{I}\Delta(n+1)(\mathbf{I}\Sigma(n+1)) \subset \mathbf{I}\Delta(n+1)(\mathbf{I}\Sigma(n+2)) \subset \dots \subset \mathbf{I}\Delta(n+1)(\mathbf{PA}) \subset \mathbf{I}\Delta(n+1)(N)$ .

2.  $\mathbf{B}*\Delta(n+1)(\mathbf{I}\Sigma(n)) \subset \mathbf{B}*\Delta(n+1)(\mathbf{I}\Sigma(n+1)) \subset \mathbf{B}*\Delta(n+1)(\mathbf{I}\Sigma(n+2)) \subset \dots \subset \mathbf{B}*\Delta(n+1)(\mathbf{PA}) \subset \mathbf{B}*\Delta(n+1)(N)$ .

Part 1 in the theorem above was previously established in [6] with a different proof. Nevertheless, Part 2 answers an open problem posed in [6].

Similar results for parameter free relativized fragments are also obtained by using Transfer Theorems and results in [5] on properties of extensions of parameter free (classical) fragments.

(B) On finite axiomatizability.

**Theorem 4.** Assume that  $\mathbf{T}$  is closed under  $\Sigma(n+1)$ -collection rule.

1.  $\mathbf{I}\Delta(n+1)(\mathbf{T})$  is finitely axiomatizable if, and only if,  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  is finitely axiomatizable.

2. If  $\mathbf{I}\Delta(n+1)(\mathbf{T})-$  is finitely axiomatizable then so is  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$ .

3. ( $n \geq 1$ )  $\mathbf{B}*\Delta(n+1)(\mathbf{T})$  is finitely axiomatizable if, and only if,  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  is finitely axiomatizable.

From Theorem 4, we deduce that if  $\mathbf{T}$  is a consistent extension of  $\mathbf{I}\Sigma(n+1)$ , then none of the following theories is finitely axiomatizable:  $\mathbf{I}\Delta(n+1)(\mathbf{T})$ ,  $\mathbf{B}*\Delta(n+1)(\mathbf{T})$ ,  $\mathbf{I}\Delta(n+1)(\mathbf{T})-$ .

Our results for parameter free minimization and collection schemes are not as general as the previous ones, but they allow us to prove that neither  $\mathbf{B}*\Delta(n+1)(\mathbf{T})-$  nor  $\mathbf{L}\Delta(n+1)(\mathbf{T})-$  is finitely axiomatizable for  $\mathbf{T} = \mathbf{Th}(N)$ ,  $\mathbf{PA}$ ,  $\mathbf{I}\Sigma(n+k)$  ( $k \geq 1$ ).

(C) On quantifier complexity properties.

**Theorem 5.** Let  $\mathbf{T}$  be a consistent theory.

1. If  $\mathbf{I}\Sigma(n+1)$  does not imply  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  then  $\mathbf{I}\Delta(n+1)(\mathbf{T})$  is  $\Pi(n+3)$ -axiomatizable but not  $\Sigma(n+3)$ . If  $\mathbf{I}\Sigma(n+1)$  implies  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  then  $\mathbf{I}\Delta(n+1)(\mathbf{T})$  is  $\Pi(n+2)$ -axiomatizable and, for  $n \geq 1$ , it is not  $\Sigma(n+2)$ -axiomatizable.
2. If  $\mathbf{I}\Sigma(n) + exp$  does not imply  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  then  $\mathbf{B}*\Delta(n+1)(\mathbf{T})$  is  $\Pi(n+3)$ -axiomatizable but not  $\Sigma(n+3)$ . Moreover, if  $\mathbf{I}\Sigma(n)$  implies  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$  then  $\mathbf{B}*\Delta(n+1)(\mathbf{T}) \equiv \mathbf{I}\Sigma n$ .

Observe that our methods provide a complete description of the quantifier complexity of  $\Delta(n+1)(\mathbf{T})$ -induction and  $\Delta(n+1)(\mathbf{T})$ -collection schemes (except for the case  $n = 0$ ). We also obtain similar (but weaker) results on the quantifier complexity of parameter free relativized fragments.

Finally, we discuss some questions left unanswered in our work. While Transfer Theorems for induction and collection schemes are best possible, we have obtained only a partial result for  $\mathbf{L}\Delta(n+1)(\mathbf{T})-$ . Namely, if  $M$  is a model of  $\mathbf{L}\Delta(n+1)(\mathbf{T})-$ , either  $M$  is a model of  $\mathbf{Th}(\Sigma(n+1) \cup \Pi(n+1))(\mathbf{T})$  or  $M$  is a model of  $\mathbf{III}(n+1)-$ .

**Problem 6.** Does Transfer Theorem for  $\mathbf{L}\Delta(n+1)(\mathbf{T})-$  hold if we replace  $\mathbf{Th}(\Sigma(n+1) \cup \Pi(n+1))(\mathbf{T})$  by  $\mathbf{Th}(\Pi(n+2))(\mathbf{T})$ ?

In addition, we obtain interesting consequences by considering relativized fragments for  $\mathbf{T} = \mathbf{I}\Delta 0 + exp$ , or  $\mathbf{III}(n+1)-$ . Taking  $\mathbf{T} = \mathbf{I}\Delta 0 + exp$ , we get a reformulation of an open problem raised by Wilkie and Paris [7] asking if every model of  $\mathbf{I}\Delta 0$  which is *not* closed under exponentiation is a model of the  $\Sigma 1$ -collection scheme. On the other hand, in [2] Beklemishev proved that, for each  $n \geq 1$ ,  $\mathbf{III}(n+1)-$  is conservative over  $\mathbf{I}\Sigma n-$  with respect to boolean combinations of  $\Sigma(n+1)$ -sentences. For  $\mathbf{T} = \mathbf{III}(n+1)-$ , the study of theories  $\mathbf{L}\Delta(n+1)(\mathbf{III}(n+1)-)-$  is related to the problem of determining whether that conservation result is best possible.

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## Bruno Courcelle

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*On Seese's Conjecture : Recent Advances (joint work with S. Oum)*

**Abstract.** D. Seese conjectured that if a set of graphs has a decidable monadic theory, then it is the image of a set of trees under a Monadic Second-order transduction, i.e. a transformation of relational structures expressed in Monadic Second-order logic. This is equivalent to saying that it has bounded clique-width. Clique-width is a graph complexity measure relevant to the construction of polynomial graph algorithms.

In my lecture in Lausanne in 2004, I explained that relativized versions of the conjecture, in particular to partial orders, to comparability graphs and to bipartite graphs, are equivalent to its full version. I presented also some provable relativizations, to line graphs, to partial orders of dimension 2 and to interval graphs.

In Fontainebleau, I will present a common work with S. Oum (Princeton) establishing the weak form of the conjecture where the hypothesis is that the satisfiability problem for monadic second-order formulas with even cardinality set predicates is decidable. The proof technique is completely different from the one used for the previously known results. It uses the notion of "vertex-minor" which parallels for clique-width the notion of "minor" intimately related to tree-width.

## Olivier Finkel

Équipe de Logique Mathématiques, Université Paris 7

*Infinite computations and highly undecidable problems*

**Abstract.**

We consider infinite computations of simple finite machines, like pushdown automata or one counter automata. The class of context free (respectively, 1-counter)  $\omega$ -languages is

the class of languages of infinite words which are accepted by pushdown (respectively, 1-counter) automata with a Büchi or a Muller acceptance condition. It is well known that any Turing machine can be simulated by a 2-counter automaton. We show the following surprising result. When considering acceptance of infinite words, from different points of view, a 1-counter automaton is sufficient to get the whole complexity of a Turing machine.

More precisely, we show first that the topological complexity of context free or of 1-counter  $\omega$ -languages is the same as that one of  $\omega$ -languages accepted by Turing machines with a Büchi or a Muller acceptance condition. In particular, for every recursive ordinal  $\alpha < \omega_1^{CK}$ , where  $\omega_1^{CK}$  is the first non recursive ordinal, there are some  $\Sigma^0_\alpha$ -complete and some  $\Pi^0_\alpha$ -complete  $\omega$ -languages accepted by Büchi 1-counter automata, [Fin04].

We show also that many classical decision problems about context free  $\omega$ -languages, or even about 1-counter  $\omega$ -languages, are located in the analytical hierarchy and are  $\Pi_2^1$ -complete. For instance, the universality problem, the inclusion problem and the equivalence problem are  $\Pi_2^1$ -complete. Some other decision problems are shown to be  $\Pi_2^1$ -hard, as the problem to determine whether a given context free  $\omega$ -language is in a given Borel class  $\Sigma^0_\alpha$ .

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**Daniel Graça**

LORIA, Nancy

*Computing with continuous-time analog circuits*

**Abstract.** We present an overview of the General Purpose Analog Computer (GPAC). This model, based on analog circuits, was introduced by C. Shannon in 1941 as a mathematical model of an analog device, the Differential Analyzer. In this talk, we will review the existing theory and discuss recent developments on this model, including results about its computational power.

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**Serge Grigorieff**

LIAFA, Université Paris 7

*Infinite computations and Kolmogorov complexities*

**Abstract.** We introduce the classes  $\text{MaxPR}[X \dashrightarrow D]$  and  $\text{MaxRec}[X \dashrightarrow D]$  of functions  $X \dashrightarrow D$  which are pointwise maximum of partial or total recursive sequences of functions where  $(D, <)$  is some computable partially ordered set and  $X$  is the set of binary words. The enumeration theorem and the invariance theorem always hold for  $\text{MaxPR}[X \dashrightarrow D]$ , leading to a variant  $\text{KmaxD}$  of Kolmogorov complexity. We characterize the orders  $D$  such that the enumeration theorem (resp. the invariance theorem) also holds for  $\text{MaxRec}[X \dashrightarrow D]$ . It turns out that  $\text{MaxRec}[X \dashrightarrow D]$  may satisfy the invariance theorem but not the enumeration theorem. Also, when  $\text{MaxRec}[X \dashrightarrow D]$  satisfies the invariance theorem then the Kolmogorov complexities associated to  $\text{MaxRec}[X \dashrightarrow D]$  and  $\text{MaxPR}[X \dashrightarrow D]$  are equal (up to a constant). Letting  $\text{KminD} = \text{kmax}[D']$ , where  $D'$  is the reverse order, we prove that either  $\text{KminD} = \text{KmaxD} = \text{K}$  (up to a constant) or  $\text{KminD}, \text{KmaxD}$  are incomparable and  $< \text{K}$  and  $> \text{K}'$ . We characterize the orders leading to each case. We also show that  $\text{KminD}, \text{KmaxD}$  cannot be both much smaller than  $\text{K}$  at any point. These results are proved in a more general setting with two orders on  $D$ , one extending the other.

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**Miki Hermann**

LIX, École Polytechnique

*Complexity of Clausal Constraints Over Chains (joint work with Nadia Creignou (Marseille), Andrei Krokhin (Durham) and Gernot Salzer (Vienna))*

**Abstract.** We investigate the complexity of the satisfiability problem of constraints over finite totally ordered domains. In our context, a clausal constraint is a disjunction of inequalities of the form  $x \leq d$  and  $x \geq d$ . We classify the complexity of constraints based on clausal patterns. A pattern abstracts away from variables and contains only information about the domain elements and the type of inequalities occurring in a constraint. Every finite set of patterns gives rise to a (clausal) constraint satisfaction problem in

which all constraints in instances must have an allowed pattern. We prove that every such problem is either polynomially decidable or NP-complete, and give a polynomial-time algorithm for recognizing the tractable cases. Some of these tractable cases are new and have not been previously identified in the literature.

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### **Emil Jerabek**

Academy of sciences (Rép. Tchèque) and Utrecht University  
*Hardness amplification in bounded arithmetic*

**Abstract.** One of the important achievements in derandomization theory is the following theorem: given a truth table of a Boolean function  $f$  on  $n$  variables, which cannot be approximated by circuits of size  $2^{\epsilon n}$  with nonnegligible advantage, one can construct in polynomial time a function  $f'$  on  $O(n)$  variables with worst-case circuit complexity  $2^{\Omega(n)}$ . We show how to formalize this statement in  $S^1_2$ . To achieve this goal, we develop in  $S^1_2$  basics of the theory of finite fields and list-decoding of Reed-Muller codes, and we present a modification of Soltys' theory  $\forall LAP$  for linear algebra.

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### **Janos Makowsky**

Technion Haifa, Israel  
*Application of Logic in Combinatorics (Joint work with Eldar Fisher)*

**Abstract.** We discuss linear recurrence relations for various combinatorial counting functions. Among these we have the density function for classes of labeled graphs, and various graph polynomials. In each of the cases discussed, a condition ensuring definability in CMSOL (Monadic Second Order Logic with modular counting) suffices to prove the existence of such recurrence relations.

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### **Anca Muscholl**

LIAFA, Université Paris 7  
*Logic with data*

**Abstract.** In a data word each position carries a label from a finite alphabet and a data value from some infinite domain. We show that two-variable logic (with successor and order relation) on such strings is decidable and that the complexity is as hard as Petri net reachability. In the case where only the successor relation is used, the complexity is shown to become tractable.

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**Paulin de Naurois**

LORIA, Nancy

*The Complexity of Semilinear Problems in Succinct Representation*

**Abstract.** We prove completeness results for twenty three problems in semilinear geometry. These results involve semilinear sets given by additive circuits as input data. If arbitrary real constants are allowed in the circuit, the completeness results are for the Blum-Shub-Smale additive model of computation. If, in contrast, the circuit is constant-free, then the completeness results are for the Turing model of computation. One such result, the  $\Delta^2_{||}$ -completeness of deciding Zariski irreducibility, exhibits for the first time a problem with a geometric nature complete in this class.

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**Joachim Niehren**

INRIA-LIFL, Lille

*Querying XML-Documents by Tree Automata*

**Abstract.** Information extraction from semi-structured documents requires to find queries in XML trees that define appropriate sets of n-tuples of nodes. In this talk, I will discuss how to represent such queries by tree automata. I will first recall notions of tree automata for unranked trees as in XML, present representation formalisms for n-ary queries by tree automata, and discuss efficiency, expressiveness, and learnability. Finally, I will illustrate an application to information extraction.

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**Romain Péchoux**

LORIA, Nancy

*Resource analysis by sup-interpretation*

**Abstract.** Sup-interpretation is a new tool to control memory resources by Static analysis which bounds from above the size of function output. This method applies to first order functional programming with pattern matching. This work is related to quasi-interpretations but we are now able to determine resources of more algorithms and it is easier to perform an analysis with this new tools.

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**Olivier Teytaud**

LRI, Université Paris 11

*Inductive-deductive systems : a statistical learning point of view*

**Abstract.** The theorems about incompleteness of arithmetic have often been cited as an argument against automatic theorem proving and expert systems. However, these

theorems rely on a worst-case analysis, which might happen to be overly pessimistic with respect to real-world domain applications.

For this reason, a new framework for a probabilistic analysis of logical complexity is presented in this paper. Specifically, the rate of non-decidable clauses and the convergence of a finitely describable set of axioms toward the target one when the latter exists in the language are studied, by combining results from mathematical logic and from statistical learning.

The theoretical analysis gives practical hints into the pros and cons of optimizing the set of axioms along learning, with respect to the convergence rates above.

**Jerzy Tomasiak**

LLAIC, Université Clermont 1

*Synthesis theories of finite structures of arithmetic (joint work with Michał Krynicki and Konrad Zdanowski (Warsaw University))*

**Abstract.**

— Experiences show that computer arithmetic is not the theory of the standard (infinite) model of arithmetic since computer may accede to a finite part of natural numbers only. Whence it is important to study the family of theories of finite fragments of natural numbers that looks like the most natural approximation of the theory the whole infinite model. More general question arises how to reconstruct the information (specification) on the whole structure from the network of its finite approximations. We examine here some examples of reconstruction operators like lower limit, upper limit ( $sl$ ) of theories as well as reduced product and ultraproduct operations on finite models.

Consider the family FIN of all theories of initial segments of a standard model of arithmetic. In this approach one replaces an actual infinity of a standard model by the (potentially) infinite family of finite models. M. Mostowski propose to study the family FIN under the operation  $sl$  which is a kind of limit operation on theories. The operation  $sl$  can be compared with some other limit operations, well known in the model theory, like reduced product and ultraproduct constructions (or limit of directed system). Our approach may find applications in theoretical foundations of computing, in information systems, in information fusion theory and in software engineering.

Let  $A = (\mathbf{N}, R_1, \dots, R_s, f_1, \dots, f_t, a_1, \dots, a_r)$  be a model having as a universe the set  $N$  of natural numbers, and s.t.  $R_1, \dots, R_s$  are relations on  $\mathbf{N}$ ,  $f_1, \dots, f_t$  are operations on  $\mathbf{N}$  and  $a_1, \dots, a_r \in \mathbf{N}$ . We consider the family  $FM(A)$  of all finite initial segments  $An = (\{0, \dots, n\}, R_1^n, \dots, R_s^n, f_1^n, \dots, f_t^n, a_1^n, \dots, a_r^n, n)$ , of the model  $A$ , where  $R_i^n$  is the restriction of  $R_i$  to the set  $\{0, \dots, n\}$ ,  $a_i^n = a_i$  if  $a_i \leq n$ , otherwise  $a_i^n = n$  and  $f_i^n$  is defined by  $f_i^n(b_1, \dots, b_{n_i}) = f_i(b_1, \dots, b_{n_i})$  if  $f_i(b_1, \dots, b_{n_i}) \leq n$  otherwise  $f_i^n(b_1, \dots, b_{n_i}) = n$ . The signature of  $An$  contains one new constant  $Max$  s.t.  $Max^n = n$ . We say that  $\phi$  is satisfied by  $b_1, \dots, b_p$  in all sufficiently large finite models of  $FM(A)$ , what is denoted by  $FM(A) \text{ models } sl\phi[b_1, \dots, b_p]$ , (or simply  $\text{models } sl\phi[b_1, \dots, b_p]$ ) if there is  $k \in \mathbf{N}$  such that for all  $n \geq k$   $An \text{ models } \phi[b_1, \dots, b_p]$ . We call  $sl$ -Theory of  $A$  the following set of sentences  $sl-Th(A) = \{\phi : \exists k \forall n \geq k An \text{ models } \phi\}$ .

The theory  $sl-Th(A)$  is a consistent theory without finite models and it can be described

in terms of limits of the sequence of sets  $(Th(A_n) : n \in N)$ .

**Theorem 1** *The following holds :*

- (i)  $sl - Th(A) = \limsup(n \rightarrow \infty)Th(A_n)$ .
- (ii) Sentence  $\phi$  is consistent with the theory  $sl - Th(A)$  iff  $\phi \in \liminf(n \rightarrow \infty)Th(A_n)$ .
- (iii)  $sl - Th(A)$  is a complete theory iff the sequence  $(Th(A_n) : n \in N)$  is convergent.

Recall that a sequence  $(A_n) : n \in N$  of sets is convergent if it has a limit:

$$\lim_{n \rightarrow \infty} A_n = \limsup(n \rightarrow \infty)(A_n) = \liminf(n \rightarrow \infty)(A_n).$$

A semantical characterization of the theory  $sl - Th(A)$  can be done in terms of ultraproducts. This theory is the intersection of all theories of ultraproducts of the family  $(A_n) : n \in N$  under nonprincipal ultrafilters over  $N$ .

**Theorem 2** *We have the following identity :*

$$sl - Th(A) = U\{Th(\prod A_n/U) : U \text{ nonprincipal ultrafilter over } N\}.$$

Another properties of the operation  $sl$  will be illustrated by examples of models  $A$  over  $N$ .

**Victor Selivanov**

Novosibirsk University (Russia)

*Some Hierarchies and Reducibilities on Regular Languages*

**Abstract.** We discuss some known and introduce some new hierarchies and reducibilities on regular sets. We establish some facts on the corresponding degree structures and relate the reducibilities to hierarchies. As an application, we characterize regular languages whose leaf-language classes (in the balanced model) are contained in the polynomial hierarchy. For any reducibility we try to give some motivation and interesting open questions, in a hope to convince the reader that study of these reducibilities is important for automata theory and computational complexity.