université PARIS-SACLAY | FACULTÉ DES SCIENCES D'ORSAY

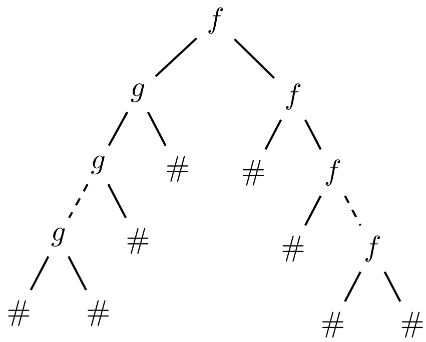
semaine du 16 janvier 2023

Automates et Applications
M1–MPRI Orsay

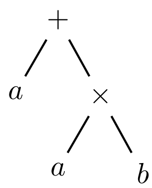
TD nº 2

1 Tree automata

1. Give a bottom-up tree automaton recognizing the language of all trees on $\mathcal{T}(\Sigma)$, with $\Sigma = \{f^2, g^2, \#\}$ for which the following holds :
   — a path with symbol g has two children f
   — the root is an g.
2. Give a bottom-up tree automaton which recognizes all trees over $\mathcal{T}(\Sigma)$, avec $\Sigma = \{f^2, g^2, \#\}$ with the following shape :



3. Let $\Sigma = \{+^2, \times^2, a, b\}$, consider the language on on Σ of non ambiguous arithmetic expressions (that is, expressions which do not require parenthesis). For instance the tree :



   is non ambiguous. Is this language regular ? Is it top-down recognizable ?
4. Let T be a tree language. Define leaves(t) as the *word* formed by taking all the symbols with arity 0 encountered during a depth first, left to right traversal of the tree. Formally, such a word can be written as the longuest sequence of symbols : $t(l_1), t(l_2), \ldots, t(l_k)$ for which the following hold :
   — $\forall i, |t(l_i)| = 0$
   — $\forall i, j, i < j \Rightarrow l_i <_{\text{lex}} l_j$ where $<_{\text{lex}}$ is the lexicographic order on paths.
   We define leaves$(T) = \{\text{leaves}(t) \mid t \in T\}$. Show that even in the case where T is a recognizable tree language, leaves(T) is not necessarily a regular word language.
5. Show that the set of perfect binary trees over $\Sigma = \{f^2, a\}$ is not regular. A perfect binary tree is a tree for which all leaves have the same depth (that is, all paths to leafs have the same length).
6. Let T be a language over $\Sigma = \{f^2, a, b\}$. Consider the congruence $f(x, y) \equiv f(y, x)$ for $x, y \in \mathcal{T}(\Sigma)$. Show that if T is regular, the set $T' = \{t' \mid \exists t \in T t \equiv t'\}$ is regular.