IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 25, 2019, accepted April 20, 2019, date of publication April 30, 2019, date of current version May 14, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2914148

On Cooperative Channel Rendezvous in Cognitive Radio Networks

KEHAO WANG[1], LIN CHEN[2,3], KAIGUI BIAN[4], WEI WANG[5], AND PAN ZHOU[6]

[1]School of Information Engineering, Wuhan University of Technology, Wuhan 430070, China
[2]Laboratoire de Recherche en Informatique (LRI), University of Paris-Sud, 91405 Orsay, France
[3]Institut Universitaire de France (IUF)
[4]Institute of Network Computing and Information System, School of EECS, Peking University, Beijing 100871, China
[5]Department of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 321000, China
[6]School of Electrical and Information Communications, Huazhong University of Science and Technology, Wuhan 430074, China

Corresponding author: Lin Chen (chen@lri.fr)

This work was supported in part by the National Natural Science Foundation of China under Grant 61672395 and Grant 61571396.

ABSTRACT In cognitive radio (CR) networks, establishing communication sessions requires the communicating pairs to meet each other on a common channel via a ''rendezvous'' process. Designing distributed rendezvous protocols without common control channels is a challenging problem due to the dynamic and opportunistic spectrum access paradigm. The existing protocols, mainly based on channel hopping, suffer from the rendezvous difficulty due to asymmetric channel perceptions and significant rendezvous delay when the system scales. In this paper, we exploit a long-neglected opportunity for enabling rendezvous in CR networks—the cooperation among cognitive users. We develop a novel cooperative rendezvous mechanism by introducing the cooperative cognitive nodes serving as ''bridges'' between communicating pairs to facilitate their rendezvous process. We establish a mathematics framework to study the rendezvous delay and robustness and derive the performance limit in asymptotic scenarios. Our analytical results are further confirmed by the simulation results showing the performance improvement of the developed cooperative rendezvous protocol with only small overhead.

INDEX TERMS Cognitive radio (CR), cooperative communication, research problems.

I. INTRODUCTION

Channel rendezvous in cognitive radio (CR) networks is the process between two communicating secondary users (SUs) of ''meeting'' each other on a common channel (a.k.a. a *rendezvous* channel) for exchanging required control information prior to data communications. In opportunistic spectrum sharing (OSS) paradigm, SUs equipped with one or multiple CRs are required to refrain from transmitting on the channels where licensed primary user (PU) signals are detected.

The OSS paradigm poses three challenges for devising channel rendezvous protocols in CR networks. First, *rendezvous failure* between a pair of SUs occurs when their rendezvous channels are blocked by PU signals. Secondly, two communicating SUs may have different views of channel availability as the PU transmission may unpredictably vary at different locations. Such asymmetry in perceptions of available channels can cause the *rendezvous difficulty* in finding a common rendezvous channel. Thirdly, a communication pair may have to sense and search through a large number of channels before successfully achieving rendezvous and thus experience significant *rendezvous delay*.

A common control channel (CCC) that simplifies the rendezvous process is widely deployed in conventional multi-channel wireless networks [1]. However, the rendezvous failure is inevitable when the PU transmission is feasible on the same band of CCC in CR networks. A number of channel hopping (CH) schemes have recently been proposed to mitigate the rendezvous failure by increasing the number of rendezvous channels—the *rendezvous diversity*—between an SU pair. For example, multiple rendezvous channels between two SUs can be guaranteed in the sequence-based CH approaches when the CH sequences are deliberately designed [2]–[19]; the random CH approach [20] can even provide the rendezvous opportunity for two SUs on every channel. However, the rendezvous difficulty in finding a common channel still remains because two SUs cannot exchange any information regarding the asymmetric perceptions of channel availability. Meanwhile, existing CH approaches

The associate editor coordinating the review of this manuscript and approving it for publication was Yan Zhang.

suffer from significant rendezvous delay when the number of channels for SUs to scan is large.

In this paper, we exploit a long-neglected opportunity for enabling channel rendezvous in CR networks—i.e., the cooperation among cognitive users. Specifically, we investigate how to improve the rendezvous performance by introducing the cooperative SUs (termed as helper nodes throughout the paper) that serve as "bridges" between neighboring SUs to facilitate their rendezvous process. A cooperative SU can simply relay the rendezvous requests of neighboring SUs on its operating channels such that the SUs that need to rendezvous can switch to a common channel by referring to the information contained in the rendezvous requests.

The cooperative SU's operation of relaying rendezvous requests brings about three benefits for channel rendezvous between neighboring SUs: 1) a greater number of channels where the rendezvous requests circulate is equivalent to the increase of rendezvous diversity; 2) the relayed rendezvous requests contain necessary information that can mitigate the rendezvous difficulty caused by the asymmetric perceptions of channel availability between two SUs; and 3) with help of the information in relayed rendezvous requests, it is easier for two SUs to find each other on a common channel at a small rendezvous latency. From the practical implementation perspective, the proposed cooperative rendezvous protocol only incurs small overhead, but has significant performance improvement. As illustrated later in the paper, the participation of even one helper node can reduce the expected rendezvous delay by 18%. The performance gain exceeds 100% when the rendezvous is assisted by 3 helper nodes.

There are a number of candidates in CR networks that can serve as helper nodes: 1) the bridge customer-premises equipments (CPE) specified by the IEEE 802.22 standard [21] located in the overlapping coverage areas of IEEE 802.22 cells; 2) the authentication nodes deployed around PUs to authenticate the primary signals [22]; 3) SUs that are willing to serve as helpers for assisting their peers to rendezvous.

This paper presents a systematic approach for devising cooperative rendezvous protocols and analyzing the resulting performance benefits. Specifically, we develop a mathematic framework to study the following optimization questions: How should helper nodes cooperate? How much performance gain can we obtain with cooperative rendezvous? What is the impact of the number of CRs per cognitive node and per helper node? Our main contributions are articulated as follows.

- *Non-cooperative rendezvous* (Sec. IV). We first analyze the non-cooperative rendezvous protocol with multiple radios per SU. We quantify the rendezvous performance and establish performance limits in asymptotic scenarios. These results serve as a theoretic basis and comparison reference for the cooperative rendezvous protocol.
- *Cooperative rendezvous* (Sec. V). We devise a novel cooperative rendezvous protocol by exploiting helper nodes to facilitate rendezvous. By characterizing rendezvous delay and robustness, we quantify the benefits for rendezvous using cooperative helper nodes.
- *Optimization of cooperative rendezvous* (Sec. VI). Based on the theoretical foundation laid in previous sections, we further study the following optimization question: how many helpers are necessary to upper-bound the expected rendezvous delay? We then develop a dynamic learning algorithm enabling a helper node to configure its cooperation level based on the target performance metric.

The rest of the paper is organized as follows. Sec. III describes the technical background materials. Sec. IV investigates the non-cooperative rendezvous protocol. Section V presents the cooperative rendezvous protocol. Sec. VI addresses the optimization of the developed cooperative rendezvous protocol. Sec. VII discusses the integration of the proposed approach with other rendezvous mechanisms and related issues. Sec. VIII presents the simulation results. Sec. II summarizes the related work. Sec. IX concludes the paper.

II. RELATED WORKS

A. SINGLE RADIO

Existing distributed rendezvous mechanisms in CR networks, most focusing on single-radio rendezvous, can be categorized into the following two classes:

- *Stationary and memoryless rendezvous* [20], [23]. Motivated by the asymmetric perceptions of channel availability at cognitive nodes and the lack of network synchronization and coordination, this class of rendezvous schemes adopt stationary and memoryless rendezvous strategies such as random channel hopping. Due to the memoryless nature, these schemes are especially robust and adapted in the ad hoc environments where no *a priori* knowledge or coordination is available. The main drawback of them is the lack of performance guarantees in terms of rendezvous delay.
- *Sequence-based rendezvous* [2]–[19]. In this category of rendezvous schemes, each SU switches across different channels based on certain CH patterns which are carefully designed to ensure that each pair of two CH sequences are overlapped within one CH period. Consequently, rendezvous delay can be bounded. However, sequence-based rendezvous schemes usually require certain coordination on the CH patterns and time synchronization, which may degrade rendezvous performance and limit their application.

B. MULTIPLE RADIOS

In [24], the authors presented an adaptive rendezvous algorithm that can function for multiple interfaces and different sizes of channel lists and an adaptive jumping pattern for different channel conditions. In [25], the authors proposed a new rendezvous algorithm, called role-based parallel sequence (RPS), in which nodes stay in a specific channel in one dedicated radio and hop on the available channels with parallel sequences in the remaining general radios. In [26], the authors proposed an Adjustable Multi-Radio Rendezvous (AMRR) algorithm in which m radios of a node is partitioned into two groups: k stay radios and $(m-k)$ hopping radios. The MSS algorithm [27] is based on the Single-radio Sunflower-Sets-based pairwise rendezvous (SSS) algorithm. Mathematical construction of sunflower sets is exploited to develop the SSS rendezvous algorithm In [28], the authors presented two novel CH schemes for multi-radio cognitive users with and without a synchronized time clock to rendezvous blindly in every time slot. In [29], the Hybrid Radios Rendezvous (HRR) algorithm is proposed to address users equipped with different numbers of radios. In [30], the authors proposed a homogeneous multi-radio channel- hopping (CH) rendezvous algorithm which achieves improved (exact) maximum-TTR and is a linear function of a number of available channels instead of quadratic functions in heterogeneous multi-radio algorithms. In [31], the authors designed a low-complexity rendezvous scheme that account for CH capability limits by treating CH patterns as random walks over spectrum graphs. In [32], the authors designed a family of CH sequences, so-called multi-MTTR asynchronous-asymmetric prime sequences (MAAPSs), for cognitive radio networks. In [33], the authors proposed a Chinese Remainder Theorem (CRT) based multi-radio rendezvous (CMR) algorithm for oblivious rendezvous problem in heterogeneous CRNs.

The cooperative rendezvous approach developed in the paper represents an orthogonal research effort to the above two directions. As a desired property, it can be integrated with both classes of rendezvous protocols (cf. Sec. VII). We expect the developed approach to bring new research perspectives on the CR rendezvous and to stimulate more profound research on this topic.

III. PRELIMINARIES

A. NETWORK MODEL

We consider a time-slotted (but not necessarily synchronized) CR network operating on a set $\mathcal{N}$ of N licensed non-overlapping channels[1]. Each SU is equipped with r ($r \geq 1$) radios and thus can access up to r channels at a time. Due to regulatory constraints posed by PUs, the available channel set of each SU is different based on its relative location to the PUs and may vary in both time and spacial domains. For each SU i, we denote $\mathcal{C}_i$ the set of channels it can access.

In order to use the spectrum in a dynamic and opportunistic way in such a multi-channel environment, a pair of SUs wishing to initiate communication need to meet each other on at least one common channel via a rendezvous process. In infrastructureless CR networks without central controllers, the rendezvous process should be carried out in a distributed fashion. In the analysis that follows, we focus on the design of distributed rendezvous protocols in an uncoordinated and efficient way without a common control channel.

B. PERFORMANCE METRICS OF RENDEZVOUS

To evaluate the performance of a rendezvous protocol, we introduce the following metrics quantifying two important aspects of a rendezvous protocol, i.e., rendezvous delay and rendezvous robustness.

- *Rendezvous delay:* The primary performance metric characterizing a rendezvous protocol is rendezvous delay. Specifically, we define the metric *expected time-to-rendezvous* (ETTR) as the expected latency (in number of time slots) before successful rendezvous on at least one channel. ETTR can be derived by calculating the *channel hitting probability*, defined as the probability that rendezvous is achieved in a given time slot.
- *Rendezvous Robustness:* The secondary performance metric is rendezvous robustness. In a CR network where the PU traffic is often unpredictable, a desirable property of a rendezvous protocol is the capability of rendezvousing on multiple channels. In this regard, we quantify the robustness of a rendezvous protocol by the probability of rendezvousing on multiple channels.

IV. NON-COOPERATIVE RENDEZVOUS

In this section, we specify and analyze a natural non-cooperative rendezvous protocol based on random channel hopping when using multiple radios per node. The analysis presented in this section also serves as a theoretical basis and comparison reference for the more sophisticated cooperative rendezvous protocol developed in Sec. V.

A. THE RENDEZVOUS MECHANISM

In a context without any *a priori* knowledge on the accessed channels of other nodes, a natural strategy is to randomize the channel choice to maximize the rendezvous probability in a non-cooperative fashion. This motivates the rendezvous protocol based on random channel hopping. Specifically, each user i randomly selects a set $\mathcal{R}_i$ of r channels from $\mathcal{C}_i$ to and tunes its r radios on them. If user s wants to initiate a communication session with another user d, it sends a rendezvous request on each channel in $\mathcal{R}_s$. The rendezvous is achieved if s and d tune their radios on at least one common channel, i.e., $\mathcal{R}_s \cap \mathcal{R}_d \neq \emptyset$. The process is repeated each slot until successful rendezvous.

In what follows, we study the performance of the non-cooperative rendezvous protocol in terms of rendezvous delay and robustness. To facilitate our analysis and concentrate on the essential properties of the studied rendezvous protocol, we start with the scenario where each user i in the system can access all the channels in $\mathcal{N}$, i.e., $\mathcal{C}_i = \mathcal{N}$. The extension to the generical scenario follows the same methodology and is presented in the appendix.

[1] Throughout this paper, for presentation convenience, we use uppercase calligraphic letters (e.g., $\mathcal{N}$) to denote sets and the correspondent uppercase letters (e.g., N) to denote their cardinalities (e.g, $N = |\mathcal{N}|$).

B. PERFORMANCE ANALYSIS: RENDEZVOUS DELAY AND ROBUSTNESS

We conduct a quantitative analysis on the rendezvous delay and robustness of the non-cooperative rendezvous mechanism with multiple radios by studying the two performance metrics defined in Sec. III-B.

To compute the ETTR (denoted as $\mathbb{E}[T_{ttr}]$), we derive the channel hitting probability (denoted as P_h), as defined in Sec. III-B. It can be noted that the two metrics are actually coupled. $\mathbb{E}[T_{ttr}]$ can be derived from P_h noticing that P_h for each slot is Bernoulli-distributed. Specifically, it holds that:

$$\mathbb{E}[T_{ttr}] = \sum_{t=1}^{\infty} t(1-P_h)^{t-1}P_h = \frac{1}{P_h}.$$

Consider the rendezvous process between s and d, both able to access all the channels. Theorem 1 establishes P_h.

Theorem 1: It holds that

$$P_h = 1 - \prod_{i=0}^{r-1}\left(1 - \frac{r}{N-i}\right). \tag{1}$$

Asymptotically, let $r = O(N^\alpha)$ $(\alpha \le 1)$, *it holds that:*

$$\lim_{N\to\infty} P_h = \begin{cases} 1 & \alpha > \frac{1}{2} \\ 1 - \frac{1}{e^c} & r = \sqrt{cN},\ c = O(1) \\ \frac{r^2}{N} = 0 & \alpha < \frac{1}{2} \end{cases}. \tag{2}$$

Proof: Please refer to Appendix. □

Theorem 1 leads us to observe the following engineering implications on the non-cooperative rendezvous when using multiple radios per user:

- When the number of radios per user r is small, P_h can be approximated in order of magnitude by r^2/N. Compared with rendezvous with a single radio ($r = 1$), P_h scales squarely in r.
- Asymptotically, $O(\sqrt{N})$ is the necessary condition to achieve $P_h \simeq O(1)$, or equivalently $\mathbb{E}[T_{ttr}] \simeq O(1)$.

We next study the rendezvous robustness by establishing the probability of rendezvousing on multiple channels in Theorem 2, whose proof is detailed in the appendix.

Theorem 2: The probability that the rendezvous can be achieved on at least two channels, denoted as P_m, is

$$P_m = 1 - \prod_{i=0}^{r-1}\left(1-\frac{r}{N-i}\right) - \frac{r^2}{N-r+1}\prod_{i=0}^{r-2}\left(1-\frac{r}{N-i}\right).$$

Asymptotically, let $R = O(N^\alpha)$ $(\alpha \le 1)$, *it holds that:*

$$\lim_{N\to\infty} P_m = \begin{cases} 1 & \alpha > \frac{1}{2} \\ 1-(1+c)e^{-c} & r = \sqrt{cN},\ c = O(1) \\ \frac{r^4}{2N^2} = 0 & \alpha < \frac{1}{2} \end{cases}.$$

Theorem 2 quantifies the rendezvous robustness:

- When r is small such that $r \simeq o(\sqrt{N})$, P_h scales quadratically in r. It can also be observed that $P_m \ll P_h$, implying that once rendezvous is achieved, in most cases it is achieved on only one channel.
- Asymptotically, when r scales to $O(\sqrt{N})$, rendezvous is predominately achieved on multiple channels.

To summarize the analytical results obtained in this section, we point out that $r \simeq O(\sqrt{N})$ is a critical point to achieve asymptotically optimal performance in order of magnitude for the non-cooperative rendezvous protocol in terms of both rendezvous delay and robustness. Below this point, compared with the single-radio case, rendezvous performance scales in r^2 and r^4 in terms of rendezvous delay and robustness, respectively.

V. COOPERATIVE RENDEZVOUS

As explained in the Introduction, exploiting cooperative helper nodes in the network can create additional rendezvous diversity, thus facilitating the rendezvous process. Motivated by this observation, we develop a cooperative rendezvous protocol in this section. We start by providing a motivating example illustrating the core idea and proceed to specify the developed protocol. A quantitative analysis on rendezvous performance is then conducted to demonstrate the benefits for rendezvous when using helper nodes.

A. A MOTIVATING EXAMPLE

Consider a CR network consisting of 12 channels and 2 SUs s, d wishing to rendezvous and a helper node m, each equipped with 3 radios and can access all the 12 channels. To rendezvous with d, s tunes its radios to channel 1, 2 and 3. The radios of d and m are tuned to channel 4, 5, 6 and 3, 5, 8, respectively. In this setting, since none of the radios of s and d is on the same channel, the rendezvous cannot be achieved. However, with the help of m, more specifically, with m capturing the rendezvous request s sends on channel 3 and relaying it on channel 5, d can be informed of the presence of s on channels 1, 2, 3 and thus can switch to one of them to rendezvous with s.

To get more quantitative insights on the benefits brought by cooperative rendezvous, we compare the channel hitting probability of the rendezvous without and with the helper m, denoted as P_h and P_h^c. By mathematic analysis (detailed in the sequel analysis in this section), we have $P_h = 0.45$ and $P_h^c = 0.55$, indicating that the participation of even one helper can increase P_h by 20%, thus reducing the expected rendezvous delay by 18%.

B. THE COOPERATIVE RENDEZVOUS MECHANISM

In the proposed cooperative rendezvous mechanism, each SU i randomly selects a set $\mathcal{R}_i$ of r channels from $\mathcal{C}_i$ and tunes its r radios on them. If user i wants to initiate a communication session, it sends a rendezvous request on each channel in $\mathcal{R}_i$. Each helper node hearing a rendezvous request on one of its operating channels c relays the rendezvous request on each channel in $\mathcal{C}_i$ except c. The rendezvous is established between s and d if they tune their radios on at least one common channel (i.e., $\mathcal{R}_s \cap \mathcal{R}_d \neq \emptyset$) or both s and d share at least one common channel with a helper m (i.e., $\mathcal{R}_s \cap \mathcal{R}_m \neq \emptyset$ and $\mathcal{R}_d \cap \mathcal{R}_m \neq \emptyset$). The process is repeated until successful rendezvous.

The core idea of the cooperative rendezvous is to let helper nodes to spread the channel information of s so as to facilitate the rendezvous. Essential to the cooperative rendezvous protocol is the cooperative helper nodes that serve as "bridges" to link the pair of nodes to be rendezvoused.

From the practical perspective, the proposed cooperative rendezvous protocol can be implemented on top of any MAC protocol (e.g., CSMA). The overhead brought by the participation of helper nodes in the rendezvous process is also limited due to the following reasons: (1) The rendezvous beacons are typically very short packets containing information related to the node and channel IDs. Their overhead on packet collisions and generated traffic is thus very limited. Moreover, they can be piggy-backed with other control packets to further limit the overhead. (2) As will be established in this section, the expected rendezvous delay of the cooperative rendezvous protocol is smaller than that of random rendezvous. Consequently, the number of rendezvous beacons sent and relayed is also smaller, thus limiting the overall protocol overhead. (3) We can further configure and limit the protocol overhead and the cooperation efforts at the helpers based on the target performance metric by incorporating the dynamic learning algorithm presented in Sec. VI.

In what follows, we study the performance of the cooperative rendezvous protocol in terms of rendezvous delay and robustness. Sec. VII further studies the rendezvous overhead. As the previous section, we focus on the scenario where every SU i in the network can access all the channels in $\mathcal{N}$. The analysis of the generic scenario is presented in the appendix.

Specifically, we consider the scenario where s and d want to rendezvous with the help of a set $\mathcal{M}$ of M helpers indexed from $m = 1$ to M, each equipped with r_c radios. Denote $\mathcal{H}^m$ the set of channels on which m tunes its r_c radios. We focus on the case where $\mathcal{H}^{m_1} \cap \mathcal{H}^{m_2} = \emptyset, \forall m_1, m_2 \in \mathcal{M}$. Without loss of generality, let $\mathcal{H}^m = \{(m-1)r_c + 1, \cdots, mr_c\}$. Our motivation of focusing on this particular scenario is to investigate the extreme case where the maximal rendezvous diversity is created by the cooperative helpers to facilitate the rendezvous process.

C. THEORETICAL PERFORMANCE ANALYSIS: RENDEZVOUS DELAY

To derive the ETTR, we study the channel hitting probability of the cooperative rendezvous mechanism, denoted as P_h^c.

Theorem 3: It holds that:

$$P_h^c = 1 - \frac{1}{\left[\binom{N}{r}\right]^2} \left[\sum_{k=0}^{M} \sum_{l=0}^{M-k} \sum_{\substack{\sum_{i=1}^{k} a_i \le r \\ a_i \ge 1, 1 \le i \le k}} \sum_{\substack{\sum_{j=1}^{l} b_j \le r \\ b_j \ge 1, 1 \le j \le l}} \binom{M}{k} \times \binom{M-k}{l} \left(\prod_{i=1}^{k} \binom{r_c}{a_i} \right) \binom{N - Mr_c}{r - \sum_{i=1}^{k} a_i} \left(\prod_{j=1}^{l} \binom{r_c}{b_j} \right) \times \binom{N - Mr_c - r + \sum_{i=1}^{k} a_i}{r - \sum_{j=1}^{l} b_i} \right],$$

Specifically, when $M = 1$, the above formula degenerates to:

$$P_h^c = 1 - \frac{\sum_{i=0}^{r} \binom{r_c}{i} \binom{N-r_c}{r-i} \binom{N-r-r_c+i}{r}}{\left[\binom{N}{r}\right]^2}.$$

Proof: Please refer to Appendix. □

Theorem 3 establishes P_h^c in close-form. Therefore, in the analysis that follows, we give an order of magnitude study by mapping the scenario with M helpers to another scenario with one *super helper*, which is more tractable, as developed in Lemma 3.

Lemma 1: Let q_1 denote the probability that s and r can rendezvous on at least one channel among the first Mr_c channels in $\mathcal{N}$ (i.e., channels covered by cooperative helpers). Let q_1^s denote this probability in a system with one super helper equipped with Mr_c radios tuning on channels 1 to Mr_c, other parameters being the same. It holds that

$$q_1^s / M \le q_1 \le q_1^s.$$

Proof: Please refer to Appendix. □

Armed with Lemma 3, we can establish the relationship between the channel hitting probability of the two systems in the following theorem.

Theorem 4: Let P_h^s denote the channel hitting probability in the system with the super helper, it holds that

$$\frac{P_h^s}{M} \le P_h^c \le P_h^s.$$

Proof: Please refer to Appendix. □

Lemma 1 and Theorem 4 illustrate that studying P_h^s, which is more tractable than P_h^c, can provide some important insights on the performance of the cooperative rendezvous mechanism. The following theorem derives P_h^s and studies its limit in the asymptotic scenario.

Theorem 5: It holds that

$$P_h^s = 1 - \frac{\sum_{i=0}^{r} \binom{Mr_c}{i} \binom{N-Mr_c}{r-i} \binom{N-Mr_c-r+i}{r}}{\left[\binom{N}{r}\right]^2}.$$

Asymptotically, let $r = O(N^\alpha)$, $r_c = O(N^{\alpha_c})$, $M = O(N^\beta)$,

$$\lim_{N\to\infty} P_h^s \simeq \begin{cases} 1 & \alpha + \frac{\alpha_c+\beta}{2} > \frac{1}{2} \\ 1 - e^{-\frac{r^2}{N}\left(1+\frac{Mr_c^2}{N}\right)} & \alpha + \frac{\alpha_c+\beta}{2} = \frac{1}{2} \\ \frac{r^2}{N}\left(1 + \frac{Mr_c^2}{N}\right) \to 0 & \alpha + \frac{\alpha_c+\beta}{2} < \frac{1}{2} \end{cases}. \quad (3)$$

Proof: Please refer to Appendix. □

Theorem 4 establishes the lower and upper bounds of P_h^c by relating it to P_h^s which is more tractable asymptotically, as shown in Theorem 5. It can be noted that when $M = 0$, meaning that there is no helper in the network, Theorem 5 degenerates to Theorem 1. It is also worth pointing out that the bounds in Theorem 4 may be too loose in some cases. This can be illustrated by the example in the proof of Theorem 4: two instances ϕ_1 and ϕ_2 can be mapped to ω_1, but only one instance ϕ_0 can be mapped to ω_0; As a result, the lower bound of P_h^c is too loose. Generically, by extensive simulations, we report that with parameter settings that we encounter in practical scenarios, P_h^c has the same order of magnitude as P_h^s by varying from $40\%P_h^s$ in the worst case to $100\%P_h^s$ in the best case.

Given the theoretic results and the above findings, we are able to quantify the performance gain brought by cooperative helpers in the rendezvous process. Specifically, in the non-cooperative rendezvous without any helper, $\alpha \geq 1/2$ is a necessary condition to achieve asymptotically non-zero P_h. In contrast, in the cooperative rendezvous, the condition to achieve asymptotically non-zero P_h^c becomes $\alpha + (\alpha_c + \beta)/2 \geq 1/2$, which is much more easier to satisfy in practical applications. As an illustrative example with $N = 64$ channels, s and r should be equipped with $r = 8$ radios in non-cooperative rendezvous to achieve non-zero P_h. However if 4 helpers participate the rendezvous, only 4 radios are required at each node ($r = r_c = 4$).

The analytical results illustrate that the participation of cooperative helpers in the rendezvous process is especially beneficial when the hardware capacity of cognitive nodes is limited (in terms of the number of radios) as it can at the best case increase M times the channel hitting probability, thus decreasing M times the expected rendezvous delay.

D. THEORETICAL PERFORMANCE ANALYSIS: RENDEZVOUS ROBUSTNESS

To study the rendezvous robustness, we first derive the probability that the rendezvous is achieved on at least two channels, denoted as P_m^c.

Theorem 6: Let P_D (P_C, respectively) denote the probability that s and d can rendezvous on exactly one channel among channels $Mr_c + 1$ to N (among channels 1 to Mr_c, respectively) and cannot rendezvous on channels 1 to Mr_c (channels $Mr_c + 1$ to N, respectively), it holds that (4) and (5), [(5), as shown at the bottom of this page]

$$P_D = \frac{1}{\left[\binom{N}{r}\right]^2}\left[\sum_{k=0}^{M}\sum_{l=0}^{M-k}\sum_{\substack{\sum_{i=1}^k a_i\leq r-1\\ a_i\geq 1, 1\leq i\leq k}}\sum_{\substack{\sum_{j=1}^l b_j\leq r-1\\ b_j\geq 1,1\leq j\leq l}}\binom{M-k}{l}\right.$$
$$\times\binom{M}{k}\left(\prod_{i=1}^{k}\binom{r_c}{a_i}\right)\binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-\sum_{i=1}^k a_i-1}$$
$$\left.\times\left(\prod_{j=1}^{l}\binom{r_c}{b_j}\right)\binom{N-Mr_c-r+\sum_{i=1}^k a_i}{r-\sum_{j=1}^l b_i-1}\right], \tag{4}$$

P_m^c can then be derived as

$$P_m^c = P_h^c - P_D - P_C.$$

Proof: Please refer to Appendix. □

Theorem 6 being too involved to derive further engineering implications, we study the order of magnitude of P_m^c by relating it to a system with a super helper, as in Sec. V-C. Specifically, Theorem 6 establishes the relationship between the two systems in terms of rendezvous robustness. The proof consists of constructing a similar mapping as that in the proof of Lemma 3 and is thus omitted here for briefly.

Theorem 7: Let P_m^s denote the probability that s and d can rendezvous on at least two channels in the system with a super helper covering Mr channels, it holds that

$$\frac{2P_m^s}{M(M+1)} \leq P_m \leq P_m^s.$$

Theorem 8 derives P_h^m and studies its limit in the asymptotic scenario.

$$P_C = \frac{1}{\left[\binom{N}{r}\right]^2}\left[\sum_{k=0}^{M-1}\sum_{l=0}^{M-k-1}\sum_{t=1}^{r}\sum_{\substack{\sum_{i=1}^k a_i\leq r-1\\ a_i\geq 1,1\leq i\leq k}}\sum_{\substack{\sum_{j=1}^l b_j\leq r-t\\ b_j\geq 1,1\leq j\leq l}}\binom{M-1}{k}\right.$$
$$\times\binom{M}{1}\binom{M-k-1}{l}\binom{r}{1}\left(\prod_{i=1}^{k}\binom{r_c}{a_i}\right)\binom{N-Mr_c}{r-\sum_{i=1}^k a_i-1}$$
$$\left.\times\binom{r}{t}\left(\prod_{j=1}^{l}\binom{r_c}{b_j}\right)\binom{N-Mr_c-r+\sum_{i=1}^k a_i+1}{r-\sum_{j=1}^l b_i-t}\right]$$
$$+\frac{1}{\left[\binom{N}{r}\right]^2}\left[\sum_{k=0}^{M-1}\sum_{l=0}^{M-k-1}\sum_{t=2}^{r}\sum_{\substack{\sum_{i=1}^k a_i\leq r-t\\ a_i\geq 1,1\leq i\leq k}}\sum_{\substack{\sum_{j=1}^l b_j\leq r-1\\ b_j\geq 1,1\leq j\leq l}}\binom{M-1}{k}\right.$$
$$\times\binom{M}{1}\binom{M-k-1}{l}\binom{r}{t}\left(\prod_{i=1}^{k}\binom{r_c}{a_i}\right)\binom{N-Mr_c}{r-\sum_{i=1}^k a_i-t}$$
$$\left.\times\binom{r}{1}\left(\prod_{j=1}^{l}\binom{r_c}{b_j}\right)\binom{N-Mr_c-r+\sum_{i=1}^k a_i+t}{r-\sum_{j=1}^l b_i-1}\right]. \tag{5}$$

Theorem 8: It holds that

$$P_m^s = P_h^s - 2\binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-1}\binom{N-r+1}{r-1} + 2\binom{Mr_c}{1}\binom{N-Mr_c}{r-1} \left[\sum_{i=2}^{r}\binom{Mr_c}{i}\binom{N-Mr_c-r+1}{r-i}\right] + \binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-1}\binom{N-Mr_c-r-1}{r-1} + \left[\binom{Mr_c}{1}\right]^2\binom{N-Mr_c}{r-1}\binom{N-Mr_c-r+1}{r-1}. \quad (6)$$

Asymptotically, let $r = O(N^{\alpha}), r_c = O(N^{\alpha_c}), M = O(N^{\beta})$,

$$\lim_{N\to\infty} P_h^s \simeq \begin{cases} 1, & \alpha + \frac{\alpha_c+\beta}{2} > \frac{1}{2} \\ 1-[1+\delta]\,e^{-\delta}, & \alpha + \frac{\alpha_c+\beta}{2} = \frac{1}{2} \\ \frac{r^4}{2N^2}\left[1+\frac{M(M+1)r_c^4}{2N^2}\right] \to 0, & \alpha + \frac{\alpha_c+\beta}{2} < \frac{1}{2} \end{cases}$$

where $\delta = \frac{r^2}{N}\left(1+\frac{Mr_c^2}{N}\right)$.

Proof: Please refer to Appendix. □

The analysis in this subsection confirms our analysis at the end of Sec. V-C on the performance of cooperative rendezvous and the benefits brought by cooperative helpers. By examining both rendezvous delay and robustness, we report the finding that the cooperative rendezvous is especially beneficial when the hardware capacity of cognitive nodes is limited and that $\alpha + (\alpha_c + \beta)/2 = 1/2$ consists of a critical point to achieve good asymptotic performance.

VI. PERFORMANCE OPTIMIZATION OF COOPERATIVE RENDEZVOUS

After specifying the cooperative rendezvous mechanism and characterizing its performance in terms of rendezvous delay and robustness, this section investigates the optimization of the cooperative rendezvous mechanism by employing the theoretic foundation laid in previous sections.

Particularly, we study the following optimization question: how many helpers are necessary to lower-bound the channel hitting probability, thus upper-bounding the ETTR? We answer the question by studying the following two scenarios.

A. RENDEZVOUS WITH COORDINATED HELPERS

We start with the case where the cooperative helpers can coordinate among them. Lemma 2 shows that to maximize the channel hitting probability P_h^c, the helpers are better off tuning their radios on different channels.

Lemma 2: P_h^c *is maximized when* $\mathcal{H}_i \bigcap \mathcal{H}_j = \emptyset$, $\forall i, j \in \mathcal{M}$.

Proof Sketch: We prove the lemma by showing that given a helpers' strategy profile Δ where $\exists i, j \in \mathcal{M}$ such that $\mathcal{H}_i \bigcap \mathcal{H}_j \neq \emptyset$, by switching a radio of j from $c_1 \in \mathcal{H}_i \bigcap \mathcal{H}_j$ to another channel $c_2 \notin \mathcal{H}_i \bigcup \mathcal{H}_j$ to construct another strategy profile Δ', we can increase P_h^c.

Recall Theorem 3, for a given threshold Θ the minimal number of helpers to achieve $P_h^c \geq \Theta$ can be derived by solving the following optimization problem:

$$M^* = \min_{M} \left\{P_h^c \geq \Theta\right\}.$$

The helpers can then coordinate among them such that there are at least M^* helper operating on different channels participating the cooperative rendezvous.

B. RENDEZVOUS WITH UNCOORDINATED HELPERS

We then proceed to the more practical while challenging scenario with uncoordinated helpers. To that end, we assume that each helper randomly tunes its radios at each slot. In Theorem 9, we derive the channel hitting probability for such generic scenario.

Theorem 9: For a set $\mathcal{T} \subseteq \mathcal{M}$, *let* $\mathcal{A}^{\mathcal{T}} \triangleq \bigcap_{j\in\mathcal{T}} \mathcal{H}^j$ *with* $A^{\mathcal{T}}$ *denoting its cardinality, the channel hitting probability under the setting* $\mathcal{H} \triangleq \{\mathcal{H}^m\}$ *is given by:*

$$P_h^c(\mathcal{H}) = 1 - \prod_{i=0}^{r-1}\left(1-\frac{r}{N-i}\right) + \sum_{\mathcal{S}\subseteq\mathcal{M}} (-1)^{|\mathcal{S}|-1} P_1\left(\bigcap_{m\in\mathcal{S}}\{m\}\right),$$

where for $\mathcal{T} \subseteq \mathcal{M}$, $P_1(\mathcal{T}) \triangleq \left[\sum_{i=1}^{A^{\mathcal{T}}} \frac{\binom{A^{\mathcal{T}}}{i}\binom{N-A^{\mathcal{T}}}{r-i}}{\binom{N}{r}}\right]^2$.

Proof: Please refer to Appendix. □

Armed with the above results, we can derive the minimal number of helpers M^* to lower-bound the average channel hitting probability by given a threshold Θ as

$$M^* = \min_{M} \left\{ \frac{1}{\left[\binom{N}{r}\right]^M} \sum_{\substack{\mathcal{H}^m\subseteq\mathcal{N},|\mathcal{H}^m|=r_c \\ m\in\mathcal{M}}} P_h^c(\mathcal{H}) \geq \Theta \right\}.$$

C. TO HELP OR NOT: DISTRIBUTED LEARNING ALGORITHM

In practical scenarios where rendezvous between s and d (or between several pairs of SUs) is repeatedly performed due to their communication pattern, it is beneficial to minimize the efforts at the helpers while ensuring that the expected channel hitting probability is lower-bounded by Θ. In this subsection, we further investigate the following question: how to ensure that there are approximately M^* helpers (M^* is the minimal number of helpers derived in the previous subsection that lower-bounds the expected channel hitting probability by Θ) participating the rendezvous? This question is important to ensure the desired rendezvous performance without bringing too much burden on the helpers. To address this question, we develop a distributed decision algorithm 1 at each helper to decide whether or not to help based on only local observation without interacting with others.

The procedure, composed of the following two steps, can gradually stabilize at the desired operating point.

Algorithm 1 Execute on Each Helper

for $t = 1, 2, \cdots, L$
  randomly generate a probability p
  if $p \leq p(t)$
    step 1: Estimating the number of helpers
      helper i observes I, S_f, and S_l
      compute $\mathcal{L}[I|m]$, $\widetilde{q(m^*)}$
    step 2: Adjusting helping probability $p(t+1)$
  end if
end for

1) ESTIMATING THE NUMBER OF PARTICIPATING HELPERS

In order to make its decision, a helper needs to estimate the number of helpers that participate the cooperative rendezvous in the current stage. The estimation should be performed locally with only limited view of the system. To this end, we apply the maximum likelihood estimation (MLE) to get accurate estimations. Specifically, we divide time into a sequence of *decision periods*, each consisting of L slots. During a single decision period, a helper chooses a fixed strategy (i.e., participate the rendezvous or not). Thus the total number of helpers participating the cooperative rendezvous does not change within a decision period, which allows helpers to learn the environment. In each rendezvous request, s includes a sequence number that increments by 1 for each rendezvous slot.

In each decision period, each helper i observes the number of slots in which it captures at least one rendezvous request (sent by s or relayed by another helper), denoted as I. It also observes the sequence numbers when the first and the last rendezvous requests are captured, denoted as S_f and S_l respectively. $\Delta_S \triangleq S_l - S_f$ approximates the number of slots in which s sends a rendezvous request. It can be noted that Δ_S is always smaller than the real value but the relative difference between them vanishes when $L \rightarrow \infty$. The helper then calculates the likelihood that there are m helpers participating the cooperative rendezvous given the observation I as:

$$\mathcal{L}[I|m] = \binom{\Delta_S}{I}[q(m)]^I[1-q(m)]^{\Delta_S - I}, \tag{7}$$

where $q(m)$ denotes the probability that a helper captures at least one rendezvous request given that m helpers participate the rendezvous. $q(m)$ is derived in Lemma 3.

Lemma 3: It holds that $q(m) = P_h^c(m-1)$, where $P_h^c(m-1)$ is the average channel hitting probability between s and the helper i with $m-1$ helpers randomly tuning their radios to participate the cooperative rendezvous. $P_h^c(m-1)$ can be calculated by a similar analysis as Theorem 3.

Proof: The proof hinges on the point that capturing at least one rendezvous request for a helper can be mapped into rendezvousing between s and the helper. □

Then the MLE of m can be computed by maximizing the log-likelihood function $\log \mathcal{L}[I|m]$, i.e.,

$$\widetilde{m^*} = max_m \ \log \mathcal{L}[I|m].$$

In our context, in order to get unbiased estimations, instead of estimating m, each helper i estimates $q(m)$ as

$$\widetilde{q(m^*)} = max_{q(m)} \ \log \mathcal{L}[I|m].$$

By the first order condition, we obtain the optimal solution

$$\widetilde{q(m^*)} = \frac{I}{\Delta_S},$$

which is the sample averaging estimation. When the length of decision period L is large, by the central limit theorem, we know that $\widetilde{q(m)} \simeq Nor\left(q(m), \frac{q(m)[1-q(m)]}{\Delta_S}\right)$, with $Nor(\cdot)$ denoting the normal distribution.

It then follows that the estimation of $q(m^*)$ at helper i is unbiased. In the following analysis, we assume that

$$\widetilde{q(m^*)} = q(m^*) + \sigma_i,$$

where $\sigma_i \in [\underline{\sigma}, \overline{\sigma}]$ is the random estimation noise with the probability density function $f(\sigma)$ satisfying

$$f(\sigma) > 0, \ \forall \sigma \in [\underline{\sigma}, \overline{\sigma}], \quad \text{and} \quad \mathbb{E}[\sigma_i] = \int_{\underline{\sigma}}^{\overline{\sigma}} \sigma f(\sigma) = 0.$$

2) ADJUSTING HELPING PROBABILITY

Given the estimation $\widetilde{q(m^*)}$, each helper i adjusts his probability of participating the cooperative rendezvous for the next decision period using the following rule:

$$p(t+1) = p(t) + \kappa[q(M^*) - \widetilde{q(m^*)}], \tag{8}$$

where $q(M^*)$ can be calculated based on the target M^*, $\kappa \ll 1$ is a smoothing factor that controls the speed of convergence. Large κ decreases the convergence delay at the price of large strategy variation. Lemma 4 establishes the convergence of the update rule (8) to M^*.

Lemma 4: Under (8), if κ is sufficiently small, the expected number of participating helpers converges to M^*.

Proof Sketch: The proof, detailed in the appendix, consists of first showing that (8) admits a unique fixed point and then establishing the convergence to the fixed point.

VII. DISCUSSION

In this section, we discuss some important issues on the practical implementation of the cooperative rendezvous protocol and its integration with other rendezvous mechanisms to further improve rendezvous performance.

A. COMPARISON AND INTEGRATION WITH OTHER CR RENDEZVOUS MECHANISMS

As analyzed in Sec. II, existing rendezvous mechanisms in CR networks can be categorized into two classes: (1) stationary and memoryless rendezvous, (2) sequence-based rendezvous. Compared with existing rendezvous approaches, the cooperative rendezvous protocol developed in the paper represents an orthogonal research effort. It can be integrated with both classes of rendezvous protocols. Throughout our analysis, we analyze the cooperative rendezvous mechanism upon the rendezvous protocol based on random channel hopping in order to quantify the performance benefits brought by cooperative helpers. Nevertheless, the cooperative rendezvous mechanism can also be integrated with any sequence-based rendezvous protocol without any modification at cognitive nodes. Such integration is especially attractive as it can combine the advantages of both schemes while limiting the side effects of them. We leave the detailed performance analysis of this proposition for future research.

B. RENDEZVOUS OVERHEAD

Besides the two performance metrics analyzed previously, the protocol overhead also has a non-negligible impact on the overall system behavior. In this subsection, we conduct an order of magnitude study on the protocol overhead of the cooperative rendezvous in terms of the expected number of rendezvous messages generated and relayed during the rendezvous process and compare it with other rendezvous protocols.

- *Non-cooperative rendezvous:* Recall that s sends a rendezvous request on r channels in each slot until rendezvous, we can derive the total number of rendezvous messages sent by s as:

$$N_m = \mathbb{E}[T_{ttr}]r = \frac{r}{P_h} \simeq \begin{cases} O(N^{\alpha}) & \alpha \geq \frac{1}{2} \\ O(N^{1-\alpha}) & \alpha < \frac{1}{2} \end{cases}.$$

- *Cooperative rendezvous:* We study the scenario where the helpers tune their radios on non-overlapping channels. Following similar analysis as Theorem 1, we can compute the probability that a helper captures the rendezvous request sent by s as $P_r = rr_c/N^2$ when $\alpha+\alpha_c < 1/2$ and $P_r \simeq O(1)$ when $\alpha + \alpha_c \geq 1/2$. Recall that each helper relays the rendezvous request if it hears the request, we can derive the total number of generated and relayed rendezvous messages as

$$N_m^c = \mathbb{E}[T_{ttr}](r + MP_r) = \frac{r + MP_r}{P_h}.$$

  After some algebraic operations, we have

$$N_m^c \simeq \begin{cases} O(N^{\max\{\alpha,\alpha_c+\beta\}}) & \alpha + \frac{\alpha_c+\beta}{2} \geq \frac{1}{2} \\ O(N^{1-\alpha}) & \alpha + \frac{\alpha_c+\beta}{2} < \frac{1}{2} \end{cases}.$$

- *Sequence-based rendezvous:* The overhead is $O(N)$ because the ETTR is $O(N)$.

We thus observe that in the worst case, the cooperative rendezvous mechanism increases the overhead from $O(N^{\alpha})$ to $O(N^{\max\{\alpha,\alpha_c+\beta\}})$ compared with the non-cooperative rendezvous. N_m^c can be further limited by adaptively adjusting the helping probability as investigated in Sec. VI.

C. IMPLEMENTATION ISSUES

We now discuss some related issues when implementing the cooperative rendezvous mechanism.

1) CHANNEL HETEROGENEITY

In CR networks, the heterogeneity among channels in terms of their availability to SUs and quality has a non-negligible impact on the performance of any cognitive protocol. Rendezvous protocol is no exception. Specifically, it is preferable to be able to rendezvous on more reliable channels less impacted by PUs. One solution is to rank the channel in terms of their availability and then relates the probability of tuning a radio on a channel to its availability in the rendezvous protocol. However, due to the unpredictability of the PU traffic and the asymmetry of the system perception at different SUs, designing a channel ranking scheme with satisfactory performance itself can be a challenging task. One solution is to rank the channels using past observations and strike a balance between accessing highly ranked channels and exploring new channels.

2) COOPERATION INCENTIVE

In our analysis, we do not address the issue of cooperation enforcement, which is another research topic and may require a separate mechanism. There are some possible solutions providing incentive to helpers to participate rendezvous. For example, a helper is paid for the rendezvous service when rendezvous is achieved with his help. Alternatively, mechanisms based on the Tit-for-Tat philosophy can be introduced such that in order to benefit the help of others, a SU should render his help to others.

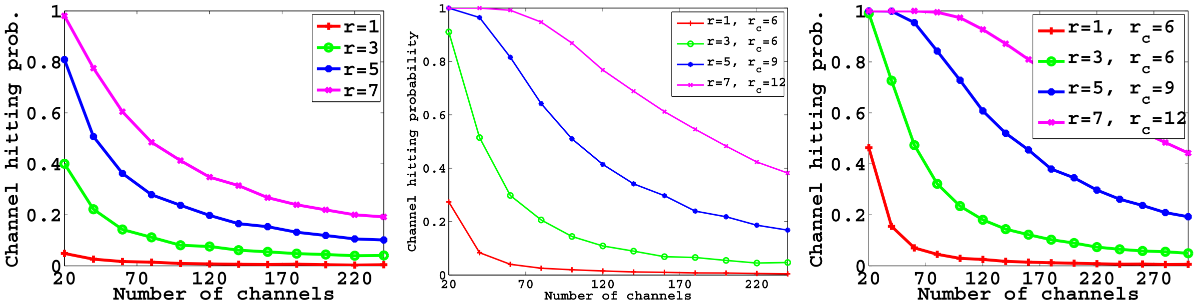
VIII. PERFORMANCE EVALUATION

In this section, we conduct a comparative numerical study on the non-cooperative and cooperative rendezvous protocols analyzed previously via a set of simulations on several representative rendezvous scenarios in CR networks.

A. NON-COOPERATIVE RENDEZVOUS

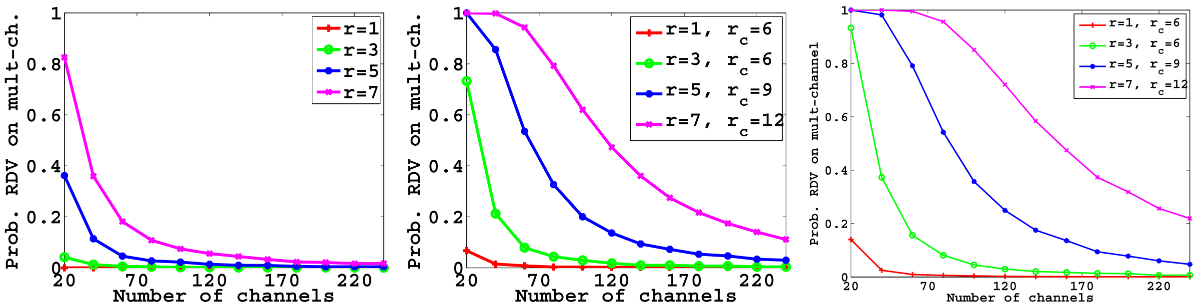
We start with the non-cooperative rendezvous protocol analyzed in Sec. IV to demonstrate its rendezvous performance and its dependence on various network parameters. The results on non-cooperative rendezvous also serve as a benchmark and comparison reference of evaluating the cooperative rendezvous approach. Specifically, we simulate an N-channel CR network with 20 SUs, each equipped with r radios, initiating communication sessions with another SU randomly chosen. Fig. 1a, Fig. 2a and Fig. 3a illustrate the rendezvous delay (via the channel hitting probability), robustness and overhead of the non-cooperative rendezvous mechanism as functions of N with different r, with each point representing the average value of a number of independent simulation runs. The required number of simulation runs is calculated using "independent replications" [34].

We make the following observations from the simulation results: 1) Rendezvous performance with single radio per SU is poor in all metrics, which is in accordance with the observations in the existing literature on the single-radio rendezvous



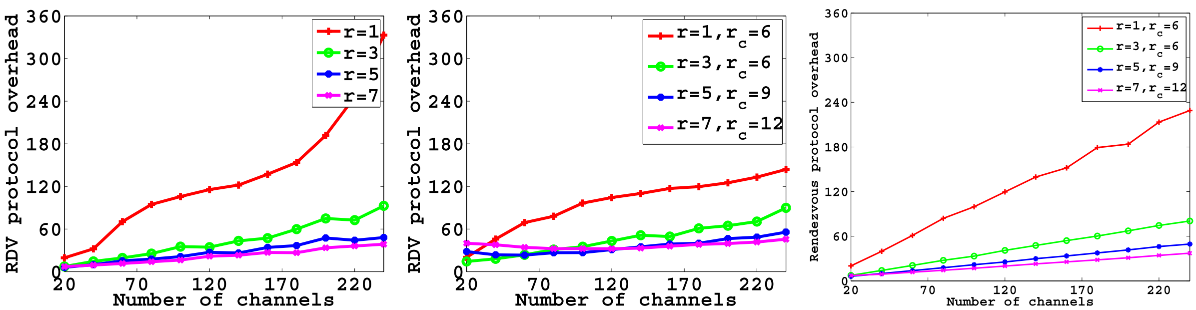
(a) (b) (c)

FIGURE 1. **Simulation result: Channel hitting probability. (a) Non-cooperative rendezvous. (b) Cooperative rendezvous: $M = 4$. (c) Cooperative rendezvous: $M = 7$.**



(a) (b) (c)

FIGURE 2. **Simulation result: Probability of rendezvousing on multiple channels. (a) Non-cooperative rendezvous. (b) Cooperative rendezvous: $M = 4$. (c) Cooperative rendezvous: $M = 7$.**



(a) (b) (c)

FIGURE 3. **Simulation result: Rendezvous overhead. (a) Non-cooperative rendezvous. (b) Cooperative rendezvous: $M = 4$. (c) Cooperative rendezvous: $M = 7$.**

based on random channel hopping. 2) Increasing the number of radios per SU can bring performance gain to certain extent. Specifically, on the rendezvous delay, when $r \leq N$, we observe that P_h scales squarely in r, as demonstrated by the analytical results. The protocol overhead also decreases significantly compared to the single-radio case. In fact, the impact of r on the protocol overhead is two-fold. On one hand, increasing r increases the number of rendezvous requests sent per slot. On the other hand, a larger r reduces the rendezvous delay. The simulation results show that the former effect outweighs the latter. The effect of increasing r is less pronounced in terms of rendezvous robustness, as illustrated by Fig. 2a that rendezvous is achieved predominantly on only one channel.

B. COOPERATIVE RENDEZVOUS

We proceed to investigate the cooperative rendezvous protocol developed in Sec. V. In our simulations, M cooperative helper nodes, each equipped with r_c radios, participate the cooperative rendezvous, the other system parameters being the same as in Sec. VIII-A. By varying M and r_c, we study several scenarios representing different cooperation resource levels in the cooperative rendezvous process.

TABLE 1. Average TTR: Non-coop. vs coop. rendezvous.

Parameter setting	Average TTR (# of slots)		
	N=20	N=100	N=200
Non-coop. RDV. r=1	20.8	112	248
Non-coop. RDV. r=5	1.23	4.22	8.40
r=1, r_c=6, M=4	3.66	51.3	137
r=5, r_c=6, M=4	1.00	1.96	4.59
r=1, r_c=6, M=7	2.12	34.3	101
r=5, r_c=6, M=7	1.00	1.37	2.90

The most important performance metric characterizing a rendezvous protocol is the rendezvous delay. Tab. 1 compares the average TTR between non-cooperative and cooperative rendezvous under different parameter settings. More detailed simulation results on the channel hitting probability, rendezvous robustness and overhead are illustrated in Fig. 1, Fig. 2 and Fig. 3. Compared with the non-cooperative rendezvous, we remark significant performance gain in both rendezvous delay and robustness, especially when the system scales (N is large). As pointed out in our theoretic analysis, the participation of cooperative helpers can create additional rendezvous diversity, whose benefits in facilitating rendezvous process are clearly demonstrated by the simulation results. We also observe from Fig. 3 that the rendezvous overhead in cooperative rendezvous is comparable to its non-cooperative peer, which shows that the overhead generated by relaying rendezvous requests can be compensated by the decrease of rendezvous delay.

C. COMPARISON OF DIFFERENT CH ALGORITHMS

We compare the ETTR of the proposed cooperative scheme, denoted by CooP, to several multi-radio-based representative CH rendezvous algorithms, including AR [24], RPS [25], AMRR [26] and MSS [27].

The numbers of radios for SUs s and d are set to be $r = 4$. The numbers of available channels for SUs s and d are set to be $0.5N$. The number of commonly available channels between SUs s and d is set to $0.2\ N$, i.e., $|\mathcal{R}_s \cap \mathcal{R}_d| = 0.2\ N$. We assume that there exists a cooperative SU m with $|\mathcal{R}_s \cap \mathcal{R}_m| = 0.1\ N$ and $|\mathcal{R}_m \cap \mathcal{R}_d| = 0.1\ N$. The number of global channels varies from 10 to 100. The simulation results in Fig. 4 are measured by the average values of 20 times simulations. From Fig. 4, we observe that the proposed cooperative scheme is better than others, which verifies the theoretical analysis on cooperative rendezvous. However, we need to point out that the shorter ETTR is achieved with consuming resource of the cooperative SU.

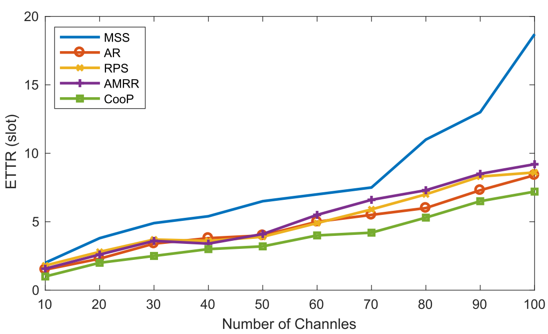


FIGURE 4. ETTR vs Number of channels.

IX. CONCLUSION AND PERSPECTIVE

We have presented a novel cooperative channel rendezvous mechanism in CR networks by introducing the cooperative cognitive nodes that serve as "bridges" between communicating pairs to facilitate their rendezvous process. To quantify the performance gain of the cooperative rendezvous, we have established a mathematic framework to study the rendezvous delay and robustness and derived the performance limit in asymptotic scenarios. Our analytical results have shed light on some important design and engineering implications of the cooperative rendezvous protocol which are further confirmed by simulation results showing that the developed cooperative rendezvous protocol outperforms existing schemes under various typical network conditions.

There are several directions for future work. First, incorporating the cooperative rendezvous with sequence-based approaches can achieve bounded TTR while benefiting the rendezvous diversity created by cooperative helpers. Performance analysis and protocol optimization therein require a systematic study. Secondly, extending the pairwise rendezvous to more sophisticated multi-user and multi-hop scenarios consists of another extension of this work.

APPENDIX
THEORETICAL PERFORMANCE ANALYSIS: GENERIC SCENARIO

This section studies the performance of the non-cooperative rendezvous and the developed cooperative rendezvous mechanisms for the generic scenario where $\mathcal{C}_i \neq \mathcal{N}$.

A. NON-COOPERATIVE RENDEZVOUS

We start with the non-cooperative rendezvous. Theorem 10 derives the probability that s and d rendezvous on exactly i channels.

Theorem 10: Let $\mathcal{C}_0 \triangleq \mathcal{C}_s \bigcap \mathcal{C}_d$ and denote θ_i ($1 \leq i \leq C_0$) the probability that s and d can rendezvous on exactly i channels, it holds that

$$\theta_i = \sum_{j=j_{min}(i)}^{j_{max}(i)} \frac{\binom{C_0}{i}\binom{C_0-i}{j}\binom{C_s-C_0}{r-i-j}\binom{C_d-i-j}{r}}{\binom{C_s}{r}\binom{C_d}{r}}$$

where $j_{min}(i) \triangleq \max\{0, r-i-(C_s-C_0)\}$, $j_{max}(i) \triangleq \min\{C_0-i, C_d-r\}$.

Proof: To compute θ_i, we study the situation where besides the i common channels with d, s tunes j radio on j channels in $\mathcal{C}_0$. The probability of this event, denoted as $\theta_i(j)$, is given by:

$$\theta_i(j) = \frac{\binom{C_0}{i}\binom{C_0-i}{j}\binom{C_s-C_0}{r-i-j}\binom{C_d-i-j}{r-i}}{\binom{C_s}{r}\binom{C_d}{r}},$$

with the following constraint on j given i:

$$\begin{cases} 0 \le j \le C_0 - i \\ 0 \le r - i - j \le C_s - C_0 \\ 0 \le r - i \le C_d - i - j \end{cases},$$

which, noticing $C_0 \le r$, leads to the following constraint on j: $j_{min}(i) \le j \le j_{max}(i)$, where $j_{min}(i) \triangleq \max\{0, r - i - (C_s - C_0)\}$, $j_{max}(i) \triangleq \min\{C_0 - i, C_d - r\}$.

By summing $\theta_i(j)$ over all possible j, we can derive θ_i as:

$$\theta_i = \sum_{j=j_{min}(i)}^{j_{max}(i)} \theta_i(j) = \sum_{j=j_{min}(i)}^{j_{max}(i)} \frac{\binom{C_0}{i}\binom{C_0-i}{j}\binom{C_s-C_0}{r-i-j}\binom{C_d-i-j}{r}}{\binom{C_s}{r}\binom{C_d}{r}},$$

which completes our proof. □

Theorem 10 can be applied to compute the rendezvous performance metrics as follows:

- *Rendezvous delay*: the channel hitting probability $P_h = \sum_{i=1}^r \theta_i$, the ETTR $\mathbb{E}[T_{ttr}] = 1/P_h$;
- *Rendezvous robustness*: the probability that s and d can rendezvous on at least two channels $P_m = \sum_{i=2}^r \theta_i$.

B. COOPERATIVE RENDEZVOUS

We now proceed to study the cooperative rendezvous. In the generic scenario, let $\mathcal{H}_s^m \triangleq \mathcal{C}_s \cap \mathcal{H}^m$, $\mathcal{H}_d^m \triangleq \mathcal{C}_d \cap \mathcal{H}^m$; for a set $\mathcal{T} \subseteq \mathcal{M}$ let $\mathcal{A}_s^T = \bigcap_{j\in\mathcal{T}} \mathcal{H}_s^j$, $\mathcal{A}_d^T = \bigcap_{j\in\mathcal{T}} \mathcal{H}_d^j$ with A_d^T and A_r^T denoting the corresponding cardinality of $\mathcal{A}_s^T$ and $\mathcal{A}_d^T$.

Theorem 11: The channel hitting probability is given by:

$$P_h^c = 1 - \left(\sum_{i=1}^r \theta_i\right) + \sum_{\mathcal{S}\subseteq\mathcal{M}} (-1)^{|S|-1} P_1\left(\bigcap_{m\in\mathcal{S}} \{m\}\right),$$

where for $\mathcal{T} \subseteq \mathcal{M}$,

$$P_1(\mathcal{T}) = \left[\sum_{i=1}^{A_s^T} \frac{\binom{A_s^T}{i}\binom{C_s - A_s^T}{r-i}}{\binom{C_s}{d}}\right]\left[\sum_{i=1}^{A_d^T} \frac{\binom{A_d^T}{i}\binom{C_d - A_d^T}{r-i}}{\binom{C_d}{r}}\right].$$

Proof: We start by establishing the probability that given a set $\mathcal{T} \subseteq \mathcal{M}$, the rendezvous is achieved with the help of at least one helper on the channels in $\bigcup_{m\in\mathcal{T}} \mathcal{H}^m$ but that it cannot be achieved by s and d directly. This probability, denoted as $P_1(\mathcal{T})$, can be computed as follows:

$$P_1(\mathcal{T}) = \left[\sum_{i=1}^{A_s^T} \frac{\binom{A_s^T}{i}\binom{C_s - A_s^T}{r-i}}{\binom{C_s}{r}}\right]\left[\sum_{i=1}^{A_d^T} \frac{\binom{A_d^T}{i}\binom{C_d - A_d^T}{r-i}}{\binom{C_d}{r}}\right].$$

In the above formula, $\frac{\binom{A_s^T}{i}\binom{C_s - A_s^T}{r-i}}{\binom{C_s}{r}}$ is the probability that each helper in $\mathcal{T}$ captures i rendezvous requests sent by s, $\sum_{i=1}^{A_d^T} \frac{\binom{A_d^T}{i}\binom{C_d - A_d^T}{r-i}}{\binom{C_d}{r}}$ is the probability that d captures the relayed rendezvous request sent by each helper in $\mathcal{T}$. The formula thus gives the probability that the rendezvous is achieved with the help of at least one helper on the channels in $\bigcup_{m\in\mathcal{T}} \mathcal{H}^m$ but that it cannot be achieved by s and d directly.

We can then derive the probability that that the rendezvous is achieved with the help of at least one helper but that it cannot be achieved by the sender and the receiver directly by using the inclusion-exclusion principle.

$$P_1 = \sum_{\mathcal{T}\subseteq\mathcal{M}} (-1)^{|T|-1} P_1\left(\bigcap_{m\in\mathcal{T}} \{m\}\right).$$

Recall Theorem 10 that the probability that the rendezvous can be achieved without the help of other nodes is $1 - \left(\sum_{i=1}^r \theta_i\right)$, the channel hitting probability can be computed as:

$$P_h^c = 1 - \left(\sum_{i=1}^r \theta_i\right) + P_1,$$

with P_1 being the gain brought by the helpers. Injecting P_1 into the above formula completes our proof. □

The calculation of P_m^c in the generic case turns out to be too involved analytically. One possible solution is to enumerate all possible combinations and check if a combination can lead to rendezvous on at least two channels.

PROOFS OF LEMMAS AND THEOREMS

C. PROOF OF THEOREM 1

We analyze the situation where rendezvous fails at a given time slot. Using the combinatorial notation $\binom{n_1}{n_2} = \frac{n_1!}{n_2!(n_1-n_2)!}$, the probability of such event, denoted as P_0, can be derived as:

$$P_0 = \frac{\binom{N}{R}\binom{N-R}{R}}{\binom{N}{R}\binom{N}{R}} = 1 - \prod_{i=0}^{r-1}\left(1 - \frac{r}{N-i}\right),$$

where $\binom{N}{R}\binom{N-R}{R}$ is the number possible combinations where there is no common channel between s and d, $\binom{N}{R}\binom{N}{R}$ is the number of total possible combinations.

P_h can then be derived by $P_h = 1 - P_0$, leading to (1). We then prove the second part of the theorem.

- When $\alpha < \frac{1}{2}$ (i.e., $r^2 \simeq o(N)$), it holds that

$$P_h \simeq 1 - \left(1 - \frac{r}{N}\right)^r \simeq 1 - \left(1 - r\frac{r}{N}\right) \simeq \frac{r^2}{N}.$$

- When $\alpha \ge \frac{1}{2}$, it holds that

$$1 - \left(1 - \frac{r}{N}\right)^r \le P_h \le 1 - \left(1 - \frac{r}{N-r}\right)^r,$$

and that

$$\begin{cases} \lim_{N\to\infty} 1 - \left(1 - \frac{r}{N}\right)^r = 1 - \left(\frac{1}{e}\right)^{\frac{r^2}{N}}, \\ \lim_{N\to\infty} 1 - \left(1 - \frac{r}{N-r}\right)^r = 1 - \left(\frac{1}{e}\right)^{\frac{r^2}{N-r}}. \end{cases}$$

Equation (2) follows readily from the above analysis.

D. PROOF OF THEOREM 2

It can be noticed that

$$P_m = \sum_{i=2}^r \theta_i = P_h - \theta_1, \tag{9}$$

where θ_1 can be derived as

$$\theta_1 = \frac{\binom{N}{1}\binom{N-1}{R-1}\binom{N-R-1}{R}}{\binom{N}{R}\binom{N}{R}},$$

where the nominator is the number of combinations that s and d choose exactly one channel in common, the denominator is the number of total possible combinations.

After some algebraic operations, we have

$$\theta_1 = \frac{r^2}{N-r+1}\prod_{i=0}^{r-2}\left(1-\frac{r}{N-i}\right).$$

By injecting P_h and θ_1 into (9), we obtain

$$P_m = 1-\prod_{i=0}^{r-1}\left(1-\frac{r}{N-i}\right)-\frac{r^2}{N-r+1}\prod_{i=0}^{r-2}\left(1-\frac{r}{N-i}\right),$$

which completes the first part of the proof.

The second part on the asymptotical scenario can be proven similarly as Theorem 1 by applying the second order approximation

$$(1-x)^n \simeq 1-nx+n^2x^2/2+o(x^2)\ (x\to 0).$$

E. PROOF OF THEOREM 3

To prove Theorem 3, we derive the probability that the rendezvous cannot be achieved, denoted as P_0^c. We can note that the rendezvous cannot be achieved if the following conditions are satisfied:

- If s tunes its radios on the channels accessed by k helpers, d should only tune its radios on the channels accessed by the rest $M-k$ helpers;
- On the rest $N-Mr_c$ channels not covered by any helper, s and d do not tune their radios on the same channels;

Consequently, P_0^c can be derived by enumerating the combinations in which the above two conditions hold as follows

$$P_0^c = \frac{1}{\left[\binom{N}{r}\right]^2}\left[\sum_{k=0}^{M}\sum_{l=0}^{M-k}\sum_{a_i\geq 1,1\leq i\leq k}^{\sum_{i=1}^k a_i\leq r}\sum_{b_j\geq 1,1\leq j\leq l}^{\sum_{j=1}^l b_j\leq r}\binom{M}{k}\right.$$
$$\times\binom{M-k}{l}\left(\prod_{i=1}^{k}\binom{r}{a_i}\right)\binom{N-Mr_c}{r-\sum_{i=1}^k a_i}\left(\prod_{j=1}^{l}\binom{r}{b_j}\right)$$
$$\left.\times\binom{N-Mr_c-r+\sum_{i=1}^k a_i}{r-\sum_{j=1}^l b_i}\right],$$

where

$$\sum_{a_i\geq 1,1\leq i\leq k}^{\sum_{i=1}^k a_i\leq r}\sum_{b_j\geq 1,1\leq j\leq l}^{\sum_{j=1}^l b_j\leq r}\left(\prod_{i=1}^{k}\binom{r}{a_i}\right)\binom{N-Mr_c}{r-\sum_{i=1}^k a_i}$$
$$\left(\prod_{j=1}^{l}\binom{r}{b_j}\right)\binom{N-Mr_c-r+\sum_{i=1}^k a_i}{r-\sum_{j=1}^l b_i}$$

is the number of combinations of the following case:

- s tunes some of its radios on the channels covered by helpers such that k helpers have at least one common channel with s, s tunes other radios on the channels not covered by any helper;
- d tunes some of its radios on the channels covered by a subset of the other $M-k$ helpers and other radios on the channels not covered by any helper or s.

Injecting P_0^c derived above into $P_h^c = 1-P_\emptyset^c$ completes the proof of general M and injecting $M=1$ into the formula completes the proof of the case $M=1$.

F. PROOF OF LEMMA 1

Let C_1, C_2 denote the number of combinations that s and d can rendezvous on at least one channel among the first Mr_c channels in the original system and in the system with a super helper, respectively, it holds that

$$q_i = \frac{C_i}{\left[\binom{N}{r}\right]^2},\quad i=1,2.$$

To prove the lemma, it suffices to show that $C_2/M \leq C_1 \leq C_2$.

It can be straightforwardly noted that $C_1 \leq C_2$ as for any instance with which s and d can rendezvous on at least one channel among the first Mr_c channels in the first system, they can rendezvous on at least one channel among the first Mr_c channels in the second system with the super helper.

We now proceed to show that $MC_1 \geq C_2$. Noticing the definition of C_1 and C_2, it follows that C_2-C_1 is the number of combinations that s and d can rendezvous on at least one channel i among the first Mr_c channels in the second system but not in the first system. Let $\Phi = \{\phi_i\}$ denote the set of instances where s and d can rendezvous on at least one channel i among the first Mr_c channels in the second system but not in the first system and let $\Omega = \{\omega_i\}$ denote the set of instances where s and d can rendezvous on at least one channel i among the first Mr_c channels in the first system, we next show that each element $\phi_i \in \Phi$ can be mapped into an element $\omega_i \in \Omega$ and that at most $M-1$ elements in Φ can be mapped to the same element in Ω. We show this by constructing the following mapping $\mathbf{T}$.

Definition 1: Let $a_i^\omega(j)$ and $b_i^\omega(j)$ $(1 \leq j \leq N)$ denote the number of radios that s and d tune on channel j in the instance ω_i. Let $a_i^\phi(j)$ and $b_i^\phi(j)$ $(1 \leq j \leq N)$ denote the number of radios that s and r tune on channel j in the instance ϕ_i. Let j_a^i and j_b^i denote the smallest channel index j with which $a_i^\phi(j)$ and $b_i^\phi(j)$ are positive, respectively. The mapping $\mathbf{T}: \Phi \to \Omega$ is defined as follows.

Each instance $\phi_i \in \Phi$ is mapped into an instance ω_i where

$$a_i^\omega(j) = a_i^\phi(j),\quad 1\leq j\leq N.$$

$$b_i^\omega(j) = \begin{cases} b_i^\phi(j_b^i) & j = j_a^i + [(j_b^i - j_a^i) \bmod r_c],\\ b_i^\phi(j_a^i) & j = j_b^i,\\ b_i^\phi(j) & 1\leq j\leq N, j\neq j_a^i+[(j_b^i-j_a^i) \bmod r_c],\ j_b^i.\end{cases}$$

We provide an example to illustrate the mapping $\mathbf{T}$.

Example 1: In a system where $N=8$, $r=r_c=2$, $M=3$, consider an instance ω_0 with $a_0^\omega(1)$, $a_0^\omega(3)$, $b_0^\omega(2)$, $b_0^\omega(6)$ being

1 and the others being 0. Consider an instance ϕ_0 with $a_0^{\phi}(1)$, $a_0^{\phi}(3)$, $b_0^{\phi}(4)$, $b_0^{\phi}(6)$ being 1 and the others being 0. According to the construction of the mapping $\mathbf{T}$, ϕ_0 can be mapped to ω_0. It can be checked that no other instance in Φ can be mapped to ω_0.

In the same system consider another instance ω_1 with $a_1^{\omega}(1)$, $a_1^{\omega}(7)$, $b_1^{\omega}(2)$, $b_1^{\omega}(8)$ being 1 and the others being 0. Consider two instances ϕ_1 and ϕ_2 with $a_1^{\phi}(1)$, $a_1^{\phi}(7)$, $a_2^{\phi}(1)$, $a_2^{\phi}(7)$, $b_1^{\phi}(4)$, $b_1^{\phi}(8)$, $b_2^{\phi}(6)$, $b_2^{\phi}(8)$ being 1 and the others being 0. According to the construction of the mapping $\mathbf{T}$, both ϕ_1 and ϕ_2 can be mapped into ω_1. No other instance in Φ can be mapped to ω_0.

Concerning the mapping $\mathbf{T}$, we can show the following two properties: (1) each element in Φ is mapped to an element in Ω; (2) at most $M-1$ elements in Φ can be mapped to the same element in Ω.

The above analysis on $\mathbf{T}$ implies that $C_2 - C_1 \leq (M-1)C_1$ and hence $MC_1 \geq C_2$. The lemma is thus proven.

G. PROOF OF THEOREM 4

It can be easily noted that $P_h^c \leq P_h^s$ as for any instance where rendezvous can be achieved in the system with the super helper, it can be achieved in the original system.

We now prove that $P_h^s/M \leq P_h^c$. To this end, we develop P_h as follows

$$P_h^c = q_1^c + q_2^c - q_3^c$$

where q_1^c is the probability that rendezvous can be achieved on at least one channel from channel 1 to Mr_c, q_2^c is the probability that rendezvous can be achieved on at least one channel from channel Mr_c+1 to N, q_3^c is the probability that rendezvous can be achieved on at least one channel from channel 1 to Mr_c and on at least one channel from channel Mr_c+1 to N. Similar development holds for P_h^s with the corresponding probabilities q_i^s $(i=1,2,3)$ in the system with the super helper.

It can be noted that $q_2^c = q_2^s$. It follows from Lemma 3 that $q_2^c \geq q_2^s/M$. It also holds that $q_3^c \leq q_3^s$. This can be shown by noticing that for any instance where rendezvous can be achieved on at least one channel from channel 1 to Mr_c and on at least one channel from channel Mr_c+1 to N in the original system, rendezvous can also be achieved on at least one channel from channel 1 to Mr_c and on at least one channel from channel Mr_c to N in the system with the super helper.

It follows from the above analysis that $P_h^s/M \leq P_h^c$.

H. PROOF OF THEOREM 5

To prove the first part of the theorem, we derive the probability that the rendezvous cannot be achieved, denoted as P_0^s. To this end, we notice that the rendezvous cannot be achieved if one of the following cases hold:

1) *Case 1:* s tunes i of its radios $(1 \leq i \leq r)$ on the channels covered by the super helper, d tunes its radios on the channels other than those covered by the super helper or s;
2) *Case 2:* s tunes all of its radios on the channels not covered by the super helper, d tunes its radios on the $N-r$ channels not covered by s.

Based on the above observation, the probability that the rendezvous cannot be achieved can be derived as follows:

$$P_0^s = \frac{1}{\left[\binom{N}{r}\right]^2}\left[\binom{N-Mr_c}{r}\binom{N-r}{r} + \sum_{i=1}^{r}\binom{Mr_c}{i}\binom{N-Mr_c}{r-i}\binom{N-Mr_c-r_c+i}{r}\right],$$

where $\binom{N-Mr_c}{r}\binom{N-r}{r}$ is the number of combinations of case 2, $\sum_{i=1}^{r}\binom{Mr_c}{i}\binom{N-Mr_c}{r-i}\binom{N-Mr_c-r+i}{r}$ is the number of combinations of case 1, $\left[\binom{N}{r}\right]^2$ is the total number of combinations.

By injecting P_0^s into $P_h^s = 1 - P_0^s$, we obtain the formula of P_h^s in the theorem.

We then proceed to prove the second part of the theorem. The proof hinges on the fact that when N is large, it holds that

$$\binom{N-Mr_c}{r}\binom{N-r}{r} \gg \sum_{i=1}^{r}\binom{Mr_c}{i}\binom{N-Mr_c}{r-i}\binom{N-Mr_c-r+i}{r}.$$

After some algebraic operations, we have

$$\lim_{N\to\infty} P_0^s \simeq \begin{cases} 0, & \alpha + \frac{\alpha_c+\beta}{2} > \frac{1}{2} \\ [1+\delta]\,e^{-\delta}, & \alpha + \frac{\alpha_c+\beta}{2} = \frac{1}{2} \\ 1 - \frac{r^4}{2N^2}\left[1 + \frac{M(M+1)r_c^4}{2N^2}\right], & \alpha + \frac{\alpha_c+\beta}{2} < \frac{1}{2} \end{cases}.$$

Noticing that $P_h^s = 1 - P_0^s$, the second part is proven.

I. PROOF OF THEOREM 6

The calculation of P_1^D can be proven noticing that

$$\left(\prod_{i=1}^{k}\binom{r}{a_i}\right)\binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-\sum_{i=1}^{k}a_i-1} \times \left(\prod_{j=1}^{l}\binom{r}{b_j}\right)\binom{N-Mr_c-r+\sum_{i=1}^{k}a_i}{r-\sum_{j=1}^{l}b_i-1}$$

is the number of combination where s and d can rendezvous on exactly one channel among channels Mr_c+1 to N and cannot rendezvous on channels 1 to Mr_c.

The calculation of P_1^C in can be proven noticing that

- the term in the first square bracket is the number of combinations where s and d can rendezvous on exactly one channel among channels 1 to Mr_c and cannot rendezvous on channels Mr_c+1 to N with s tuning 1 radio and d tuning t $(t \geq 1)$ radios on the channels covered by a helper;
- the term in the second square bracket is the number of the same combinations with s tuning t $(t \geq 2)$ radios and d tuning 1 radio on the channels covered by a helper.

Finally, the probability of rendezvousing on multiple channels P_m^c can be derived by the channel hitting probability minus the probability of rendezvousing on one channel:

$$P_m^c = P_h^c - P_D - P_C,$$

which completes the proof.

J. PROOF OF THEOREM 8

It can be noted that P_m^s equals to P_h^s minus the probability that s and d rendezvous on exactly one channel. We now derive the second probability. To that end, we note that s and d rendezvous on exactly one channel can be categorized into the following four cases:

- *Case 1:* The rendezvous is achieved on a channel i not covered by the super helper and s does not tune any of its radios on the channels covered by the super helper. The number of combinations in this case is
$$\binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-1}\binom{N-r+1}{r-1}.$$
- *Case 2:* The rendezvous is achieved on a channel i not covered by the super helper and s tunes at least one of its radios on the channels covered by the super helper. The number of combinations in this case is
$$\binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-1}\binom{N-r+1}{r-1} - \binom{N-Mr_c}{1}\binom{N-Mr_c-1}{r-1}\binom{N-Mr_c-r-1}{r-1}.$$
- *Case 3:* The rendezvous is achieved on a channel i covered by the super helper and s tunes one radio on the channels covered by the super helper. The number of combinations in this case is
$$\binom{Mr_c}{1}\binom{N-Mr_c}{r-1}\left[\sum_{i=1}^{r}\binom{Mr_c}{i}\binom{N-Mr_c-r+1}{r-i}\right].$$
- *Case 4:* The rendezvous is achieved on a channel i covered by the super helper and s tunes more than one radios on the channels covered by the super helper. In this case, d must tune one radio on on the channels covered by the super helper. The number of combinations in this case is
$$\binom{Mr_c}{1}\binom{N-Mr_c}{r-1}\left[\sum_{i=2}^{r}\binom{Mr_c}{i}\binom{N-Mr_c-r+1}{r-i}\right].$$

Combining the results in the above four cases completes the proof of the first part of the theorem. The second part can be shown by performing algebraic operations by neglecting terms of lower order of magnitude.

K. PROOF OF LEMMA 2

To prove the lemma, we show that given a helpers' strategy profile Δ where $\exists i, j \in \mathcal{M}$ such that $\mathcal{H}_i \bigcap \mathcal{H}_j \neq \emptyset$, by switching a radio of j from $c_1 \in \mathcal{H}_i \bigcap \mathcal{H}_j$ to another channel $c_2 \notin \mathcal{H}_i \bigcup \mathcal{H}_j$ to construct another strategy profile Δ', we can increase P_h^c. To show this, we construct the following mapping to map any instance ω with strategy Δ to another instance ω' with strategy Δ': if in ω, s (d, respectively) tunes one radio on c_1 and d (s) tunes one radio on a channel in $\mathcal{H}_j$, then switch the radio of s (d) on c_1 to c_2 to construct ω'; otherwise let $\omega' = \omega$.

It can be noted that if s and d can rendezvous in ω, they can rendezvous in ω'. Hence the channel hitting probability is increased by switching from Δ to Δ'. By iteratively performing the above switching, we can show that P_h^c is maximized when $\mathcal{H}_i \bigcap \mathcal{H}_j = \emptyset, \forall i, j \in \mathcal{M}$.

L. PROOF OF THEOREM 9

We start by establishing the probability that given a set $\mathcal{T} \subseteq \mathcal{M}$, the rendezvous is achieved with the help of at least one helper on the channels in $\bigcup_{m\in\mathcal{T}} \mathcal{H}^m$ but that it cannot be achieved by the sender and the receiver directly. This probability, denoted as $P_1(\mathcal{T})$, can be computed as follows:

$$P_1(\mathcal{T}) = \left[\sum_{i=1}^{A^T} \frac{\binom{A^T}{i}\binom{N-A^T}{r-i}}{\binom{N}{r}}\right]^2.$$

In the above formula, $\left[\sum_{i=1}^{A^T} \frac{\binom{A^T}{i}\binom{N-A^T}{r-i}}{\binom{N}{r}}\right]$ is the probability that each helper in $\mathcal{T}$ captures i rendezvous requests sent by s; it is also the probability that d captures the relayed rendezvous request sent by each helper in $\mathcal{T}$. The formula thus gives the probability that the rendezvous is achieved with the help of at least one helper on the channels in $\bigcup_{m\in\mathcal{T}} \mathcal{H}^m$ but that it cannot be achieved by s and d directly.

We can then derive the probability that that the rendezvous is achieved with the help of at least one helper but that it cannot be achieved directly by using the inclusion-exclusion principle.

$$P_1 = \sum_{\mathcal{T}\subseteq\mathcal{M}} (-1)^{|\mathcal{T}|-1} P_1\left(\bigcap_{m\in\mathcal{T}} \{m\}\right).$$

Recall the proof of Theorem 1 that the probability that the rendezvous can be achieved without the help of other nodes is $1 - P_0$, the channel hitting probability can be computed as:

$$P_h^c(\mathcal{H}) = 1 - P_0 + P_1,$$

with P_1 being the gain brought by the helpers. Injecting P_0 and P_1 into the above formula completes our proof.

M. PROOF OF LEMMA 4

Denote $m^*(t)$ the number of participating helpers in decision period t, by summing (8) for all helpers i, we have

$$m^*(t+1) = m^*(t) + \kappa M \left(q(M^*) - q(m^*(t)) \sum_{i\in\mathcal{N}} -\mathbb{E}[\sigma_i]\right).$$

Noticing that $\mathbb{E}[\sigma_i] = 0$, we have

$$m^*(t+1) = m^*(t) + \kappa M \left[q(M^*) - q(m^*(t))\right]. \quad (10)$$

It can be noted that (10) admits a unique fixed point $m^*(t) = M^*$. We next show that starting from any state $m^*(0)$, $\lim_{t\to\infty} = M^*$ under (10). To this end, we first state the following properties:

- If $\leq m^*(t) \leq M^*$, noticing that $q(m^*(t)) \leq q^*(M)$, it follows from (10) that $m^*(t+1) \geq m^*(t)$. Moreover, if κ is sufficiently small such that $\kappa \leq \min_{0 \leq x \leq M} \frac{q(M^*)-q(x)}{M(M^*-x)}$, it follows from (10) that

$$M^* - m^*(t+1) = M^* - m^*(t) - \kappa M \left[q(M^*) - q(m^*(t))\right] \geq 0.$$

- If $m^*(t) \geq M^*$, similarly, we have

$$m^*(t) \geq m^*(t+1) \geq M^*.$$

Now consider an arbitrary sequence of update steps commencing from an initial vector $m^*(0)$, we distinguish the following two cases:

- *Case 1:* $0 \leq m^*(0) < M^*$. In this case, we have

$$m^*(0) \leq m^*(1) \leq \cdots \leq m^*(t) \leq \cdots \leq M^*.$$

  We thus obtain a non-decreasing sequence upper-bounded by M^*. It follows that it must converge to a limit. Since there is no other fixed point other than M^*, this limit must be M^*.
- *Case 2:* $M^* \leq m^*(0) \leq M$. Similarly, we obtain a non-increasing sequence $m^*(t)$ converging to M^*.

Combine the above analysis, we establish the convergence of $m^*(t)$ to M^*.

REFERENCES

[1] J. So and N. H. Vaidya, "Multi-channel MAC for ad hoc networks: Handling multi-channel hidden terminals using a single transceiver," in *Proc. MobiHoc*, May 2004, pp. 222–233.

[2] K. Bian, J.-M. Park, and R. Chen, "A quorum-based framework for establishing control channels in dynamic spectrum access networks," in *Proc. MOBICOM*, Sep. 2009, pp. 25–36.

[3] K. Bian and J.-M. Park, "Asynchronous channel hopping for establishing rendezvous in cognitive radio networks," in *Proc. INFOCOM*, Apr. 2011, pp. 236–240.

[4] Z. Lin, H. Liu, X. W. Chu, and Y.-W. Leung, "Jump-stay based channel-hopping algorithm with guaranteed rendezvous for cognitive radio networks," in *Proc. INFOCOM*, Apr. 2011, pp. 2444–2452.

[5] M. J. Abdel-Rahman and M. Krunz, "Game-theoretic quorum-based frequency hopping for anti-jamming rendezvous in DSA networks," in *Proc. IEEE DYSPAN*, Apr. 2014, pp. 248–258.

[6] S. Chen, A. Russell, A. Samanta, and R. Sundaram, "Deterministic blind rendezvous in cognitive radio networks," in *Proc. IEEE ICDCS*, Jun./Jul. 2014, pp. 358–367.

[7] Z. Gu, Q.-S. Hua, and W. Dai, "Fully distributed algorithms for blind rendezvous in cognitive radio networks," in *Proc. ACM MobiHoc*, 2014, pp. 155–164.

[8] S.-H. Wu, C.-C. Wu, W.-K. Hon, and K. G. Shin, "Rendezvous for heterogeneous spectrum-agile devices," in *Proc. IEEE INFOCOM*, Apr./May 2014, pp. 2247–2255.

[9] Z. Gu, H. Pu, Q.-S. Hua, and F. C. M. Lau, "Improved rendezvous algorithms for heterogeneous cognitive radio networks," in *Proc. IEEE INFOCOM*, Apr./May 2015, pp. 154–162.

[10] L. Chen, K. Bian, L. Chen, C. Liu, J.-M. Park, and X. Li, "A group-theoretic framework for rendezvous in heterogeneous cognitive radio networks," in *Proc. ACM MobiHoc*, 2014, pp. 165–174.

[11] C.-M. Chao and H.-Y. Fu, "Supporting fast rendezvous guarantee by randomized quorum and latin square for cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, pp. 8388–8399, Oct. 2016.

[12] C.-M. Chao, C.-Y. Hsu, and Y.-T. Ling, "Efficient asynchronous channel hopping design for cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 6888–6900, Sep. 2016.

[13] C.-S. Chang, G.-C. Yang, M.-H. Chiang, and W. C. Kwong, "Construction of synchronous-symmetric channel-hopping sequences over galois extension field for cognitive radio networks," *IEEE Commun. Lett.*, vol. 21, no. 6, pp. 1425–1428, Jun. 2017.

[14] W.-C. Chen, G.-C. Yang, M.-K. Chang, and W. C. Kwong, "Construction and analysis of shift-invariant, asynchronous-symmetric channel-hopping sequences for cognitive radio networks," *IEEE Trans. Commun.*, vol. 65, no. 4, pp. 1494–1506, Apr. 2017.

[15] C.-S. Chang, C.-Y. Chen, D.-S. Lee, and W. Liao, "Efficient encoding of user IDs for nearly optimal expected time-to-rendezvous in heterogeneous cognitive radio networks," *IEEE/ACM Trans. Netw.*, vol. 25, no. 6, pp. 3323–3327, 2017.

[16] X. J. Tan, C. Zhou, and J. Chen, "Symmetric channel hopping for blind rendezvous in cognitive radio networks based on union of disjoint difference sets," *IEEE Trans. Veh. Technol.*, vol. 66, no. 11, pp. 10233–10248, Nov. 2017.

[17] J. Li, H. Zhao, J. Wei, D. Ma, and L. Zhou, "Sender-jump receiver-wait: A simple blind rendezvous algorithm for distributed cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 17, no. 1, pp. 183–196, Jan. 2018.

[18] C. Chang, S. Wang, and Y. Liu, "A jamming-resistant channel hopping scheme for cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 10, pp. 6712–6725, Oct. 2017.

[19] B. Yang, M. Zheng, and W. Liang, "A time-efficient rendezvous algorithm with a full rendezvous degree for heterogeneous cognitive radio networks," in *Proc. INFOCOM*, Apr. 2016, pp. 1–9.

[20] M. D. Silvius, F. Ge, A. Young, A. B. MacKenzie, and C. W. Bostian, "Smart radio: Spectrum access for first responders," *Proc. SPIE*, Apr. 2008, Art. no. 698008.

[21] (2018). *IEEE 802.22 Working Group*. [Online]. Available: http://www.ieee802.org/22/

[22] Y. Liu, P. Ning, and H. Dai, "Authenticating primary users' signals in cognitive radio networks via integrated cryptographic and wireless link signatures," in *Proc. IEEE Symp. Secur. Privacy*, May 2010, pp. 286–301.

[23] Y. Azar, O. Gurel-Gurevich, E. Lubetzky, and T. Moscibroda, "Optimal discovery strategies in white space networks," in *Proc. 19th Eur. Conf. Algorithms (ECA)*, 2011, pp. 713–722.

[24] R. Paul and Y. Choi, "Adaptive rendezvous for heterogeneous channel environments in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7753–7765, Nov. 2016.

[25] L. Yu, H. Liu, Y. W. Leung, X. Chu, and Z. Lin, "Multiple radios for fast rendezvous in cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 14, no. 9, pp. 1917–1931, Sep. 2015.

[26] L. Yu, H. Liu, Y.-W. Leung, X. Chu, and Z. Lin, "Adjustable rendezvous in multi-radio cognitive radio networks," in *Proc. IEEE Global Commun. Conf.*, San Diego, CA, USA, Dec. 2015, pp. 1–7.

[27] B. Yang, W. Liang, M. Zheng, and Y. Liang, "Fully distributed channel-hopping algorithms for rendezvous setup in cognitive multiradio networks," *IEEE Trans. Veh. Technol.*, vol. 65, no. 10, pp. 8629–8643, Oct. 2016.

[28] X. J. Tan, J. Wang, and Y. Yuan, "Difference-set-based channel hopping for minimum-delay blind rendezvous in multi-radio cognitive radio networks," *IEEE Trans. Veh. Technol.*, to be published.

[29] A. Li, G. Han, and T. Ohtsuk, "Multiple radios for fast rendezvous in heterogeneous cognitive radio networks," *IEEE ACCESS*, vol. 7, pp. 37342–37359, 2019.

[30] T.-H. Lin, G.-C. Yang, and W. C. Kwong, "A homogeneous multi-radio rendezvous algorithm for cognitive radio networks," *IEEE Commun. Lett.*, vol. 23, no. 4, pp. 736–739, Apr. 2019.

[31] S. A. Pmbudi, W. Wang, and C. Wang, "Fast rendezvous for spectrum-agile IoT devices with limited channel hopping capability," in *Proc. INFOCOM*, Apr. 2018, pp. 1385–1393.

[32] Y.-T. Wang, G.-C. Yang, S.-H. Huang, M.-K. Chang, and W. C. Kwong, "Multi-MTTR asynchronous-asymmetric channel-hopping sequences for scalable cognitive radio networks," *IEEE Trans. Cogn. Commun. Netw.*, vol. 4, no. 4, pp. 692–703, Dec. 2018.

[33] J. Sheu and J. Lin, "A multi-radio rendezvous algorithm based on Chinese remainder theorem in heterogeneous cognitive radio networks," *IEEE Trans. Mobile Comput.*, vol. 17, no. 9, pp. 1980–1990, Sep. 2018.

[34] W. Whitt, "The efficiency of one long run versus independent replications in steady-state simulation," *Manage. Sci.*, vol. 37, no. 6, pp. 645–666, 1991.