Typed Iterators for XML

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Dealing with XML

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- ⇒ Tree organized data
- ⇒ Pervasive (XHTML, Ajax, Web Services, ...)

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- ⇒ Regular tree grammars (a.k.a. regular types)
- ⇒ Describe sets of documents *very precisely*

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Programs ? (iterators)

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- ⇒ "remove every <a> element ocurring in the input"
- ⇒ "convert an xhtml document from *transitional* to *strict*"

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<foo>[ a* b* ]   ⇝ <foo>[ b* ]

<bar>[ b a* b* ] ⇝ <bar>[ b+ ]

<baz>[ c* b?  ]  ⇝ <baz>[ c* b?  ]
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Neither parametric polymorphism (à la ML) nor regular expression types (à la XDuce/ℂDuce) are up to the task

Example: List concatenation

type of ℓ_1	type of ℓ_2	type of (concat ℓ_1 ℓ_2)
[Int*]	[Bool+]	[Int* Bool+]
[Int]	[Int* Char?]	[Int+ Char?]
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val concat : ...
let ℓ = concat ℓ1  ℓ2 in ...
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must have the same type

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val concat : [ Any* ] → [ Any* ] → [ Any* ]
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ℓ has type [Any*]

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$\underbrace{\texttt{concat}\ \ell_1\ \ \ell_2}_{\text{type (concat } \ell_1\ \ \ell_2)}$

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type (concat ℓ_1 ℓ_2)

“Execute the transformation at the type level”

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type t = [] | [ a t b ]
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flatten t
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$\texttt{flatten t} \rightsquigarrow \{ \texttt{[a}^n \texttt{ b}^n \texttt{]} \mid n \geq 0 \}$

- In general there isn't a best regular approximation

```
[ (a | b)* ]
[ a* b* ]
[] | [ a+ b+ ] ...
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- The language must be expressive enough to express flattening, reversal, XPath,...

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type t = [] | [ a t b ]
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[] | [ a+ b+ ] ...
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Contributions : Filters

Small sub-language of combinators :

- grafted into an host language
  - ⇒ are used to define XML transformations
  - ⇒ the host language is used for non-XML stuff
  - ⇒ implementation with ℂDuce as host
- can express XPath and XSLT-like transformations
- is precisely typed
- relies on some type annotations for "non-regular cases"
  - ⇒ annotations are sparse and well-localized
  - ⇒ completeness result up-to annotations

Host language

Filters : iterate expressions of the host language over a data-structure (list, tree, XML document,. . .)

Requirements for the host language :

- type algebra with a product constructor

Filters

Definition (filter)

A filter is a regular (possibly infinite) production of :

$$
\begin{array}{rcll}
f & ::= & e & \text{(expression of the host language)} \\
& | & p \rightarrow f & \text{(pattern)} \\
& | & (f, f) & \text{(product)} \\
& | & f | f & \text{(union)} \\
& | & f ; f & \text{(composition)}
\end{array}
$$

$$f(v) \leadsto r$$

with some restrictions

Examples

$$id = x \to x$$

Examples

$$id \;=\; x \rightarrow x$$

$$id(\texttt{"foo"}) \rightsquigarrow$$

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$$(x \rightarrow x)(\texttt{"foo"})$$

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x

x	"foo"

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$id($`"foo"`$) \leadsto$

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$bump(\texttt{[1 2 3]}) \leadsto$

$\quad bump(\texttt{(1,(2,(3,[])))})$

Examples

$$id = x \to x$$

$$bump = [\,] \to [\,] \mid (x \to x + 1, bump)$$

$bump(\texttt{[1 2 3]}) \rightsquigarrow$

$\texttt{(2,(}bump\texttt{(2,(3,[]))))}$

x	1

Examples

$$
\begin{aligned}
id &= x \to x \\[2em]
bump &= [\,] \to [\,] \\
&\mid\; (x \to x + 1, bump)
\end{aligned}
$$

$bump([1\ 2\ 3]) \rightsquigarrow$

$(2,(3,(bump(3,[]))))$

x	2

Examples

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$bump($`[1 2 3]`$) \leadsto$

`(2,(3,(4,(`$bump$` []))))`

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$concat(([1\ 2\ 3],[4\ 5])) \leadsto$

$concat((1,(2,(3,[]))),(4,(5,[])))$

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$(1,(2,(3,\ {\color{red}y}\ ))))$

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`(1,(2,(3,(4,(5,[])))))`

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Example:

$$bad = x \to (x,x);bad$$

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Example:

$bad \; = \; x \rightarrow (x, x)\mathbf{;}bad \quad bad(\texttt{(0,0)})$

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$bad = x \to (x,x);bad \quad bad(((0,0),(0,0)))$

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Example:

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Theorem

The evaluation of a filter on a finite value terminates.

Typing example (1/2)

$$\begin{array}{lll} \mathit{bump} & = & [\,] \to [\,] \\ & \mid & (x \to x + \mathit{1}, \mathit{bump}) \end{array} \qquad t = \texttt{[]} \,|\, (\texttt{Int}, t) \quad (\equiv \texttt{[Int*]})$$

Typing example (1/2)

$$bump \;=\; [\,] \to [\,] \quad | \quad (x \to x + 1, bump) \qquad t = \texttt{[]} | (\texttt{Int}, t) \;\; (\equiv \texttt{[Int*]})$$

Let us compute $bump(t)$:

$$bump(t) \;=$$

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Let us compute $bump(t)$:

$$bump(t) \;=\; \texttt{[]} \mid (\texttt{Int}, bump(t))$$

Typing example (1/2)

$$bump \; = \; [\,] \rightarrow [\,] \qquad\qquad t = \texttt{[]} \,|\, (\texttt{Int}, t) \;\; (\equiv \texttt{[Int*]})$$
$$\qquad\quad | \;\; (x \rightarrow x + 1, bump)$$

Let us compute $bump(t)$:

$$\begin{aligned} bump(t) &= \texttt{[]} \mid (\texttt{Int}, bump(t)) \\ &\equiv \texttt{[Int*]} \end{aligned}$$

Typing example (2/2)

flatten $t = \texttt{[]} \mid \texttt{[}\ a\ t\ b\ \texttt{]}$

"flattens nested lists"

Typing example (2/2)

flatten $t = $ `[] | [ ` a t b ` ]`

"flattens nested lists"

Let us compute $\mathit{flatten}(t)$:

$$\mathit{flatten}(t) \ =$$

Typing example (2/2)

flatten $t = \texttt{[]} \mid \texttt{[}\ a\ t\ b\ \texttt{]}$

"flattens nested lists"

Let us compute $\mathit{flatten}(t)$:

$$\mathit{flatten}(t) = \texttt{[]}$$

Typing example (2/2)

flatten $t = [] \mid [\ a\ t\ b\ ]$

"flattens nested lists"

Let us compute *flatten*(t):

$$\begin{aligned} flatten(t) &= [] \\ &\mid [\ \mathtt{a}\ \mathtt{b}\ ] \end{aligned}$$

Typing example (2/2)

flatten $t = \texttt{[]} \mid \texttt{[} \ a \ t \ b \ \texttt{]}$

"flattens nested lists"

Let us compute $\mathit{flatten}(t)$:

$$\begin{aligned} \mathit{flatten}(t) &= \texttt{[]} \\ &\mid \texttt{[a b]} \\ &\mid \texttt{[a a b b]} \end{aligned}$$

Typing example (2/2)

flatten $t = $ `[]` | `[` a t b `]`

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 & | & \ldots \\
 & \leq & \texttt{[(a|b)*]}
\end{array}
$$

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&\mid \ \ldots \\
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Theorem

Subject reduction for filters

Typing algorithm

- Possible to erase the subsumption rule "almost everywhere"
  - ⇒ The subsumption is only necessary to type the left-hand side of a "**;**": this is where we put the annotation.

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- Algorithm is complete up-to annotations
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Implementation and examples

Added the filters as a sublanguage of $\mathbb{C}$Duce:

Definition (Concrete syntax)

$$
\begin{array}{rcll}
f & ::= & e \mid p\ \texttt{->}f \mid f\ \texttt{;}f \mid \texttt{(}f\texttt{,}f\texttt{)} \mid f\texttt{|}f & \text{unchanged} \\
 & \mid & \texttt{<}f\ f\texttt{>}f & \text{xml} \\
 & \mid & \texttt{let filter}\ \underline{x}\texttt{=}f\ [\ \texttt{and}\ \underline{x}\texttt{=}f \ldots] & \text{binding} \\
 & \mid & \underline{x} & \text{variable} \\
e & ::= & \ldots \mid \texttt{apply}\ f\ \texttt{to}\ e\ [\ \texttt{where}\ a\ ] & \text{application} \\
a & ::= & \underline{x}\texttt{=}\{t_1\texttt{,} \ldots \texttt{,}t_n\}\ [\ \texttt{and}\ a\ ] & \text{annotation}
\end{array}
$$

Examples (1)

Pattern matching:

```
match e with
 | p1 -> e1
     ...
 | pn -> en
```

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Pattern matching:

```
apply (p1 -> e1) | ... | (pn -> en) to e
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Tree mapping:

```
let filter up = < ( `section -> `chapter
                  | `subsection -> `section
                  | `paragraph -> `subsection
                  |  x -> x ) >uplist
               | x -> x
and filter uplist = [] -> [] | (up,uplist)
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Pattern matching:

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apply (p1 -> e1) | ... | (pn -> en) to e
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Tree mapping:

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               | x -> x
and filter uplist = [] -> [] | (up,uplist)
```

If e : <doc>[<section>[(<subsection>[Char+]|Char)*]+] then:

```
apply up  to e
```

has type: <doc>[<chapter>[(<section>[Char+]|Char)*]+]

Examples (2)

```
let filter flatten =  [] -> []
        | ([Any*] -> flatten, flatten);concat
        | (x->x, flatten)

type t = [ ‘a t ‘b ] | []
type s = [ ‘c s ‘d ] | [‘c ‘d ]

let u : t = ...
let v : s = ...

apply flatten to u where {| flatten = { [ (‘a|‘b)* ] } |}
apply flatten to v where {| flatten = { [ (‘c|‘d)+ ] } |}
```

XPath encoding

`//a/b` : “returns exactly all `<b>`s which are under an `<a>`”

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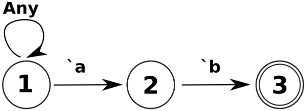

XPath encoding

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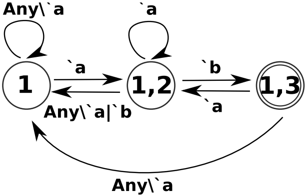

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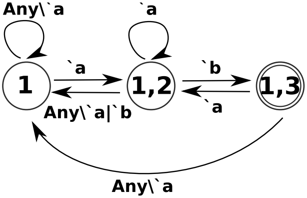


```
let filter f1 =
      <(‘a -> ‘a)> I_f12; <_>x -> x
    | <( x -> x )> I_f1;  <_>x -> x
    | _ -> []

and filter f12 =
<(‘a -> ‘a)> I_f12; <_> x -> x
|  res -> <(‘b -> ‘b)>I_f13; <_> x -> (res,x)
| <(x -> x)> I_f1; <_> x -> x
| _ -> []

and filter f13 =
    <(‘a -> ‘a)> I_f12; <_> x -> x
  | <(x -> x)> I_f1; <_> x -> x
  | _ -> []

and filter I_f1 =
   [] -> []
 | (f1,I_f1);concat
```

XPath encoding and typing

- XPath encoding:
  - `self`, `child` and `descendant-or-self` axes
  - handles some predicates by rewriting to patterns
  - respects XPath semantics (document order, no duplicates,...)

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- XPath encoding:
  - `self`, `child` and `descendant-or-self` axes
  - handles some predicates by rewriting to patterns
  - respects XPath semantics (document order, no duplicates,...)
- XPath typing:
  - Only need *one* annotation
  - Use of an ad-hoc algorithm to compute the annotation

⇒ Automatic type inference for a non-trivial subset of XPath

Conclusion

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Filters :

- provide a way to define
  - ⇒ expressive ($\mathbb{C}$Duce iterators, XSLT, XPath,. . .)
  - ⇒ precisely typed (esp. typing products, see paper)
  - ⇒ modular

  transformations within an host language
- implementation
  - ⇒ integrated with $\mathbb{C}$Duce
  - ⇒ encoding and automatic typing of an XPath fragment
  - ⇒ other syntactical constructs ("parametrized filters", . . .)

Future work

- How to infer annotations in the general case ?

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Real typing rules

$$\frac{\texttt{type}(\Gamma, e) = s}{\Gamma \vdash e(t) = s} \qquad \frac{\Gamma \cup t/p \vdash f(t) = s \qquad t \leq \wr p \int \mathbin{\&} \wr f \int}{\Gamma \vdash (p \to f)(t) = s}$$

$$\frac{\pi(t) = \{(t_1^1, t_2^1), \ldots, (t_1^n, t_2^n)\} \qquad \Gamma \vdash f_1(t_1^i) = s_1^i \qquad \Gamma \vdash f_2(t_2^i) = s_2^i}{\Gamma \vdash (f_1, f_2)(t) = \bigvee_{i \in 1..n} (s_1^i, s_2^i)}$$

$$\frac{\begin{array}{l} t \leq \wr f_1 \int \mathbin{|} \wr f_2 \int \\ t_1 = t \mathbin{\&} \wr f_1 \int \\ t_2 = t \setminus \wr f_1 \int \qquad \Gamma \vdash f_1(t_1) = s_1 \qquad \Gamma \vdash f_2(t_2) = s_2 \end{array}}{\Gamma \vdash (f_1 | f_2)(t) = \bigvee_{\{i | t_i \neq \mathsf{Empty}\}} s_i}$$

$$\frac{\begin{array}{l} t \leq \wr f_1 \int \\ s_1 \leq \wr f_2 \int \qquad \Gamma \vdash f_1(t) = s_1 \qquad \Gamma \vdash f_2(s_1) = s_2 \end{array}}{\Gamma \vdash (f_1 ; f_2)(t) = s_2}$$

$$\frac{\Gamma \vdash e(t) = s' \quad s' \leq s}{}$$

Typing the union

$$bump \;=\; [\,] \rightarrow [\,] \;\mid\; \left(x \rightarrow x + 1, bump\right) \qquad t \;=\; \texttt{[Int]} \,\big|\, \left(\texttt{Int}, t\right) \quad (\equiv \texttt{[Int+]}\,)$$

Typing the union

$$
\begin{array}{rcl}
bump & = & [\,] \to [\,] \\
& | & (x \to x + 1, bump)[\texttt{Int+}]\,)
\end{array}
\qquad
t \;=\; [\texttt{Int}] | (\texttt{Int}, t) \qquad (\equiv
$$

$$
\dfrac{\dfrac{\varnothing \vdash [\,]([\,]) = [\,]}{\varnothing \vdash [\,] \to [\,]([\,]) = [\,]} \quad \dfrac{\dfrac{\{x : \texttt{Int}\} \vdash x(\texttt{Int}) = \texttt{Int}}{\varnothing \vdash x \to x + 1(\texttt{Int}) = \texttt{Int}} \quad \dfrac{\vdots}{\varnothing \vdash bump(t) =}}{\varnothing \vdash (x \to x + 1, bump)((\texttt{Int}, t)) = (\texttt{Int}, s}}{\varnothing \vdash bump(t) = s}
$$

With the simple typing rule:

$$s \;=\; [\texttt{Int*}]$$

With the precise typing rule:

$$s \;=\; [\texttt{Int+}]$$

Product decomposition

In general, if $t \leq$ (Any,Any), $t = (t_1^1, t_2^1) | \ldots | (t_1^n, t_2^n)$ for some n.

Problem: there is more than one way to decompose t.

The decomposition affects the properties of the type-system.

Product decomposition

In general, if $t \leq (\texttt{Any},\texttt{Any})$, $t = (t_1^1,t_2^1)| \ldots |(t_1^n,t_2^n)$ for some n.

Problem: there is more than one way to decompose t.

The decomposition affects the properties of the type-system.

Consider:

$$f_1 = 0..3 \to \texttt{A} | 4..7 \to \texttt{B} \qquad f_2 = 0..4 \to \texttt{C} | 0..6 \to \texttt{D} \qquad f = (f_1,f_2)$$

and the types t and s:

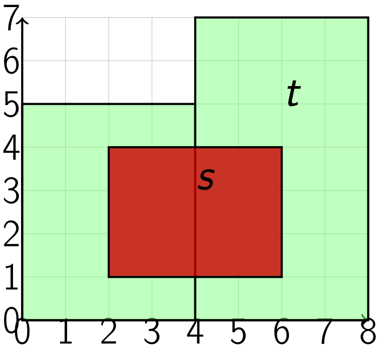
$$t = (0..3,0..4)|(4..7,0..6) \qquad s = (2..5,1..3)$$

We can prove that:

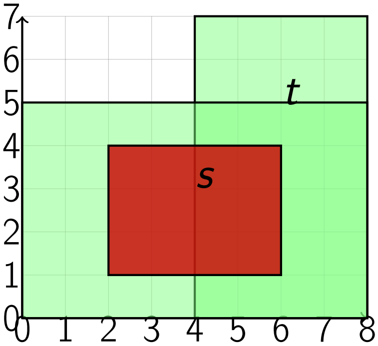
$$\varnothing \vdash f(t) = (\texttt{A},\texttt{C})|(\texttt{B},\texttt{D})$$

but also:

Maximal product decomposition



Two disjoint components: **(**0..3**,**0..4**)** and **(**4..7**,**0..6**)**. *s* overlaps both.



Two non-disjoint components: **(**0..7**,**0..4**)** and **(**4..7**,**0..6**)**. *s* is included in **(**0..7**,**0..4**)**.