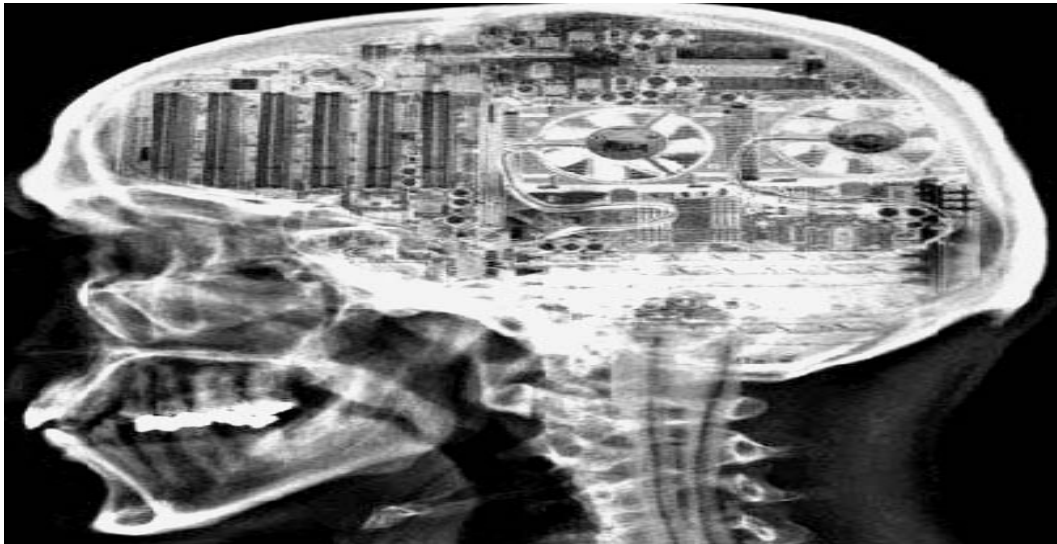


Licence Mention Informatique - L3/S6 - 2009/10
Introduction au Génie Logiciel
Partie3: Test: Static White-Box Test



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Difficulties with Static Unit Tests so far

- ❑ The **generation of test-data** is left open; however, this is the CORE of the problem
- ❑ Setup of drivers and stubs necessary (but that can be automated ...)
- ❑ Random-testdata generators are usually **very ineffective** (why ???) ☹
writing „simple“ versions compliant to the post-conditions results in additional programming labour ... ☹

Difficulties with Static Unit Tests so far

- ❑ When do we have tested enough ?
When is our test “adequate” ?

We have to decide on adequacy criteria in advance.

This can be:

- criteria on the coverage of the spec of the program
 - criteria on statistical models and an error model
-
- ❑ Some empirical observations:
 - No relation between detection order and detection difficulty
 - No relation between detection difficulty and correction
 - The more errors you found, the more you find more...
 - The quality of a test set is independent of its size.

Idea:

- ❑ Lets exploit the structure of the program !!!

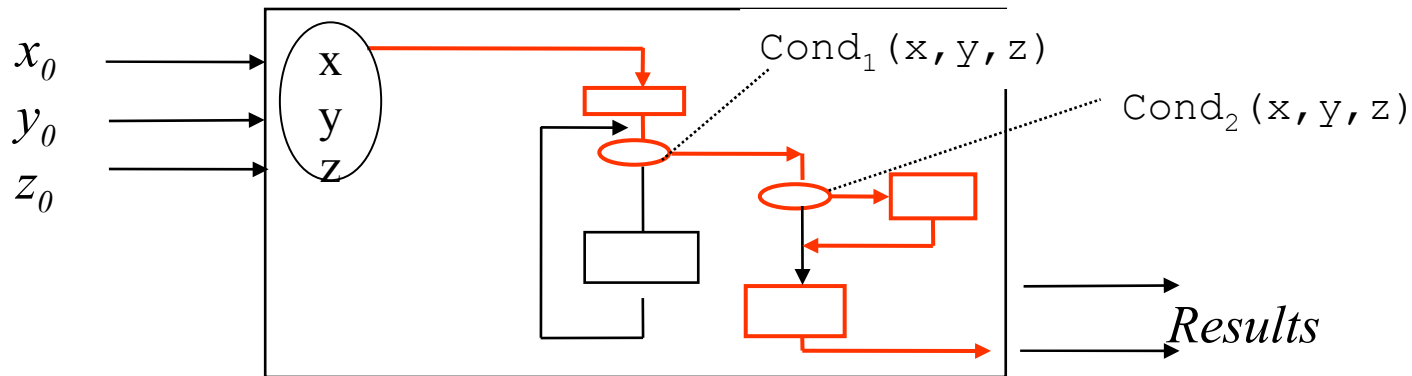
(and not, as before in specification based tests („black box“-tests), depend entirely on the spec).

Assumption: Programmers make most likely errors in branching points of a program (Condition, While-Loop, ...).

Lets develop a test method that tries to check against this !

Static Structural (“white-box”) Tests

- ❑ we select “critical” paths
- ❑ specification used to verify the obtained results



what the program does and how ...

*A path corresponds to one logical expression over x_0, y_0, z_0 .
corresponding to one test-case (comprising several test data ...)*

$$\neg Cond_1(x_0, y_0, z_0) \wedge \neg Cond_2(x_0, y_0, z_0)$$

We are interested either in edges (control flow), or in nodes (data flow)

A Program for the triangle example

```
procedure triangle(j,k,l : positive) is
  eg: natural := 0;
begin
  if j + k <= l or k + l <= j or l + j <= k then
    put("impossible");
  else if j = k then          eg := eg + 1; end if;
    if j = l then          eg := eg + 1; end if;
    if l = k then          eg := eg + 1; end if;
    if eg = 0 then put("arbitrary");
    elsif eg = 1 then put("isocele");
    else          put("equilateral");
    end if;
end if;
end triangle;
```

What are tests adapted to this program ?

- ❑ try a certain number of execution "paths"
(which ones ? all of them ?)
- ❑ find input values to stimulate these paths
- ❑ compare the results with expected values
(i.e. the specification)

Functional-test vs. structural test?

Both are complementary and complete each other:

- ❑ Structural Tests have weaknesses in principle:
 - if you forget a condition, the specification will most likely reveal this !

- ❑ Structural Tests have weaknesses in principle:
for a given specification, there are several possible implementations (working more or less differently from the spec):
 - *sorted arrays : linear search ? binary search ?*
 - *$(x, n) \rightarrow x^n$: successive multiplication ? quadratic multiplication ?*

Each implementation demands for different test sets !

Equivalent programs ...

Program 1 :

```
S:=1; P:=N;
```

```
while P >= 1 loop S:= S*X; P:= P-1; end loop;
```

Program 2 :

```
S:=1; P:= N;
```

```
while P >= 1 loop
```

```
  if P mod 2 /= 0 then P := P -1; S := S*X; end if;
```

```
  S:= S*S; P := P div 2;
```

```
end loop;
```

Both programs satisfy the same spec but ...

- one is more efficient, but more difficult to test.
- test sets for one are not necessarily “good” for the other, too !

Control Flow Graphs

A graph with oriented edges root E and an exit S,

- the nodes be either “elementary instruction blocs” or “decision nodes” labelled by a predicate.
- the arcs indicate the control flow between the elementary instruction blocs and decision nodes (control flow)
- all blocs of predicates are accessible from E and lead to S (otherwise, dead code is to be suppressed !)

elementary instruction blocs: a sequence of

- assignments
 - update operations (on arrays, ..., not discussed here)
 - procedure calls (not discussed here !!!)
- conditions and expressions are assumed to be side-effect free

Computing Control Flow Graphs

- Identify longest sequences of assignments

Computing Control Flow Graphs

- Identify longest sequences of assignments

Example:

```
S:=1;
```

```
P:=N;
```

```
while P >= 1
```

```
  loop S:= S*X;
```

```
    P:= P-1;
```

```
  end loop;
```

Computing Control Flow Graphs

- Identify longest sequences of assignments

Example:

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S:=1;  
P:=N;
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```
while P >= 1  
loop S:= S*X;  
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end loop;
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Computing Control Flow Graphs

- ❑ Identify longest sequences of assignments
- ❑ Erase if_then_elses by branching

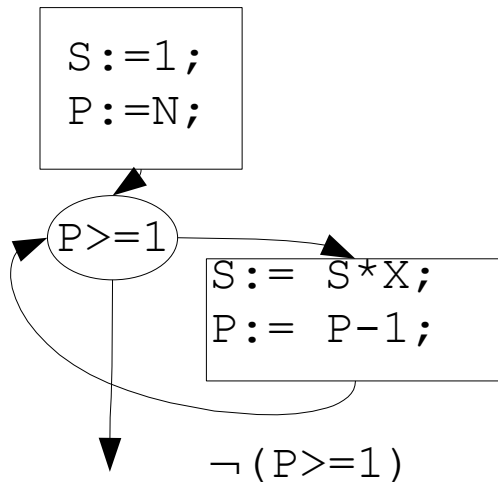
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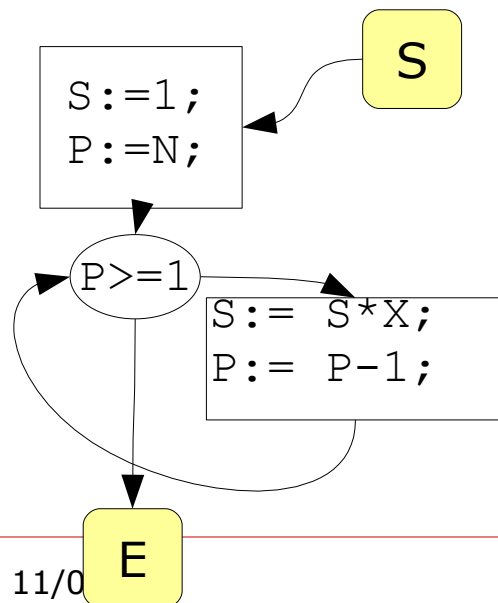
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Computing Control Flow Graphs

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Example:



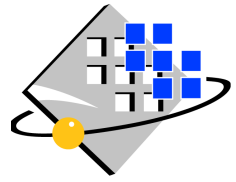
Computing Control Flow Graphs

- ❑ Identify longest sequences of assignments
- ❑ Erase if_then_elses by branching
- ❑ Erase while_loops by loops
- ❑ Add entry node and exit loop-arc, entry-arc, exit-arc

A Control-Flow-Graph (CFG) is usually a by-product of a compiler ...

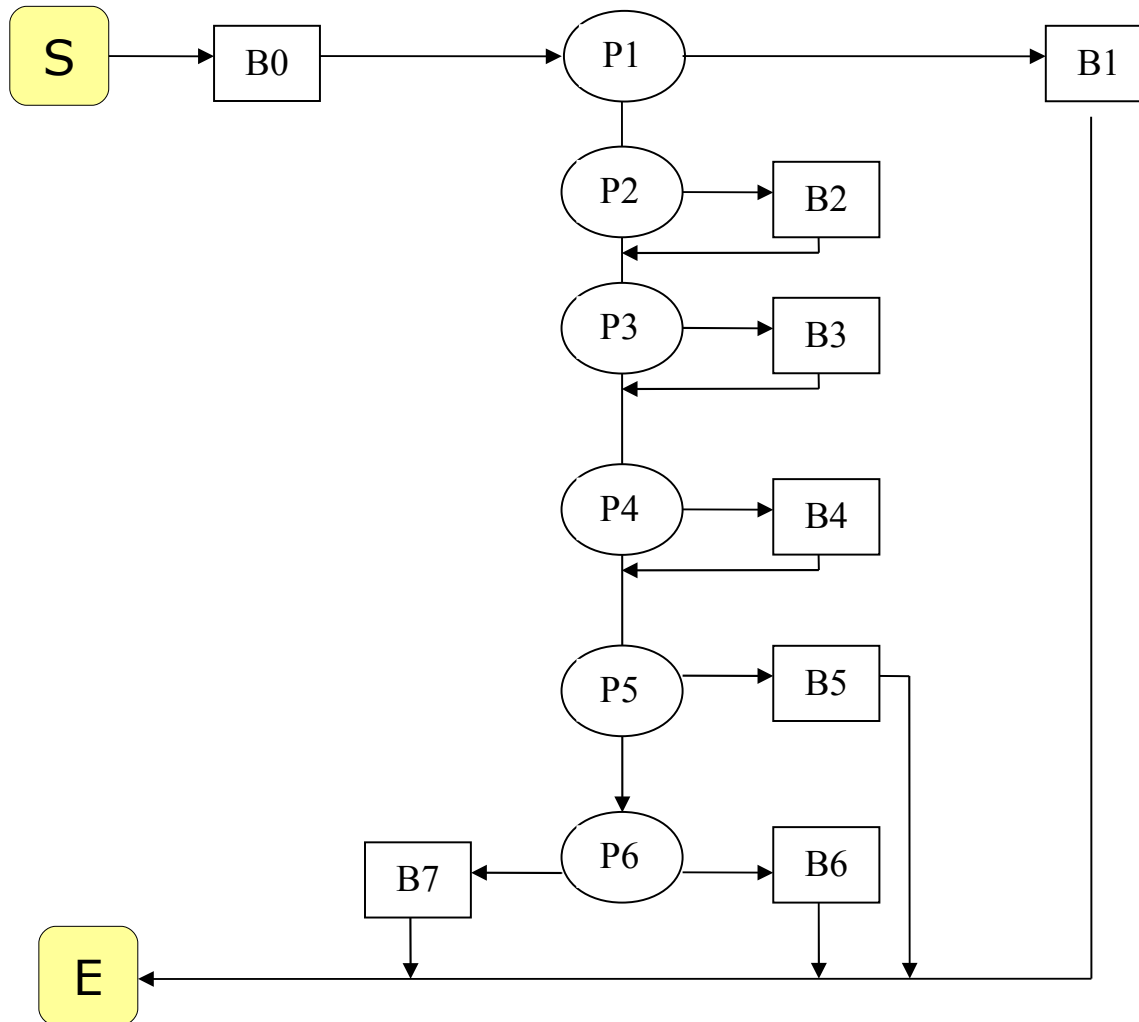
Revisiting our triangle example ...

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procedure triangle(j,k,l : positive) is
  eg: natural := 0;
begin
  if j + k <= l or k + l <= j or l + j <= k then
    put("impossible");
  else if j = k then          eg := eg + 1; end if;
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    if eg = 0 then put("quelconque");
    elsif eg = 1 then put("isocele");
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    end if;
end if;
end triangle;
```



Q: What is the CFG of the body
of triangle ?

Le graphe de flot de contrôle du programme



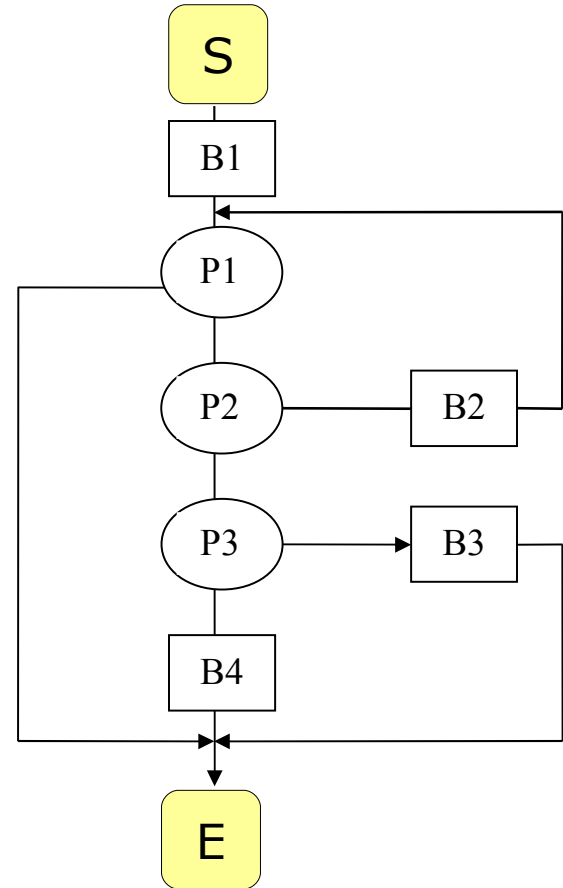
A procedure with loop and return

```
procedure supprime (T: in out Table; p: in out integer;  
                    x: in integer) is  
  
    i: integer := 1;  
  
begin  
    while      i <> p loop  
        if      T[i].val <> x then  i := i + 1;  
        elsif   i = p - 1          then p := p - 1; return;  
        else    T[i] := T[p-1]; p := p - 1; return;  
        end if;  
    end loop;  
end supprime;
```

... and its control flow graph

What are the feasible paths ?

How to describe this ?



Paths and Path Conditions

- ❑ Let M a procedure to test, and G its control-flow graph.
Terminology:
 - sub-path of M = path of G
 - initial path of M = path of G starting at S
 - path of M = path of G starting at S and leading to E
*i.e. a **complete** execution of the procedure*
 - a given path is associated to predicate (over parameters and state):
a condition over the **initial values initiales** of parameters
(and global variables) to achieve **exactly** this execution path
 - faisable paths = a path of M pour a set for all parameters and global
variables *exists* such that the path is executable.
i.e. the path condition is satisfiable

Computing Path Conditions by Symbolic Execution

Let P be an initial path in M .

- we give symbolic values for each variable x_0, y_0, z_0, \dots
- we set the path condition Φ initially “true”
- We follow the path, block for block, along P :

If the block is an instruction block B :

we execute symbolically B by memorizing the new values by expressions (symbolically) dependent on x_0, y_0, z_0, \dots

If the block is a decision block $P(x, \dots, z)$

if we follow the « true » arc we set $\Phi := \Phi \wedge P(\underline{x}, \dots, \underline{z})$,

if we follow the « false » arc we set $\Phi := \Phi \wedge \neg P(\underline{x}, \dots, \underline{z})$.

(The $\underline{x}, \dots, \underline{z}$ are the symbolic values for x, \dots, z .

This effect is produced by a substitution to be discussed later.)

Execution

- Execution (in imperative languages) is based on the notion of *state*.

A state is a table (or: function) that maps a variable V to some value of a domain D .

$$\text{state} = V \rightarrow D$$

As usual, we denote (finite) functions as follows:

$$\{ x \mapsto 1, y \mapsto 5, x \mapsto 12 \}$$

Symbolic Execution

- In static program analysis, it is in general not possible to infer concrete values of D.

However, it can be inferred **a set of possible values.**

For example, if we know that

$$x \in \{1..10\}$$

and we have an assignment $x := x + 2$, we know:

$$x \in \{3..12\}$$

afterwards.

Symbolic Execution

- This gives rise to the notion of a *symbolic state*.

$$\text{state}_{\text{sym}} = V \rightarrow \text{Set}(D)$$

As usual, we denote sets by

$$\{ x \mid E \}$$

where E is a boolean expression.

In our concrete technique, sets will always have the form $\{ x_0 \mid x_0 = E \}$ where E is an arithmetic expression (possibly containing variables of V).

Symbolic States and Substitutions

- Since in our concrete technique, sets have the form $\{x_0 \mid x_0 = E\}$, we can abbreviate:

$$\{x \mapsto \{x_0 \mid x_0 = E_1\}, y \mapsto \{y_0 \mid y_0 = E_2\}, z \mapsto \{z_0 \mid z_0 = E_3\}\}$$

to

$$\{x \mapsto E_1, y \mapsto E_2, z \mapsto E_3\}$$

and treat them as substitutions - all variables in an expression were subsequently replaced by their substituands ...

Symbolic States and Substitutions

- Example substitution:

$$(x + 2 * y) \{x \mapsto 1, y \mapsto x_0\}$$

$$= 1 + 2 * x_0$$

- An *initial symbolic state* is a state of the form:

$$\{ x \mapsto x_0, y \mapsto y_0, z \mapsto z_0 \}$$

Basic Blocks *as* Substitutions

Symbolic Pre-State

$x \mapsto x_0$
$y \mapsto y_0 + 3 * x_0$
$z \mapsto z_0$
$i \mapsto i_0$

Block

$i := x + y + 1$ $z := z + i$

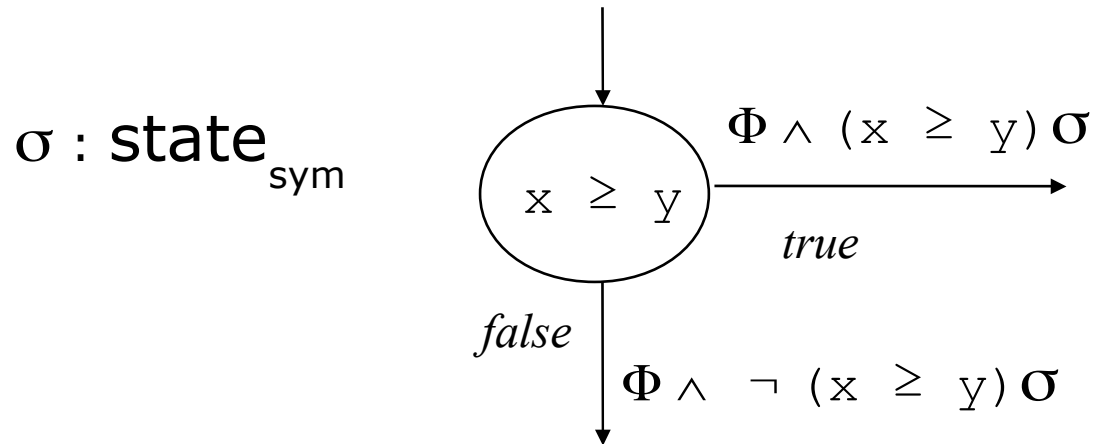
Symbolic Post-State

$x \mapsto x_0$
$y \mapsto y_0 + 3 * x_0$
$z \mapsto z_0 + y_0 + 4 * x_0 + 1$
$i \mapsto y_0 + 4 * x_0 + 1$

x_0 , y_0 and z_0 represent the initial values of x , y et z .

i is supposed to be a local variable (not initialized at the beginning).

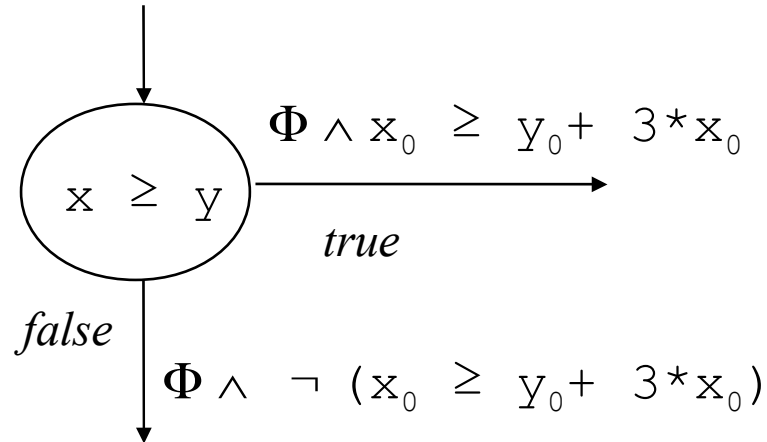
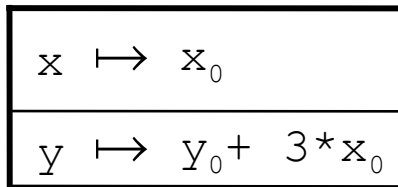
Symbolic Execution



Thus, we execute symbolically and transform the symbolic state in order to obtain an expression depending on the **initial values of the parameters**, **(accesses to undefined local variables are treated by exception)**

Thus, we can construct for a given path the path-condition. For reasoning **GLOBALLY** over a loop, we would have to invent an « invariant » (corresponding to an induction scheme).

Symbolic Execution



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Example: A Symbolic Path Execution

Recall

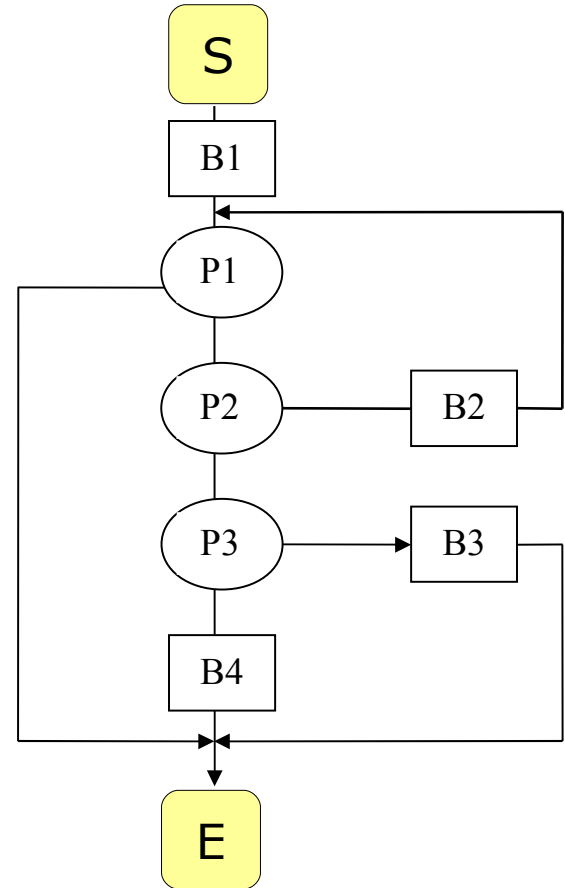
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        else    T[i] := T[p-1]; p := p - 1; return;
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Example: A Symbolic Path Execution

... and the corresponding control flow graph.

We want to execute the path:

[S,B1,P1,E]



Example: A Symbolic Path Execution

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$\Phi := \text{True}$

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$\Phi := \neg (i <> p) \sigma_{B1}$

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$\Phi := p_0 = 1$

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$i \mapsto 1$

Example: A Symbolic Path Execution

Result:

Test-Case:

Path : [S,B1,P1,E]

Path Condition: $\Phi := p_0 = 1$

A concrete Test:

T	↦	mtTab
p	↦	95
x	↦	17
i	↦	1

Paths and Test Sets

*In (this version of) program-based testing
a test case with a (feasible) path*

- a test case \approx an initial path in M
 - = a collection of values for variables (params and global)
(+ the output values described by the spécification)

- a test case set \approx a finite set of paths of M
 - = (by assuming a uniformity hypothesis)
a finite set of input values and
a set of expected outputs.

Unfeasible paths and decidability

- ❑ In general, it is undecidable if a path is feasible ...
- ❑ In general, it is undecidable if a program will terminate ...
- ❑ In general, equivalence on two programs is undecidable ...
- ❑ In general, a first-order formula over arithmetic is undecidable ...
- ❑ ...

Indecidable = it is known (mathematically proven) that there is no algorithm; this is worse than “we know none” !

BUT: for many relevant programs, practically good solutions exist (Z3 -> Pex, Simplify, JavaPathfinder-SE) ...

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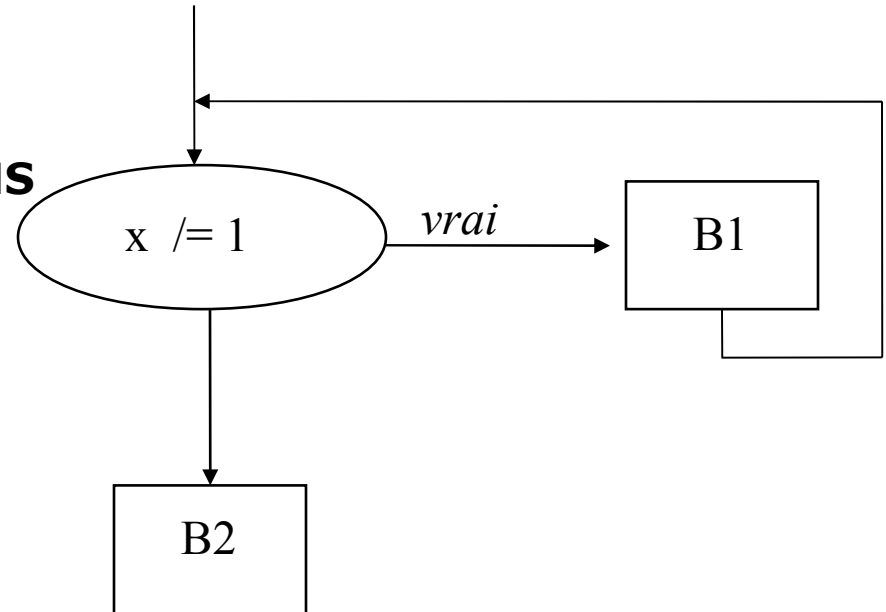
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BUT: for many relevant programs, practically good solutions exist (Z3 -> Pex, Simplify, JavaPathfinder-SE) ...

A Challenge-Example (Collatz-Function):

... **ALTHOUGH FOR SOME SPECTACULARLY SIMPLE PROGRAMS THESE SYSTEMS FAIL:**

```
while x <> 1 loop  
  if pair(x) then x := x / 2;  
  else x := 3 * x + 1;  
  end if;  
end loop;
```

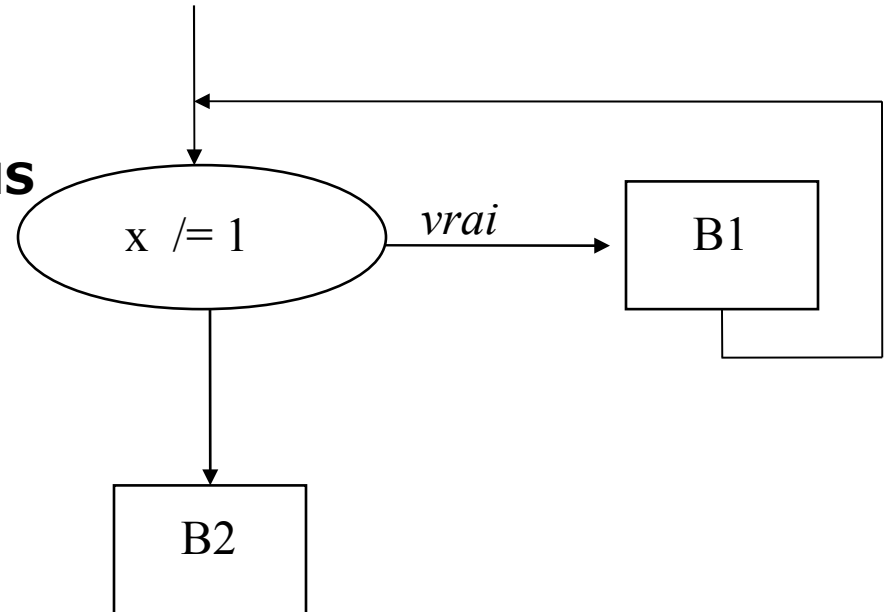


- does this function terminate for all x ?
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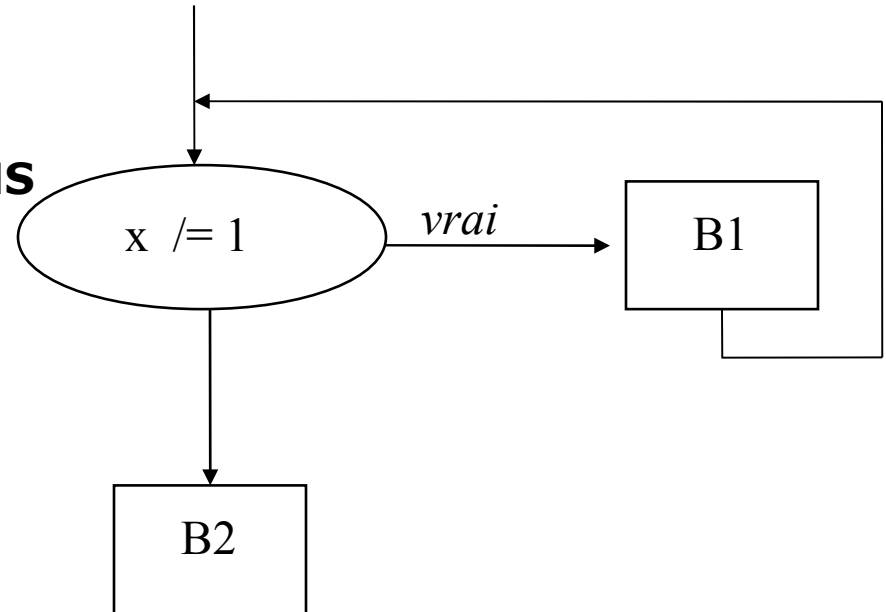


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end loop;
```



- does this function terminate for all x ?
- or equivalently: is B2 reached for all x ? ANSWER:unknown
- this implies that we can not know in advance that there exist infeasible paths !

The Triangle Prog without Unfeasible Paths

```
procedure triangle(j,k,l)
begin
  if j k<=l or k+1<=j or l+j<=k then put("impossible");
  elsif j = k and k = l then put("equilateral");
  elsif j = k or k = l or j = l then put("isocele")
  else put("quelconque");
end if;
end;
```

- ☞ If we find a path for which we do not know that it is feasible (maybe for deep mathematical reasons, maybe simply because our prover is too weak), however, it is likely in practice that there is an error ...

The notion of a “coverage criteria”

A coverage criterion is a predicate on CFG characterizing a particular subset of its paths ...

M = a procedure (with associated CFG G)

T = a test case set = a finite set of **feasible** paths in M

C = a coverage criterion (= a “set of paths”)

C(M, T) is true iff T satisfies the criterion C

Examples

- all nodes appear at least once in T
- all arcs appear at least once in T
- ...

Well-known Coverage Criteria I

Criterion AllInstructions(M,T):

For all nodes N (basic instructions or decisions)
in the CFG of M exists a path in T that contains N

Well-known Coverage Criteria II

Criterion AllTransitions(M,T):

For all arcs A in the CFG of M exists a path in T that uses A

Well-known Coverage Criteria III

Criterion AllPaths(M,T):

All possible paths ...

☹ Whenever there is a loop, T is usually infinite !

Variant: AllPaths_k(M,T).

We limit the paths through a loop to maximally k times ...

☞ we have again a finite number of paths

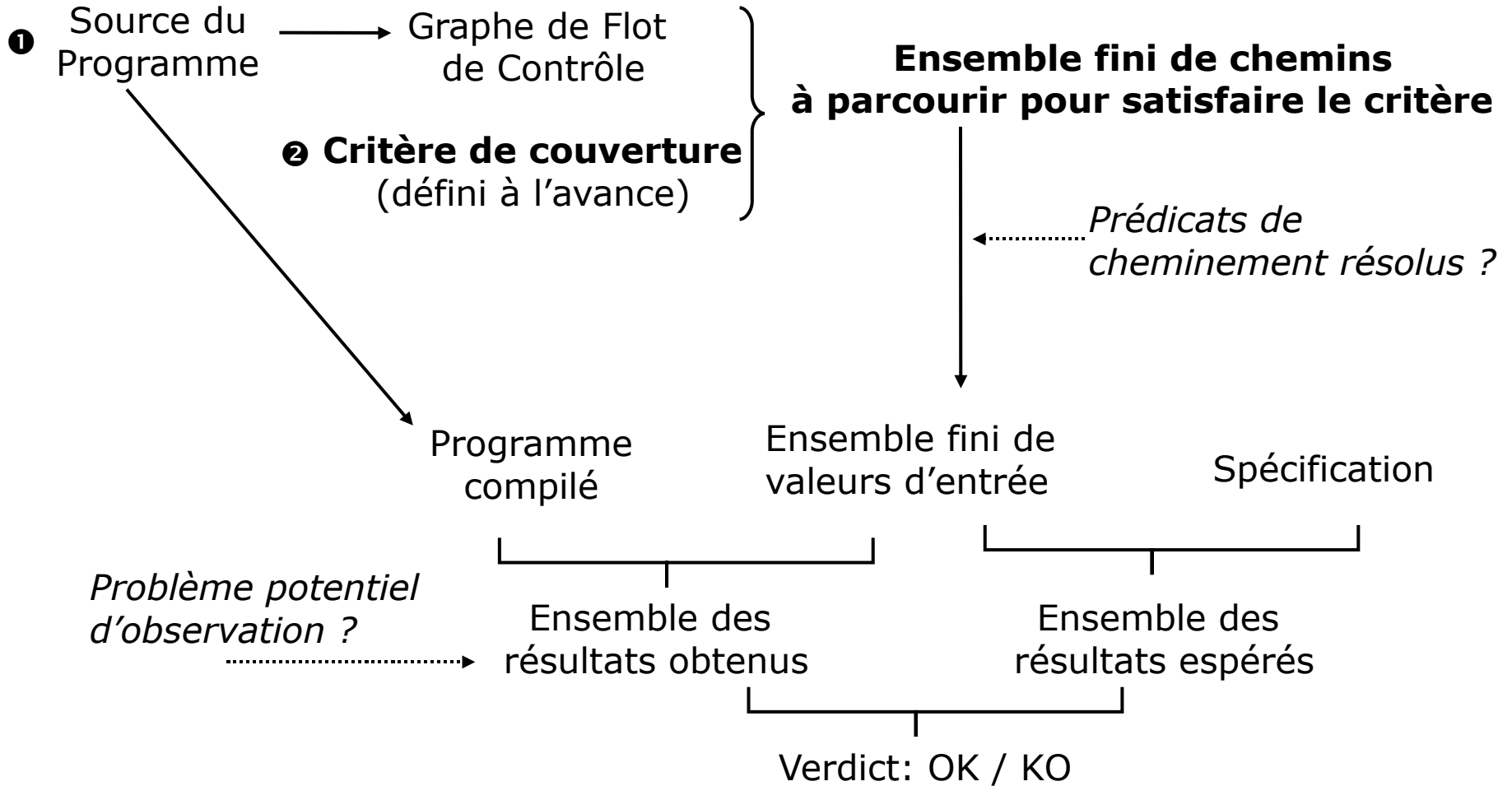
☞ the criterion is less constraining than AllTransitions_k(M,T)

A Hierarchy of Coverage Criteria

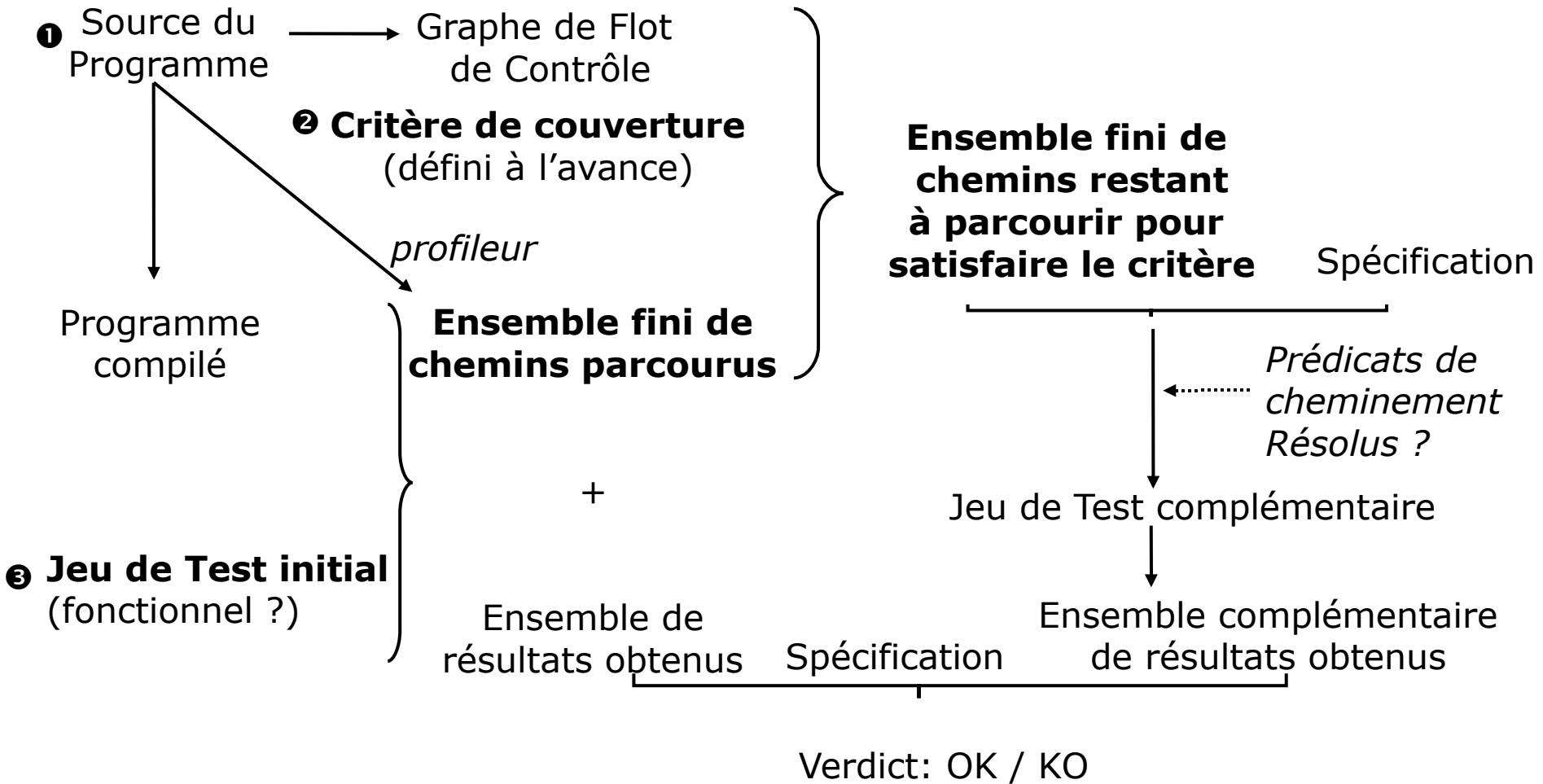
- AllPaths(M,T) \Rightarrow
 AllPaths_k(M,T) \Rightarrow
 AllTransitions(M,T) \Rightarrow
 AllInstructions(M,T)

- Each of these implications reflects a proper containment; the other way round is never true.

Using Coverage Criteria 1



Using Coverage Criteria 2



Summary

- ❑ We have developed a technique for program-based tests
- ❑ ... based on symbolic execution
- ❑ ... used in tools like JavaPathFinder-SE or Pex
- ❑ Core-Concept:
Feasible Paths in a Control Flow Graph
- ❑ Although many theoretical negative results on key properties, good practical approximations are available
- ❑ CFG based Coverage Criteria give rise to a Hierarchy