Unbiased Online Recurrent Optimization

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Supervised by

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GT Deepnet

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UORO

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- ... at the same cost as just running the system,
  ... but with (reasonable) additional noise.
- Not relevant if you have a large number of short training sequences
- Relevant if you have a small number of long training sequences, or only one training sequence (e.g., life).

Algorithms to train dynamical systems

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RTRL	$O(p^2)$	No	No
?	$O(p)$	No	No

Recurrent nets as dynamical systems

Problem: how to train a dynamical system defined by

$$s_{t+1} = F(x_{t+1}, s_t, \theta)$$

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Examples: RNNs, LSTMs, GRUs, ...

Simple strategy: online gradient descent over the loss at time t,

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Standard approach to compute $\frac{\partial \ell_t}{\partial \theta}$: backpropagation through time (BPTT).
Problems: goes back in time, keep track of past history...

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Same forward-backward structure in many problems: hidden Markov models (EM), reinforcement learning and optimal control (Bellman equations)...

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Algorithms that go forward in time must maintain the gradient of the current state with respect to the parameters:

$$G_t := \frac{\partial s_t}{\partial \theta}$$

and then compute the gradient of the loss via the chain rule

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Sometimes, cannot even store G_t.

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- The estimates are noisy but unbiased $\implies$ over time the parameter evolves in the correct direction.

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Problem: Even if $\tilde{G}_t = \tilde{s}_t \tilde{\theta}_t^\top$ is rank-one, $\tilde{G}_{t+1}$ is full-rank again.

The rank-one trick

Proposition

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Then $(A\nu)\,\nu^\top$ is an unbiased, rank-one estimate of A:

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Proof: $\mathbb{E}\,\nu\,\nu^\top = I$.

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Corollary: UORO is unbiased, $\mathbb{E}\tilde{s}_t\tilde{\theta}_t^\top = G_t$.

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Then $\theta \leftarrow \theta - \alpha \frac{\partial \ell_t}{\partial s_t} \cdot \tilde{s}_t \cdot \tilde{\theta}_t^\top - \frac{\partial \ell_t}{\partial \theta}\Big|_{s_t}$ is an unbiased gradient descent on θ.

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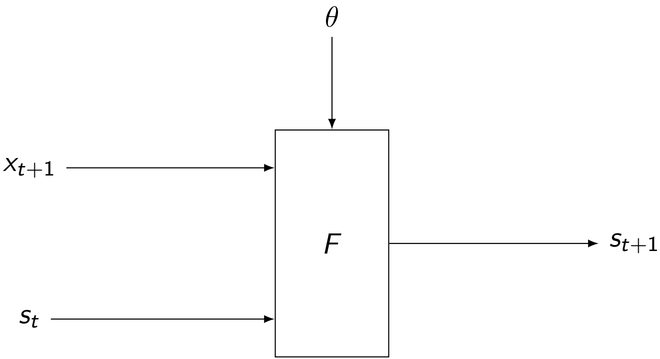
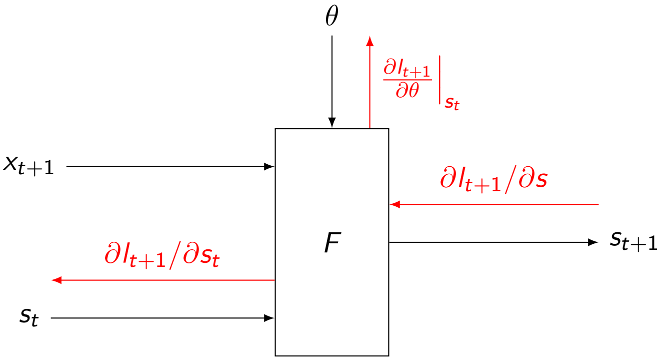
For RNNs, LSTMs, GRUs: same computational cost as running the RNN itself.

UORO graphically



UORO graphically

- Backpropagate the loss.



Gradient estimate: $\tilde{g}_{t+1} = \left(\frac{\partial l_{t+1}}{\partial s_t} \cdot \tilde{s}_t\right) \tilde{\theta}_t^\top + \left.\frac{\partial l_{t+1}}{\partial \theta}\right|_{s_t}$

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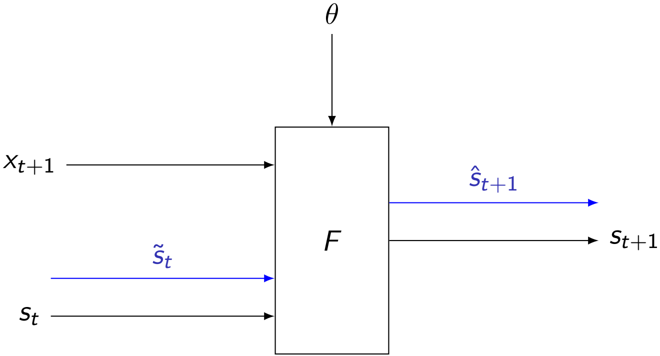
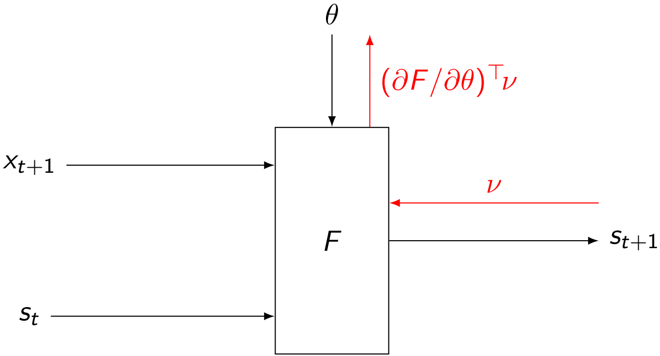
- Backpropagate the loss.
- Forward differentiate $\tilde{s}_t$ (left multiplication by Jacobian).



UORO graphically

- Backpropagate the loss.
- Forward differentiate $\tilde{s}_t$ (left multiplication by Jacobian).
- Backpropagate random signs.



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- $\tilde{\theta}_{t+1} = \frac{1}{\rho_0} \, \tilde{\theta}_t + \frac{1}{\rho_1} \, \left(\frac{\partial F}{\partial \theta}\right)^{\top} \nu$

UORO: Results

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  - distant brackets
  - $a^n b^n$ problem for long dependencies

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- Datasets:
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  - $a^n b^n$ problem for long dependencies
- Model: LSTMs or GRUs, log-loss over probabilistic prediction of next character for character predictions.
- Baselines: Comparision with Truncated backpropagation through time

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- Linear model.
- Learn a parameter that has a positive influence in the short term, but a negative influence in the long go.

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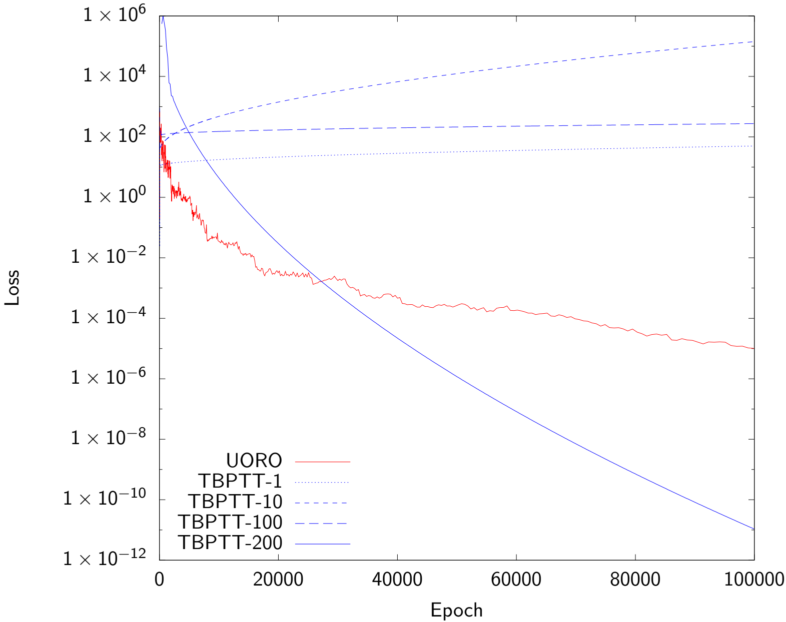
- Learn a parameter that has a positive influence in the short term, but a negative influence in the long go.
- UORO succeeds in balancing dependencies correctly.
- TBPTT fails even when truncation is far above the inherent time range of the model.

Influence balancing, positive influence on 10 steps, negative influence on the 13 next steps.



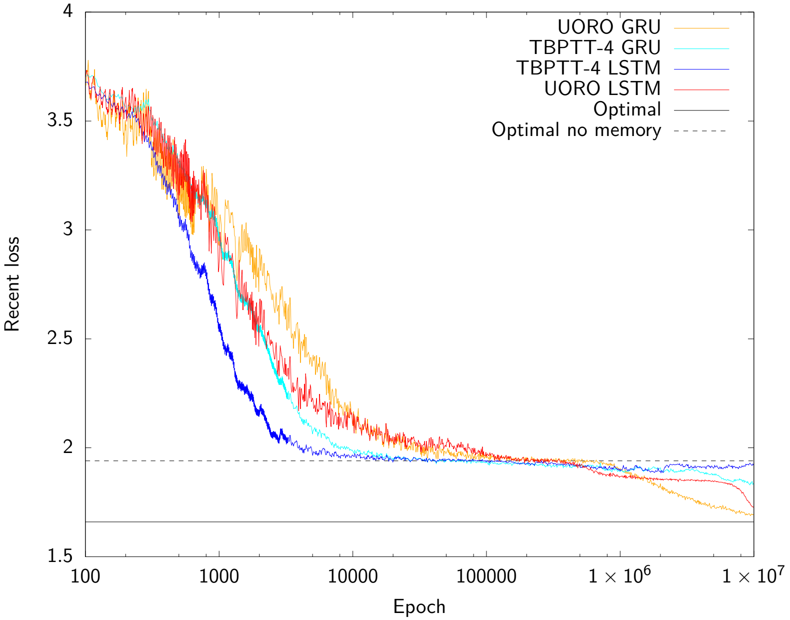
Distant brackets

Sample:

```
[a]lmsle[a]
[c]kopas[c]
[d]llses[d]
[l]oksse[l]
```

Correct prediction requires storing the first character between bracket.

Learning curves on distant brackets



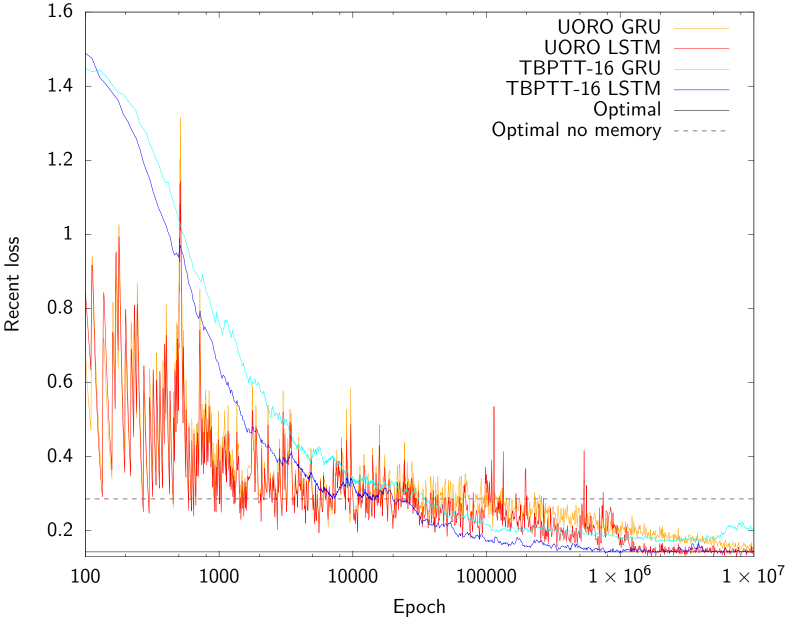
$a^n b^n$

Sample:

```
aaaaa
bbbbb
a
b
aaaaaaaaaaaaaaaa
bbbbbbbbbbbbbbbb
```

Correct prediction requires counting the number of a's.

Learning curves on $a^n b^n$:



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