A Game Theoretic Framework of Distributed Power and Rate Control in IEEE 802.11 WLANs

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I. Introduction

In IEEE 802.11 WLANs where the wireless channel is shared by all network participants, one challenge for the network participants is to achieve maximum throughput with minimum energy consumption by choosing appropriate transmission power and data rate.

Such power and rate control problem have been widely investigated in cellular networks under both optimization and game theoretic frameworks. However, in the context of IEEE 802.11 WLANs, the problem is by nature different: 1). In cellular networks, the transmission of one node interferes those of others, but IEEE 802.11 WLANs are interference-free environments with no interference during a successful transmission; 2). The contention based medium access control of IEEE 802.11 creates an important feature which we refer to as *data rate interference* that does not exist in cellular networks, i.e., the data rate (PHY layer) of one node not only determines its own throughput (MAC layer), but also influences the throughput of others. Therefore, there is a need for a quantitative model for the power and rate control in such environments as IEEE 802.11 WLANs.

In this paper, motivated by the need of a quantitative model for the power and rate control in IEEE 802.11 WLANs and the lack of related work in the literature, we address the problem by establishing a quantitative game theoretic framework. Our motivation of using game theoretic approach rather than global optimization approach is two-fold: 1) Game theory is a powerful tool to model selfish behaviors and their impact on the system performance in distributed environments with self-interested players; 2) Game theory can model the features or constraints of IEEE 802.11 WLANs such as lack of coordination and network feedback.

II. Problem Formulation

A. IEEE 802.11 Medium Access Control

We consider a saturated IEEE 802.11 WLAN of n nodes using IEEE DCF with RTS/CTS dialog. Recent work [1] has shown that (under decoupling assumptions) the expected throughput of node i can be calculated as:

$$S_i = \frac{\beta(1-\beta)^{n-1}L}{T_{slot}} = \frac{1}{\frac{q_2}{q_1} + \sum_{j\in\mathcal{N}}\frac{1}{C_j}}$$

where $q_1 = n\beta(1-\beta)^{n-1}L$, $q_2 = 1+n\beta(1-\beta)^{n-1}(T_o-T_c) + (1-(1-\beta)^n)\,T_c$ where β is the average attempt probability, T_{slot} is the average virtual slot length, L is packet size, C_i is the data rate of i, T_o is the transmission overhead related to a frame transmission (SIFS/DIFS, etc), T_c is the fixed overhead for an RTS collision.

Furthermore, let P_s denote the frame success rate (FSR), the probability of correct reception of a frame at destination. The effective throughput of i S_i^{eff} can be expressed as $S_i^{eff} = P_s S_i$. Assuming perfect error detection and no error correction, we have $P_s = (1-P_e)^L$, where P_e is the bit error rate (BER). P_e is a function of E_b/N_0, the bit-energy-to-noise ratio of the received signal. Assuming an additive white Gaussian noise (AWGN) channel, in our context, the bit-energy-to-noise ratio of i of the received signal is derived from the SNR (Signal-to-Noise Ratio) as follows:

$$\frac{E_b}{N_0} = SNR\frac{B_t}{C_i} = \frac{h_iP_i}{\sigma^2}\frac{B_t}{C_i} = \frac{h_iB_t}{\sigma^2}\frac{P_i}{C_i}$$

where C_i is the bit rate of the modulation scheme and B_t is the unspread bandwidth of the signal, P_i is the transmission power of i, h_i is the channel gain from the sender i to the receiver and σ^2 is the AWGN power at the receiver. Let $\gamma_i = \dfrac{h_iB_t}{\sigma^2}\dfrac{P_i}{C_i}$, we can express the FSR of i as a function of γ_i: $P_s = f_i(\gamma_i)$. In our work, in order to perform a closed-form analysis, we suppose that each user applies the non-coherent FSK modulation scheme where $f_i(\gamma_i) = f(\gamma_i) = (1-\frac{1}{2}e^{-\gamma_i/2})^L$.

To compute the energy consumption, consider a virtual slot T_{slot}, the possibility of transmitting a frame with success for i is $P_{tr} = \beta(1-\beta)^{n-1}$, the frame transmission time is L/C_i; the collision possibility is

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$P_c = 1-(1-\beta)^n - n\beta(1-\beta)^{n-1}$, the collision duration is the RTS frame length $t_{RTS} \ll L/C_i$. Thus the expected energy consumption per unit time for node i is

$$Q_i = \frac{P_i P_{tr}\frac{L}{C_i} + P_c t_{RTS}}{T_{slot}} \simeq \frac{P_i P_{tr}\frac{L}{C_i}}{T_{slot}} = \frac{\beta(1-\beta)^{n-1} L P_i}{C_i T_{slot}}$$

B. Utility Function and Game Formulation

In game theory, utility function is used to describe the satisfaction level of the player as a result of its actions. In IEEE 802.11 WLANs, the objective of a node is to achieve maximum effective throughput, meanwhile minimize the energy consumption. We define the utility function $U_i = P_s S_i - \zeta_i Q_i$. ζ_i represents the player's individual preference between reward and energy cost.

We model the power and rate control in IEEE 802.11 WLANs as a non-cooperative game G_{NPRC} where players choose their transmission power and data rate to maximize their utility functions.

III. Solving the Game

It is clear that G_{NPRC} is not decomposable, i.e., $\max_{C_i,P_i} U_i \neq \max_{C_i}\max_{P_i} U_i$ and $\max_{C_i,P_i} U_i \neq \max_{P_i}\max_{C_i} U_i$. We thus consider the following equivalent game G'_{NPRC}:

$$G'_{NPRC}: \max_{\gamma_i \geq 0, C_i^{min} \leq C_i \leq C_i^{max}} U_i = \frac{f(\gamma_i) - k_i\gamma_i}{\frac{q_2}{q_1} + \sum_{j\in\mathcal{N}} \frac{1}{C_j}}$$

where $k_i = \frac{\sigma^2}{h_i B_t}\zeta_i$.

Theorem 1: There is a unique Nash Equilibrium (NE) in G'_{NPRC} $\{C_i^{max}\}$, $\{\gamma_i^*\}$ (γ_i^* is the root of $f'(\gamma_i) = k_i$).

It is further easy to show that the NE of G'_{NPRC} is efficient, i.e., social optimal. We study how to approach this efficient NE. One might propose to trivially set $C_i = C_i^{max}$ and $\gamma_i = \gamma_i^*$. However, in practice, it is not easy for a player to calculate γ_i^*. Moreover, this often leads to large fluctuations that may cause temporary system instability. Hence we propose an alternative update scheme to approach the NE: the subgradient update scheme, consisting of 1). setting $C_i = C_i^{max}$; 2). updating γ_i as $\gamma_i^{t+1} = \gamma_i^t + \lambda\frac{\partial U_i(\gamma_i)}{\partial\gamma_i}\Big|_{\gamma_i=\gamma_i^t}$, where λ is the step size usually sufficiently small.

At each iteration, each player takes a step in the direction of the positive subgradient. The engineering implication is that if the marginal effective throughput outweighs the price, player i increases its γ_i, otherwise it decreases γ_i. By setting the step size sufficiently small, the subgradient update experiences a smooth trajectory.

One issue left is how to calculate $\frac{\partial U_i(\gamma_i)}{\partial\gamma_i}\Big|_{\gamma_i=\gamma_i^t}$. In fact, assuming after sufficient long time, each player operates on C_i^{max} according to the subgradient update scheme, $\frac{\partial U_i(\gamma_i)}{\partial\gamma_i}\Big|_{\gamma_i=\gamma_i^t}$ can be estimated by $\frac{U_i(\gamma_i^t) - U_i(\gamma_i^{t-1})}{\gamma_i^t - \gamma_i^{t-1}}$. Noticing $P_i = \frac{\sigma^2}{h_i B_t} C_i\gamma_i$, we have:

Theorem 2: In G'_{NRPC}, if $k_i > \frac{1}{2ln(L/2)}, \forall i \in \mathcal{N}$, the subgradient update scheme (both synchronous and asynchronous version) converges to the efficient NE $C_i = C_i^{max}$, $P_i = \frac{\sigma^2}{h_i B_t} C_i \gamma_i^*$: 1). Set $C_i = C_i^{max}$; 2). At iteration $t+1$, set $P_i^{t+1} = P_i^t + \lambda\frac{U_i(P_i^t) - U_i(P_i^{t-1})}{P_i^t - P_i^{t-1}}$.

Algorithm 1 Game Theoretic Power and Rate Control Procedure

Sender side:

INITIATION: Set the initial power to random value P_i^0. Schedule the power update at time $t_1, t_2, \cdots$.

Set $C_i = C_i^{max}$

At t_m, calculate $U_i(P_i^{m-1})$ as $U_i(P_i^{m-1}) = N_{suc} * L - \zeta_i P_i^{m-1} * N_{send} * L/C_i$

Update P_i as $P_i^m = P_i^{m-1} + \lambda\frac{U_i(P_i^{m-1}) - U_i(P_i^{m-2})}{P_i^{m-1} - P_i^{m-2}}$. Set the flag in RTS frame at the transmission of next frame informing the receiver the new iteration begins.

Receiver side:

When receiving the RTS with the flag set, include N_{suc} in the CTS frame, then set N_{suc} to 0

At the reception of each frame, if the frame is not erroneous, $N_{suc} \leftarrow N_{suc} + 1$

The proposed power and rate control scheme, as shown above, has the following desirable properties: 1). No system-dependent parameters are needed to approach the NE, such as $f(\gamma_i)$, h_i; 2). Only minor changes are needed to incorporate the proposed procedure into the IEEE 802.11 MAC protocol. The proposed scheme is thus totally transparent for users; 3). Each user only needs to temporarily buffer the number of frames sent during last iteration N_{send}, the utility of the last two iterations and the number of frames received without error during the last iteration N_{suc}; 4). The update can be performed asynchronously among users. No system-wide synchronization is required.

References

[1] A. Kumar, E. Altman, D. Miorandi and M. Goyal, "New Insights from a Fixed Point Analysis of Single Cell IEEE 802.11 WLANs". Proc. IEEE Infocom, Miami, USA, March, 2005.