

Theorem-prover based Testing with HOL-TestGen

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M2R: Test des Systemes Informatiques
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Outline

- 1 Introduction to Sequence Testing
- 2 Foundation: State-Monads
- 3 Connecting Specifications and Test-Sequences
- 4 Test-Case Generation
- 5 Summing Up

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Motivation: Sequence Test

- So far, we have used HOL-TestGen only for test specifications of the form:

$$pre\ x \rightarrow post\ x\ (prog\ x)$$

- This seems to limit the HOL-TestGen approach to **UNIT**-tests.
- This seems to exclude testing of systems with internal state.

Motivation: Sequence Test Example I

Example: A little Bank - Account System.

internal var register : table[client, nat]integer

op deposit (c : client, no : account_no, amount:nat) : unit
pre (c,no) : dom(register)
post register'=register[(c,no) := register(c,no) + amount]

op balance (c : client, no : account_no) : int
pre (c,no) : dom(register)
post register'=register and result = register(c,no)

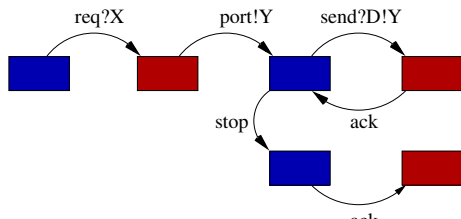
op withdraw(c : client, no : account_no, amount:nat) : unit
pre (c,no) : dom(register) and register(c,no) >= amount
post register'=register[(c,no) := register(c,no) - amount]

Motivation: Sequence Test Example II

- ❶ **Problem:** Only the public interface (i. e. the operations deposit, balance and withdraw. The internal (hidden) state is not accessible.
- ❷ **Problem:** we can therefore only control the state by *sequences* of operation calls, not just produce data and leave it to one operation call as in unit tests.
- ❸ **Problem:** The spec does not speak about the initial states.

Motivation: A Reactive System Example I

- A toy client-server system:



a channel is requested within a bound X , a channel Y is chosen by the server, the client communicates along this channel ...

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Motivation: A Reactive System Example II

Observation:

X and Y are only known at runtime!

- a test-driver is needed that manages a serialization of tests at test run time.
- ... including use an environment that keeps track of the instances of X and Y ?
- **Infrastructure:** An **observer** maps **abstract events** (req X , port Y , ...) in traces to **concrete events** (req 4, port 2, ...) in runs!

Apparent Limitations of HOL-TestGen

So far, we have used HOL-TestGen only for test specifications of the form:

$$pre\ x \rightarrow post\ x\ (prog\ x)$$

- **No Non-determinism.**

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- **No Automata** - No Tests for Sequential Behaviour.

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- HOL has lists and recursive predicates; thus sets of lists, thus languages . . .

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- No possibility to describe **reactive tests**.

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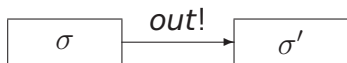
- post must indeed be executable; however, the pre-post style of specification represents a *relational* description of *prog*.
- HOL has lists and recursive predicates; thus sets of lists, thus languages . . .
- HOL has Monads. And therefore means for IO-specifications.

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The core of state-based computations:

state transitions from state σ to σ' emitting output $out!$



Such state-transitions can be modeled in various ways:

- as total functions: $\sigma \Rightarrow (o \times \sigma)$
- as partial functions: $\sigma \Rightarrow (o \times \sigma)$ option
- as relations: $\sigma \Rightarrow (o \times \sigma)$ set
- as finite series relation: $\sigma \Rightarrow (o \times \sigma)$ list
- as infinite series relation: $\sigma \Rightarrow (o \times \sigma)$ sequence
- ...

We write for this form of type scheme $(o, \sigma)Mon_\phi$ for ϕ in $\{option, set, list, \dots\}$. Note that $(o, \sigma)Mon_\phi$ in itself is not a type in the Isabelle type-system (only the instances thereof).

Background:

If a type $(\sigma, \sigma)Mon_\phi$ is completed to an algebraic structure with two operations :

$$\text{bind}_\phi :: [(\alpha, \sigma)Mon_\phi, \alpha \Rightarrow (\beta, \sigma)Mon_\phi] \Rightarrow (\beta, \sigma)Mon_\phi$$

and

$$\text{unit}_\phi :: \alpha \Rightarrow (\alpha, \sigma)Mon_\phi$$

satisfying the associativity and both neutrality laws:

❶ **associativity:**

$$\text{bind}_\phi F (\lambda y. \text{bind}_\phi G H) = \text{bind}_\phi (\text{bind}_\phi F (\lambda y. \text{bind}_\phi G) H)$$

❷ **neutrality_left:** $\text{bind}_\phi (\text{unit } F) G = G$

❸ **neutrality_right:** $\text{bind}_\phi F (\text{unit } G) = F$

What is the Relevance for Computing?

- 1 Monads talk of the sequential “glue”, the `_;` and `resulte` in imperative languages.
- 2 Monads are an abstraction of “computational structures” arranging computations based on an underlying state. This can be used in (for example):
 - 1 computations based on state
 - 2 computations based on state involving exceptions
 - 3 computations based on state involving backtracking
 - 4 computations based on state involving altogether
 - 5 ...
- 3 They have intensively used for the study of programming and specification language semantics
- 4 ... some of them are executable and were intensively used in purely functional languages such as Haskell.

A basic case for “imperative programming”: the *state-exception-Monad* Mon_{SE} based on the type $(o, \sigma)Mon_{SE} = \sigma \Rightarrow (o \times \sigma)option$.

- 1 It composes partial functions
- 2 In case a function evaluation fails (which can be viewed as “an exception occurred”), the execution is stopped and the state remains unchanged (pretty much like Java or SML),
- 3 ... otherwise the execution continues with the new state.
- 4 $unit_{SE}$ corresponds to the usual “result” operation.

We define:

1 **definition** $\text{bind}_{SE} :: [(o, \sigma) \text{MON}_{SE}, o \Rightarrow (o, \sigma) \text{MON}_{SE}] \Rightarrow (o, \sigma) \text{MON}_{SE}$
where " $\text{bind}_{SE} f g \equiv \lambda \sigma . \text{case } f \sigma \text{ of}$
 None \Rightarrow None
 | Some (out, σ') \Rightarrow g out σ' "

2 **definition** $\text{unit}_{SE} :: "o \Rightarrow (o, \sigma) \text{MON}_{SE}"$
where " $\text{unit}_{SE} e \equiv \lambda \sigma . \text{Some}(e, \sigma)"$

where we use the syntax

$$x \leftarrow f; g x$$

for $\text{bind}_{SE} f (\lambda x.g)$ and return e for $\text{unit}_{SE} e$.

Test Sequences as Monadic Compositions

In the state exception monad, we can already represent a particular form of test-driver equivalent to a *test sequence*:

- 1 A **test sequence** has the form:

$$x_1 \leftarrow put_1; x_2 \leftarrow (\lambda _ . put_2); \dots; x_n \leftarrow (\lambda _ . put_n); \\ \text{return}(post\ x_1 \ \dots \ x_n)$$

i. e. the program steps under test put_i do not depend from output of prior steps.

- 2 A **reactive test sequence** has the form:

$$x_1 \leftarrow put_1; x_2 \leftarrow put_2\ x_1; \dots; x_n \leftarrow put_n\ x_1 \ \dots \ x_{n-1}; \\ \text{return}(post\ x_1 \ \dots \ x_n)$$

i. e. the program steps under test put_i **may depend** from output of prior steps.

In order to make test-sequences amenable to HOL-TestGen, we need to represent them as data-types (so: lists of put_i). We introduce a *multi – bind* combinator taking **a list of io-stepping functions** (i. e., in particular, put_i 's) and executes them while taking exceptions into account:

consts mbind :: [ι list, $\iota \Rightarrow (o, \sigma) \text{MON}_{SE}$] $\Rightarrow (o \text{ list}, \sigma) \text{MON}_{SE}$
primrec

"mbind [] iostep $\sigma = \text{Some}([], \sigma)$ "

"mbind (a#H) iostep $\sigma =$

(**case** iostep a σ **of**

None $\Rightarrow \text{Some}([], \sigma)$

|Some (out, σ') \Rightarrow (**case** mbind H iostep σ' **of**

None $\Rightarrow \text{Some}([out], \sigma')$

|Some(outs, σ'') $\Rightarrow \text{Some}(out\#\text{outs}, \sigma''$)

Note that mbind has a slightly different behaviour than bind_{SE} wrt. exceptions!

On this level, we can now state **valid test sequences** as a test specification of the form:

$$\sigma_0 \models (os \leftarrow (\text{mbind } \iota s \text{ } ioprogram); \text{return}(post \ os))$$

where the σ_0 is the initial state and the *validity statement* $_ \models _$ means: start computation *ioprogram* in the initial state and run it sequentially over the input sequence ιs and transfer all outputs *os* to the post condition. Sequences are *valid* iff the postcondition is true. The *validity statement* is defined as follows:

definition $\text{valid} :: \sigma \Rightarrow (\text{bool}, \sigma) \text{MON}_{SE} \Rightarrow \text{bool}$ (infix \models)
where $\sigma \models m \equiv (m \ \sigma \neq \text{None} \wedge \text{fst}(\text{the } (m \ \sigma)))$

Remark: From valid test sequence, HOL-TestGen test were generated by exploring the data-structure *input sequence* ιs up to given depths k by the standard mechanisms used in unit-tests.

However, it may be convenient to specify constraints on ιs , let it be by automata, by regular expressions, by temporal formulas or by other means. In the literature, these constraints were also called **test purposes** (TP).

$$TP(\iota s) \implies \sigma_0 \models (os \leftarrow (\text{mbind } \iota s \text{ } ioprogram); \text{return}(post\ os))$$

A basic case for the “state transition system specification”: the *state-relation-Monad* Mon_{SB} based on the type $(o, \sigma)Mon_{SB} = \sigma \Rightarrow (o \times \sigma)set$.

- 1 It composes relations on states (involving input and output)
- 2 In case a function evaluation fails (which can be viewed as “an exception occurred”), the execution is stopped and the state remains unchanged (roughly like PROLOG),
- 3 ... otherwise the execution continues with the new state.
- 4 $unit_{SB}$ corresponds to the usual “result” operation.

We define:

❶ **definition** $\text{bind}_{\text{SB}}::[(\alpha, \sigma)\text{MON}_{\text{SB}}, \alpha \Rightarrow (\beta, \sigma)\text{MON}_{\text{SB}}] \Rightarrow (\beta, \sigma)\text{MON}_{\text{SB}}$
where " $\text{bind}_{\text{SB}} f g \sigma \equiv \bigcup ((\lambda(\text{out}, \sigma). (g \text{ out } \sigma)) \text{ ' } (f \sigma))$ "

❷ **definition** $\text{unit}_{\text{SB}}:: \text{o} \Rightarrow (\text{o}, \sigma)\text{MON}_{\text{SB}}$
where " $\text{unit}_{\text{SB}} e \equiv \lambda \sigma. \{(e, \sigma)\}$ "

where we use the syntax

$$x \leftarrow f;; g x$$

for $\text{bind}_{\text{SB}} f (\lambda x.g)$ and returns e for $\text{unit}_{\text{SB}} e$.

In contrast to MON_{SE} , the operations of MON_{SB} are not executable in general (**why?**).

On the other hand, concepts like pre- and post conditions can be easily expressed in terms of MON_{SB} .

Example: The post-condition of the operation `balance` is directly expressed in HOL as:

$$\text{post}(c :: \text{client}, no :: \text{account_no}) = \\ \lambda \sigma. \{(\text{result}, \sigma') \mid \sigma = \sigma' \wedge \text{result} = \text{the}(\text{register}(c, no))\}$$

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Revisiting the Little Bank Example I

Example: A Little Bank - Account System.

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pre (c,no) : dom(register)
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op balance (c : client, no : account_no) : int
pre (c,no) : dom(register)
post register'=register and result = register(c,no)

op withdraw(c : client, no : account_no, amount:nat) : unit
pre (c,no) : dom(register) and register(c,no) >= amount
post register'=register[(c,no) := register(c,no) - amount]

In order to formalize input and output implicit in such a specification, such that we can consider it uniformly as “a list of input data” and “a list of output data”, we need to **convert** the given interface into

- 1 a type for the internal state,
- 2 a uniform data-type containing all inputs, and
- 3 a uniform data-type containing all outputs.

This so-called **interface encapsulation** is a syntactic transformation and could in principle be done automatically. (Not supported yet in HOL-TestGen).

Example: Interface Encapsulation

For “Little Bank”, we have:

- 1 a type for the internal state register:

$(\text{client} \times \text{nat}) \rightarrow \text{int}$

- 2 the inputs data-type:

datatype $\text{in}_c =$ deposit client account_no nat
 | withdraw client account_no nat
 | balance client account_no

- 3 a uniform data-type containing all outputs:

datatype $\text{out}_c =$ depositO | balanceO nat | withdrawO

This so-called **interface encapsulation** is a syntactic transformation and could in principle be done automatically. (Not supported yet in HOL-TestGen).

Also pre-and post-conditions of “Little Bank” were encapsulated, such that we have now a typed state transition system on σ (= register), \mathbf{in}_c and \mathbf{out}_c .

consts precondition :: "register $\Rightarrow \mathbf{in}_c \Rightarrow \mathbf{bool}$ "

primrec

"precond σ (deposit c no m) = ((c,no) \in dom σ)"

"precond σ (balance c no) = ((c,no) \in dom σ)"

"precond σ (withdraw c no m) = ((c,no) \in dom σ
 \wedge (int m) \leq the(σ (c,no)))"

The post-condition looks as follows:

consts postcond :: "register \Rightarrow $\mathbf{in}_c \Rightarrow \mathbf{out}_c \times$ register \Rightarrow bool"

primrec

"postcond σ (deposit c no m) =
 $(\lambda (n, \text{env}'). (n = \text{depositO}$
 $\quad \wedge \sigma' = \sigma ((c, \text{no}) \mapsto \text{the}(\text{env}(c, \text{no})) + \text{int } m)))$ "

"postcond σ (balance c no) =
 $(\lambda (n, \text{env}'). (\sigma = \sigma' \wedge (\exists x. \text{balanceO } x = n$
 $\quad \wedge x = \text{nat}(\text{the}(\sigma(c, \text{no}))))))$ "

"postcond σ (withdraw c no m) =
 $(\lambda (n, \text{env}'). (n = \text{withdrawO}$
 $\quad \wedge \sigma' = \sigma ((c, \text{no}) \mapsto \text{the}(\text{env}(c, \text{no})) - \text{int } m)))$ "

The following combinators — based on the Hilbert-Operator — hold the key for a conversion between monads:

definition `impl :: [[σ, ι] \Rightarrow bool, $\iota \Rightarrow (o, \sigma) \text{MON}_{\text{SB}}$] $\Rightarrow \iota \Rightarrow (o, \sigma) \text{MON}_{\text{SE}}$`

where "impl pre post $\iota =$
 $(\lambda \sigma. \text{if pre } \sigma \ \iota$
 $\quad \text{then Some(SOME(out, } \sigma'). \text{ post } \iota \sigma(\text{out, } \sigma'))$
 $\quad \text{else arbitrary})"$

definition `strong_impl :: [[σ, ι] \Rightarrow bool, $\iota \Rightarrow (o, \sigma) \text{MON}_{\text{SB}}$] $\Rightarrow \iota \Rightarrow (o, \sigma) \text{M}$`

where "strong_impl pre post $\iota =$
 $(\lambda \sigma. \text{if pre } \sigma \ \iota$
 $\quad \text{then Some(SOME(out, } \sigma'). \text{ post } \iota \sigma(\text{out, } \sigma'))$
 $\quad \text{else None})"$

definition `is_strong_impl` :: "[$\sigma \Rightarrow \iota \Rightarrow \text{bool}$,
 $\iota \Rightarrow ('o, \sigma)\text{MON}_{\text{SB}}$,
 $\iota \Rightarrow ('o, \sigma)\text{MON}_{\text{SE}}$] $\Rightarrow \text{bool}$ "

where "is_strong_impl pre post ioprogram =

$$(\forall \sigma \iota. (\neg \text{pre } \sigma \iota \wedge \text{ioprogram } \iota \sigma = \text{None}) \vee$$

$$(\text{pre } \sigma \iota \wedge (\exists x. \text{ioprogram } \iota \sigma = \text{Some } x)))$$

This results in the following:

theorem "is_strong_impl pre post (strong_impl pre post)"

This following characterization of implementable specifications gives the key for turning specs into programs. First, we define the concept of an **implementable** specification, i. e. the fact that there is a function that maps legal input to output/state pairs, that satisfy the postcondition:

definition `implementable::[$\sigma \Rightarrow \iota \Rightarrow \text{bool}$, $\iota \Rightarrow (\text{o}, \sigma) \text{MON}_{\text{SB}}$] $\Rightarrow \text{bool}$`
where "implementable pre post =
 $(\forall \sigma \iota. \text{pre } \sigma \iota \longrightarrow (\exists \text{out } \sigma'. \text{post } \iota \sigma (\text{out}, \sigma')))$ "

This results in the following characterization theorem:

theorem `implementable_c_harn:`

" $\llbracket \text{implementable pre post; pre } \sigma \iota \rrbracket \implies$
 $\text{post } \iota \sigma (\text{the}(\text{strong_impl pre post } \iota \sigma))$ "

It is now straight-forward to “convert” our (interface encapsulated) specification into a program. Simply:

```
strong_impl precondition postcond
```

does the trick.

This program will report violations of pre- and postconditions as exceptions which were then treated at run-time.

Problem: How can we use the specification to *generate* test-sequences symbolically?

Observation: Our specification is *state-deterministic*, i. e. for each observable output, there is at most one corresponding state.

For this type of specification, we can use HOL-TestGen as follows: we state:

$$\sigma_0 \models s \leftarrow \text{mbind } S \text{ (strong_impl precondition postcond); return}(s = x)$$

as a constraint, let HOL-TestGen find solutions for x , and use these solutions in the generated test drivers.

For this, we need the generic symbolic evaluation rules:

$$(\sigma \models (s \leftarrow \text{return } x ; \text{return } (P \ s))) = P \ x$$

$$(\sigma \models (s \leftarrow \text{mbind } (a\#S) \ \text{ioprogram} ; \text{return } (P \ s))) =$$

(**case** ioprogram a σ **of**

 None $\Rightarrow (\sigma \models (\text{return } (P \ [])))$

 | Some(b, σ') $\Rightarrow (\sigma' \models (s \leftarrow \text{mbind } S \ \text{ioprogram} ; \text{return } (P \ (b\#s))))$)

The introduced case-statements were eliminated in the case-splitting of the test-case-generation phase.

... and the program specific symbolic evaluation rules (where $H = (\text{strong_implprecond postcond})$):

$$\begin{aligned}
 (\sigma \models (s \leftarrow \text{mbind } ((\text{deposit } c \text{ no } m)\#S) H; \text{return } (P \ s))) = \\
 & (\text{if } (c, \text{no}) \in \text{dom } \sigma \\
 & \quad \text{then } (\sigma((c, \text{no}) \mapsto \text{the } (\sigma(c, \text{no})) + \text{int } m)) \\
 & \quad \quad \models (s \leftarrow \text{mbind } S H; \text{return } (P(\text{depositO}\#s))) \\
 & \quad \text{else } (\sigma \models (\text{return } (P \ []))))
 \end{aligned}$$

$$\begin{aligned}
 (\sigma \models (s \leftarrow \text{mbind } ((\text{balance } c \text{ no})\#S) H; \text{return } (P \ s))) = \\
 & (\text{if } (c, \text{no}) \in \text{dom } \sigma \\
 & \quad \text{then } (\sigma \models (s \leftarrow \text{mbind } S H; \\
 & \quad \quad \text{return } (P(\text{balanceO}(\text{nat}(\text{the } (\sigma(c, \text{no}))))\#s)))) \\
 & \quad \text{else } (\sigma \models (\text{return } (P \ []))))
 \end{aligned}$$

...

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Generating all possible input sequences is far too general: there would be a lot of superfluous attempts to access a wrong account with a wrong account number, far too many initial states.

In order to reduce the number of possible input sequences, we define a *test purpose*, i. e. a predicate that constrains the number of possible input traces for one given client with an account which is initially empty.

This raises a particular *testability assumption* (at the beginning, the system is in particular initial state) which results from our lacking `init` method in our interface.

This test-purpose is formalized as follows:

consts test_purpose :: "[client, account_no, \mathbf{in}_c list] \Rightarrow bool"
primrec

"test_purpose c no [] = False"

"test_purpose c no (a#R) = (**case** R **of**
 [] \Rightarrow a = balance c no
 | a'#R' \Rightarrow (((\exists m. a = deposit c no m) \vee
 (\exists m. a = withdraw c no m)) \wedge
 test_purpose c no R))"

This test-purpose formalizes that the input sequences belong to the language expressed as regular expression:

(withdraw c no _ | deposit c no _)* balance c no

The test-specification is formalized as follows:

test_spec test_balance:

assumes account_defined: "(c,no) ∈ dom σ_0 "

and test_purpose : "test_purpose c no ιs "

and symbolic_run_yields_x :

" $\sigma_0 \models (s \leftarrow \text{mbind } \iota s \text{ (strong_impl precondition postcond);}$
return (s = x))"

shows " $\sigma_0 \models (s \leftarrow \text{mbind } \iota s \text{ SUT; return (s = x))$ "

The resulting test-theorem for $k=5$ looks follows:

1. $(\lambda a. \text{Some } 2) \models$
 $(s \leftarrow \text{mbind } [\text{balance } ?X1 \ ?X2] \text{ SUT}; \text{return } s = [\text{balanceO } 2])$
2. THYP ...
3. $(\lambda a. \text{Some } 5) \models$
 $(s \leftarrow \text{mbind}$
 $\quad [\text{deposit } ?X3 \ ?X4 \ ?X5, \text{balance } ?X3 \ ?X4]$
 $\quad \text{SUT}; \text{return } s = [\text{depositO}, \text{balanceO } (\text{nat } (5 + \text{int}$
 $\quad \ ?X5))])$
4. THYP ...
5. THYP
6. $\text{int } ?X6 \leq 7 \implies$
 $(\lambda a. \text{Some } 7) \models (s \leftarrow \text{mbind}$
 $\quad [\text{withdraw } ?X7 \ ?X8 \ ?X6, \text{balance } ?X7 \ ?X8] \text{ SUT};$
 $\quad \text{return } s = [\text{withdrawO}, \text{balanceO } (\text{nat } (7 - \text{int } ?X6))])$

Caution: Which are the underlying *Testability Hypothesis* (to be clear: *not* Test-Hypotheses) of this problem ???

Well, we made two (more or less explicit) testability hypothesis underlying our test-construction, that must be assured by other means than just running the test:

- 1 **initialization condition** (reflected by the assumption $(c, no) \in \text{dom } \sigma_0$). We must assume that a concrete user and accountnumber is defined.
- 2 **determinism condition** (reflected by the assumption that SUT has type $\mathbf{in}_c \Rightarrow (\text{out}_c, \text{register}) \text{Mon}_{SE}$). We assume that SUT behaves indeed like a function in a state in the sense of our model; we assume it is deterministic and will not have *hidden* state or engage in *hidden* state-transitions (like clocks, etc.)

Pragmatically: if we detect violations against these hypotheses during testing, we must refine our model ...

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- 1 Test-Sequence generation can be formalized as a constraint-resolution problem, too.
- 2 Reason: We have data-types (this lists and languages) and Monads in HOL
- 3 Test-drivers can be generated as well
- 4 Handling of Testability hypotheses implicit (control over the init-state, *PUT* a function in the sense of the specification)

Bibliography I