On the Power and Rate Control in IEEE 802.11 WLANs – A Game Theoretical Approach

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***Abstract*—We present a non-cooperative game-theoretical study of the power and rate control problem in IEEE 802.11 WLANs where network participants choose appropriate transmission power and data rate to achieve maximum throughput with minimum energy consumption. In such game-theoretical study, the central question is whether a Nash equilibrium (NE) exists, if so, whether the network operates efficiently at the NE. In this paper, we show the existence and uniqueness of the NE and the convergence to the NE under best response strategy. However, the unique NE is inefficient. Motivated by this fact, we propose both linear and non-linear pricing scheme to improve efficiency. We demonstrate that by wisely choosing the parameters, the game converges to an efficient NE. Finally, we examine the convergence to the NE under a practical rate update scheme: the subgradient rate update.**

I. Introduction

In IEEE 802.11 WLANs, the wireless channel is shared by all network participants. The contention based medium access control (e.g., CSMA/CA) is used to access the shared medium among contending nodes. In such distributed environment, increasing physical data transmission rate increases the throughput at the price of higher energy consumption level. Thus one challenge for the network participants is to achieve maximum throughput with minimum power consumption by choosing appropriate data transmission rate.

Such power and rate control problems have been studied in cellular networks under both optimization and game theoretical frameworks where the transmission of one user interferes other users due to the power interference among users. The problem is different in IEEE 802.11 WLANs, an interference-free environment in terms of transmission power due to the contention based medium access control mechanism. However, the contention based medium access control creates another important feature which we refer to as data rate interference, i.e., the data rate of one user not only determines its own throughput, but also influences the throughput of other users. In this paper, we perform a cross-layer (PHY and MAC layers) study on the power and rate control problem in IEEE 802.11 WLANs taking into account the transmission rate interference. Since IEEE 802.11 WLANs are by nature distributed and non-cooperative environments, we argue that the game theoretical model is more appropriate to study the above problem than the global optimization model. The main contribution of our work is as follows:

- We model the power and rate control problem as non-cooperative rate control game where each user tries to maximize its throughput while minimizing its energy consumption, which is a function of the transmission rate. We show the uniqueness and the inefficiency of the Nash equilibrium (NE) and the convergence to the NE under best response strategy.
- We then propose both linear and non-linear pricing schema to improve efficiency. We show that by wisely choosing the parameters, the game converges to an efficient NE. We also examine the convergence to the NE under a practical rate update schema: the subgradient rate update.

II. Related Work

The problem of power and rate control is widely addressed in the context of cellular networks where game theory is exploited as a powerful tool to model the user behaviors [4], [7]. However, very little work has been done on modeling the power and rate control in wireless networks with contention based medium access mechanism such as IEEE 802.11 WLANs although the same problem in latter context is by nature different to that in cellular networks due to the transmission rate interference caused by the medium access contentions. [5] shows via both simulation and analytical model that in a non-cooperative environment under IEEE 802.11 DCF (Distributed Coordination Function), a selfish node may achieve higher throughput by using a lower data transmission rate at the expense of reduced overall network throughput. In [5], the power consumption is not taken into account when maximizing the node's throughput. [6] restudy the problem using both cooperative and non-cooperative approach. Their emphasis is on the cooperative control and little analysis is done on the non-cooperative control. Neither [5] and [6] considers the concept of pricing in their work.

Our work differs with the existing work in that: 1) we conduct a more in-depth analysis on the non-cooperative power and rate control, including the existence, uniqueness, convergence and efficiency of the Nash equilibrium; 2) based on our analysis, we propose both linear and non-linear pricing schemas to derive an efficient Nash equilibrium maximizing the overall network utility.

III. System Model

We base our analysis on the results of [1]. We consider a single-hop IEEE 802.11 WLAN of n nodes. Each node uses the IEEE 802.11 DCF protocol with RTS/CTS frame exchange to access the channel and each node has an equal channel allocation probability. The network is saturated such that every node always has packets to transmit when having chance to do so. We assume that that all nodes use the same back-off parameters and the packets are of the same size L. With these assumptions, [1] shows the throughput of node i is

$$S_i = \frac{\beta(1-\beta)^{n-1}L}{1+n\beta(1-\beta)^{n-1}(T_o - t_c + \frac{1}{n}\sum_{j=1}^{n}\frac{L}{C_j}) + (1-(1-\beta)^{n-1})T_c}$$

where β is the long run average attempt rate per node per slot in back-off time, C_i is the data rate of i. T_o is the transmission overhead in slots related to a frame transmission (SIFS/DIFS, etc), T_c is the fixed overhead for an RTS collision in slots. We direct readers to [1] for more detail.

IV. Utility Function

In game theory, the utility function is used to describe the satisfaction level of the player as a result of its actions. In our context, network participants try to maximize their throughput while minimizing the power consumption. We adopt the utility function in [6] defined as the difference between the throughput and the power consumption:

$$U_i := S_i - \zeta_i Q_i(C_i)$$

where $\zeta_i > 0$ is the relative importance weight (energy consumption versus throughput) of i, $Q_i(C_i)$ is the power consumption and can be approximated by a linear function of C_i: $Q_i(C_i) = a_i C_i$, where a_i is a constant that may depend on the path attenuation under given channel conditions. We can write U_i as

$$U_i(C_i) = \frac{q_1}{nq_2 + q_1\sum_{j=1}^{n}\frac{1}{C_j}} - \zeta_i a_i C_i$$

where $q_1 = n\beta(1-\beta)^{n-1}L$, $q_2 = 1+n\beta(1-\beta)^{n-1}(T_0 - T_c) + (1-(1-\beta)^n)T_c$.

Lemma 1: Let $A_i = \frac{1}{\sqrt{\zeta_i a_i}} - 1$, $B = n\frac{q_2}{q_1}$. It holds that

- The utility function U_i is concave w.r.t C_i.
- If $A_i > 0$, then $U_i(C_i)$ admits a unique positive maximizer $\tilde{C}_i = \frac{A_i}{B+\sum_{j=1,j\neq i}^{n}\frac{1}{C_j}}$. $U_i(C_i)$ is monotonously increasing w.r.t C_i in $(0, \tilde{C}_i)$ and monotonously decreasing in $(\tilde{C}_i, +\infty)$.
- If $A_i \leq 0$, then $U_i(C_i)$ is monotonously decreasing w.r.t C_i in $(0, +\infty)$

V. Non-cooperative Rate Control Game

We formulate the power and rate control problem as a non-cooperative rate control game G_{NRC}. Our motivation of using game theoretical approach is two-fold: 1) It is a powerful tool to model selfish behaviors and their impact on the system performance in distributed environments with self-interested players; 2) It can model the features or constraints of WLANs such as lack of coordination and network feedback. In fact in such environments, its distributed nature, selfish behavior is much more robust and scalable than any centralized cooperative control, which is very expensive or even impossible to implement.

Definition 1: The non-cooperative rate control game G_{NRC} is a 3-tuple $\big(\mathcal{N}, \{P_i\}, \{U_i(\cdot)\}\big)$, where $\mathcal{N}$={1, 2, $\cdots$, n} is the player set, P_i is the strategy set of player i, $U_i(\cdot)$ is the utility function of player i defined previously. Each player i selects its rate $C_i \in P_i = [C_{min}, C_{max}]$ to maximize its utility U_i. Formally, G_{NRC} is expressed as:

$$G_{NRC}: \quad \max_{C_i \in P_i} U_i(C_i, C_{-i}), \quad i \in \mathcal{N} \tag{1}$$

In G_{NRC}, the data rate optimizing player's individual utility U_i depends on both its own transmission rate C_i and that of its opponents, denoted as C_{-i}. Generally, for non-cooperative games as G_{NRC}, in some cases, the game may reach an equilibrium where no player has incentive to deviate from its current strategy. Such equilibrium is called Nash equilibrium in game theory, which can be seen as optimal "agreements" between the opponents of the game.

VI. Solving the Game

In the non-cooperative game, one of the most important questions is whether there exists a NE or not. The NE is defined as follows in the case of G_{NRC}.

Definition 2: A data rate vector $C = (C_1, \cdots, C_n)$ is said to be a NE of G_{NRC} if no player can improve its utility by unilaterally deviating from NE:

$$U_i(C_i, C_{-i}) \geq U_i(C_i', C_{-i}), \forall C_i' \in [C_{min}, C_{max}]$$

The concept of NE offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point where no player wishes to deviate. However, such a point does not necessarily exist. First, we investigate the existence of NE in G_{NRC}.

Theorem 1: G_{NRC} admits at least one NE.

Proof: In G_{NRC}, the strategy space of each user is a compact, convex set with minimum and maximum rate constraints C_{min} and C_{max}, respectively. The utility function U_i is concave w.r.t C_i and bounded. G_{NRC} is a n-person game defined in [3] and admits at least a NE. ■

A. Best Response Strategy

At NE, the data rate chosen by a rational self-interested player constitutes a best response to the data rate currently chosen by other players. Formally, player i's best response $r_i : C_{-i} \to C_i$ is defined as follows:

$$r_i = \underset{C_{min} \leq C_i \leq C_{max}}{\operatorname{argmax}} U_i(C_i, C_{-i}) \tag{2}$$

In G_{NRC}, applying Lemma 1, the best response of each player i is as follows:

$$C_i = \begin{cases} C_{min} & \tilde{C}_i < C_{min} \\ \tilde{C}_i = \dfrac{A_i}{B+\sum_{j\in\mathcal{N}, j\neq i}\frac{1}{C_j}} & C_{min} \leq \tilde{C}_i \leq C_{max} \\ C_{max} & \tilde{C}_i > C_{max} \end{cases} \tag{3}$$

The following corollary is immediate.

Corollary 1: If $\{C_i\}$ updated according to best strategy response converges to $\{C_i^*\}$, where $C_{min} \leq C_i^* \leq C_{max}$, then $\{C_i^*\}$ is a NE.

Consider G_{NRC} is played repeated, choosing the best response at each stage consists of a natural and rational strategy called the best response strategy where each player updates its transmission rate for the next time stage such that it maximizes its utility based on the transmission rate of opponents in the current time stage. It is commonly used to study stability of NE. Next we investigate uniqueness of NE and the convergence of the best response strategy.

B. Uniqueness and Convergence of NE

Theorem 2: G_{NRC} admits a unique NE. Start from any initial point, the iteration defined by best response function converges to the unique NE.

Proof: By Theorem 1, we know that there exists at least one NE. Let C denote the NE. By definition, the NE has to satisfy $C = r(C)$, where $r(C) = (r_1(C), r_2(C), \cdots, r_n(C))$ is the best response vector of all players. The key point to prove the uniqueness is to show that the best response function $r(C)$ is standard. A function is said to be standard if it satisfies the following properties:

- positivity: $r(C) > 0$;
- monotonicity: if $C \geq C'$, then $r(C) \geq r(C')$;
- scalability: for all $\mu > 1$, $\mu r(C) > r(\mu C)$.

In above properties, the vector inequality $C > C'$ is defined as a strict inequality in all components. These properties can be easily verified for $r(C)$ in G_{NRC}. It is shown in [6] that the fixed point $C = r(C)$ is unique for a standard function and that start from any point, the iteration defined by the standard function converges to the unique fixed point. Therefore, the NE of G_{NRC} is unique. The global convergence to the unique NE under the best response strategy is also guaranteed. ■

Remark: Theorem 2 is a powerful result since it not only establishes the uniqueness of NE, but also guarantees the convergence of the NE under the best response strategy. It follows straightforwardly that the unique NE is also stable in that any deviated point from the NE will be dragged back to the NE under best response strategy.

Let $C^* = \{C_i^*\}$ denote the unique NE of G_{NRC}, solving C^* equals to finding the fixed point of the best response function i.e., solving $r(C) = C$. In the unconstraint case where all players can attain the unique maximizer of U_i, we obtain $C_i^* = \dfrac{(A_i+1)\left(1 - \sum_{j\in\mathcal{N}} \frac{1}{A_j+1}\right)}{B}$.

In general cases, some players may not be able to reach the unconstraint maximizer of its utility function. It follows from the best response function that $\forall i, j \in \mathcal{N}$, if $A_i < A_j$, $C_i^* < C_j^*$. Hence, at the NE: a subset $\mathcal{N}_1$ of nodes with small A_i values operate on C_{min}; a subset $\mathcal{N}_2$ of nodes with large A_i values operate on C_{max}; the rest nodes $k \in \mathcal{N} - \mathcal{N}_1 - \mathcal{N}_2$ operate on the unconstraint maximizer $C_k^* = \dfrac{(A_j+1)\left(1 - \sum_{r\in\mathcal{N}-\mathcal{N}_1-\mathcal{N}_2} \frac{1}{A_r+1}\right)}{B + \frac{|\mathcal{N}_1|}{C_{min}} + \frac{|\mathcal{N}_2|}{C_{max}}}$. Note that the subsets $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N} - \mathcal{N}_1 - \mathcal{N}_2$ may be empty, e.g., if $\mathcal{N}_1 = \mathcal{N}_2 = \Phi$, it is the unconstraint case. With the above guidelines in mind, we propose the following algorithm to find the unique NE (the correctness proof of the algorithm is omitted here due to space limitation).

Algorithm 1 Find the NE

Sort the players according to their A_i values such that after the sorting, $A_1 \leq \cdots \leq A_n$
$\mathcal{N}_1 \leftarrow \Phi$, $\mathcal{N}_2 \leftarrow \Phi$, $i \leftarrow 1$, $j \leftarrow n$, $changed \leftarrow true$

while $change = true$ **do**
  if $\dfrac{(A_i+1)\left(1 - \sum_{r\in\mathcal{N}-\mathcal{N}_1-\mathcal{N}_2} \frac{1}{A_r+1}\right)}{B + \frac{|\mathcal{N}_1|}{C_{min}} + \frac{|\mathcal{N}_2|}{C_{max}}} < C_{min}$ **then**
    $C_i^* \leftarrow C_{min}$, $\mathcal{N}_1 \leftarrow i$, $i{+}{+}$
  else
    $changed = false$
  end if
  if $\dfrac{(A_j+1)\left(1 - \sum_{r\in\mathcal{N}-\mathcal{N}_1-\mathcal{N}_2} \frac{1}{A_r+1}\right)}{B + \frac{|\mathcal{N}_1|}{C_{min}} + \frac{|\mathcal{N}_2|}{C_{max}}} > C_{max}$ **then**
    $C_j^* \leftarrow C_{max}$, $\mathcal{N}_2 \leftarrow j$, $j{-}{-}$
  else
    $changed = false$
  end if
end while

for $k = i$ to j **do**
  $C_k^* = \dfrac{(A_j+1)\left(1 - \sum_{r\in\mathcal{N}-\mathcal{N}_1-\mathcal{N}_2} \frac{1}{A_r+1}\right)}{B + \frac{|\mathcal{N}_1|}{C_{min}} + \frac{|\mathcal{N}_2|}{C_{max}}}$
end for
return C^*

Remark: We consider the unconstraint case to gain a more in-depth insight on the NE. $\dfrac{\partial \zeta_i Q_i}{\partial C_i} = \dfrac{1}{(A_i+1)^2} = \zeta_i a_i$ can be regarded as the price for player i operating on C_i. The NE is thus the point where the marginal throughput gain $\dfrac{\partial S_i}{\partial C_i}$ equals to the price. From the players's point of view, operating at higher transmission rate increases the throughput at the expense of paying more in terms of energy. Hence, search the NE is actually to seek a compromised point between the gain (throughput) and the cost (energy consumption).

VII. Inefficiency of the NE – Price of Anarchy

The NE discussed in last section provides a solution where no player can increase its utility any further through individual effort. It is an outcome obtained as a result of distributed decision taking which may be less efficient than cooperation rate configuration among players. Indeed, it is well known that in general the NE are inefficient. A natural question we pose here is whether the obtained NE of G_{NRC} is efficient (social optimal), i.e., whether the network achieves the optimum performance at the NE. In this section, we focus on this question by investigating the degradation of network performance due to the selfish behavior of users by comparing the social utility on the NE with the maximum one.

We borrow the concept of Price of Anarchy (PoA) widely applied to qualify the NE in routing game and congestion control game to study the efficiency of the NE in G_{NRC}. To this end, we define PoA in our context as the ratio of the system utility at the unique NE and the optimal social utility. The PoA in our context measures the upper bound of the amount of suffering to the network as a whole due to lack of coordination.

Let $\widehat{C} = \{\widehat{C_i}\}$ denote the social optimal point of G_{NRC} maximizing the global network utility $\sum_{i\in\mathcal{N}} U_i$, by imposing $\frac{\partial \sum_{i\in\mathcal{N}} U_i}{\partial C_i} = 0$, we have,

$$\widehat{C_i} = \begin{cases} C_{min} & \widetilde{C_i'} < C_{min} \\ \widetilde{C_i'} = \dfrac{A_i'}{B + \sum_{j\in\mathcal{N}, j\neq i}\frac{1}{C_j}} & C_{min} \le \widetilde{C_i'} \le C_{max} \\ C_{max} & \widetilde{C_i'} > C_{max} \end{cases} \quad (4)$$

where $A_i' = \sqrt{\dfrac{n}{\zeta_i a_i}} - 1$.

Compare (4) with the best response function of G_{NRC} (3), we notice that by substituting A_i by A_i' in (3), we obtain a standard function $r'(C)$ whose unique fixed point is exactly the social optimal point.

There are several interesting engineering implications from the above analysis: 1) If each player updates its data rate according to $r'(C)$, the network will converge to the social optimal point; 2) With slight modification, the algorithm proposed in last section can be applied to calculate the social optimal point; 3) It holds that $C^* \le \widehat{C}, \forall i \in \mathcal{N}$ (for any vectors C and C', $C \le C'$ is defined such that for any component C_i and C_i', $C_i \le C_i'$.). This can be shown by noticing that $A_i < A_i', \forall i \in \mathcal{N}$, thus start from the same initial point C^0, at each iteration t, it holds that $r^t(C^0) \le (r')^t(C^0)$.

We study the unconstraint case to show the inefficiency of the NE of G_{NRC}. In such circumstance, $\widehat{C_i} = \dfrac{(A_i'+1)\left(1 - \sum_{j\in\mathcal{N}}\frac{1}{A_j'+1}\right)}{B}$, all players transmit at lower rate than the social optimal value. This implicates that in such cases the NE is inefficient. The price of anarchy PoA$=\dfrac{\sum_{i\in\mathcal{N}} U_i(C_i^*)}{\sum_{i\in\mathcal{N}} U_i(\widehat{C_i})} < 1$. We resummarizes the result in the following theorem:

Theorem 3: In unconstraint case, the obtained unique NE is inefficient. PoA$=\dfrac{\sum_{i\in\mathcal{N}} U_i(C_i^*)}{\sum_{i\in\mathcal{N}} U_i(\widehat{C_i})} < 1$

Remark: In fact, the NE is not Pareto optimal either. If all players switch from the NE to the global optimal point, both the individual and the social utility increase. This is due to the fact of lack of cooperation and the incentive to operate at social optimal point.

To further illustrate how inefficient the unique NE can be, we study the PoA for the symmetric unconstraint case where $\zeta_i = \zeta$, $a_i = a$ for all nodes i. In such cases, $C_i^* = \frac{q_1}{q_2}\left(\frac{1}{n\sqrt{\zeta a}} - 1\right)$, $\widehat{C_i} = \frac{q_1}{q_2}\left(\frac{1}{\sqrt{n\zeta a}} - 1\right)$.

$$PoA = \frac{U_i(C_i^*)}{U_i(\widehat{C_i})} = \frac{\frac{q_1}{nq_2 + \sum_{j\in\mathcal{N}}\frac{1}{C_j}} - \zeta a C_i\Big|_{C_j = C_j^*}}{\frac{q_1}{nq_2 + \sum_{j\in\mathcal{N}}\frac{1}{C_j}} - \zeta a C_i\Big|_{C_j = \widehat{C_j}}} = \frac{1 - n\sqrt{\zeta a}}{1 - \sqrt{\zeta a}}$$

We have omitted some trivial mathematical operations in the above manipulation. One necessary condition is that $C_i^*, \widehat{C_j} > 0$, thus $n\sqrt{\zeta a} < 1$. We observe that in cases where $C_{min} \to 0$ and $n\sqrt{\zeta a} \to 1$, $PoA \to 0$. In such cases, the NE corresponds to a quasi collapse of the network. The above analysis motivates us to seek incentive mechanism to encourage players to approach the social optimal point.

VIII. Non-cooperative Rate Control Game With Pricing

Pricing is a powerful technique in game theory to motivate selfish players to adopt a social optimal behavior. In our context, we turn to pricing to let the network converge to an efficient NE. We implicitly encourage cooperation via pricing in the non-cooperative environment. More specifically, noticing that at non-cooperative environment, players tend to transmit at lower rate than the data rate at the social optimal point, we encourage the players to increase their data rate via pricing to approach the NE to the social optimal point. In this new context, we develop a non-cooperative game with pricing denoted by $G_{NRCP}=(\mathcal{N}, \{P_i\}, \{U_i'(\cdot)\})$, where the utility function $U_i'(\cdot)$ is defined as $U_i' = U_i + \tau_i(C_i)$, where $\tau_i : P_i \to \mathbb{R}$ is the general form of the pricing function. In this paper, we focus on two pricing schemas in G_{NRCP}: linear and non-linear pricing.

A. Linear Pricing Function

We first concentrate on the linear pricing schema where $\tau_i(C_i) = b_i C_i$. Motivated by the fact that in unconstraint case, at NE of G_{NRC}, players transmit at lower rate than the social optimal case, we impose a pricing function monotonously increasing w.r.t. C_i by setting $b_i > 0$ to encourage the players to transmit at a higher rate. $b_i C_i$ can be regarded as extra gain to players imposed by the pricing policy. The non-cooperative rate control game with linear pricing G_{NRC-LP} is thus formally expressed as $G_{NRC}^{LP} : \max_{C_i\in P_i} U_i^L(C_i, C_{-i}) = U_i(C_i, C_{-i}) + b_i C_i, i \in \mathcal{N}$

In this paper, we do not specify how to realize the pricing in practice. Possible approaches include virtual currency etc. We are now ready to discuss how to tune the pricing factor b_i to establish the efficient NE in G_{NRC}^{LP}.

In G_{NRC}^{LP}, the concavity of the utility function is maintained and the existence of NE is thus guaranteed. Moreover, player i's best response $r_i^L : C_{-i} \to C_i$ is as follows:

$$C_i = \begin{cases} C_{min} & \tilde{C_i^L} < C_{min} \\ \tilde{C_i^L} = \dfrac{A_i^L}{B + \sum_{j\in\mathcal{N}, j\neq i}\frac{1}{C_j}} & C_{min} \le \tilde{C_i^L} \le C_{max} \\ C_{max} & \tilde{C_i^L} > C_{max} \end{cases} \quad (5)$$

where $A_i^L = \frac{1}{\sqrt{\zeta_i(a_i - b_i)}} - 1$.

It is easy to show that $r^L(C)$ is standard. The following theorem follows immediately.

Theorem 4: G_{NRC}^{LP} admits a unique NE. Start from any initial point, the iteration defined by best response $r^L(C)$ converges to the unique equilibrium which is the unique fixed point of $r^L(C)$.

Compare (5) with (4), if $A_i^L = A_i'$, $r^L(C)$ becomes the same as $r'(C)$. As consequence, the NE of G_{NRC}^{LP} coincides with the social optimal point. This implicates that to make the NE of G_{NRC}^{LP} efficient, it suffices to set $A_i^L = A_i'$, which can be achieved by setting $b_i = \frac{(n-1)}{n} a_i$.

Theorem 5: In G_{NRC}^{LP}, if $b_i = \frac{(n-1)}{n} a_i$, then the unique NE is efficient (socially optimal).

In the analysis of G_{NRC}, we can interpret $\frac{\partial \zeta_i Q_i}{\partial C_i} = \frac{1}{(A_i+1)^2} = \zeta_i a_i$ as the price for player i operating on C_i. Here in G_{NRC}^{LP}, the above price becomes $\frac{1}{(A_i^L+1)^2} = \zeta_i(a_i - b_i) = \zeta_i a_i/n$. As the price decreases, each player i tends to increase its transmission rate C_i at NE.

A desirable property of the linear pricing schema is that $b_i = \frac{(n-1)}{n} a_i$ is independent to ζ_i, which is inaccessible to any others except player i. However, since b_i depends on a_i, any error on a_i influences the effectiveness of the pricing scheme. We consider the following unconstraint case to study the above influence: if i maliciously reports a_i as $a_i' = g a_i$ where $g > 1$, the new NE of G_{NRC}^{LP} $C^{L'} = \{C_i^{L'}\}$ will deviate from the original NE $C^L = \{C_i^L\}$, which is also the social optimal point. More specifically, we have $b_i' = \frac{n-1}{n} a_i'$, $A_i^{L'} < A_i^L$, thus $C_j^{L'} < C_j^L$ for all players j. We also have $U_i(C^{L'}) > U_i(C^L)$ and $U_j(C^{L'}) < U_j(C^L)$ for other players j. This implies that by reporting a larger value of a_i, a malicious selfish player i can get more payoff at the expense of others and the sub-optimality of the network as a whole.

With the above vulnerability of the linear pricing scheme in mind, we propose the following non-linear pricing scheme.

B. Non-linear Pricing Function

Note $\frac{\partial \sum_{j=0}^n U_j}{\partial C_i} = \frac{\partial n S_i - \sum_{j=0}^n \zeta_j Q_j}{\partial C_i} = \frac{\partial (n S_i - \zeta_i Q_i)}{\partial C_i}$, we reconfigure the utility function such that $U_i^N = nS_i - \zeta Q_i$ $(\tau_i = (n-1)S_i)$. In this case, the best response function of the non-cooperative rate control game with non-linear pricing G_{NRC}^{NP} is the same as $r'(C)$ in (4). We immediately have the following results:

Theorem 6: The game G_{NRCP} admits a unique NE. Start from any initial point, the iteration defined by best response converges to the unique equilibrium.

Theorem 7: In G_{NRC}^{NP}, the unique NE is efficient.

C. Discussion

In fact, the linear and the non-linear pricing scheme represent different efforts of encouraging players two increase their data rate. The linear pricing scheme achieves the goal by decreasing the cost of the transmission from $a_i C_i$ to $(a_i - b_i)C_i$. Consequently players tend to transmit at higher rates than in G_{NRC} to maximize the new utility; On the other hand, the non-linear scheme attain the same goal by increasing the gain of the transmission from S_i to nS_i. Again consequently players tend to transmit at higher rates than in G_{NRC}. An interesting property of the non-linear pricing scheme is its independence to both ζ_i and a_i. Thus it can be deployed in a more general context and is more robust to malicious attacks.

D. Subgradient Rate Update

Till now, we have studied the convergence to the NE under the best response strategy. In some cases, the best response strategy may lead to unbounded variation in strategy spaces. Thus in this section, we consider an alternative strategy which is widely used in system control and is of practical importance: the subgradient rate update. In the following part of this section, we study the convergence of the subgradient update schema to the unique NE in G_{NRC}^{NP}. The analysis is also applicable in the linear pricing scheme.

Consider the subgradient rate update schema $C^{t+1} = T(C^t) = \{T_i(C^t)\}$ defined as follows

$$C_i^{t+1} = T(C^t) = C_i^t + \lambda \frac{\partial U_i^N}{\partial C_i} = C_i^t + \lambda \phi_i(C^t), \forall i \in \mathcal{N}, t \geq 0$$

where $\phi_i(x) = \frac{\partial U_i^N}{\partial x_i} = \frac{n}{\left(B + \Sigma_{j \in \mathcal{N}} \frac{1}{x_j}\right)^2 (x_i)^2} - \zeta_i a_i$.

At each iteration of the subgradient update scheme, each player takes a step in the direction of the positive subgradient. The engineering implication is that if the marginal throughput gain in G_{NRC}^{NP} $\frac{\partial n S_i}{\partial C_i}$ is greater than the price $\zeta_i a_i$, player i increases its C_i, otherwise it decreases C_i. By setting the step size λ sufficiently small, the subgradient update schema experiences much less variation and is much smoother than the best response strategy.

We now study the sufficient conditions for the convergence of the above scheme to the unique NE. Our analysis follows the classical techniques in [8]. We first define a function $g_i(\tau) : [0,1] \rightarrow \mathbb{R}$ for player i as

$$g_i(\tau) = \tau C_i^t + (1-\tau)\widetilde{C_i^N} + \lambda \phi_i(\tau C_i^t + (1-\tau)\widetilde{C_i^N})$$

where $\widetilde{C_i^N} = \{\widetilde{C_i^N}\}$ is the unique NE. It follows that

$$|T_i(C^t) - T_i(\widetilde{C_i^N})| = |g_i(1) - g_i(0)| = \left| \int_0^1 \frac{dg_i(\tau)}{d\tau} d\tau \right|$$

$$\leq \int_0^1 \left| \frac{dg_i(\tau)}{d\tau} \right| d\tau \leq \max_{\tau \in [0,1]} \left| \frac{dg_i(\tau)}{d\tau} \right|$$

where $\widetilde{C^N}$ is the fixed point of the mapping T. Noticing that λ is usually sufficiently small, we have

$$\left|\frac{dg_i(\tau)}{d\tau}\right| = \left|C_i^t - \widetilde{C_i^N} + \lambda \sum_{j\in\mathcal{N}} \frac{\partial \phi_i}{\partial C_j^t}\left(C_j^t - \widetilde{C_j^N}\right)\right|$$

$$\leq \left|1 + \lambda \frac{\partial \phi_i}{\partial C_i^t}\right| \cdot \left|C_i^t - \widetilde{C_i^N}\right| + \left|\lambda \sum_{j\in\mathcal{N}, j\neq i} \frac{\partial \phi_i}{\partial C_j^t}\right| \cdot \left|C_j^t - \widetilde{C_j^N}\right|$$

Noticing that $\frac{\partial \phi_i}{\partial C_j^t} > 0$, we have

$$\left|\frac{dg_i(\tau)}{d\tau}\right| \leq \left|1 + \lambda \sum_{j\in\mathcal{N}} \frac{\partial \phi_i}{\partial C_j^t}\right| \cdot \left|\left|C_j^t - \widetilde{C_j^N}\right|\right|$$

where $||C^t|| = \max_i |C_i^t|$ is the maximum norm. By injecting $\frac{\partial \phi_i}{\partial C_i^t}$ into the above equation, we obtain

$$\left|\frac{dg_i(\tau)}{d\tau}\right| \leq \Big[1 - \frac{2\lambda a_i n}{\left(BC_i^t + C_i^t \sum_{j\in\mathcal{N}} \frac{1}{C_j^t}\right)^3}\Big(B + \sum_{j\in\mathcal{N}, j\neq i}\left(\frac{1}{C_j^t} - \frac{C_i^t}{(C_j^t)^2}\right)\Big)\Big] \cdot \left|\left|C^t - \widetilde{C_i^N}\right|\right|$$

Define

$$k_i = 1 - \frac{2\lambda a_i n}{\left(BC_i^t + C_i^t \sum_{j\in\mathcal{N}} \frac{1}{C_j^t}\right)^3}\left(B + \sum_{j\in\mathcal{N}, j\neq i}\left(\frac{1}{C_j^t} - \frac{C_i^t}{(C_j^t)^2}\right)\right)$$

and $k = \max_i k_i$, the sufficient condition for the convergence of the subgradient rate update scheme is $k < 1$. Imposing this condition, we get

$$B > \frac{(n-1)(C_{max} - C_{min})}{C_{min}^2}$$

Under the above condition, it follows that start from any initial value C^0, $\lim_{n\to+\infty} T^n(C^0) = \widetilde{C_i^N}$. We summarize the above result in the following theorem.

Theorem 8: In G_{NRCP} with non-linear pricing function, if $B > \frac{(n-1)(C_{max} - C_{min})}{C_{min}^2}$, then the subgradient rate update scheme converges to the unique NE $\widetilde{C_i^N}$.

Remark: Theorem 8 provides a guideline for the convergence of the subgradient rate update. However, the condition is only sufficient, not necessary and may be too stringent in some cases. Hence, it is possible that even the above condition is not met, the subgradient update schema also converges to the NE.

In the above subgradient rate update scheme, each player updates its data rate at the same time instance. A natural and more practical generalization is the asynchronous subgradient update scheme where only a random subset of players update their data rate at a given time instance. This scheme is actually more realistic in that it is difficult for the players to synchronize their update in a practical implementation. In our context, concerning the convergence of the asynchronous subgradient update scheme, we have the following theorem. The proof of the theorem consists of proving two well known conditions sufficient for asynchronous convergence of a non-linear iterative mapping. The detailed proof is omitted here since it is done in [7] for the same problem but in different context.

Theorem 9: In G_{NRCP} with non-linear pricing function, if $B > \frac{(n-1)(C_{max} - C_{min})}{C_{min}^2}$, then the asynchronous subgradient rate update scheme converges to the unique NE.

IX. Numerical Results

In this section, we present illustrative numerical results for our non-cooperative game model of rate control. The following set of parameters are used in our experiments: $L = 12000$ bits (1500 bytes), $a_i = 0.001$, $b_0 = 16$, $b_k = 2^k b_0$, the data frame transmission overhead $T_o = 52$ slots, the RTS collision overhead $T_c = 17$ slots, the slot size is $20\mu s$ and $K = 10$ (see [6] for detailed description of the above parameters). C_{min} and C_{max} are set to 1Mbps and 100Mbps.

We study a network of 2 nodes with $\zeta_i = 5$. We plot the trajectory of the data rate under (3), i.e., the best response strategy. It can be proven that the trajectory of the data rate converges to the unique NE under the best response strategy, which is confirmed in this numerical study in Figure 1.

Figure 2 shows the node's individual utility U_i as a function of transmission rate C_i. As shown both analytically and numerically in the figure, the unique NE is not the social optimal point.

We then study the inefficiency of the NE in unconstraint case ($C_{min} \to 0$, $C_{max} \to +\infty$) by plotting the PoA as a function of n and ζ_i in Figure 3.

We next study the non-cooperative game model with pricing scheme using the same scenario as in Figure 1. The trajectory of the data rate with linear and non-linear pricing scheme under the best response strategy is plotted in Figure 4 and 5. Comparing with Figure 2, we can see that the numerical result confirms our analytical result in that the trajectory converges to the unique NE which is also the social optimal point under the best response strategy.

We also study the trajectory of the data rate with non-linear pricing scheme under subgradient update scheme in Figure 6 and 7. We can check that the sufficient condition of the convergence under the subgradient update scheme does not hold in our scenario. However, The subgradient algorithm converges, indicating that in some cases, the sufficient condition for the convergence under subgradient update scheme may be too stringent. Under the subgradient update scheme, the convergence to the NE achieves in a much smoother way than the best response strategy. The convergence is smoother with smaller λ, as a price, the convergence rate is slower accordingly.

Finally, we study a more realistic scenario: a WLAN of 10 nodes with the same parameters as the previous illus-

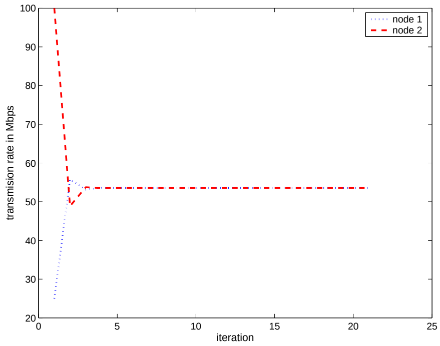

Fig. 1. Rate trajectory under best response

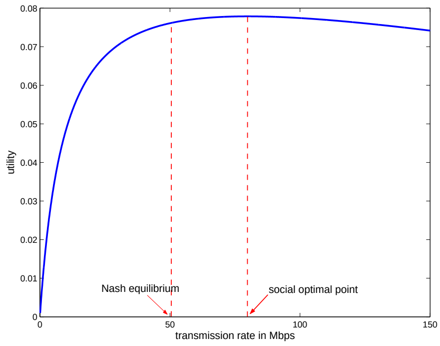

Fig. 2. NE and social optimal point

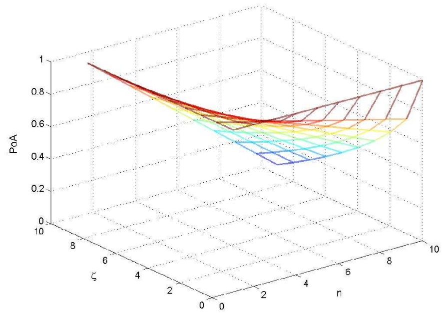

Fig. 3. Price of anarchy (PoA)

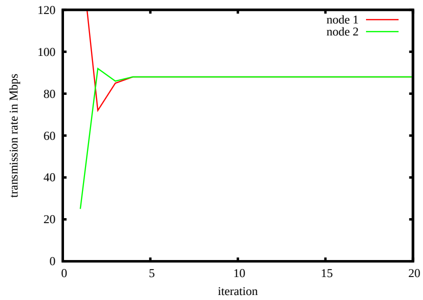

Fig. 4. Rate trajectory under best response: linear pricing

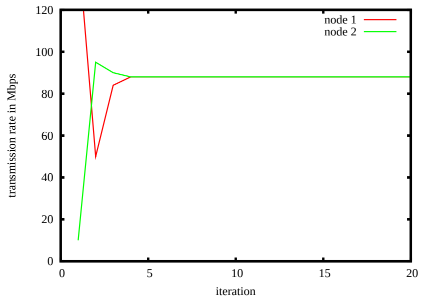

Fig. 5. Rate trajectory under best response: non-linear pricing

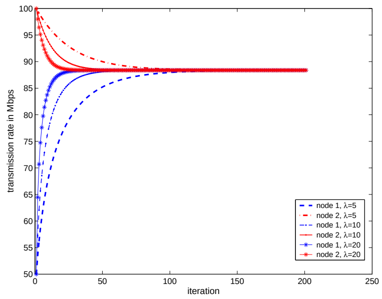

Fig. 6. Rate trajectory under synchronous subgradient update: non-linear pricing

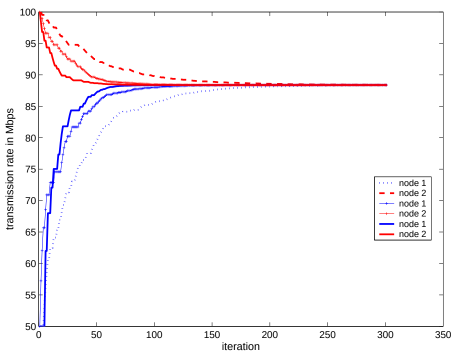

Fig. 7. Rate trajectory under asynchronous subgradient update: non-linear pricing

trative scenario of 2 nodes. We report that the network converges to the NE. Table 1 compares the analytical result with the simulation result. We can see from the table that the simulation result matches the analytical result quite well. Besides, we can see that in this scenario, the rate control is necessary or even indispensable to achieve network-wide high performance.

	Analytical result	Numerical result
NE (no pricing)	8.42Mbps	8.39Mbps
NE (non-linear pricing)	70.71Mbps	70.82Mbps
Social optimal data rate	70.71Mbps	70.85Mbps
PoA	0.315	0.312

TABLE I

NE analysis: a WLAN of 10 players

X. Conclusion

In this paper we formulated the power and rate control problem in IEEE 802.11 WLANs as a non-cooperative game. We showed analytically the existence and uniqueness of the NE and the convergence to the NE under best response strategy. However, the unique NE is inefficient. Motivated by this fact, we proposed both linear and non-linear pricing scheme to improve efficiency. We demonstrated that by wisely choosing the parameters, the game converged to an efficient NE. In our future work, we plan to extend our work for the multi-hop ad hoc networks.

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