TD 8

Exercice 1

1. $\vdash \{x \leq 0\}$ `y := x+2` $\{y \leq 2\}$
2. $\vdash \{x \leq 0\}$ `x := x-1` $\{x < 0\}$
3. $\vdash \{x \geq 0\}$ `WHILE x` $\geq$ `0 DO x := x-1` $\{x = -1\}$
4. $\vdash \{a = x \wedge b = y\}$ `a := a + b; b := a - 2*b; a := a * b` $\{a = x^2 - y^2\}$
5. $\vdash \{i = 8\}$ `WHILE i < 5 DO i := 2*i` $\{i \geq 5\}$

Figure 1: Some simple proof tasks

Ex 1 (Variante 1)

$$\dfrac{x \leqslant 0 \to y \leqslant 2[y \mapsto x+2] \qquad \dfrac{}{\vdash \{y \leqslant 2[y \mapsto x+2]\}\ y := x+2 \{y \leqslant 2\}}\text{(ass)} \qquad y \leqslant 2 \to y \leqslant 2}{\vdash \{x \leqslant 0\}\ y := x+2\ \{y \leqslant 2\}}\text{(cons)}$$

avec la preuve du contrainte:

$x \leqslant 0 \to y \leqslant 2[y \mapsto x+2]$

≡ $x \leqslant 0 \longrightarrow x+2 \leqslant 2$

≡ $x \leqslant 0 \longrightarrow x \leqslant 0$

≡ True

Ex 1 Variante 2)

$$\dfrac{}{\vdash \{x \leqslant 0\}\ y := x+2 \{y \leqslant 2\}}\text{(ass)*}$$

avec la preuve du contrainte (*):

$y \leqslant 2[y \mapsto x+2]$

≡ $x+2 \leqslant 2$

≡ $x \leqslant 0$

Ex 2

$$\dfrac{x \leqslant 0 \to x<0[x \mapsto x-1]^{(*)} \qquad \dfrac{}{\vdash \{x<0[x \mapsto x-1]\}\ x:= x-1\{x<0\}}\text{(ass)} \qquad x<0 \to x<0}{\vdash \{x\leqslant 0\}\ x:= x-1\{x<0\}}\text{(cons)}$$

avec la preuve du contrainte (*):

$x \leqslant 0 \to x<0[x \mapsto x-1]$

$\equiv \quad x \leqslant 0 \to x-1<0$

$\equiv \quad x \leqslant 0 \to x<1$

$\equiv$ True

Ex 3

$$\dfrac{x \geqslant 0 \to x\geqslant -1 \qquad \dfrac{\dfrac{x\geqslant -1 \ \wedge\ x\geqslant 0 \to x\geqslant -1[x \mapsto x-1]^{(*)} \qquad \dfrac{}{\vdash \{x\geqslant -1[x \mapsto x-1]\}\ x := x-1\ \{x\geqslant -1\}}\text{(ass)} \qquad x\geqslant -1 \to x\geqslant -1}{\vdash \{x\geqslant -1 \ \wedge\ x\geqslant 0\}\ x := x-1\ \{x\geqslant -1\}}\text{(cons)}}{\vdash \{x\geqslant -1\}\ \text{WHILE } x\geqslant 0 \text{ DO } x := x-1\ \{\neg(x\geqslant 0) \ \wedge\ x\geqslant -1\}}\text{(while)} \qquad \neg(x\geqslant 0) \ \wedge\ x\geqslant -1 \to x = -1^{(**)}}{\vdash \{x\geqslant 0\}\ \text{WHILE } x\geqslant 0 \text{ DO } x := x-1\ \{x=-1\}}\text{(cons)}$$

avec la preuve du contrainte (*):

$x\geqslant -1 \ \wedge\ x\geqslant 0 \to x\geqslant -1[x \mapsto x-1]$

$\equiv x\geqslant -1 \ \wedge\ x\geqslant 0 \to x-1 \geqslant -1$

$\equiv x\geqslant 0 \to x \geqslant 0$

$\equiv$ True

et la preuve du contrainte (**)

$\neg(x\geqslant 0) \ \wedge\ x\geqslant -1 \ \to x = -1$

$\equiv x < 0 \ \wedge\ x\geqslant -1 \ \to x = -1$

$\equiv$ True

Ex 4

(aff)
|- {B} a:=a+b {(A[a ↦ a*b])[b ↦ a-2*b]}

(ass)
|- {(A[a ↦ a*b])[b ↦ a-2*b]} b:=a-2*b{A[a ↦ a*b]}

(seq)
|- {B} a:=a+b; b:=a-2*b {A[a ↦ a*b]}

(ass)
|- {A[a ↦ a*b]} a:=a*b {A}

(seq)
|- {B} a:=a+b; b:=a-2*b; a:=a*b {A}

a=x ∧ b=y →(*) B   A → A

(cons)
|- {a=x ∧ b=y} a:=a+b; b:=a-2*b; a:=a*b {A}

Abbreviations:

A ≡ a=x^2- y^2

B ≡ ((A[a ↦ a*b])[b ↦ a-2*b])[a ↦ a+b]

et la preuve du contrainte (*)

a=x ∧ b=y → B

≡ a=x ∧ b=y → ((A[a ↦ a*b])[b ↦ a-2*b])[a ↦ a+b])

≡ a=x ∧ b=y → ((a*b=x^2- y^2)[b ↦ a-2*b])[a ↦ a+b])

≡ a=x ∧ b=y → (a*(a-2*b)=x^2- y^2)[a ↦ a+b]

≡ a=x ∧ b=y → (a+b)*((a+b)-2*b)=x^2- y^2

≡ a=x ∧ b=y → a^2 + 2ab + b^2 - 2ab - 2*b^2 = x^2 - y^2

≡ a=x ∧ b=y → a^2 - b^2 = x^2 - y^2

≡ True

Ex 5

(falseE)
|- {i = 8 ∧ i < 5} ... {i=8}

(while)
|- {i = 8} WHILE i < 5 DO ... {i⩾5 ∧ i=8}

i=8 → i=8   i⩾5 ∧ i=8 → i⩾5

(cons)
|- {i = 8} WHILE i < 5 DO ... {i⩾5}