A Distributed Market Framework for Mobile Data Offloading

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***Abstract*—We develop a distributed market framework to price the offloading service, and conduct a detailed analysis of the incentives for offloading service providers and conflicts arising from the interactions of different participators. Specifically, we formulate a multi-leader multi-follower Stackelberg game (MLMF-SG) to model the interactions between the offloading service providers and the offloading service consumers in the considered market framework, and investigate the cases where the offloading capacity of APs is unlimited and limited, respectively. For the case without capacity limit, we decompose the followers' game of the MLMF-SG (FG-MLMF-SG) into a number of simple follower games (FGs), and prove the existence and uniqueness of the equilibrium of the FGs from which the existence and uniqueness of the FG-MLMF-SG also follows. For the leaders' game of the MLMF-SG, we also prove the existence and uniqueness of the equilibrium. For the case with capacity limit, by considering a symmetric strategy profile, we establish the existence and uniqueness of the equilibrium of the corresponding MLMF-SG, and present a distributed algorithm that allows the leaders to achieve the equilibrium. Finally, extensive numerical experiments demonstrate that the Stackelberg equilibrium is very close to the corresponding social optimum for both considered cases.**

I. Introduction

The data traffic in cellular networks has seen a tremendous growth over the past few years due to the explosion of mobile devices, e.g. smart phones, tablets, laptops etc. The increasing data traffic in cellular networks suggests that traffic from cellular networks should be offloaded so as to alleviate traffic congestion and improve user satisfaction. Thus, mobile data offloading emerged as a promising approach to utilize certain complementary transmission technologies to deliver data traffic originally transmitted over cellular networks to the users. Recently, a large number of studies have investigated the potential benefits of mobile data offloading and various innovative schemes have been proposed to better manage data traffic including WiFi [1]–[4], femtocells [5]–[9], and opportunistic offloading [10], [11]. In fact, these studies have shown that data offloading is a cost-effective and energy-prudent approach to resolve network congestion and improve network capacity.

However, the merit of mobile data offloading does not always guarantee that offloading is adopted by the offloading service providers (OSPs) and offloading service consumers (OSCs), i.e., mobile data flows, in practice. One of the most important reasons for not adopting mobile data offloading is the lack of economic incentives, i.e., OSPs may be reluctant to make their resources available for offloading data traffic without permission or appropriate economic reimbursement since offloading data traffic will consume their limited wireless resources and reduce broadband connection capacity. Thus, it is of significant importance to analyze the economic implications of mobile data offloading from the perspective of both OSPs and OSCs. For ease of presentation, in this paper, we focus on WiFi offloading in which OSPs and OSCs represent Access Points (APs) and cellular data flows, respectively.

Motivated by [12]–[14], we consider a typical offloading scenario where a number of cellular data flows offload their data traffic to a number of APs in their vicinity, e.g., hotspots near base stations. In particular, we propose a pricing framework based on the concept of 'paying for offloading' to ensure efficient use of the offloading APs. Under this framework each cellular data flow corresponding to a mobile source-destination pair offers a payment to incentivize APs to participate in offloading, and then the payment is shared in proportion to the amount of data offloaded to each AP. Hence, the utility of an AP is its share of received payment minus its own offloading cost. For a cellular data flow, its utility is defined as a generic concave function of the sum of the utilities from offloading on the APs minus the cost paid to these offloading APs. We model the interaction of the APs and the cellular data flows as an MLMF-SG, where the APs are the followers who respond to the payment offered by the cellular data flows (i.e., each AP offloads a part of the data of some flows such that its utility is maximized, given the payment offered by the flows and the actions of its competing peers); and the cellular data flows are the leaders who set the payment to maximize their own utility in anticipation of the Nash equilibrium (NE) response of the followers. Notwithstanding our interest in the mobile data offloading context, the considered model is generic enough to be applied any other scenario where a set of 'jobs' compete for the services of a pool of 'workers', such that the jobs set their payment rates, workers are free to choose the job they will attempt, and payment from each job is eventually shared according to certain allocation rules among all the workers that serve the job.

Unlike most pricing methods in the existing literature that involve only one type of selfish players [12] or two types of selfish players without competition between them [13], [14], our framework features two types of players, each of which competes not only with its peers but also with the players of the other type. This property distinguishes our work from the scenario considered in [13], [14], where only players of the same type can compete with each other although there exist two types of selfish players. This difference cause the utility functions of players in this paper to be completely different from those in [13], [14] as far as concavity is concerned. Concretely, with the strategy profile in [13], [14], the utility functions of both followers and leaders are concave, which ensures that there exists an equilibrium in the followers' game and the leaders' game, respectively. However, in our case, the payment from a flow is shared proportionally among all APs according to the amount of data offloaded to each AP. As a consequence, an AP's utility depends not only on its own strategy but also on the strategies of its peers, which leads to complex interactions among the APs. Accordingly, the sharing of payment causes the utility functions to be non-concave, which necessitates a completely new and original study of the game's equilibrium.

The main contributions of this paper can be summarized as follows:

- We develop a distributed market pricing framework for mobile data flows to price the offloading service.
- We formulate a Stackelberg game to model the interactions between offloading service providers and offloading service consumers under the market framework, and investigate the cases where the offloading capacity of APs is limited and unlimited, respectively. For both cases, we establish the existence and uniqueness of the equilibrium of the proposed Stackelberg game, obtain the Stackelberg equilibrium in closed form when the offloading capacity of the APs is not limited, and further propose a distributed pricing algorithm to ensure that the game converges to an equilibrium when the offloading capacity of the APs is limited.
- We conduct a large number of simulations to verify our theoretical analysis on the proposed Stackelberg game for the two considered cases. As a noteworthy property of the developed framework, simulation results demonstrate that the Stackelberg equilibrium is very close to the social optimum.

II. Problem Formulation

In this section, we first provide the system model of WiFi offloading, and then introduce the pricing market framework. Subsequently, we formulate the problem to a Stackelberg game.

A. System Model

We consider a set $\mathcal{F}$ of mobile data flows (or data traffics) in a cellular network where each flow f transmits a number of data packets from the source S_f to the destination D_f. A set $\mathcal{R}$ of potential offloading APs (with $|\mathcal{R}| = R \geq 2$) in the vicinity of the flows, may help flow f to offload its data packets to the destination via another transmission network, e.g. WiFi. In return, the APs may obtain a certain reimbursement from flow f. The APs are assumed to be WiFis operating on different carriers, and accordingly the APs' signals do not mutually interfere with each other. Assume that time is slotted, and there is a network-wide slot synchronization. We focus on how the packets of flow f should be priced such that the APs have an incentive to offload data packets of flow f.

B. Pricing Framework

For a selfish AP i ($i \in \mathcal{R}$), to incentivize offloading, it must receive some reimbursement that is greater than its offloading cost. For this purpose, each flow f offers a payment of C^f to incentivize APs to offload data traffic, where C^f is determined by the flow itself, i.e., C^f is the strategy of flow f. We denote by r_i^f the amount of data offloaded by AP i for flow f. Hence, the utility of flow $f \in \mathcal{F}$ is defined as the net payoff that f gets per slot:

$$U_f \triangleq u_f\Big(\sum_{i\in\mathcal{R}} \log(1 + r_i^f)\Big) - C^f, \tag{1}$$

where the $\log(1 + r_i^f)$ term[1] reflects the diminishing utility of flow f from r_i^f. Function $u_f(\cdot)$ represents the total utility from the assistance of all APs. We assume $u_f(w)$ is continuously differentiable, strictly increasing, and weakly concave in w, i.e., $u_f^{'}(w) > 0$ and $u_f^{''}(w) \leq 0$, with $u_f(0) = 0$.

Next, we consider the utility of the APs. For flow f ($f \in \mathcal{F}$), the payment of C^f is shared in accordance with the level of cooperation, i.e., the amount of data offloaded by the APs that offload packets of flow f. The vector $\mathbf{r}_i = \{r_i^f, f \in \mathcal{F}\}$ is the strategy of AP i where $\sum_{f\in\mathcal{F}} r_i^f \leq B$ reflects the limited offloading capacity B of AP i. We denote the cost (e.g. in terms of energy) for AP i to offload a packet of flow f by e_i^f. Thus, the expected payoff per slot for AP i is

$$V_i \triangleq \sum_{f\in\mathcal{F}} V_i^f = \sum_{f\in\mathcal{F}} \Big[C^f \frac{r_i^f}{\sum_{j\in\mathcal{R}} r_j^f} - e_i^f r_i^f\Big], \tag{2}$$

where $V_i^f \triangleq C^f \frac{r_i^f}{\sum_{j\in\mathcal{R}} r_j^f} - e_i^f r_i^f$.

The payoff function of AP i has the following property.

Lemma 1. *V_i is not a concave function in r_j^f ($j \in \mathcal{R}, j \neq i$).*

Proof: It is easy to show $\frac{\partial^2 V_i}{\partial^2 r_j^f} \geq 0$, which means that V_i is not a concave function in r_j^f ($j \in \mathcal{R}, j \neq i$). ∎

C. Stackelberg Game

We model the offloading problem with pricing as a S-tackelberg game which includes two roles (leader and follower) and two stages. In the first stage, each flow f (as a leader) announces its reimbursement C^f, and the reimbursement from all flows are collected in a reimbursement vector $\mathbf{C} = (C^1, C^2, \cdots, C^{|\mathcal{F}|})$. In the second stage, each offloading AP i (as a follower) in $\mathcal{R}$ choose its offloading

[1] We adopt $\log(1 + r_i^f)$ only for presentation purpose. This term can be replaced by other types of utility functions as long as they reflect the diminishing utility of flow f in terms of r_i^f

size $\mathbf{r}_i = (r_i^1, r_i^2, \cdots, r_i^{|\mathcal{F}|})$ for different flows to maximize its own utility. Hence, the flows are the leaders and the APs are the followers in this Stackelberg game. For convenience, let $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_{|\mathcal{R}|})$ denote the strategy profile of all APs where $\mathbf{r}_i$ is the strategy profile of AP i. Let $\mathbf{r}_{-i}$ denote the strategy profile excluding $\mathbf{r}_i$ and $\mathbf{r}_{-i}^f$ be the profile excluding AP i given f. Then, $\mathbf{r} = (\mathbf{r}_i, \mathbf{r}_{-i})$ and $\mathbf{r}_i = (r_i^f, \mathbf{r}_{-i}^f)$.

1) Followers' Game: Given $\mathbf{r}_{-i}$, each follower (AP i) chooses its strategy $\mathbf{r}_i$ to maximize its utility in response to the leaders' strategies $\mathbf{C} \triangleq (C^f, \mathbf{C}^{-f}) = (C^1, C^2, \cdots, C^{|\mathcal{F}|})$. Thus, the objective of AP i is to solve the following optimization problem:

$$\widetilde{\mathbf{r}}_i(\mathbf{C}) = \underset{\mathbf{r}_i}{\operatorname{argmax}}\, V_i(\mathbf{r}_i, \mathbf{r}_{-i}, \mathbf{C}) \tag{3}$$

$$s.t. \sum_{f\in\mathcal{F}} r_i^f \le B, \quad \forall i \in \mathcal{R} \tag{4}$$

$$r_i^f \ge 0, \quad \forall i \in \mathcal{R},\ \forall f \in \mathcal{F}. \tag{5}$$

Then, we have $\widetilde{\mathbf{r}}(\mathbf{C}) = \Big(\widetilde{\mathbf{r}}_1(\mathbf{C}), \cdots, \widetilde{\mathbf{r}}_{|\mathcal{R}|}(\mathbf{C})\Big)$. Note that the followers' game itself can be considered as a non-cooperative game [?].

2) Leaders' game: Given $\mathbf{C}^{-f}$, each leader (flow f) chooses its strategy C^f to maximize its utility function $U_f(\cdot)$ anticipating that the followers will eventually respond with a collection of strategies that constitute an NE according to (3). Thus, the leaders' problem is

$$\widetilde{C}^f = \underset{C^f}{\operatorname{argmax}}\, U_f(C^f, \mathbf{C}^{-f}, \widetilde{\mathbf{r}}(C^f, \mathbf{C}^{-f})). \tag{6}$$

The solution of the Stackelberg game is characterized by a Stackelberg Nash Equilibrium (SNE), that is a strategy profile from which no player has incentive to deviate unilaterally.

III. Stackelberg Game Equilibrium Analysis Without Capacity Bound

In this section, we investigate the existence and uniqueness of an SNE for the considered Stackelberg game if the capacity of the APs is not limited (corresponding to omitting the constraint (4)).

A. Followers' Game

Since the capacity of the APs is much larger than that of mobile devices, it is reasonable to assume that there is no offloading capacity limit for the APs. Under this assumption, the following proposition decomposes the complicated followers' game defined in Section II into a number of simpler games.

Preposition 1. *If the capacity of the APs is not limited, FG-MLMF-SG can be decomposed into $|\mathcal{F}|$ followers' games $\Big(FG(1), \cdots, FG(|\mathcal{F}|)\Big)$.*

Proof: If the capacity of the APs is not limited, according to (2) and (3), FG-MLMF-SG, denoted by $\widetilde{\mathbf{r}}_i(\mathbf{C}) = \operatorname{argmax}_{\mathbf{r}_i} V_i(\mathbf{r}_i, \mathbf{r}_{-i}, \mathbf{C})$, can be decomposed into $|\mathcal{F}|$ followers games $\Big(FG(1), \cdots, FG(|\mathcal{F}|)\Big)$, where $FG(f),\ f \in \mathcal{F}$ corresponds to the optimization problem $\widetilde{r}_i^f(C^f) = \operatorname{argmax}_{r_i^f} V_i^f(r_i^f, \mathbf{r}_{-i}^f, C^f)$. ■

Definition 1. *Given C^f and $\mathbf{r}_{-i}^f$, a strategy is the best response strategy of AP i for $FG(f)$, denoted by $\Gamma_i^f(\mathbf{r}_{-i}^f)$, if it maximizes $V_i^f(r_i^f, \mathbf{r}_{-i}^f)$ over $r_i^f \ge 0$.*

From $\frac{\partial V_i^f}{\partial r_i^f} = 0$, we obtain $\widetilde{r}_i^f = \sqrt{\frac{C^f \sum_{j\in\mathcal{R}\setminus\{i\}} \widetilde{r}_j^f}{e_i^f}} - \sum_{j\in\mathcal{R}\setminus\{i\}} \widetilde{r}_j^f$. Therefore, the best response $\Gamma_i^f(\mathbf{r}_{-i}^f)$ of follower i for flow f is

$$\Gamma_i^f(\mathbf{r}_{-i}^f) = \begin{cases} \sqrt{\frac{C^f \sum_{j\in\mathcal{R}\setminus\{i\}} \widetilde{r}_j^f}{e_i^f}} - \sum_{j\in\mathcal{R}\setminus\{i\}} \widetilde{r}_j^f, & \text{if } e_i^f \sum_{j\in\mathcal{R}\setminus\{i\}} \widetilde{r}_j^f \le C^f \\ 0, & \text{otherwise}. \end{cases} \tag{7}$$

The best responses of follower i for $\Big(FG(1), \cdots, FG(|\mathcal{F}|)\Big)$ are collected in the best response vector $\Gamma_i(\mathbf{r}_{-i}) = \Big(\Gamma_i^1(\mathbf{r}_{-i}^1), \cdots, \Gamma_i^{|\mathcal{F}|}(\mathbf{r}_{-i}^{|\mathcal{F}|})\Big)$.

The following theorem states that the best response strategy leads to an NE of the FG-MLMF-SG.

Theorem 1. *The strategy profile $\widetilde{\mathbf{r}} = (\widetilde{\mathbf{r}}^1, \widetilde{\mathbf{r}}^2, \cdots, \widetilde{\mathbf{r}}^{|\mathcal{F}|})$ is an NE of the FG-MLMF-SG, where $\widetilde{\mathbf{r}}^f = (\widetilde{r}_1^f, \widetilde{r}_2^f, \cdots, \widetilde{r}_{|\mathcal{R}|}^f)$ is an NE of $FG(f)$, where*

1) *the optimal sets of offloading APs, denoted by $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_F)$, are computed by Algorithm 1;*
2) *$\widetilde{r}_i^f = \frac{(|\mathcal{S}_f|-1)C^f}{\sum_{j\in\mathcal{S}_f} e_j^f}\Big(1 - \frac{(|\mathcal{S}_f|-1)e_i^f}{\sum_{j\in\mathcal{S}_f} e_j^f}\Big)$ if $i \in \mathcal{S}_f$; $\widetilde{r}_i^f = 0$ otherwise.*

Algorithm 1 Computation of optimal sets of offloading APs

```
1: for f ∈ F do
2:     Sort APs according to their offloading costs: e^f_{σ1} ≤ e^f_{σ2} ≤ ··· ≤ e^f_{σR};
3:     S_f = {σ1, σ2}, i = 3;
4:     while i ≤ R and e^f_{σi} < (Σ_{j∈S_f} e^f_j)/(|S_f|−1) do
5:         S_f = S_f ∪ {σi}, i = i + 1;
6:     end while
7: end for
8: return S = (S_1, S_2, ··· , S_F).
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Proof: Please refer to Appendix A of [15] . ■

After proving the existence of an NE of the FG-MLMF-SG, we next prove the uniqueness of the NE.

Theorem 2. *Given C^f, denote the strategy profile of an NE by $\hat{\mathbf{r}} = (\hat{\mathbf{r}}^1, \hat{\mathbf{r}}^2, \cdots, \hat{\mathbf{r}}^{|\mathcal{F}|})$, where $\hat{\mathbf{r}}^f = (\hat{r}_1^f, \hat{r}_2^f, \cdots, \hat{r}_{|\mathcal{R}|}^f)$, and define $\hat{\mathcal{S}}_f = \{i \in \mathcal{R} : \hat{r}_i^f > 0\}$. Then, we have*

1) *$\hat{r}_i^f = \frac{(|\hat{\mathcal{S}}_f|-1)C^f}{\sum_{j\in\hat{\mathcal{S}}_f} e_j^f}\Big(1 - \frac{(|\hat{\mathcal{S}}_f|-1)e_i^f}{\sum_{j\in\hat{\mathcal{S}}_f} e_j^f}\Big)$ if $i \in \hat{\mathcal{S}}_f$; $\hat{r}_i^f = 0$ otherwise;*
2) *We sort $\{e_j^f : j \in \mathcal{R}\}$ to $e_{\sigma_1}^f \le e_{\sigma_2}^f \le \cdots \le e_{\sigma_R}^f$, then $\hat{\mathcal{S}}_f = \{\sigma_1, \cdots, \sigma_i\}$, where $\sigma_1, \cdots, \sigma_R$ is a permutation of $\mathcal{R}$ given f, $e_{\sigma_{i+1}}^f \ge \frac{\sum_{j=1}^{i} e_{\sigma_j}^f}{i-1}$, and $i \ge 2$.*

These statements imply that the FG-MLMF-SG has a unique NE.

Proof: Please refer to Appendix B of [15]. ■

Theorem 1 and Theorem 2 imply that there exists a unique NE in the FG-MLMF-SG.

B. Leaders' Game

According to the above analysis, the flows, which are the leaders in the MLMF-SG, know that there exists a unique NE for the APs for any given pricing vector $\mathbf{C}$. Hence, each flow f can maximize its benefit by setting C^f.

Given a specific flow f, feeding back into (1), we have

$$U_f = u_f\Big(\sum_{i\in\mathcal{R}} \log(1+r_i^f)\Big) - C^f = u_f\Big(\sum_{i\in\mathcal{S}_f} \log\big(1+C^f k_i\big)\Big) - C^f,$$

where $k_i = \frac{|\mathcal{S}_f|-1}{\sum_{j\in\mathcal{S}_f} e_j^f}\Big(1-\frac{(|\mathcal{S}_f|-1)e_i^f}{\sum_{j\in\mathcal{S}_f} e_j^f}\Big)$.

Theorem 3. *There exists a unique NE of the leaders' game in the MLMF-SG.*

Proof: Please refer to [15]. ■

Thus far, we have established the existence and uniqueness of the NE for the MLMF-SG when the offloading capacity of the APs is not limited.

IV. Stackelberg Game Equilibrium Analysis With Capacity Bound

In this section, we consider the game if a capacity constraint on the APs is present, and characterize the properties of the NE.

A. Followers' Game

To make the analysis of the game tractable, we assume that the offloading cost of a specific flow does not depend on the APs, that is, $e_i^f = e^f$ for any AP $i \in \mathcal{R}$ given f. Note that this assumption is reasonable as all APs are assumed to be located in the vicinity of flow f.

We commence our discussion of the properties of the equilibrium by considering the best response of AP i using the strategy $\mathbf{r}_i = (r_i^1, \cdots, r_i^{|\mathcal{F}|})$. The corresponding optimization problem from the perspective of AP i can be stated as:

$$\max_{\mathbf{r}_i} V_i(\mathbf{r}_i, \mathbf{r}_{-i}) \quad s.t. \quad \sum_{f\in\mathcal{F}} r_i^f \le B, \quad r_i^f \ge 0, \quad \forall f \in \mathcal{F}. \tag{8}$$

Thus, the corresponding Lagrangian function is given by:

$$L(\mathbf{r}_i, \lambda_i, \nu) = V_i(\mathbf{r}_i, \mathbf{r}_{-i}) - \lambda_i \cdot \Big(\sum_{f\in\mathcal{F}} r_i^f - B\Big) + \sum_{f\in\mathcal{F}} \nu_i^f r_i^f. \tag{9}$$

Since V_i is continuously differentiable in r_i^f, it follows that the Karush-Kuhn-Tucker (KKT) conditions corresponding to problem (9) are necessary for optimality. On the other hand, we note from (2) that, for a fixed $\mathbf{r}_{-i}$, function $V_i(\mathbf{r}_i, \mathbf{r}_{-i})$ is concave in $\mathbf{r}_i$ although it is not concave in $\mathbf{r}$ according to Lemma 1. This implies that the KKT conditions are sufficient for optimality as well. Thus, we conclude that a strategy profile is an equilibrium if and only if (i.i.f) there exist $\lambda_i \ge 0$ and $\{\nu_i^f \ge 0, f \in \mathcal{F}\}$ such that the following conditions are satisfied:

$$(A_1): \quad \frac{\partial V_i}{\partial r_i^f} = \lambda_i - \nu_i^f, \quad \forall f \in \mathcal{F}$$
$$(A_2): \quad \lambda_i \cdot \Big(\sum_f r_i^f - B\Big) = 0$$
$$(A_3): \quad \nu_i^f r_i^f = 0, \quad \forall f \in \mathcal{F}.$$

For ease of further discussion, we introduce the concept of strictly interior equilibrium which is formally defined as follows:

Definition 2. *We say that an equilibrium is a strictly interior equilibrium if the offloading size of any AP $i \in \mathcal{R}$ for any flow $f \in \mathcal{F}$ is strictly positive, i.e., $r_i^f > 0$.*

Now, we are ready to provide the following theorem, which guarantees the symmetry of a strictly interior equilibrium.

Theorem 4. *If a strictly interior equilibrium exists in the followers' game, then it is symmetrical, i.e., $r_i^f = r^f$ for any $i \in \mathcal{R}$.*

Proof: Please refer to Appendix C of [15]. ■

Thus, in the following, we focus on symmetric strategy profiles, that is, all nodes use a symmetric strategy, i.e., $r_i^f = r^f$ for any $i \in \mathcal{R}$. To this end, we define the function

$$g^f(r^f) \triangleq \frac{\partial V_i}{\partial r_i^f}\Big|_{r_j^f = r^f,\ \forall j\in\mathcal{R}} = C^f \frac{R-1}{R^2 r^f} - e^f = C^f h^f(r^f) - e^f,$$

where $h^f(r^f) \triangleq \frac{R-1}{R^2 r^f}$.

Given a symmetric strategy profile, by Theorem 4, the KKT conditions for (9) can be refined to the existence of $\lambda_i \ge 0$ and $\{\nu_i^f = 0, f \in \mathcal{F}\}$ such that (A1)-(A3) are satisfied.

Theorem 5. *For any vector of flow price $\mathbf{C}$, there exists a unique set of $\{\rho^f, f \in \mathcal{F}\}$ such that the symmetric strategy profile $\{r_j^f = \rho^f, j \in \mathcal{R}\}$ is a Nash equilibrium. Furthermore, there exist $\lambda \ge 0$ and $\{\nu^f = 0, f \in \mathcal{F}\}$, such that*

$$(B_1): \quad g^f(\rho^f) = \lambda - \nu^f, \quad \forall f \in \mathcal{F}$$
$$(B_2): \quad \lambda\Big(\sum_{f\in\mathcal{F}} \rho^f - B\Big) = 0$$
$$(B_3): \quad \nu^f \rho^f = 0, \quad \forall f \in \mathcal{F}.$$

Proof: Please refer to Appendix D of [15]. ■

Based on Theorem 5, we obtain that the solution of the following convex optimization problem is the NE of the followers' game in the MLMF-SG.

$$\max_{\rho^1 \cdots \rho^{|\mathcal{F}|}} \sum_{f\in\mathcal{F}} \Big(C^f \frac{R-1}{R^2} \log(\rho^f) - e^f \rho^f\Big)$$
$$s.t. \quad \sum_{f\in\mathcal{F}} \rho^f \le B, \quad \rho^f > 0 \ \ \forall f \in \mathcal{F},$$

which can be solved by software packages, such as Matlab.

B. Leaders' Game

In this subsection, we study the effect of the payment rate C^f of a specific flow $f \in \mathcal{F}$ on the followers' symmetric equilibrium when all other rates $\mathbf{C}^{-f}$ remain fixed. To streamline the discussion, we express the value of ρ^f of the equilibrium corresponding to a given C^f as a function $\rho^f = \Psi(C^f)$ (since we focus only on ρ^f and are not interested in the strategy values for other flows). Also, we define the value of λ that satisfies condition (B1)-(B3) in the equilibrium as a function $\lambda = \Lambda(C^f)$.

We begin by exploring these functions for extreme values of C^f. Clearly, for $C^f = 0$, the utility of any AP cooperating with flow f is non-positive, implying $\rho^f = \Psi(C^f = 0) = 0$. However, from the KKT conditions (B1)-(B3), we know $\rho^f > 0$, which implies $C^f > 0$. Thus, we assume that ρ^f must be larger than a infinitesimal positive value, i.e., $\rho^f = 0^+$. Define $C^f = \Psi^{-1}(\rho^f = 0^+) \triangleq \underline{C}^f$ and $\underline{\lambda} = \Lambda(C^f = \underline{C}^f)$. $\Lambda(C^f)$ and $\Psi(C^f)$ have the following properties.

Lemma 2. *$\Lambda(C^f)$ and $\Psi(C^f)$ have the following properties:*

1) *$\lambda = \Lambda(C^f)$ is continuous and non-decreasing in C^f;*
2) *$\rho^f = \Psi(C^f)$ is continuous, and strictly increasing in $C^f \in (0, \infty)$;*
3) *$\rho^f = \Psi(C^f)$ is concave in $C^f \in (0, \infty)$;*

Proof: Please refer to Appendix E of [15]. ■

Lemma 3. *For a fixed $\mathbf{C}^{-f}$, the function $U_f(C^f, \mathbf{C}^{-f})$ is concave in C^f.*

Proof: Please refer to [15]. ■

Lemma 4. *The best-response function $\Upsilon^f(\mathbf{C}^{-f})$ of flow f is bounded by $0 \le \Upsilon^f(\mathbf{C}^{-f}) \le u_f\Big(R\log(1+B)\Big)$.*

Proof: Notice that $U_f = u_f\Big(R\log(1+\rho^f)\Big) - C^f$. Obviously, for the best response, the utility is nonnegative (utility 0 can always be obtained by $C^f = 0$). Hence, $0 \le \Upsilon^f(\mathbf{C}^{-f}) \le \max_{\rho^f} u_f\Big(R\log(1+\rho^f)\Big) = u_f\Big(R\log(1+B)\Big)$. ■

Due to the concavity of U_f in C^f (Lemma 3), a unique solution is guaranteed; furthermore, we observe that if u_f is continuously differentiable, the best response function is continuous as well.

Theorem 6. *If the followers always respond with their symmetrical NE, then an equilibrium of the leaders' game, i.e., an SNE of the overall system, exists and is unique.*

Proof: Please refer to Appendix F of [15]. ■

Thus far, we have obtained the static characteristics of the leaders' game, i.e., the existence and uniqueness of the equilibrium. Next, we analyze the dynamic behavior of the leaders' game, i.e., how the game converges to the equilibrium from any initial strategy profile by best-response strategy updates. Before delving into the convergence analysis, we discuss the monotonicity of $\Upsilon(\mathbf{C}^{-f})$ of flow f.

Lemma 5. *The best response $\Upsilon(\mathbf{C}^{-f})$ of flow f is monotonic and non-decreasing in $C^{f'}$ for any $f' \in \mathcal{F} \setminus \{f\}$.*

Proof: Please refer to Appendix G of [15]. ■

Now, we are ready to state the following theorem which characterizes the dynamic behavior of the leaders' game.

Theorem 7. *Given some initial price vector $\mathbf{C}(0)$, if each flow f responds according to Algorithm 2, that is, flow $f \in \mathcal{F}$ updates its strategy as $C^f(n+1) = \Upsilon(\mathbf{C}^{-f}(n))$, then $\lim_{n\to\infty} \mathbf{C}(n) = \mathbf{C}^*$, where $\mathbf{C}^*$ is the equilibrium of the leaders' game.*

Proof: Please refer to Appendix H of [15]. ■

Distributed Algorithm 2 computes the price $C^f(n+1)$ of flow f ($f \in \mathcal{F}$) at $n+1$, where the price $C^f(n+1)$ of flow f depends on $\Big(\rho^1(n), \cdots, \rho^{|\mathcal{F}|}(n)\Big)$ rather than the price of other flows, i.e., $C^{f'}(n)$ ($f' \neq f$).

Algorithm 2 Computing price for flow f

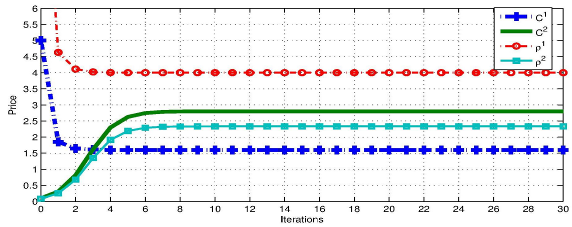
input: $\rho^1(n), \cdots, \rho^{|\mathcal{F}|}(n)$;
if flow $f \in \mathcal{F}$ updates its strategy **then**
  if $\rho^f(n) + \sum_{f' \neq f} \rho^{f'}(n) < B$ **then**
    $C^f(n+1) = u'_f(R\log(1+\rho^f(n)))\frac{R\rho^f(n)}{1+\rho^f(n)}$;
  else
    $\lambda = [\frac{u'_f(R\log(1+\rho^f(n)))}{\rho^f(n)(1+\rho^f(n))}\frac{R-1}{R} - \sum_{f'\in\mathcal{F}}\frac{e^{f'}}{\rho^{f'}(n)}]\frac{1}{\sum_{f'\in\mathcal{F}}\frac{1}{\rho^{f'}(n)}}$;
    $C^f(n+1) = \rho^f(n)(\lambda + e^f)\frac{R^2}{R-1}$ for flow f;
  end if
end if
ouput: $C^f(n+1)$.

V. Numerical Simulation

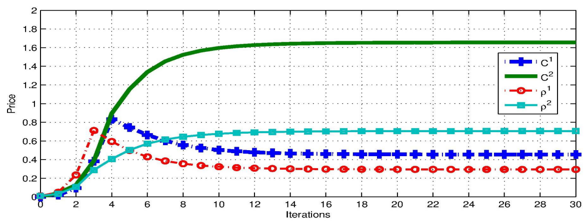
Here, we demonstrate some of the theoretical results derived in this paper, and gain further insight into the behavior of the game for different scenarios via a numerical study.

A. Convergence with Offloading Capacity Limit

We first consider the simplest scenario with two cellular traffic flows $|\mathcal{F}| = 2$ and two APs $|\mathcal{R}| = 2$, which allows us to illustrate the interactions between flows and APs. Specifically, for the cellular traffic flow $f \in \mathcal{F}$, we adopt a linear utility function $U_f = \omega_f \sum_{i\in\mathcal{R}} \log(1+r_i^f) - C^f$. The parameters are set as follows: offloading costs $e^1 = 0.1$ and $e^2 = 0.3$, weight coefficients $w_1 = 1$ and $w_2 = 2$, and capacity limit $B = 7$ in Fig. 1(a) and $B = 1$ in Figs. 1(b)–(d), respectively. We obtain $\rho^1 = 4$ and $\rho^2 = 2.33$, and further, $C^1 = 1.6$ and $C^2 = 2.8$ from $C^f = e^f\rho^f\frac{R^2}{R-1}$ according to Algorithm 2. Note that $\rho^1 + \rho^2 = 6.33 < 7$ implying that the condition $\rho^1 + \rho^2 < B$ holds, which is shown in Fig. 1(a). On the other hand, for $B = 1$, $\rho^1 + \rho^2 = 1$ must be satisfied at the NE, which is illustrated in Figs. 1(b)–(d). Moreover, we observe from Figs. 1(b)–(d) that the price vector and the strategy profile converge from different initial price vectors $\mathbf{C}(0) = (0.01, 0.01)$, $\mathbf{C}(0) = (5, 0.01)$, and $\mathbf{C}(0) = (10, 10)$, respectively, which validates the proposed Algorithm 2.



(a)



(b)

Fig. 1. Convergence of price vector and policy vector to NE: $e^1 = 0.1$, $e^2 = 0.3$, $w_1 = 1$, $w_2 = 2$, and $B = 7$ in (a), and $B = 1$ in (b).

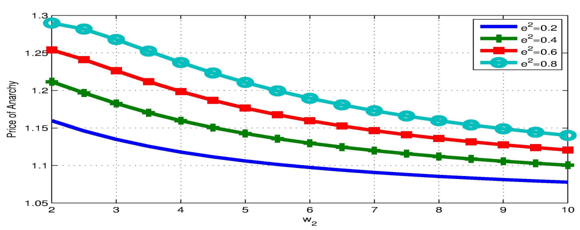


Fig. 2. The impact of the offloading cost and heterogeneity of traffic flows on PoA. $w_1 = 1$, $e_1^1 = e_2^1 = 0.2$, and $e_1^2 = e_2^2 = e^2$.

B. Offloading Cost with Offloading Capacity Limit

In this scenario, we consider two symmetric APs and two traffic flows. In particular, the offloading cost of the APs for flow f is homogeneous, i.e., $e_1^1 = e_2^1 = 0.2$ and $e_1^2 = e_2^2 = e^2$. Meanwhile, the utility function of each flow is assumed to be a linear function, i.e, $u_f(\sum_{i\in\mathcal{R}} \log(1+r_i^f)) = w_f \sum_{i\in\mathcal{R}} \log(1+r_i^f)$ where $w_1 = 1$. Fig. 2 show how the PoA is affected by the offloading cost and the heterogeneity of flows. We observe that as w_2 increases, corresponding to an increasing heterogeneity of flows, the PoA tends to decrease and approaches 1; on the other hand, as e^2 increases from 0.2 to 0.8, PoA tends to increase. For example, when $w_2 = 2$, the APs are more reluctant to offload flow $f = 2$ for its larger offloading cost, and accordingly, the two traffic flows are not treated equally. In this case, flow $f = 2$ cannot participate in the market pricing to the same extent as its counterpart $f = 1$, which leads to an increase of the PoA.

VI. Conclusions

In this paper, we have proposed a pricing framework for cellular networks to offload mobile data traffic with the assistance of WiFi network. We have modeled the pricing mechanism as a multi-leader multi-follower Stackelberg game in which the offloading service providers are the followers and the offloading service consumers are the leaders. For the case where the APs do not have an offloading capacity limit, we have decomposed the followers' game of the multi-leader multi-follower Stackelberg game into a fixed number of followers' games, and proved the existence and uniqueness of the equilibrium, and obtained an efficient algorithm to compute the equilibrium. For the case with offloading capacity limit, by considering the symmetric strategy profile, we have established some structural results for the equilibrium, and further proved the existence and uniqueness of the equilibrium of the Stackelberg game. Finally, extensive numerical experiments were provided to demonstrate that the Stackelberg equilibrium is very close to the corresponding social optimum for both considered cases.

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