IET Communications

Research Article

ISSN 1751-8628
Received on 5th October 2018
Revised 26th January 2019
Accepted on 6th March 2019
doi: 10.1049/iet-com.2018.5956
www.ietdl.org

Multi-radio channel rendezvous in cognitive radio networks

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Abstract: In decentralised cognitive radio (CR) networks, establishing communication sessions between a communicating pair requires them to meet each other on a common channel via a 'rendezvous' process. Devising distributed CR rendezvous protocol is a challenging task as cognitive nodes are not necessarily synchronised, and may have different perceptions of channel availability. In this study, the authors present *M-Rendezvous*, an order-optimal rendezvous protocol exploiting the performance gain brought by having multiple radios at cognitive nodes. As a distinguished feature, M-Rendezvous is a unified rendezvous protocol that can operate in both *homogenous* case where both of the rendezvous nodes are equipped with only one radio or multiple radios, and *heterogeneous* case where one of the rendezvous nodes has single radio and the other has multiple radios. In both cases, by rigorous analysis, the authors demonstrate that M-Rendezvous can guarantee rendezvous over *every* channel with bounded and order-minimal delay even when rendezvous nodes have asynchronous clocks and asymmetrical channel perceptions.

1 Introduction

Cognitive radio (CR) [1] has emerged in recent years as a promising technology to enable more efficient spectrum utilisation by allowing unlicensed cognitive users to access the spectrum of licensed primary users (PUs) in an opportunistic way. In decentralised CR networks (CRN), establishing communication sessions between a communicating pair requires two nodes to meet each other on a common channel via a rendezvous process. Moreover, with a high probability, more than one communicating pairs would stay on a common channel, which requires identifying multiple available common channels for possible alternative if collision happened. Therefore, multi-radio rendezvous is presented as a natural scheme to alleviate the possible collision among multiple communications pairs in which different radios act as different roles, i.e. anchor radio or scan radio.

However, designing distributed channel rendezvous protocols is a challenging problem under the opportunistic spectrum access paradigm for the following reasons:

(1) *Asymmetrical perceptions of channel availability*: Two communicating cognitive nodes may have different perceptions of channel availability as the PU transmission may unpredictably vary at different locations. Such channel perception asymmetry increases the difficulty in finding a common rendezvous channel free of PU signals.
(2) *Lack of clock synchronisation*: In decentralised CR systems, it is difficult to maintain tight synchronisation among the local clocks of cognitive nodes. Without clock synchronisation, rendezvous protocols may fail when using pre-scheduled channel hopping (CH) sequences.
(3) *System scalability versus rendezvous delay*: When the system scales, a rendezvous pair (two nodes wishing to establish a communication session) need to search through a large number of channels before successfully achieving rendezvous, thus suffering from significant rendezvous delay.

A widely adopted approach to facilitate the rendezvous process in conventional multi-channel wireless networks is to deploy a common control channel (CCC) that can be either static [2, 3] or dynamic [4, 5]. However, rendezvous failure is inevitable when the CCC is temporarily occupied by PU transmissions or when the CCC becomes congested.

To mitigate the rendezvous failure in CCC-based approaches, rendezvous protocols based on random CH have been proposed in which each node hops randomly among the available channels to rendezvous with others in an uncoordinated fashion [6, 7]. The random CH-based rendezvous protocol fails to bound the worst-case rendezvous latency (more precisely, the maximal time-to-rendezvous, MTTR) and thus suffers from the long-tail rendezvous latency problem in which two rendezvous nodes may experience extremely long delay before rendezvousing on a common channel.

Recently, a number of sequence-based CH rendezvous approaches (cf. [8–40]) have been developed to achieve bounded MTTR for any pair of rendezvous nodes. However, the rendezvous difficulty problem for the discovery of a common channel between two rendezvous nodes still remains because they cannot exchange any information regarding the asymmetric perceptions of channel availability. Meanwhile, existing CH approaches incur a significant rendezvous delay in terms of both average and worst-case rendezvous delay when the number of channels for cognitive users to scan becomes large.

1.1 Related works

With the rapid development of the wireless communication technology and the significant decreasing prices of radios, it is nowadays feasible to equip a wireless device with multiple radios, each operating on a separated spectrum channel. Equipping all or some nodes with multiple radios can significantly boost the network capacity by enabling simultaneous operations over multiple channels and mitigating interferences through proper channel assignment.

Hence, multi-radio rendezvous has attracted many researchers [33–40]. In [33, 34], the authors proposed a new rendezvous algorithm, called role-based parallel sequence (RPS), in which nodes stay in a specific channel in one dedicated radio and hop on the available channels with parallel sequences in the remaining general radios. In [35], the authors proposed an adjustable multi-radio rendezvous algorithm in which m radios of a node are partitioned into two groups: k stay radios and $(m-k)$ hopping radios. In [36], the authors proposed a deterministic multi-grid-quorum CH which exploits multiple overlapped grid quorums to map the available channels at each device to its radios. In [37], the authors followed a systematic approach by first proposing a role-based algorithm that ensures maximum rendezvous diversity and then extending it to a common strategy through the use of multiples radios. In [38], the authors exploited the mathematical construction of sunflower sets to develop a single-radio sunflower set-based pairwise rendezvous algorithm and proposed an approximation algorithm to construct disjoint sunflower sets. However, Yang *et al.* [38] cannot provide guaranteed rendezvous when two users start rendezvous process at the same time. In [39], the authors presented a quorum-based CH system which can be used for implementing rendezvous protocols in dynamic spectrum access networks that are robust against link breakage caused by the appearance of incumbent user signals. However, Li *et al.* [39] require that the numbers of radios in different nodes are even. In [40], the authors proposed a chinese remainder theorem based multi-radio rendezvous (CMR) algorithm for heterogeneous CRNs. However, CMR requires that there exist at least two different prime CH sequence lengths.

In addition, Banerjee *et al.* [41] studied an energy harvesting (EH) based two-hop relay assisted CR system with minimising secrecy outage probability under some constraints. Banerjee and Maity [42] studied a residual energy maximisation problem on a two-hop decode-and-forward relay-assisted CRN with an eavesdropper. Banerjee *et al.* [43] studied the maximisation problem of the sum secondary throughput in a cooperative CRN with an eavesdropper as well as EH capability.

1.2 Major contributions

In this paper, we propose an order-optimal multi-radio CR channel rendezvous protocol, called *M-Rendezvous* only requiring different IDs for different nodes [14], by exploiting the benefits brought by the rendezvous diversity created by multiple radios in minimising the rendezvous delay and increasing the rendezvous robustness. As a notable feature, M-Rendezvous is a unified channel rendezvous protocol that can operate in both *homogenous* case where both of the rendezvous nodes are equipped with only one radio or multiple radios, and *heterogeneous* case where one of the rendezvous nodes has single radio and the other has multiple radios. In both cases, by rigorous analysis, we demonstrate the following properties of M-Rendezvous.

Guaranteed rendezvous in fully decentralised environments: M-Rendezvous operates in a fully decentralised fashion without any CCC, and ensures that any two nodes can rendezvous within a finite number of time slots.

Maximum rendezvous diversity: M-Rendezvous guarantees rendezvous over *each* channel. The ability to rendezvous over all channels minimises the probability of rendezvous failures that are caused by the unpredictable presence of PU signals.

Minimal worst-case rendezvous delay: The worst-case rendezvous latency of M-Rendezvous is linear to the number of channels in the system and decreases squarely with the number of radio per node. Note that the linear rendezvous delay is the optimal bound for any channel rendezvous protocol achieving full rendezvous diversity.

Robustness against asymmetrical channel perceptions: M-Rendezvous does not require cognitive nodes to have the same view on the accessible channel set and the channel index and can guarantee rendezvous even when the rendezvous nodes have only one commonly accessible channel and asymmetrical channel index mappings.

Robustness against clock drift: M-Rendezvous achieves the same rendezvous performance even when the clocks of two rendezvous nodes are not synchronised.

The above properties make M-Rendezvous especially suitable for the decentralised CR environment. To see this more formally, consider a rendezvous pair a and b who can access a set $\mathcal{N}$ of channels. Due to PU activity, the accessible channel set at node i ($i \in \{a, b\}$) is $\mathcal{N}_i = \mathcal{N} \backslash \mathcal{P}_i$ with $\mathcal{P}_i$ being the set of channels occupied by PU at node i, and hence $\mathcal{N}_i$ being the set of channels available for node i's access. M-Rendezvous guarantees rendezvous on every channel $h \in \mathcal{N}_a \bigcap \mathcal{N}_b$ as long as $\mathcal{N}_a \bigcap \mathcal{N}_b \neq \emptyset$.

Compared to [33–40], our proposition M-Rendezvous is a unified channel rendezvous protocol that can operate in both homogenous case and heterogeneous case. Moreover, M-Rendezvous can guarantee full rendezvous diversity and is robust against both channel perception asymmetry on both accessible channel set and channel index.

One point worth commenting is that a naive rendezvous solution of using a single-radio rendezvous protocol for single-radio nodes and a multi-radio rendezvous protocol if multiple radios are available cannot solve the rendezvous problem because this approach either fails to provide bounded rendezvous delay or cannot achieve full rendezvous diversity.

1.3 Paper organisation

The rest of the paper is organised as follows. Section 2 describes the system model and formulates the optimal channel rendezvous problem. Section 3 establishes the performance bound for any channel rendezvous protocol by relating the two important performance metrics, rendezvous delay and diversity. Section 4 presents the design of M-Rendezvous for multi-radio and performs a theoretical analysis on its performance in both homogenous and heterogeneous cases. Section 5 further investigates M-Rendezvous under the more challenging environment where rendezvous nodes have asymmetrical channel perceptions. Section 6 presents the simulation results. Section 7 concludes the paper.

2 System model and problem formulation

In this section, we describe the system model and introduce the performance metrics, based on which the optimal channel rendezvous problem is further formulated.

2.1 System model and design metrics

We consider a time-slotted CRN operating on a set $\mathcal{N}$ of N licensed orthogonal channels. Each cognitive node i is equipped with $r_i \geq 1$ radios allowing it to exploit r_i channels simultaneously. In such dynamic and opportunistic spectrum sharing paradigm, channel rendezvous is a crucial process that enables a communication pair to meet each other on a common channel before any effective data exchange.

We are interested in devising channel rendezvous protocols that can enable pairwise rendezvous on *every* available channel for any *asynchronous* pair with the *minimal* worst-case rendezvous delay. In what follows, we introduce three relevant performance metrics to access any CR rendezvous protocol, based on which we formulate the optimal channel rendezvous problem.

2.2 Optimal channel rendezvous problem formulation

A commonly adopted rendezvous solution is CH where each node hops its radios across different channels based on random or specific CH patterns so as to rendezvous with its peers. A CH sequence determines the order with which a radio visits all available channels. In the following, we define the CH pattern of a cognitive node characterising the way the node hops its radios across channels.

Definition 1 (CH pattern): The CH pattern of a cognitive node with r radios is defined as a set of CH sequences $\boldsymbol{u} \triangleq \{u_m\}_{1 \leq m \leq r}$ where u_m is the CH sequence of the mth radio, defined as $u_m \triangleq \{u_m^t\}_{1 \leq t \leq T}$, where T is the period of the sequence [A random CH sequence can be regarded as a special case where $T \to \infty$.],

$u_m^t \in \mathcal{N}$ is the channel index of the sequence u_m in time slot t of a CH period. In the case $r = 1$, the CH pattern degenerates to the CH sequence u_1.

Given two CH patterns $\boldsymbol{u}$ and $\boldsymbol{v}$, if $\exists t$ and m, m' such that $u_m^t = v_{m'}^t = h \in \mathcal{N}$, we say that $\boldsymbol{u}$ and $\boldsymbol{v}$ can *rendezvous* in slot t on channel h. Slot t is called the rendezvous slot and channel h is called the rendezvous channel between $\boldsymbol{u}$ and $\boldsymbol{v}$. Let $\mathscr{C}(\boldsymbol{u}, \boldsymbol{v})$ denote the set of *rendezvous channels* between $\boldsymbol{u}$ and $\boldsymbol{v}$. It holds that $|\mathscr{C}(\boldsymbol{u}, \boldsymbol{v})| \le N$.

Example 1: To illustrate the above definition, consider a system with $N = 6$ and a rendezvous pair u and v equipped with $r_u = r_v = 2$ radios and whose CH patterns are: $u_1 = \{1, 2, 3\}$, $u_2 = \{4, 5, 6\}$ and $v_1 = \{1, 3, 5\}$, $v_2 = \{2, 4, 6\}$. We can observe that u and v can rendezvous on channels 1 and 6 at slots 1 and 3, respectively, i.e. $\mathscr{C}(\boldsymbol{u}, \boldsymbol{v}) = \{1, 6\}$.

To model the situation where the clocks of different nodes are not synchronised, we apply the concept of *cyclic rotation* to the CH sequences. Specifically, given a CH sequence w, we denote $w(k)$ a cyclic rotation of w by k time slots, where k is referred to as the *cyclic rotation phase*. Given a CH pattern $\boldsymbol{u}$, we denote $\boldsymbol{u}(k)$ a cyclic rotation of all the CH sequences $u_m \in \boldsymbol{u}$ by k time slots, i.e. $\boldsymbol{u}(k) \triangleq \{u_m(k)\}_{m \in [1, r]}$. Consider Example 1, we have $\boldsymbol{u}(2) \triangleq \{u_1(2), u_2(2)\}$ where $u_1(2) = \{3, 1, 2\}$, $u_2(2) = \{6, 4, 5\}$.

We now formally express the worst-case rendezvous delay and rendezvous diversity defined in Section 2.1 when nodes' clocks are not synchronised.

- Maximal time-to-rendezvous: for CH patterns $\boldsymbol{u}$ and $\boldsymbol{v}$, we define $D(\boldsymbol{u}, \boldsymbol{v})$ as the first rendezvous slot between them and we define $\Gamma(\boldsymbol{u}, \boldsymbol{v}) = \max_{k,l} D(\boldsymbol{u}(k), \boldsymbol{v}(l))$ as the worst case MTTR between $\boldsymbol{u}$ and $\boldsymbol{v}$ among all possible cyclic rotation phases k and l.
- Rendezvous diversity: for CH patterns $\boldsymbol{u}$ and $\boldsymbol{v}$, we define the worst case rendezvous diversity as $\Delta((\boldsymbol{u}, \boldsymbol{v})) \triangleq \min_{k,l} |\mathscr{C}(\boldsymbol{u}(l), \boldsymbol{v}(k))|$.
- Full rendezvous diversity: given perception channel sets $\mathcal{N}_a$ and $\mathcal{N}_b$ of nodes a and b, if there exists CH sequences $\boldsymbol{u}$ and $\boldsymbol{v}$ for a and b such that $|\mathscr{C}(\boldsymbol{u}, \boldsymbol{v})| = |\mathcal{N}_a \bigcap \mathcal{N}_b|$, then nodes a and b can achieve full rendezvous diversity under $\boldsymbol{u}$ and $\boldsymbol{v}$.
- Rendezvous channel load: for X rendezvous pairs, denote x_i the number of pairs rendezvousing on channel i, $L \triangleq (\sum_{i \in \mathcal{N}} x_i)^2 / (N \sum_{i \in \mathcal{N}} x_i^2)$ quantifies the degree to which rendezvous are distributed among the channels.

We are now ready to formulate the optimal channel rendezvous problem.

Problem 1: The optimal channel rendezvous problem is defined as follows:

$$\begin{aligned} &\text{minimise} && T, \\ &\text{subject to} && \forall t_a^0 \in [0, T_a - 1], t_b^0 \in [0, T_b - 1], \exists t \le T \\ &\text{such that} && x_a^t(t_a^0) = x_b^t(t_b^0) = h, \forall h \in \mathcal{N}_a \bigcap \mathcal{N}_b. \end{aligned}$$

That is, devising CH patterns to minimise T, the worst-case rendezvous delay while achieving full rendezvous diversity between any pair of nodes a and b for any initial time offsets t_a^0 and t_b^0 and any channel perception $\mathcal{N}_a$ and $\mathcal{N}_b$.

3 Protocol-independent performance bound

Rendezvous delay and diversity are the keys performance metrics in evaluating any channel rendezvous protocol. It is insightful to note that there exists an intrinsic trade-off between reducing the rendezvous latency and increasing the rendezvous diversity. Intuitively, focusing on a subset of channels reduces the rendezvous latency but also limits the rendezvous diversity. The following theorem analytically quantifies this trade-off to establish the performance bound of channel rendezvous in the generic protocol-independent context.

Theorem 1 (Protocol-independent rendezvous performance bound): For any CH-based channel rendezvous protocol achieving full rendezvous diversity, the worst-case rendezvous latency (MTTR) between two nodes a and b is lower-bounded by $N / r_a r_b$, i.e.

$$\Gamma(\boldsymbol{u}, \boldsymbol{v}) \ge \frac{N}{r_a r_b}, \quad \forall \boldsymbol{u}, \boldsymbol{v},$$

where $\boldsymbol{u}$ and $\boldsymbol{v}$ denote the CH patterns of a and b, r_a and r_b denote the number of radios of a and b.

Proof: We denote the period of the CH patterns $\boldsymbol{u}$ and $\boldsymbol{v}$ as T_u and T_v. Without loss of generality, we fix $\boldsymbol{u}$ and cyclically rotate $\boldsymbol{v}$ by l where $l = 0, 1, \ldots, T_u - 1$. Now consider $\boldsymbol{u}$ and $\boldsymbol{v}(l)$. Recall the definition of MTTR that $\Gamma((\boldsymbol{u}, \boldsymbol{v}))$ is the worst case rendezvous delay, there must be at least one rendezvous slot between $\boldsymbol{u}$ and $\boldsymbol{v}(l)$ each $\Gamma((\boldsymbol{u}, \boldsymbol{v}))$ slots, resulting a minimal number of rendezvous slots $T_v / \Gamma(\boldsymbol{u}, \boldsymbol{v})$ between them during T_v. Let A denote the total number of accumulated rendezvous between $\boldsymbol{u}$ and $\boldsymbol{v}(l)$ as l is incremented from 0 to $T_u - 1$, we have

$$A \ge \frac{T_u T_v}{\Gamma(\boldsymbol{u}, \boldsymbol{v})}. \tag{1}$$

On the other hand, let $n(u_i, h)$ ($n(v_j, h)$, respectively) denote the number of time slots in sequence u_i (v_j, respectively) that are assigned with channel h. We can express the period T_u for any $1 \le i \le r_a$ as $T_u = \sum_{h=1}^{N} n(u_i, h)$. Symmetrically, we can express the period T_v for any $1 \le j \le r_b$ as $T_v = \sum_{h=1}^{N} n(v_j, h)$. It then follows that

$$\begin{aligned} T_u T_v &= T_v \sum_{h=1}^{N} n(u_i, h) = T_u \sum_{h=1}^{N} n(v_j, h) \\ &= \sum_{h=1}^{N} \frac{T_v n(u_i, h) + T_u n(v_j, h)}{2}. \end{aligned} \tag{2}$$

Since $\boldsymbol{u}$ and $\boldsymbol{v}$ achieve maximal (full) rendezvous diversity, for any channel h, the total number of rendezvous that involve a given time slot, with v_j is $n(u_i, h)$. Since there are $n(v_j, h)$ time slots in v_j that are assigned channel h, the total accumulated number of rendezvous between u_i and $v_j(l)$, as l is incremented from 0 to $T_u - 1$, in which the rendezvous channel is h, is $n(u_i, h) \cdot n(v_j, h)$.

Hence, the total number of accumulated rendezvous as l is incremented from 0 to $T_u - 1$ is

$$A = \sum_{1 \le i \le r_a, 1 \le j \le r_b} n(u_i, h) \cdot n(v_j, h).$$

Noticing that

$$\left(\frac{n(u_i, h) + n(v_j, h)}{2}\right)^2 \ge n(u_i, h) \cdot n(v_j, h),$$

it follows from (2) that

$$A T_u T_v = \sum_{1 \le i \le r_a, 1 \le j \le r_b} T_v n(u_i, h) \cdot T_u n(v_j, h) \le \frac{r_a r_b T_u^2 T_v^2}{N}.$$

It then follows from (1) that

$$\frac{r_a r_b T_u T_v}{N} \ge \frac{T_u T_v}{\Gamma(\boldsymbol{u}, \boldsymbol{v})},$$

which leads to $\Gamma(\boldsymbol{u}, \boldsymbol{v}) \geq N/r_a r_b$. □

Theorem 1 leads to the following observations:

- Asymptotically, when $r_a \simeq r_b \simeq r$, for any rendezvous protocol achieving full rendezvous diversity, the lower-bound of the rendezvous delay scales linearly in the number of channels N while decreases squarely in r, i.e. $\Gamma \simeq O(N/r^2)$.
- When $r_a = r_b = 1$, Theorem 1 characterises the rendezvous delay of the single radio case, which has been extensively explored in the literature. However, it is worth noting that most of the existing work on single-radio channel rendezvous achieves the rendezvous delay lower-bound $O(N)$ without ensuring rendezvous on every channel.
- In the heterogeneous case where a has multiple radios while b has only one radio, Γ decreases linearly in r_a, meaning that having only one node equipped with multiple radios can still bring rendezvous performance gain linear to r_a.

In what follows, we develop an order-optimal multi-radio channel rendezvous protocol, termed as M-Rendezvous, that has $O(N/r_a r_b)$ rendezvous delay with full rendezvous diversity.

4 M-Rendezvous: multi-radio nodes

This section presents the design of M-Rendezvous for multi-radio nodes. We start by specifying the M-Rendezvous design and proceed to establish its performance in the homogeneous case where both of the rendezvous nodes a and b have multiple radios, i.e. $r_a, r_b > 1$. We then relate the rendezvous delay and diversity to a number of protocol parameters to further fine-tune M-Rendezvous to balance the design metrics.

4.1 Protocol description

Our proposed rendezvous protocol, M-Rendezvous, is an asynchronous CH-based channel rendezvous protocol that can achieve order-minimal worst-case rendezvous delay with full rendezvous diversity. The idea of M-Rendezvous comes from the observation that given two nodes each equipped with at least two radios, if each node keeps one radio on a fixed channel and another radio scanning sequentially across the channels, the two nodes are ensured to rendezvous on some channel, as illustrated in Fig. 1.

Specifically, each node i running M-Rendezvous classifies its radios into two groups, r_a^i anchor radios (indexed from 1 to r_a^i) and $r_s^i = r - r_a^i$ scan radios (indexed from 1 to r_s^i). M-Rendezvous operates according to a periodic slot-based CH sequences in which a period consists of $F_i \triangleq N/r_a^i$ frames (indexed from 1 to F_i), each composed of $S_i \triangleq (N - r_a^i)/r_s^i$ slots (indexed from 1 to S_i) <DIFadd>[To streamline our presentation and make the analysis clear, we assume that both F_i and S_i are integers. In generical cases when this is not the case, the following operations can be performed before executing the protocol: expand the channel set from $\{1, \cdots, N\}$ to $\{1, N = k_0 r_a^i\}$ where $N \leq k_0 r_a^i$. In the expanded channel set, channels 1 to N are the original channels, channel $N + 1$ to $N' = k_0 r_a^i$ correspond to the channels randomly chosen from channels 1 to N. After the expansion, F_i becomes an integer. We now show that by appropriately choosing k_0, we can ensure that S_i is also integer. In this regard, let k denote the integer such that $(k-1)r_a^i < N \leq k r_a^i$. We can find k_0, with $k \leq k_0 \leq k + r_s^i - 1$, such that $k_0 - 1$ is divisible by r_s^i; hence $S_i = (N - r_a^i)/r_s^i = (k_0 r_a^i - r_a^i)/r_s^i = (k_0 - 1)r_a^i/r_s^i$ is integer. Moreover, since in practice r_a^i, $r_s^i \ll N$, N' is close to N, thus the expansion operation will not degrade significantly the rendezvous performance.]</DIFadd>. The CH pattern of each node is repeated each $F_i S_i$ time slots, as illustrated in Fig. 2 for $N = 8$, $r_s^i = r_a^i = 2$ and $F_i = 4$, $S_i = 3$. Generically, the CH pattern of node i is specified as follows:

- *Anchor radio*: the anchor radio m_i ($1 \leq m_i \leq r_a^i$) operates on channel $f \oplus (N(m_i - 1))/r_a^i$ in frame f ($1 \leq f \leq F_i$), where $\oplus$ and $\ominus$ denote the operations of addition and substraction modulo N, respectively. There are two properties hinging behind such CH pattern: (i) an anchor radio scans sequentially the N channels by staying on one channel for S_i slots (one frame duration), and (ii) two neighbouring anchor radios are separated by F_i channels.
- *Scan radio*: the r_s scan radios scan from channel 1 to N (by keeping each scan radio scanning one channel each slot) except the channels on which operate the anchor radios. There are two properties hinging behind the scan CH pattern: (i) the scan radios never scan the channels of the anchor radios so as to maximise the number covered channels, and (ii) within each frame, all channels are scanned by either an anchor radio or a scan radio.

It can be noted that *anchor-anchor*, *scan-scan*, *anchor-scan* overlaps all result in rendezvous.

Slot index		*1*	*2*	*3*	*4*	*5*	...
Node *a*:	**anchor**	6	6	6	6	6	...
	scan	1	2	3	4	5	...
Node *b*:	**anchor**	1	1	1	1	1	...
	scan	2	3	4	5	6	...

Fig. 1 *Example illustrating the idea of M-Rendezvous: a rendezvous pair a and b, both equipped with $r = 2$ radios, let one radio (anchor) stay on a fixed channel and let the other radio scan sequentially across the channels; rendezvous is achieved in slots 1 and 5*

Frame index		*1*			*2*			*3*			*4*		
Node *i*:	**anchor radio 1**	1	1	1	2	2	2	3	3	3	4	4	4
	anchor radio2	5	5	5	6	6	6	7	7	7	8	8	8
Node *i*:	**scan radio 1**	2	4	7	1	4	7	1	4	6	1	3	6
	scan radio2	3	6	8	3	5	8	2	5	8	2	5	7

Fig. 2 *Example illustrating M-Rendezvous for multi-radio nodes: $N = 8$, $r_s^i = r_a^i = 2$*

4.2 Rendezvous performance analysis

This subsection studies the rendezvous performance of M-Rendezvous between two nodes equipped with multiple radios. We begin by studying the two structural properties of the CH pattern of M-Rendezvous, which, on one hand, bring more insight on the anchor and scan radio operation, on the other hand, serve as building blocks to establish performance bounds. Readers are referred to Fig. 2 to better understand the properties.

Lemma 1 (Structural properties of CH pattern of M-Rendezvous): The following structural properties of the CH pattern of M-Rendezvous hold for each node i.

- Pseudo-monotonicity: at any slot t, if a channel h is covered by a scan radio, then the next channel $h \oplus 1$ is covered (by either an anchor radio or a scan radio) either at the current slot t or the next slot $t + 1$;
- Pseudo-continuity: each channel $h \in \mathcal{N}$ is covered by either an anchor radio or a scan radio for any consecutive S_i slots;

Proof: The pseudo-monotonicity follows readily from the CH pattern in M-Rendezvous. To prove the pseudo-continuity, it

Node a :	anchor radio	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
	scan radio	2	3	4	5	1	3	4	5	1	2	4	5	1	2	3	5	1	2	3	4
Node b :	anchor radio	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	1	1	1	1	2
	scan radio	3	4	5	1	2	4	5	1	2	3	5	1	2	3	4	2	3	4	5	1

Fig. 3 *Example illustrating the performance bound of M-Rendezvous: $N = 5$, $r = 2$ with a clock drift of 5 slots*

suffices to show that starting from any slot of any frame, each channel h is ensured to be covered by a radio within S_i slots. If the S_i slots starting from $t = 1$ to S_i belong the same frame, the pseudo-continuity follows directly from the CH pattern within a frame. Otherwise the S_i slots span across two frames, denoted as f and $f + 1$. In this case, let c_m $(1 \le m \le r_a^i)$ denote the index of the channel covered by the anchor radio m in the current frame and let $c_0 = 0$ and $c_{r_a^i+1} = N + 1$. Let s denote the first channel covered by the scan radios at the current time slot $t = 1$. There exists $0 \le m_0 \le r_a^i + 1$ such that $c_{m_0} < s < c_{m_0+1}$.

By examining the CH pattern of M-Rendezvous, we can notice that for the remaining slots of frame f, the scan radios need to scan $N - (s - 1) - (r_a^i - m_0)$ channels, which requires $\tau = [N - (s - 1) - (r_a^i - m_0)]/r_s^i$ slots. Now consider the remaining $S_i - \tau$ slots in frame $f + 1$, it follows from the CH pattern that the anchor radios cover channels $c_m \oplus 1$ $(1 \le m \le r_a^i)$ in frame $f + 1$. Noticing that $c_{m_0} < s < c_{m_0+1}$, we have $c_{m_0} + 1 \le s \le c_{m_0+1}$. It follows from the CH pattern in frame $f + 1$ that in the first $S_i - \tau$ slots of frame $f + 1$, the channels from 1 to h are covered, where

$$\begin{aligned} h &= r_s^i \tau + m_0 \\ &= r_s^i\left[S_i - \frac{N - (s-1) - (r_a^i - m_0)}{r_s^i}\right] + m_0 \\ &= r_s^i\left[\frac{N - r_a^i}{r_s^i} - \frac{N - (s-1) - (r_a^i - m_0)}{r_s^i}\right] + m_0 \\ &= s - 1. \end{aligned}$$

Hence, every channel is covered within S_i slots, with channels s to N covered in frame f and 1 to $s - 1$ in frame $f + 1$. We thus complete the proof. □

Armed with Lemma 1, we next study the rendezvous delay and diversity of M-Rendezvous. Specifically, consider a rendezvous pair a and b, we show that if $r_s^i, r_a^i \simeq O(r)$ $(i \in \{a, b\})$, the worst-case MTTR is bounded by $O(N/r^2)$ time slots (Theorem 2) and the full rendezvous diversity is achieved within at most $O(N^2/r^2)$ slots (Theorem 3). Before delving into the technical details of the analysis, we provide the intuitions of the theorems and their proofs, as illustrated in Fig. 3:

- Rendezvous delay: Consider the anchor radios of a that stay in the same channels for the entire frame of $O(N/r)$ slots and consider the scan radios of b that scan sequentially across the channels within one frame. Let $O(r)$ be the number of scan radios of b, then if node b using $O(r)$ scan radios chases the anchor radios of a, we know that a and b will rendezvous on some channel within at most $O(N/r^2)$ slots;
- Rendezvous diversity: Following the above explanation, a rendezvous is guaranteed on the channels covered by the anchor radios of a within at most $O(N/r^2)$ slots. Moreover, within $O(N/r)$ frames each channel is covered by an anchor radio of a for one entire frame, leading to a rendezvous on every channel within at most $O(N/r)$ frames, i.e. $O(N^2/r^2)$ slots.

Theorem 2 (Rendezvous delay): The worst case rendezvous delay (MTTR) of M-Rendezvous Γ between a and b is

$$\min\left\{\left\lceil\frac{F_a - 1}{r_s^b}\right\rceil, \left\lceil\frac{F_b - 1}{r_s^a}\right\rceil\right\}$$

slots where $F_i = N/r_a^i$ $(i \in \{a, b\})$. Asymptotically, if $r_s^i, r_a^i \simeq O(r)$ $(i \in \{a, b\})$, it holds that $\Gamma \simeq O(N/r^2)$.

Proof: Recall the CH pattern of the anchor radios in M-Rendezvous that any two neighbouring anchor radios of a are separated by F_a channels, it follows from Lemma 1 (pseudo-monotonicity) that there exists channel h covered by an anchor radio m of node a such that after at most $\lceil (F_a - 1)/r_s^b \rceil - 1$ slots, one radio of b (either an anchor radio or a scan radio) will cover channel h. Let t_h denote the index of such slot, it holds that $t_h \le \lceil (F_a - 1)/r_s^b \rceil - 1$.

Now consider the anchor radio m of a at slot t_h, it covers either channel h or channel $h \oplus 1$. If it covers channel h, then a rendezvous is achieved at slot t_h on channel h. We now prove the case where it covers channel $h \oplus 1$. On one hand, it follows from the CH pattern of the anchor radios that it will still cover channel $h \oplus 1$ in slot $t_h + 1$. On the other hand, it follows from the pseudo-monotonicity in Lemma 1 that channel $h \oplus 1$ is covered by a radio of b in either slot t_h or slot $t_h + 1$, both leading to a rendezvous with the delay of t_h and $t_h + 1$, thus upper-bounding the MTTR by $\lceil (F_a - 1)/r_s^b \rceil$.

Symmetrically, we can upper-bound the MTTR by $\lceil (F_b - 1)/r_s^a \rceil$. Hence, the MTTR upper-bound is

$$\min\left\{\left\lceil\frac{F_a - 1}{r_s^b}\right\rceil, \left\lceil\frac{F_b - 1}{r_s^a}\right\rceil\right\}.$$

Asymptotically, if $r_s^i, r_a^i \simeq O(r)$ $(i \in \{a, b\})$, we have $\Gamma \simeq O(N/r^2)$. □

Theorem 3 (Rendezvous diversity): M-Rendezvous can guarantee rendezvous on all the N channels between a and b (assume $F_a \ge F_b$) within at most $S_a(F_a + 1) - 1$ slots. Asymptotically, if $r_s^i, r_a^i \simeq O(r)$ $(i \in \{a, b\})$, the full rendezvous diversity is achieved within $O(N^2/r^2)$ slots.

Proof: Recall Lemma 1 (pseudo-continuity), within a frame where the anchor radio l of node a $(1 \le l \le m)$ covers channel h_l for S_a slots, a rendezvous is ensured to occur in the frame between the anchor radio l and a radio of node b on channel h_l. It then follows from the CH pattern of node a that within F_a frames, i.e. $S_a F_a$ slots, each channel is covered by an anchor radio of a for one frame, resulting in the full rendezvous diversity. Due to the initial clock drift of a, the first entire frame where every anchor radio of i covers a channel for S_a slots must occur at most after $S_a - 1$ slot. Hence, the full rendezvous diversity is guaranteed within at most $S_a(F_a + 1) - 1$ slots, which approximates to $O(N^2/r^2)$ asymptotically. □

Theorem 2 guarantees that a rendezvous pair with any clock drift between them can rendezvous within at most $O(N/r^2)$ slots. Theorem 3 further establishes full rendezvous diversity is achieved within at most $O(N^2/r^2)$ slots. This capability to rendezvous on every channel significantly improves rendezvous robustness by minimising the impact of PU activities.

4.3 Optimality and optimisation of M-Rendezvous

Armed with the theoretical results established in the previous subsection, we investigate the optimality and optimisation of M-Rendezvous in this subsection. Specifically, we study the following natural questions:

- How to set the number of anchor and scan radios r_a^i and r_s^i to optimise the performance of M-Rendezvous?

- Under what circumstances the exact optimality (rather than order optimality) derived in Theorem 1 is reached?

In this regard, we consider the rendezvous between a and b by distinguishing the following two cases.

4.3.1 Approaching exact optimality: system with pre-assigned roles.: Firstly, it can be deduced from Theorem 2 that the MTTR Γ is minimised in the degenerated case where $r_a^a = N$, $r_s^b = 0$ (or symmetrically $r_a^a = 0$, $r_s^b = N$) with the minimum approaching asymptotically N/r^2, the theoretical protocol-independent delay upper-bound established in Theorem 1. Intuitively, setting $r_a^a = N$ minimises the space between two neighbouring anchor radios of a, while setting $r_s^b = 0$ maximises the scan capacity of the scan radios of b, thus minimising the rendezvous delay as a whole. At the system level, this situation corresponds to the configuration where a and b operate in different modes: a operates in the anchor mode [We need to slightly modify M-Rendezvous in the way that each anchor radio, separated by N/r channels to the neighbour anchor radios, stays on a channel for N slots before moving to the next channel.] as all of its r radios are configured as anchor radios; b operates in the scan mode as all of its N radios are configured as scan radios. We can notice that if each node in the network has a pre-assigned role as either a sender or a receiver (e.g. in half-duplex communication systems or Bluetooth pairing), this operation setting can be implemented by letting the senders operate on the anchor mode and the receivers on the scan mode to approach the exact optimality.

4.3.2 Achieving $\frac{1}{4}$-optimality: system with non-preassigned roles.: In systems where the role of a node as sender/receiver cannot be pre-assigned, the above operation setting may fail to function as two nodes operating on the same mode may fail to rendezvous with each other. With straightforward algebraic operations, we can show that by setting $r_a^a = r_s^a = r/2$, we can minimise the worst-case rendezvous delay without the knowledge of the configuration of b (r_s^b and r_a^b). The intuition is that without knowing the configuration of its rendezvous peer, the best strategy of a node is to evenly distribute its capacity into scan and anchor efforts. The MTTR Γ in this situation has the same order of magnitude $O(N/r^2)$ but with a discount factor 4 as the price of not having pre-assigned roles. More generically, more flexible configurations can be chosen for node i by varying r_a^i and r_s^i but keeping them the same order. For example, a larger r_a^i (r_s^i, respectively) can be attributed to nodes more likely to be senders (receivers). Under this setting, we can still achieve the order of magnitude minimum for the worst-case MTTR with a smaller discounting factor in average.

From the designer's perspective, the above two operation settings can be programmed by configuring the number of anchor and scan radios to gear M-Rendezvous to target applications and achieve further design tradeoff.

4.4 Tuning anchor pattern to balance rendezvous channel load: the case of multiple rendezvous pairs

In this subsection, we extend our study on M-Rendezvous with multiple rendezvous pairs. In this context, a desirable property is to balance the rendezvous channel load such that the rendezvous of different communication pairs are evenly distributed among all the N channels. Unfortunately, the baseline version of M-Rendezvous may lead to unbalanced load among different rendezvous channels when the clocks drifts between different rendezvous pairs fall in some particular pattern.

To illustrate the problem of unbalanced rendezvous channel load, we consider an example by extending the example shown in Fig. 3 to the case of multiple rendezvous pairs. Specifically, consider x rendezvous pairs, each composed of two nodes a_l, b_l ($1 \le l \le x$) having the same radio configuration as the nodes a and b in Fig. 3. The clocks of a_l and b_l are synchronised with the clock of a and b in Fig. 3, respectively. This scenario represents the situation where nodes in $\{a_l\}$ ($\{b_l\}$, respectively) are geographically close to each other while significantly more distant to nodes in $\{b_l\}$ ($\{a_l\}$). Under this context, it can be noted that the first rendezvous of all rendezvous pairs is achieved on channel 2, resulting in an extremely unbalanced situation with the rendezvous channel load $L = 1/N$ that may lead to congestion on channel 2.

The rendezvous channel load unbalancing problem occurs when the CH patterns of the radios, particularly that of the anchor radios, of the rendezvous pairs are 'synchronised' with a constant offset due to particular clock drift patterns as illustrated in the example analysed above. To mitigate this problem, we propose to 'desynchronise' the CH patterns of the anchor radios of the rendezvous pairs. We illustrate our idea in an example of one scan radio and one anchor radio, while the extension to the generic cases is trivial. For each CH period of M-Rendezvous, instead of starting from channel 1, the anchor radio starts from a random channel $h \in \mathcal{N}$. The CH pattern of the scan radios remains the same as that of baseline M-Rendezvous. Reconsider the example of the previous paragraph, it can be easily shown that the rendezvous channel load becomes $L = 1$ which corresponds to a balanced situation. To summarise, the introduction of such randomness in the anchor radio CH pattern desynchronises the anchor radios and thus balances the rendezvous channel load without degrading the performance in terms of rendezvous delay and diversity.

5 M-Rendezvous under asymmetrical channel perceptions

In previous analysis, we implicitly assume that the rendezvous pair a and b can access all the N channels, i.e. they have the same perception on the accessible channel set. In this section, we relax this assumption to show how M-Rendezvous can be adapted to the situation where the rendezvous pair have asymmetrical channel perceptions so as to iron out a version of M-Rendezvous that works in practice.

Specifically, the channel perception asymmetry can be characterised at the following two levels:

- *Asymmetry on accessible channel set:* The rendezvous pair a and b may have different accessible channel set, denoted as $\mathcal{N}_a$ and $\mathcal{N}_b$, both subsets of $\mathcal{N}$. For example, in a system with $\mathcal{N} = \{1, 2, 3, 4\}$, we may have $\mathcal{N}_a = \{1, 2, 3\}$ and $\mathcal{N}_b = \{2, 4\}$.
- *Asymmetry on channel index:* Every rendezvous node (say node a) may have its own channel labelling function to assign each physical channel in its accessible channel set (say $\mathcal{N}_a$) with a channel index. To formalise the channel index asymmetry, we define the channel index function as follows.

Definition 2: The channel index function Φ_i for node i is a bi-injective mapping:

$$\Phi_i : \mathcal{N}_i \to \{1, \cdots, N_i\},$$

where $\forall h_1, h_2 \in \mathcal{N}_i$, $\Phi_i(h_1) = \Phi_i(h_2) \Rightarrow h_1 = h_2$. The inverse mapping of $\Phi_i(h_1)$ is denoted as Φ_i^{-1}.

Reconsider the example with $\mathcal{N} = \{1, 2, 3, 4\}$ and $\mathcal{N}_b = \{2, 4\}$, we have $\Phi_b(2) = 1$ and $\Phi_b(4) = 2$.

To study the design of M-Rendezvous under such asymmetrical channel perceptions, we bring the asymmetry to its extreme by focusing on the case where $\mathcal{N}_a$ and $\mathcal{N}_b$ overlap on only one channel h, i.e. $\mathcal{N}_a \cap \mathcal{N}_b = \{h\}$, indexed as h_a by a and h_b by b.

5.1 Protocol-independent bound on rendezvous delay

We start by establishing the lower-bound of the worst-case rendezvous delay for any channel rendezvous protocol under asymmetrical channel perceptions.

Theorem 4 (Protocol-independent worst-case rendezvous delay bound under asymmetrical channel perceptions): The worst-case rendezvous delay among all possible channel index functions of any rendezvous protocol cannot be lower than N_aN_b/r_ar_b.

Proof: We prove the theorem by contradiction. Consider $(N_aN_b/r_ar_b) - 1$ consecutive slots from slot 0 to $(N_aN_b/r_ar_b) - 2$. Since the total number of channel index combinations (h_a, h_b) with $h_a \in \mathcal{N}_a$ and $h_b \in \mathcal{N}_b$ is N_aN_b, there must exist one pair $\{h_a^0, h_b^0\}$ such that we cannot find a slot $0 \le t \le (N_aN_b/r_ar_b) - 2$ such that one of the r_a radios of a operates on channel h_a^0 and one of the r_b radios of b operates on h_b^0.

Let h^* denote the unique channel that both a and b can access. We show that there exists channel mappings Φ_a and Φ_b under which a and b cannot rendezvous within $(N_aN_b/r_ar_b) - 1$ slots. To this end, we consider a pair of random channel index mappings Φ_a and Φ_b and consider the following cases:

- If $\Phi_a(h^*) = h_a^0$ and $\Phi_b(c^*) = h_b^0$, rendezvous cannot be achieved within $(N_aN_b/r_ar_b) - 1$ slots since there does not exist a slot $0 \le t \le (N_aN_b/r_ar_b) - 2$ such that a radio of a operates on channel h_a^0 and a radio of b operates on h_b^0.
- Otherwise, we construct the following telephone label functions for a and b, denoted as Φ_a' and Φ_b':

$$\Phi_a'(h) = \begin{cases} \Phi_a(h) & h \ne h^*, \Phi_a^{-1}(h_a^0), \\ h_a^0 & h = h^*, \\ \Phi_a(h^*) & h = \Phi_a^{-1}(h_a^0); \end{cases}$$

$$\Phi_b'(h) = \begin{cases} \Phi_b(h) & h \ne h^*, \Phi_b^{-1}(h_b^0), \\ h_b^0 & h = h^*, \\ \phi_b(h^*) & h = \Phi_b^{-1}(h_b^0)\,. \end{cases}$$

Again, since there does not exist a slot $0 \le t \le (N_aN_b/r_ar_b) - 2$ such that a radio of a operates on channel h_a^0 and a radio of b operates on h_b^0, rendezvous cannot be achieved within $(N_aN_b/r_ar_b) - 1$ slots.

The analysis in the above two cases contradicts the assumption that the worst-case rendezvous delay is at most $(N_aN_b/r_ar_b) - 1$ and completes our proof. □

5.2 Adaptation of M-Rendezvous under asymmetrical channel perceptions

We assume that each node has an ID which is globally unique, e.g. its MAC address. We use the mechanism proposed in [14] to generate padded binary sequences that are cyclic rotationally distinct one to the other.

We adapt M-Rendezvous for each node i as follows to make it robust against asymmetrical channel perceptions:

Case 1: i has one radio.

- Add a bit 0 at the end of its ID $\boldsymbol{i}$ to form a new ID: $\boldsymbol{i} \leftarrow \boldsymbol{i} \parallel 0$;
- If N_i is not a power multiple of 2, let N_i' denote the smallest power multiple of 2 larger than N_i, expand $\mathcal{N}_i$ to $\mathcal{N}_i' \triangleq \{1, \dots, N_i'\}$ where the first N_i elements denote the channels in $\mathcal{N}_i$ and any $N_i + 1 \le h \le N_i' - 1$ denotes a random channel in $\mathcal{N}_i$;
- Construct the CH sequence based on $\boldsymbol{i}$ and $\mathcal{N}_i$ if N_i is a power multiple of 2 or $\mathcal{N}_i'$ otherwise.

Case 2: i has multiple radios.

- Add a bit 0 at the end of its ID $\boldsymbol{i}$ to form a new ID: $\boldsymbol{i} \leftarrow \boldsymbol{i} \parallel 0$;
- If there exists k such that $2^k(2n+1) < S_i < 2^{k+1}(2n+1)$ where n denotes the length of new IDs after adding the bit 0, then expand $\mathcal{N}_i$ to $\mathcal{N}_i'$ where $N_i' = 2^{k+1}(2n+1)r_s^i + r_a^i$ (i.e. $S_i' = 2^{k+1}(2n+1)$ [Please refer to Section 4 for the definition on S_i.]) by following the same procedure as that in case 1; construct the CH sequence based on $\mathcal{N}_i'$;
- Otherwise construct the CH sequence based on $\mathcal{N}_i$.

The following theorem establishes the worst-case rendezvous delay of the adapted M-Rendezvous under asymmetrical channel perceptions.

Theorem 5 (Rendezvous delay of adapted M-Rendezvous under asymmetrical channel perceptions): Let h^* denote the unique channel accessible by both a and b, rendezvous can be guaranteed on h^* within max $\{O(N_a^2), O(N_b^2)\}$ slots by using the adapted M-Rendezvous.

Proof: We prove the theorem by distinguishing the following three cases. By slightly abusing notations without introducing ambiguity, we use N_i to denote N_i' after adaptation if necessary.

Case 1: both a and b have multiple radios. Without loss of generality, assume that $S_a \ge S_b$. Recall the CH pattern of the anchor radios, within at most $(F_a + 1)S_a - 1$ slots, there must be a complete frame where an anchor radio of a is on channel h. The bound $(F_a + 1)S_a - 1$ is achieved when starting by an incomplete frame of $S_a - 1$ slots with an anchor radio on channel h. Recall Lemma 1 (pseudo-continuity), for any consecutive S_b slots, channel h is covered by a radio of b. It then follows from $S_a \ge S_b$ that a rendezvous is guaranteed to happen on channel h within at most $(F_a + 1)N_a - 1$ slots, i.e. $O(N_a^2)$ slots.

Case 2: only one of a and b has multiple radio and the other has one radio. Without loss of generality, assume that a has one radio and b has $r_b \ge 2$ radios. We further consider the following two subcases.

- *Subcase 2.1:* $2nN_a \ge S_b$. Recall the CH sequence of a, within $N_a + 1$ periods, there must exist an entire period in which a operates on channel h^* during the anchor frames. More specifically, since there are n consecutive bits 1 in the padded ID α, there exist $2nN_a$ consecutive slots in which a operates on h^*. It follows from Lemma 1 (pseudo-continuity) that each channel $h \in \mathcal{N}_b$ is covered by either an anchor radio or a scan radio within the frame (S_b slots). Hence, a and b can achieve rendezvous on h^* within at most N_a periods, i.e. $O(N_a^2)$ slots.
- *Subcase 2.2:* $2nN_a < S_b$. Recall that after adaptation, we have (i) N_a is a power multiple of 2 and (ii) there exists k such that $S_b = 2^k(2n+1)$. Hence, it holds that $S_b \ge (2n+1)N_a$. Recall the CH sequence of b, within at most $\frac{N_b}{r_a^b}$ frames, there must exist one frame during which an anchor radio of b operates on channel h^*. Since a frame lasts S_b slots, it holds that within N_bS_b slots, there are S_b consecutive slots during which an anchor radio of b operates on channel h^*. Now consider the padded ID of a $\boldsymbol{a} \parallel \mathbf{1} \parallel \mathbf{0}$, since $\boldsymbol{a}$ ends with a bit 0 after adaptation, there must exist at least one 0 bit within any $n+1$ consecutive bits. Hence, there must exist at least one scan frame within any consecutive $n+1$ frames. It follows that there must exist at least one slot within any consecutive $(2n+1)N_a$ slots during which a operates on channel h^*. It then follows from $S_b \ge (2n+1)N_a$ that a rendezvous must occur on channel h^* within N_bS_b slots, i.e. $O(N_b^2)$ slots.

Case 3: both a and b have single radio. Without loss of generality, assume that $N_a \ge N_b$. We consider the following two subcases:

- *Subcase 3.1:* $N_a = N_b$. Without loss of generality, suppose that the clock of a is i slots ahead of the clock of b, where i is an arbitrary non-negative integer. Let $\boldsymbol{u}$ and $\boldsymbol{v}$ denote the CH sequences of a and b,, noticing that the period of $\boldsymbol{u}$ and $\boldsymbol{v}$ is $6nN$, it suffices to consider the case with $i \le 6nN - 1$. Let $i = 2Ni_1 + i_2$ where $i_1 \triangleq \lceil \frac{i}{2N} \rceil - 1 \in [0, 3n - 1]$ and $i_2 \triangleq i \bmod 2N \in [0, 2N - 1]$.

- If $i_2 \in [0, N_a - 1]$. Let j denote the bit index such that $a(i_1)_j$ differs b_j. Without loss of generality, suppose that $a(i_1)_j = 0$ and $b_j = 1$. Recall the CH sequence of b, within $N_b + 1$ periods, there must exist an entire period in which b operates on channel h^* during the anchor frames. We now consider such anchor frame corresponding to b_j. Note that frame j is a scan frame at node a that scans channel h^* each N_a slots and that the frame j of a and b overlap during consecutive $2N_a - i_2 \ge N_a + 1$ slots, it holds that a and b can rendezvous on channel h^* within $N_b + 1$ periods, or $O(N_b^2)$ slots.
- If $i_2 \in [N_a, 2N_a - 1]$. Let j denote the bit index such that $a(i_1 + 1)_j$ differs b_j. By following the same analysis, we can show that the rendezvous is ensured on channel h^* within $O(N_b^2)$ slots.

• *Subcase 3.2:* $N_a > N_b$. Recall that after adaptation, both N_a and N_b are power multiples of 2, it holds that $N_a \ge 2N_b$. Recall the CH sequence of a, within at most $N_a + 1$ periods, there must exist one period during which a operates on channel h^* in anchor frames. Consider such a period and notice the padded ID of a $\boldsymbol{a} \parallel \boldsymbol{1} \parallel \boldsymbol{0}$, there must exist at least n consecutive anchor frames lasting $2nN_a$ slots during which a operates on channel h^*. Now consider the padded ID of b $\boldsymbol{b} \parallel \boldsymbol{1} \parallel \boldsymbol{0}$, since $\boldsymbol{b}$ ends with a bit 0, there must exist at least one 0 bit within any $n + 1$ consecutive bits. Hence, there must exist at least one scan frame within any consecutive $n + 1$ frames. It follows that there must exist at least one slot within any consecutive $(2n + 1)N_b$ slots during which b operates on channel h^*. It then follows from $N_a \ge 2N_b$ that a rendezvous must occur on channel h^* within $O(N_a^2)$ slots.

Combining the analysis on the three cases completes the proof.
□

The above analysis demonstrates a notable property of the adapted M-Rendezvous on the robustness of rendezvous against asymmetrical channel perceptions.

6 Performance evaluation

In this section, we simulate the baseline scenario of a multi-channel CRN of $N = 200$ channels, where two rendezvous nodes are equipped with r radios ($r \ge 2$). We simulate the cases where nodes have symmetrical and asymmetrical channel perceptions and the rendezvous channel load. We compare the performance of M-Rendezvous with RPS and CMR.

6.1 Symmetrical channel perceptions

We first simulate the scenario where the rendezvous nodes have symmetrical channel perceptions and can access all the channels. We study the rendezvous delay and diversity by plotting the worst-case rendezvous delay (MTTR) and the worst-case delay to achieve full rendezvous diversity by varying r in Figs. 4 and 5.

We make the following observations from the simulation results:

(1) The MTTR is bounded for both protocols (with and without pre-assigned role), meaning that they can both guarantee rendezvous when the clocks of the rendezvous pair are not synchronised. In terms of MTTR, M-Rendezvous (without pre-assigned roles) has comparable performance with CMR [40] and performs slightly better than RPS [34]. This can be explained as follows: RPS does not take into account full rendezvous diversity in essence while CMR needs two prime CH sequence lengths as a prerequisite.

(2) In terms of rendezvous robustness, M-Rendezvous can achieve rendezvous on every channel, which is not the case with RPS since RPS cannot guarantee full rendezvous diversity in essence.

(3) Having more radios per node brings performance gain in terms of both rendezvous delay and diversity (note the logarithmic scale of the y-axis). The benefit of increasing r on the performance is more significant with small r. By carefully examining the results in both figures, we observe that the performance gain goes squarely w.r.t. r, which confirms our theoretical results established in Section 4.

(4) In the case with pre-assigned roles, M-Rendezvous achieves better performance compared with the case without pre-assigned roles. The result is also in accordance to our theoretical finding that this setting has the minimal inter-anchor radio distance in terms of the number of channels at the sender side and the maximal scan capability at the receiver side, thus minimising the MTTR and the delay to achieve full rendezvous diversity.

Under the opportunistic spectrum sharing paradigm, rendezvous may be significantly affected by the primary traffic. To evaluate the impact of primary traffic on rendezvous performance, we conduct a set of simulations under different primary activities. A commonly used model to characterise the primary activity is to model it as an i.i.d. Bernoulli random variable with the busy probability λ [44, 45]. In our simulation, we generate a primary traffic on each channel by varying λ.

Fig. 6 plots the average time to rendezvous (ATTR) with different λ. We make the following observations: (i) the ATTR decreases rapidly when the number of radio increases and drops below 50 slots for both $\lambda = 0.2$ and $\lambda = 0.4$ when r reaches 4 for M-Rendezvous, RPS, and CMR; (ii) M-Rendezvous outperforms RPS with a larger gap in ATTR which demonstrates the robustness of M-Rendezvous under the presence of PUs because of full rendezvous diversity of M-Rendezvous.

6.2 Asymmetrical channel perceptions

We now investigate the scenario where rendezvous nodes have asymmetrical channel perceptions, i.e. they do not have the same knowledge on $\mathcal{N}$. Specifically, we simulate the following three scenarios:

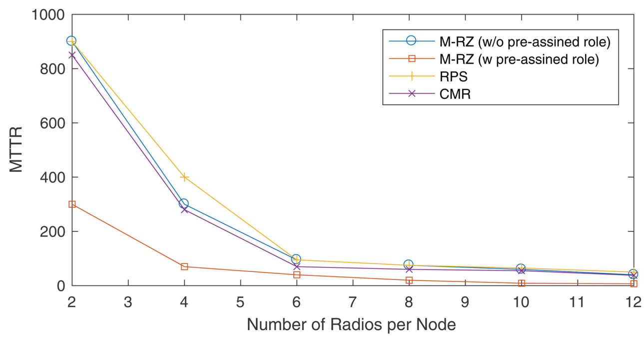

Fig. 4 *Worst-case rendezvous delay (MTTR): multi-radio scenario, symmetrical channel perceptions*

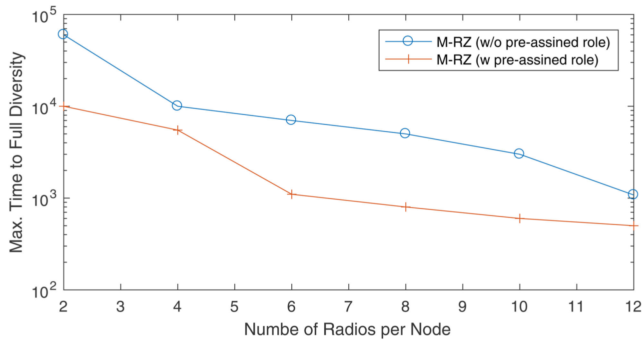

Fig. 5 *M-Rendezvous: worst-case delay to achieve full rendezvous diversity: multi-radio scenario, symmetrical channel perceptions*

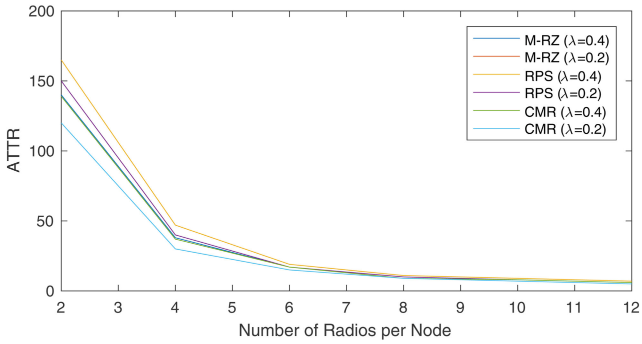

Fig. 6 *ATTR under different PU activities*

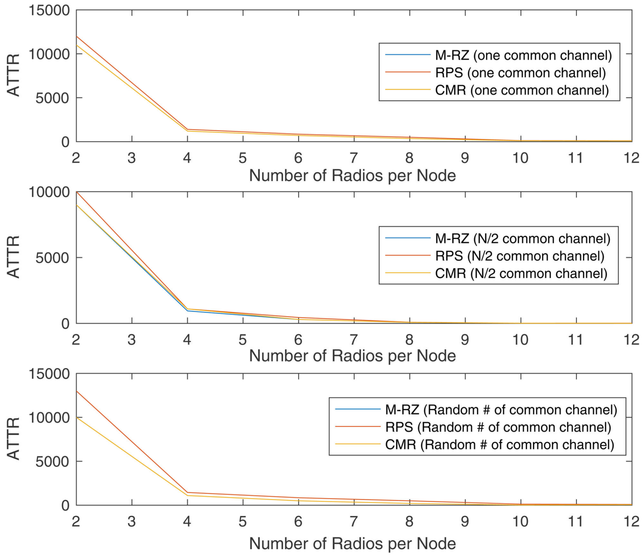

Fig. 7 *ATTR under asymmetrical channel perceptions*

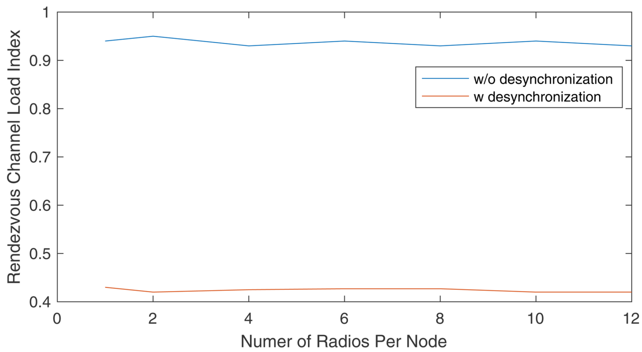

Fig. 8 *Rendezvous channel load index L*

- There is only one common channel between them;
- There are $N/2$ common channels;
- The number of common channels is randomly distributed in $[1, N]$.

We observe that all simulated runs result in rendezvous and that rendezvous are achieved in every channel accessible to both of the rendezvous nodes under M-Rendezvous. In other words, M-Rendezvous ensures bounded MTTR with full rendezvous diversity. To further quantify rendezvous performance, we plot ATTR in Fig. 7. We observe that M-Rendezvous outperforms RPS (note the logarithmic scale of the y-axis), which is due to the fact that M-Rendezvous can achieve full rendezvous diversity and thus is more robust against channel perception asymmetry.

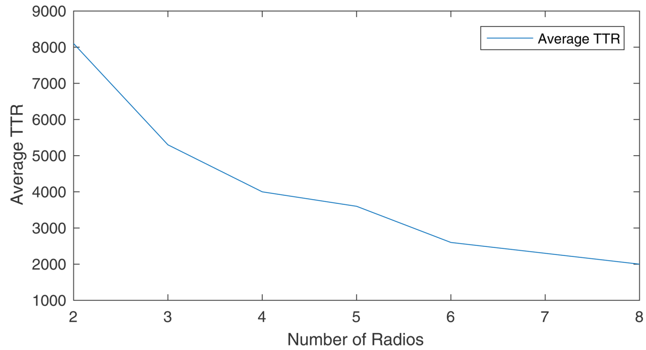

Fig. 9 *ATTR: heterogeneous scenario*

6.3 Rendezvous channel load balancing

We also evaluate the rendezvous channel load of M-Rendezvous by incorporating the load balancing mechanism developed in Section 4.4.

We have pointed out in Section 4.4 that the baseline M-Rendezvous may lead to unbalanced load among different rendezvous channels when the clocks drifts between different rendezvous pairs fall in some particular fashion. Specifically, the rendezvous channel load index L drops to $1/N$ in these cases. To mitigate the problem, we have developed a mechanism to 'desynchronise' the CH patterns of the anchor radios of the rendezvous pair. In Fig. 8, we plot the rendezvous channel load index L of M-Rendezvous by incorporating the proposed load balancing mechanism with 200 rendezvous pairs. For comparison, we also trace L by running a number of simulation runs with random asynchonised clocks in both ACH and M-Rendezvous without implementing the load balancing mechanism. It can be observed that in all the simulated settings, L is close to its maximum 1 when the anchor radio CH pattern desynchronisation is implemented, resulting in significant performance gain compared with the baseline setting without such desynchronisation. Consequently, with the proposed rendezvous channel load balancing approach implemented, M-Rendezvous can evenly distribute the rendezvous load among different channels. This property makes M-Rendezvous especially adapted in the decentralised CR environment in which CR nodes are densely deployed.

6.4 Rendezvous between single-radio and multi-radio nodes: heterogeneous case

We simulate a scenario with asymmetrical channel perceptions where the multi-radio node can access all the channels in $\mathcal{N}$ while the single-radio node can only access $\lfloor N/2 \rfloor$ channels in $\mathcal{N}$. We observe that all simulated runs result in rendezvous and that rendezvous are achieved in every channel accessible to both of the rendezvous nodes, which demonstrates the capability of M-Rendezvous of achieving bounded MTTR with full rendezvous diversity even in the heterogeneous case.

Fig. 9 further traces average TTR (ATTR) as a function of r, from which we observe a decrease of ATTR as r increases which demonstrates the performance benefits brought by having multiple radios as even one of the rendezvous peers.

7 Conclusion

In this paper, we have presented M-Rendezvous, an order-optimal rendezvous protocol exploiting the potential performance gain brought by having multiple radios at cognitive nodes. As a distinguished feature, M-Rendezvous is a unified rendezvous protocol that can operate in both homogenous case where both of the rendezvous nodes are equipped with only one radio or multiple radios, and heterogeneous case where one of the rendezvous nodes has single radio and the other has multiple radios. In both cases, by rigorous analysis, we have demonstrated that M-Rendezvous can guarantee rendezvous over every channel with bounded and order-minimal delay even when rendezvous nodes have asynchronous clocks and asymmetrical channel perceptions.

Our analysis also sheds light on the theoretical performance bound of any channel rendezvous protocol by relating the two important performance metrics, rendezvous delay and diversity. For any rendezvous protocol with full rendezvous diversity, the lower-bound of the rendezvous delay scales linearly in the number of channels while decreases squarely in the number of radios per node. We believe that this is a fundamental result that can guide the design of other channel rendezvous protocols in the future research.

8 Acknowledgement

This work is partly supported by National Natural Science Foundation of China under Grant no. 61672395.

9 References

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