Periodic Metro Scheduling

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Introduction

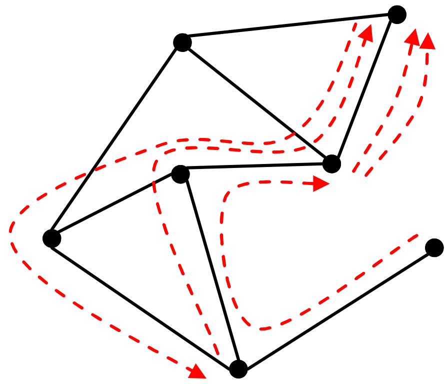
- Periodic Metro Scheduling (PMS): Maximize the safety distance between trains that use the same edge.
- Path Coloring (PC): Minimize the number of wavelengths used to accommodate a set of communication requests in an all-optical network.

Outline

We study PMS in the following topologies:

- Chains and spiders: we present exact algorithms that utilize exact algorithms for PC as subroutines.
- Special classes of rings and trees: we prove NP-hardness results by reduction from PC, and present $\frac{1}{\rho}\frac{L}{L+1}$-approximation algorithms.
- General rings: we present an $\frac{1}{6}$-approximation algorithm.

Periodic Metro Scheduling (PMS)

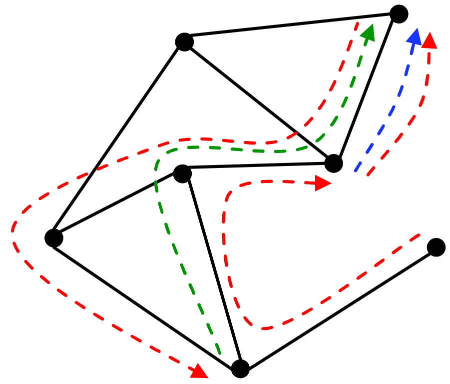


- Maximize safety distance: the minimum time distance between successive trains that pass from the same edge in the same direction.
- This results in a safer and more delay-tolerant system.

PMS in more detail

- We are given a railway network graph with edges representing directed railway lines.
- We are given a set of simple paths representing train routes.
- All trains move at the same speed. The travel time for each edge is given.
- All routes are re-executed periodically (e.g. every hour). The period T is given.
- The waiting time at stations is negligible.
- Goal: maximize safety distance.

Path Coloring (directed)

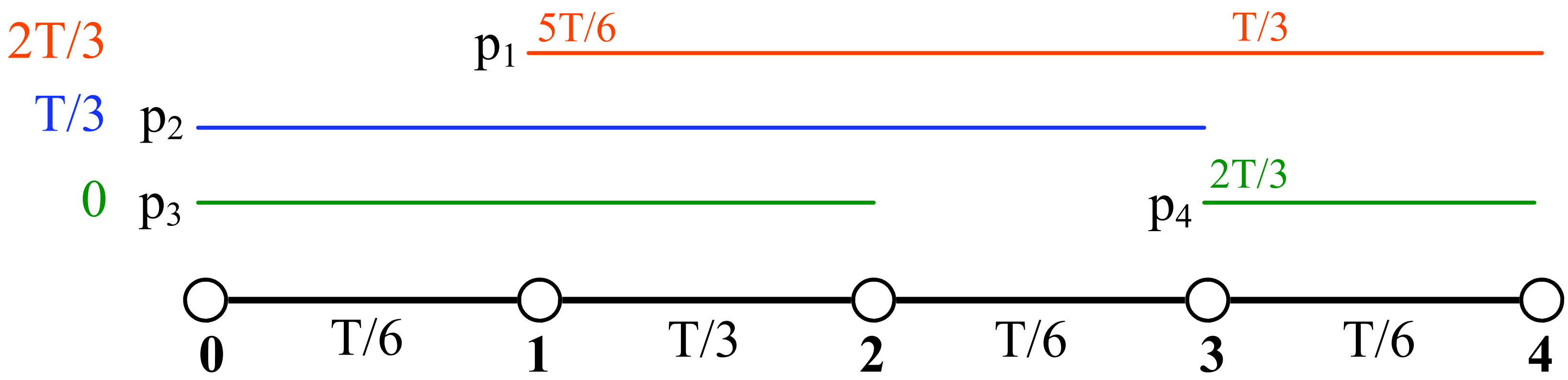


- Assign different colors to paths sharing a directed edge.
- Minimize the number of colors used.

An algorithm for PMS in chains

- Solve the corresponding PC instance with L colors, where L is the maximum load on any edge.
- Divide the period into L timeslots: $0, \frac{T}{L}, 2\frac{T}{L}, \ldots, (L-1)\frac{T}{L}$. Assign the i-th timeslot to paths colored with the i-th color.
- For each path, set starting time $s\_time = timeslot +$ (starting dist from 0).
- Upper bound: safety distance $\leq \frac{T}{L}$.
- The solution obtained is optimal.

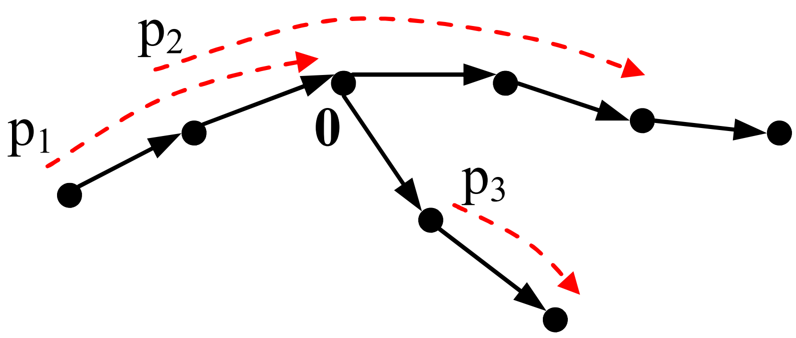
An example for chains



Stars and spiders

- Solve the corresponding PC instance using L colors and assign timeslots accordingly.
- Compute starting times:
  - For paths passing through 0, or directed towards it, set $s\_time = timeslot - (\text{starting dist from } 0)$
  - For paths directed away from 0, set

    $s\_time = timeslot + (\text{starting dist from } 0)$



PMS in rings is NP-hard

- PC is NP-hard [Garey, Johnson, Miller, Papadimitriou, 1980].
- Given a PC instance with n nodes, construct a PMS instance with time distances 1 and $T = n$.
- A coloring with k colors yields a schedule with safety distance $\frac{T}{k}$ by synchronizing each route with respect to an arbitrarily chosen node.

PMS in rings is NP-hard (cont'd)

- A schedule with distance $\frac{T}{k}$ yields a k-coloring by following the reverse procedure:
  - For each route r compute the value

    $$t'(r) = s\_time(r) - (\text{starting dist from } 0)$$

    Assign to r the color i, where i is the maximum integer s.t. $i \cdot \frac{T}{k} \leq t'$.
  - For two overlapping routes r and r', the difference between $t'(r)$ and $t'(r')$ is $\geq \frac{T}{k}$. Therefore, the coloring is valid.

Relation between PC and PMS in rings

- $\text{OPT}_{\text{PMS}} < \frac{T}{\text{OPT}_{\text{PC}} - 1}$
  otherwise, we would obtain a coloring with $\text{OPT}_{\text{PC}} - 1$ colors.

- $\text{OPT}_{\text{PMS}} \leq \frac{T}{L}$
  upper bound.

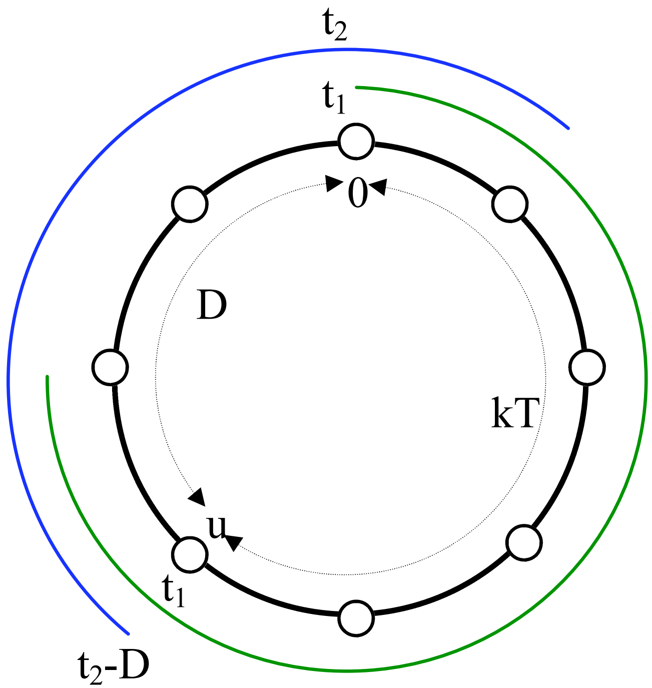
- $\text{SOL}_{\text{PMS}} = \frac{T}{\text{SOL}_{\text{PC}}}$
  by using SOL_{PC} timeslots.

PMS in rings ($C \equiv_T 0$)

- Theorem: A ρ-approximation for PC yields a $\frac{1}{\rho}\frac{L}{L+1}$-approximation for PMS.
- Corollary: There is a $\frac{2}{3}\frac{L}{L+1}$-approximation algorithm [Karapetian, 1980] and a $0.73\frac{L}{L+1}$-approximation randomized algorithm [Kumar, 1998] for PMS in rings where $C \equiv_T 0$.

Rings ($C \not\equiv_T 0$)

- When $C \not\equiv_T 0$, $\frac{T}{\mathrm{OPT_{PC}}}$ may not be achievable.



- The time distance of the two routes is the minimum of their time distances at nodes 0 and u.

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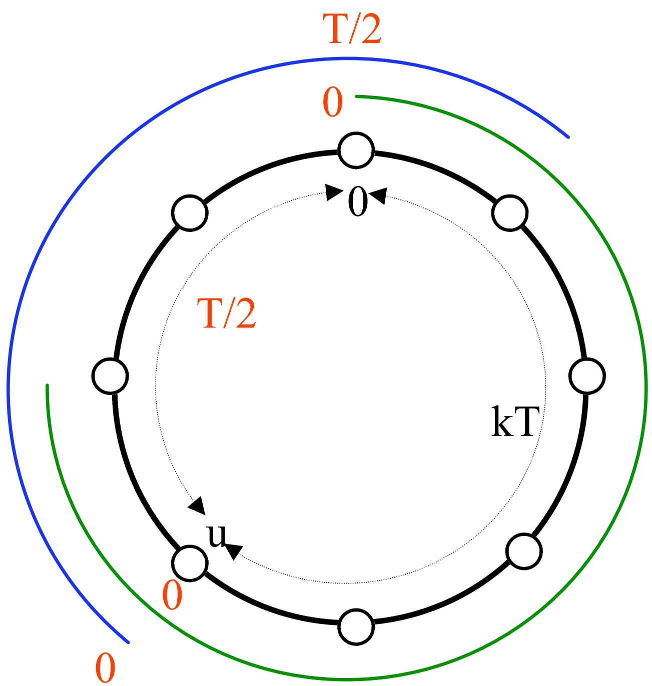

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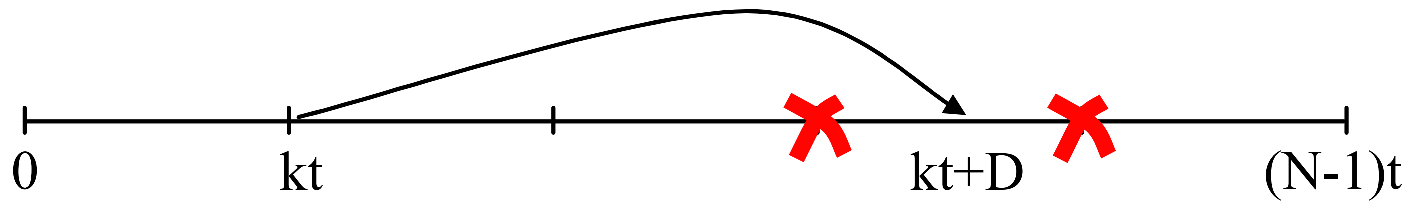


- When a timeslot kt is assigned, timeslots close to $kt + D$ are excluded, where $D = C \bmod T$.
- As a result we need more timeslots.

An algorithm for PMS in rings ($C \not\equiv_T 0$)

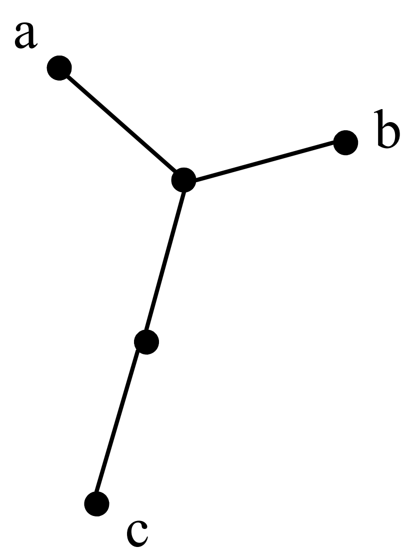
- Split the paths into two sets $\mathcal{P}_0$ and $\mathcal{P}_c$.
- Solve the chain in $\mathcal{P}_c$ optimally, excluding timeslots as needed.
- Sort paths in $\mathcal{P}_0$ in non-increasing order of ending nodes.
- Assign timeslots to those paths, excluding timeslots as needed.
- Set s_time for $p \in \mathcal{P}_0$ s.t. p arrives at 0 at time $timeslot$.

Rings ($C \not\equiv_T 0$)

- We need $6L'$ timeslots, where $L' = \max\{L_c, L_0\}$.
- We have achieved a time distance of $\frac{T}{6L'}$.
- $\frac{T}{L'}$ is an upper bound on the optimal solution.
- Therefore, we obtain a $\frac{1}{6}$-approximation algorithm.

Trees with $\tau(e) \equiv_{\frac{T}{2}} 0$

- We study trees where the time distance for every edge is an integer multiple of $\frac{T}{2}$.
- In this case for the time distances τ between any three nodes a, b, c we have $\tau(a,b) + \tau(b,c) \equiv_T \tau(a,c)$.

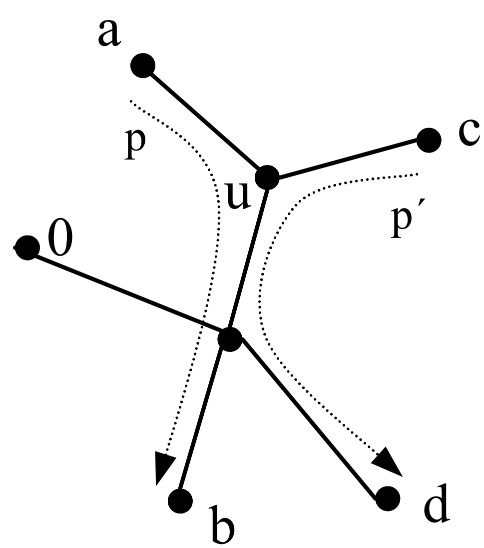


PMS in trees is NP-hard

- PC is NP-hard [Mihail, Kaklamanis, Rao, 1995].
- Given a PC instance, set the time distances equal to 1 and the period $T = 2$.
- A coloring with k colors exists iff there is a schedule with safety distance $\frac{T}{k}$.
  - proof: by similar arguments as for rings.

Trees ($\tau(e) \equiv_{\frac{T}{2}} 0$)

- Suppose paths $p : a \longrightarrow b$ and $p' : c \longrightarrow d$ overlap and u is their first common node.
- p arrives at u at $timeslot(p)+\tau(0,a)+\tau(a,u) \equiv_T timeslot(p) + \tau(0,u)$.
- p' arrives at u at $timeslot(p')+\tau(0,c)+\tau(c,u) \equiv_T timeslot(p') + \tau(0,u)$.



Trees ($\tau(e) \equiv_{\frac{T}{2}} 0$)

- Theorem: Given a ρ-approximation for PC in trees, a $\frac{1}{\rho}\frac{L}{L+1}$-approximation can be achieved for PMS in this class of trees.
- Corollary: There is a $\frac{3}{5}\frac{L}{L+1}$-approximation algorithm for PMS in this class of trees [Erlebach, Jansen, Kaklamanis, Mihail, Persiano, 1999].

Conclusions

- Exact algorithms for PMS in chains and spiders.
- Hardness results and $\frac{1}{\rho}\frac{L}{L+1}$-approximation algorithms for rings with $C \equiv_T 0$ and trees with $\tau(e) \equiv_{\frac{T}{2}} 0$.
- A $\frac{1}{6}$-approximation algorithm for general rings.

Further work:

- General trees, other topologies.
- Other techniques (without using PC).