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EULERIAN SUBGRAPH CONTAINING GIVEN VERTICES

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Eulerian subgraph containing given vertices

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Abstract

A vertex set $S \subseteq V(G)$ is k-weak-edge-connected if for every $C \subset S$ and $x \in S-C$ there are $\min\{k, |C|\}$ edge-disjoint (x, C)-paths in G. For a graph G and a k-weak-edge-connected vertex set $S \subset V(G)$ with $k \geq 3$ and $4 \leq |S| \leq 2k$, we show that G has an eulerian subgraph containing all vertices in S.

1 Introduction

All graphs considered in this paper are undirected and simple. A graph is *eulerian* if it is a connected even graph, *i.e.*, each vertex has even degree. A graph is *supereulerian* if it contains a spanning eulerian subgraph. Since a 3-regular graph is supereulerian if and only if it is hamiltonian, and the hamiltonian problem is NP-complete even for 3-regular graphs, the problem of determining whether a graph is supereulerian is NP-complete [8, 9, 13]. So, it is interesting to ask what is the maximum order of an eulerian subgraph in a given graph [12, 14]. In this paper we study a more general problem as follows: for a given vertex set $S \subseteq V(G)$, is there an eulerian subgraph of G containing all vertices in S. Note that in the particular case of $S = V(G)$, G contains an eulerian subgraph containing all vertices in S if and only if G is supereulerian.

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We begin by introducing some results on cycles containing a given subset of vertices Two basic and classic results due to Dirac are the followings.

Theorem 1. *[4] : If G is a 2-connected graph on $n \geq 3$ vertices, then $c(G) \geq \min\{n, 2\delta\}$, where $c(G)$ is the circumference of G.*

Theorem 2. *[3] If G is a k-connected graph then it has a cycle through any k vertices.*

Flandrin *et al.* [6] generalized Theorem 2 by limiting the connected condition of the graph to a connected condition on the subset of vertices considered. Let G be an arbitrary graph and $S \subseteq V(G)$ be a set of at least two vertices. S is k-connected if any two vertices of S can not be seperated in G by deleting at most $k-1$ vertices.

Theorem 3. *[6] If S is a k-connected subgraph of G, then G has a cycle through any k vertices of S.*

The above theorems are sharp (see for example the complete bipartite graphs $K_{k,k+1}$).

It has been proved by Győri and Plummer [10] and independently Favaron and Jackson [7] that 3-connected $K_{1,3}$-free graphs are 9-cyclable, *i.e.* any nine vertices is in a cycle. For $K_{1,4}$-free graphs, Flandrin *et al.* obtained

Theorem 4. *[5] Let G be a $K_{1,4}$-free graph and S be a k-connected subset of vertices in G with $k \geq 4$ and $4 \leq |S| \leq 2k$. Then there exists a cycle containing S.*

Corollary 1. *Let G be a k-connected $K_{1,4}$-free graph and S be a subset of vertices in G such that $k \geq 4$ and $4 \leq |S| \leq 2k$. Then there exists a cycle containing S.*

Let C be a vertex set in G, and x a vertex in $G-C$. A path P is called an *(x, C)-path* if the two ends of P are x and y respectively, where y is the only vertex in $P \cap C$, which will be called the *attachment of P on C*. A vertex set $S \subseteq V(G)$ is *k-weak-edge-connected* if for every $C \subset S$ and $x \in S - C$ there are $\min\{k, |C|\}$ edge-disjoint (x, C)-paths in G. By Menger's Theorem, if G is k-edge-connected, then every vertex set of $V(G)$ is k-weak-edge-connected. Conversely, if $|V(G)| \geq k+1$, then $V(G)$ is k-weak-edge-connected implies that G is k-edge-connected. For $k \leq 2$, k-weak-edge-connectedness is equivalent to k-edge-connectedness when $|S| \geq k+1$. The case $k=1$ is trivial. In the case $k=2$, for any three vertices in S, there are two edge disjoint paths between one of them and the other two, hence they are 2-edge-connected. When $k \geq 3$, k-weak-edge-connectedness is indeed 'weaker' than k-edge-connectedness, as can be seen from the graph in Figure 1, where S is the set of the blackened vertices.

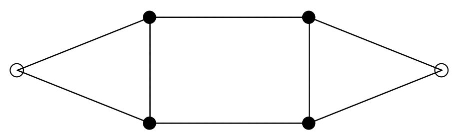


Figure 1

Let G be a graph, $k \geq 3$ an integer, and S a k-weak-edge-connected subset of $V(G)$ with $4 \leq |S| \leq 2k$. The main result of this paper is that G has an eulerian subgraph containing all vertices in S. We will give a proof of this result in the next section. As a consequence of the main theorem, we have

Corollary 2. *Let G be a graph and S a k-edge-connected subset of $V(G)$ with $k \geq 3$ and $|S| \leq 2k$. Then G has an eulerian subgraph containing all vertices of S.*

Given an eulerian cycle, we can start from any vertex, traverse every edge exactly once, and then come back to the starting point. So, for an eulerian cycle, we can associate with it a direction. Let x, y be two vertices in an eulerian cycle C. Denote by xCy the segment from x to y traversed in the previously fixed direction, and $x\overline{C}y$ the segment from x to y traversed in the reversing direction. Similar notation is used for paths. Furthermore, for a path P and a vertex x on P, xP denotes the segment of P from x to its end, and Px denotes the segment of P from its initial to x.

For simplicity, we will use the graph itself to denote its vertex set. We follow [1] or [2] for notations or terminology not defined here.

2 Main Result

Similar to the result of exercise 6.42 in [11], which concerns with vertex-disjoint paths, we have the following lemma dealing with edge-disjoint paths.

Lemma 1. *Let M be a vertex set in a graph G, and x a vertex in $G-M$. Suppose there are ℓ edge-disjoint (x,M)-paths $Q_1,...,Q_\ell$ and $\ell+r$ edge-disjoint (x,M)-paths $P_1,...,P_{\ell+r}$. Then there are ℓ edge-disjoint (x,M)-paths $R_1,...,R_\ell$ and r integers $1\le j_1<...<j_r\le \ell+r$, such that*

(i) R_i has the same attachment as Q_i $(i=1,...,\ell)$, and

(ii) $R_1,...,R_\ell,P_{j_1},...,P_{j_r}$ are edge-disjoint.

Proof. Let $\mathcal{R}=\{R_1,...,R_\ell,...,R_t\}$ be a set of edge-disjoint paths such that

(1) R_i has the same attachment as Q_i $(i=1,...,\ell)$, and

(2) subject to (1), $|(\bigcup\limits_{R\in\mathcal{R}}E(R))\cap(\bigcup\limits_{i=1}^{\ell+r}E(P_i))|$ is as large as possible.

Note that such $\mathcal{R}$ exists by the existence of $Q_1,...,Q_\ell$.

Claim. If the begining edge of P_j is different from any begining edge of R_i $(i=1,...,\ell)$, then P_j is edge-disjoint from $\{R_i\}_{i=1}^{\ell}$.

Suppose the claim is not true. Let yz be the first edge on P_j which also belongs to some path in $\{R_i\}_{i=1}^{\ell}$, say R_{i_0}. Replace $xR_{i_0}y$ with xP_jy, and denote by $\mathcal{R}'$ the new set of paths. Then $\mathcal{R}'$ is also a set of edge-disjoint paths satisfying (1). But $|(\bigcup\limits_{R\in\mathcal{R}'}E(R))\cap(\bigcup\limits_{i=1}^{\ell+r}E(P_i))|>|(\bigcup\limits_{R\in\mathcal{R}}E(R))\cap(\bigcup\limits_{i=1}^{\ell+r}E(P_i))|$, which contradicts (2).

The existence of $j_1,...,j_r$ follows from the above claim and the observation that there are at least r paths in $\{P_j\}_{j=1}^{\ell+r}$ whose begining edges are different from those of $\{R_i\}_{i=1}^{\ell}$. □

As a corollary, we have

Lemma 2. *Suppose G is a graph, k is an integer with $k\ge 2$, S is a k-weak-edge-connected subset of $V(G)$, M is a vertex set in G with $M\cap S=C$, x is a vertex in $S-M$, Q_1,Q_2 are two edge-disjoint (x,M)-paths with attachments x_1 and x_2 respectively. Then there* *exist* $c = \min\{k, |C|\}$ *edge-disjoint* (x, M)*-paths* $P_1, ..., P_c$ *with attachments* $y_1, ..., y_c$ *such that* $\{x_1, x_2\} \subseteq \{y_1, ..., y_c\}$.

Proof. Since S is k-weak-edge-connected, there are c edge-disjoint (x, C)-paths $R_1, ..., R_c$ in G. Suppose the directions of these paths are from x to C. Denote by z_i the first vertex of R_i in $R_i \cap M$. Set $R'_i = R_i z_i$. Then $R'_1, ..., R'_c$ are c edge-disjoint (x, M)-paths and the result follows from Lemma 1. $\square$

The following is our main result:

Theorem 5 (the main theorem). *Let* G *be a graph of order at least 3, and* S *a subset of vertices in* G*. Then* G *has an eulerian subgraph containing all vertices of* S *if one of the following conditions is satisfied:*

(1) For $k = 2, 3$*,* S *is* k*-edge-connected with* $|S| \leq k$ *and*

(2) For $k \geq 3$*,* S *is a* k*-weak-edge-connected subset of* $V(G)$ *with* $4 \leq |S| \leq 2k$*.*

Remark 1. The requirement that $k \geq 3$ in (2) is necessary, as can be seen from $G = K_{2,2n+1}$ $(n \geq 2)$, where S is the partite set containing $2n + 1$ vertices which is 2-weak-edge-connected but not 3-weak-edge-connected. The requirement $|S| \geq 4$ in (2) is also necessary, as can be seen from $K_{2,3}$, where S is the partite set containing three vertices which is k-weak-edge-connected for any k.

Proof of the main theorem:

Let C be an eulerian subgraph of G with $|C \cap S|$ as large as possible. By our discussion after the definition of k-weak-edge-connectedness, S is 2-edge-connected, and thus $|C \cap S| \geq 2$ by Menger's Theorem. As a consequence, the theorem holds when $|S| \leq 2$. So, suppose $|S| \geq 3$ in the following.

Assume that this theorem is not true, we shall derive a contradiction by showing that C is 'augmentable', *i.e.*, there is another eulerian subgraph C' of G containing more S-vertices than C. Let x be a vertex in $S - C$.

If S satisfies condition (1) with $k=3$, then $|C \cap S| = 2$, and there are three edge disjoint (x, C)-paths Q_1, Q_2, Q_3 with attachments x_1, x_2, x_3 on C, where x_1, x_2, x_3 lie on C in this order. For each i, there is at least one S-vertex in (x_i, x_{i+1}) ('+' is comprehended as modulo 3) since otherwise C can be augmented by setting $C' = xQ_{i+1}x_{i+1}Cx_i\overline{Q}_ix$. But then $|C \cap S| \geq 3$, a contradiction.

Next we assume that S is a k-weak-edge-connected subset of $V(G)$ with $k \geq 3$ and $|S| \leq 2k$. In this case,

$$|C \cap S| < |S| \leq 2k. \tag{1}$$

Since S is k-weak-edge-connected, there are $c = \min\{k, |C \cap S|\}$ edge-disjoint (x, C)-paths $Q_1, ..., Q_c$ with attachments $x_1, ..., x_c$, where $x_1, ..., x_c$ lie on C sequentially in this order. Denote by $X = \{x_1, ..., x_c\}$. Call the segments (x_i, x_{i+1}) $(i = 1, ..., c)$ on C as X-segments, where '+' is comprehended as modulo c. Similar to the above, we have

Claim 1. For each i, there is at least one S-vertex in (x_i, x_{i+1}). □

If $|C \cap S| = 2$, since $|S| \geq 4$, we have $y_i \in S - \{x\}$, $1 \leq i \leq 3$ such that $y_1 \in x_1Cx_2$ and $y_2 \in x_2Cx_1$. Then there are three edge disjoint $(y_3, \{x, y_1, y_2\})$-paths, and hence three edge disjoint $(y_3, C \cup Q_1 \cup Q_2)$-paths with attachments z_i $(1 \leq i \leq 3)$ respectively. It is easy to verify that if $|\{z_1, z_2, z_3\} \cap (Q_1 \cup x_1Cy_1 \cup x_2Cy_2)| \geq 2$, there will be an eulerian subgraph containing at least three S-vertices, a contradiction. So $|\{z_1, z_2, z_3\} \cap (Q_1 \cup x_1Cy_1 \cup x_2Cy_2)| \leq 1$. Similarly $|\{z_1, z_2, z_3\} \cap (Q_2 \cup y_1Cx_2 \cup y_2Cx_1)| \leq 1$. But this is again a contradiction. Therefore we may assume that $|C \cap S| \geq 3$.

Claim 2. There is an X-segment containing exactly one S-vertex.

In fact, if $c < k$, then each X-segment contains exactly one S-vertex. Whereas when $c = k$, the claim follows from the hypothesis that C contains less than $2c$ S-vertices. □

Suppose, without loss of generality, that (x_1, x_2) is an X-segment which contains exactly one S-vertex y. By taking $M = x_2Cx_1 \cup Q_1 \cup Q_2$ in Lemma 2, we obtain c edge-disjoint (y, M)-paths $P_1, ..., P_c$ with attachments $y_1, ..., y_c$ lying sequentially on C, in which $y_1 = x_1$ and $y_2 = x_2$.

Claim 3. *$P_1, ..., P_c$ are vertex-disjoint from $Q_1, ..., Q_c$ except that $P_1 \cap Q_1 = \{x_1\}$ and $P_2 \cap Q_2 = \{x_2\}$.*

First, it can be seen from the choice of M that P_2 is vertex-disjoint from Q_1 and internally disjoint from Q_2, P_1 is vertex-disjoint from Q_2 and internally disjoint from Q_1.

Suppose that $P_j \cap (Q_1 \cup Q_2) \neq \emptyset$ for some $j \geq 3$. That is to say, without loss of generality, $y_j \in Q_1$. But then C is augmentable to $C' = xQ_2x_2Cx_1\overline{P_1}yP_jy_j\overline{Q_1}x$ (see Figure 2). So $P_j \cap (Q_1 \cup Q_2) = \emptyset$ for any $j \geq 3$.

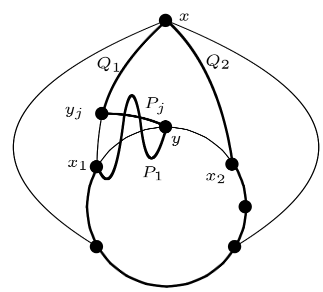


Figure 2

Suppose $Q_i \cap (\bigcup_{j=1}^{c} P_j) \neq \emptyset$ for some $i \geq 3$. Let u be the first vertex of Q_i which also belongs to $\bigcup_{j=1}^{c} P_j$, say $u \in P_j$. Then C is augmentable by taking $C' = xQ_iu\overline{P_j}yP_2x_2Cx_1\overline{Q_1}x$ if $j \neq 2$ and $C' = xQ_iu\overline{P_2}yP_1x_1\overline{C}x_2\overline{Q_2}x$ if $j = 2$ (see Figure 3). $\square$

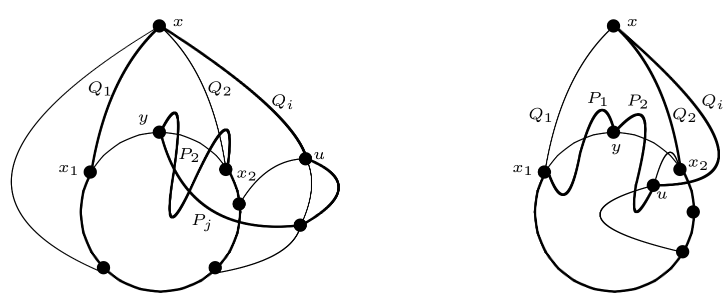


Figure 3

Set $Y = \{y_3, ...y_c\}$. As a consequence of Claim 3, $Y \subseteq C$ and $Y \cap X = \emptyset$. Moreover, we have the following

Claim 4. *For each pair of vertices $u, v \in X \cup Y$, there is at least one S-vertex between them.*

It has been shown in Claim 1 that $\{u, v\} \not\subseteq X$. If there is a Y-segment (y_i, y_{i+1}) not containing any S-vertex, then C is augmentable by setting $C' = xQ_2x_2Cy_i\overline{P}_iyP_{i+1}y_{i+1}Cx_1$

$\overline{Q}_1 x$. So $\{u, v\} \nsubseteq Y$.

Next, suppose $u = x_i \in X$ and $v = y_j \in Y$. Without loss of generality we assume that x_2, u, v, x_1 are in this order on C. Then C is augmentable by setting $C' = xQ_i x_i \overline{C} x_2 \overline{P_2} y P_j y_j C x_1 \overline{Q_1} x$ if there is no S-vertex in $x_i C y_j$ (see Figure 4). $\square$

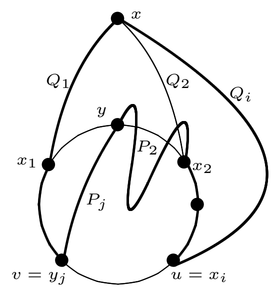


Figure 4

Claim 5. $c = k < |C \cap S|$.

By the definition of c, we have $c = |C \cap S|$ or $c = k$. If $c = |C \cap S|$, then $c \geq 3$. But by Claim 4, $|C \cap S| \geq |X| + |Y| = 2c - 2 > c$, a contradiction. So, $c < |C \cap S|$, and thus $c = k$. $\square$

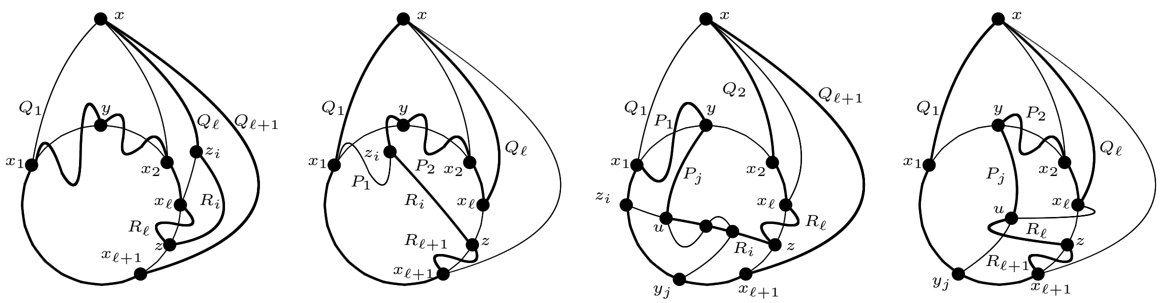
Claim 6. *There is only one X-segment on C containing exactly one S-vertex.*

Suppose $S \cap (x_\ell C x_{\ell+1}) = \{z\}$ for some $\ell \neq 1$. By taking $M = (C - (x_1 C x_2 \cup x_\ell C x_{\ell+1})) \cup Q_1 \cup Q_2 \cup Q_\ell \cup Q_{\ell+1} \cup P_1 \cup P_2$ in Lemma 2, we obtain c edge-disjoint (z, M)-paths $R_1, ..., R_c$ with attachments $z_1, ..., z_c$ lying on C sequentially, and $z_\ell = x_\ell$, $z_{\ell+1} = x_{\ell+1}$.

Subclaim 6.1. $R_1, ..., R_c$ are vertex disjoint from $Q_1, ..., Q_c$ and $P_1, ..., P_c$, except for the obvious common ends.

First, by the choice of M, we see that R_ℓ and $R_{\ell+1}$ are vertex-disjoint from P_1, P_2, Q_1, Q_2 and internally disjoint from $Q_\ell, Q_{\ell+1}$. Suppose R_i intersects $P_1 \cup P_2 \cup Q_1 \cup Q_2 \cup Q_\ell \cup Q_{\ell+1}$ for some $i \neq \ell, \ell+1$. If R_i comes across Q_ℓ or $Q_{\ell+1}$ first, say Q_ℓ, then $z_i \in Q_\ell$, and $C' = xQ_{\ell+1} x_{\ell+1} C x_1 \overline{P_1} y P_2 x_2 C x_\ell \overline{R_\ell} z R_i z_i \overline{Q_\ell} x$ is an augmentation of C (see Figure 5 (a)). If R_i comes across P_1, P_2, Q_1 or Q_2 first, say P_1, then $z_i \in P_1$, and $C' = xQ_\ell x_\ell \overline{C} x_2 \overline{P_2} y P_1 z_i \overline{R_i} z R_{\ell+1} x_{\ell+1} C x_1 \overline{Q_1} x$ is an augmentation of C (see Figure 5 (b)). So, $R_i \cap (P_1 \cup P_2 \cup Q_1 \cup Q_2 \cup Q_\ell \cup Q_{\ell+1}) = \emptyset$ for any i. Suppose $P_j \cap R_i \neq \emptyset$ for some $j \geq 3$ and some i. Let u be the first vertex on P_j which also belongs to some R_i. Then C can be augmented by setting $C' = xQ_{\ell+1} x_{\ell+1} C x_1 \overline{P_1} y P_j u \overline{R_i} z R_\ell x_\ell \overline{C} x_2 \overline{Q_2} x$ if $i \neq \ell$ (see Figure

5 (c)) or $C' = xQ_\ell x_\ell \overline{C} x_2 \overline{P_2} y P_j u \overline{R_\ell} z R_{\ell+1} x_{\ell+1} C x_1 \overline{Q_1} x$ if $i = \ell$ (see Figure 5 (d)). Similar contradiction arises if some Q_j intersects some R_i. $\square$



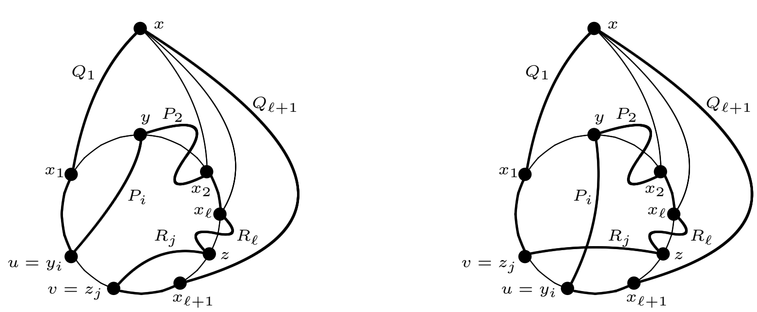
(a) (b) (c) (d)

Figure 5

Set $Z = \{z_1, ..., z_{\ell-1}, z_{\ell+2}, ..., z_c\}$. It follows from Subclaim 6.1 that $Z \subseteq C$ and $Z \cap (X \cup Y) = \emptyset$.

Subclaim 6.2. For every pair of vertices u, v in $X \cup Y \cup Z$, there is at least one S-vertex between them.

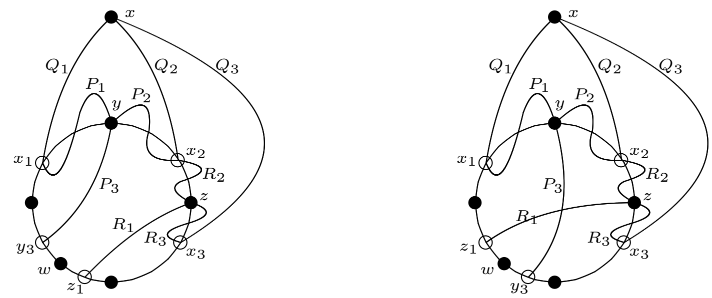
In view of Claim 4 and by symmetry, we only show the case that $u = y_i \in Y$ and $v = z_j \in Z$. Suppose there is no S-vertex between them. We assume by symmetry that $y_i, z_j \in x_{\ell+1} C x_1$. Then $i \neq 2$ and $j \neq \ell$. Set $C' = xQ_{\ell+1} x_{\ell+1} C z_j \overline{R_j} z R_\ell x_\ell \overline{C} x_2 \overline{P_2} y P_i y_i C x_1 \overline{Q_1} x$ if z_j precedes y_i on C (see Figure 6 (a)), and $C' = xQ_{\ell+1} x_{\ell+1} C y_i \overline{P_i} y P_2 x_2 C x_\ell \overline{R_\ell} z R_j z_j C x_1 \overline{Q_1} x$ if y_i precedes z_j on C (see Figure 6 (b)). Then C can be augmented to C'. $\square$



(a) (b)

Figure 6

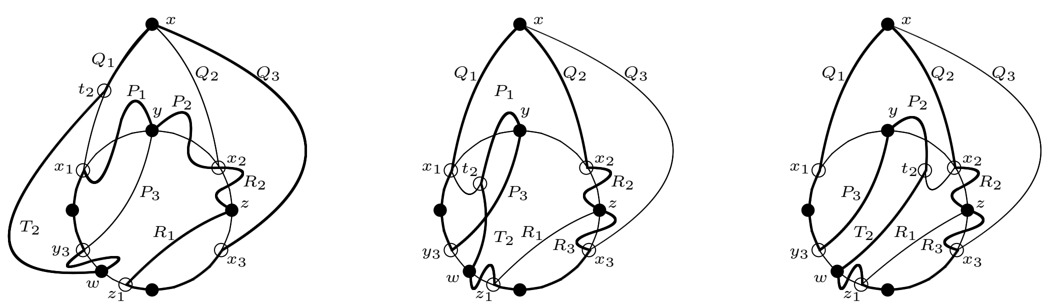
It follows from Subclaim 6.2 and Claim 5 that $|C \cap S| \geq |X|+|Y|+|Z| = 3c-4 = 3k-4$. By inequality (1), this happens only when $c = k = 3$ and G has a sub-structure as in Figure 7 (a) or (b), where the blackened vertices are in S.



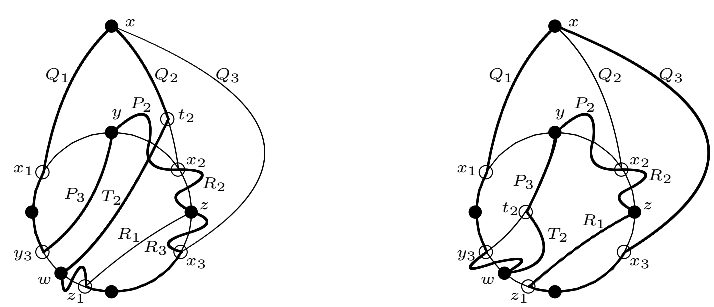
(a) (b)

Figure 7

Let w be the S-vertex between y_3 and z_1 (as indicated in Figure 7). For simplicity, we only consider the case that G has a sub-structure in Figure 7 (a). By taking $M = x_3Cz_1 \cup y_3Cx_1 \cup Q_1 \cup Q_2 \cup Q_3 \cup P_1 \cup P_2 \cup P_3 \cup R_1 \cup R_2 \cup R_3$ in Lemma 2, we have three edge-disjoint (w, M)-paths T_1, T_2, T_3 with attachments z_1, t_2 and y_3 respectively. Similar to the above arguments, T_1, T_2, T_3 are disjoint from $Q_1 \cup Q_2 \cup Q_3 \cup P_1 \cup P_2 \cup P_3 \cup R_1 \cup R_2 \cup R_3$ except for the obvious common ends (see Figure 8).



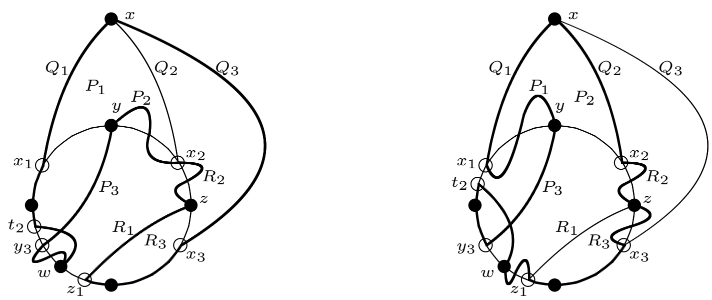
(a) (b) (c)



(d) (e)

Figure 8. By the choice of M, T_1 and T_3 obviously satisfies our requirement. If T_2 first comes across (a) Q_1, (b) P_1, (c) P_2, (d) Q_2, (e) P_3, then C can be augmented to the blackened eulerian subgraph. By symmetry, the cases that T_2 first comes across Q_3, R_3, R_2, R_1 are similar.

It follows that $t_2 \in x_3Cz_1 \cup y_3Cx_1$. Suppose by symmetry that $t_2 \in y_3Cx_1$. Then C can be augmented as indicated in Figure 9. $\square$



(a) (b)

Figure 9

By Claim 5, Claim 6, and inequality (1), we see that $c = k$, and for each $i \neq 1$, there are exactly two S-vertices in (x_i, x_{i+1}). Denote them by u_i and u_i' (u_i precedes u_i' on C). By Claim 4, there are at least $c - 2$ X-segments, such that for each such a segment (x_i, x_{i+1}) there is a Y-vertex between u_i and u_i'. So, we may suppose that (x_2, x_3) is such a segment. Let y_3 be the Y-vertex lying between u_2 and u_2'. By taking $M = (C \setminus (x_1, y_3)) \cup P_1 \cup P_2 \cup P_3 \cup Q_1 \cup Q_2$ in Lemma 2, we obtain c edge-disjoint (u_2, M)-paths $R_1, ..., R_c$ with attachments $z_1, ..., z_c$, such that $z_2 = x_2$ and $z_3 = y_3$. By the choice of M, R_2 and R_3 are internally disjoint from $P_1 \cup P_2 \cup P_3 \cup Q_1 \cup Q_2$. Write $C'' = xQ_2x_2\overline{R_2}u_2R_3y_3Cx_1\overline{Q_1}x$. Then y is a vertex in $S - C''$ with two Y-segments ($x_1C''x_2$ and $x_2C''y_3$) on C'' having exactly one S-vertex each. Since $|C'' \cap S| = |C \cap S|$, Claim 6 is also applicable to y and C'', which incurs a contradiction. $\square$

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