Oblivious Neighbor Discovery for Wireless Devices with Directional Antennas

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***Abstract*—Neighbor discovery, the process of discovering all neighbors in a device's communication range, is one of the bootstrapping networking primitives of paramount importance and is particularly challenging when devices have directional antennas instead of omni-directional ones. In this paper, we study the following fundamental problem which we term as *oblivious neighbor discovery*: How can neighbor nodes with heterogeneous antenna configurations and without clock synchronization discover each other within a bounded delay in a fully decentralised manner without any prior coordination? We first establish a theoretical framework on oblivious neighbor discovery and establish the performance bound of any neighbor discovery protocol achieving oblivious discovery. Guided by the theoretical results, we then design an oblivious neighbor discovery protocol and prove that it achieves guaranteed oblivious discovery with order-minimal worst-case discovery delay in the asynchronous and heterogeneous environment. We further demonstrate how our protocol can be configured to achieve a desired trade-off between average and worst-case performance.**

I. Introduction

Directional antennas have been widely used in emerging wireless networks given the capability in limiting interference, enlarging transmission range and hence boosting network capacity and reducing energy consumption. For example, direction antennas are particularly attractive in the 60GHz networks to ensure high transmission quality and acquire sufficient link budget to cater Gbps data rate. In spite of significant performance gain brought by directional antennas, their deployment brings specific design challenges for many fundamental communication and networking functionalities, some of which require a complete rethinking or redesign.

In this paper, we focus on *neighbor discovery*, a supporting primitive that discovers all the neighbors in a device's communication range. It is one of the bootstrapping primitives supporting many basic network functionalities, such as topology control, clustering, medium access control, etc. Compared to the traditional omni-direction antenna paradigm, neighbor discovery with directional antennas is intuitively more challenging as directional antennas can only cover a fraction of the azimuth. Hence, neighbor discovery protocols need to be carefully designed in order to guarantee that any pair of neighbor nodes can eventually steer their antennas toward each other at certain time instance. Moreover, nodes may not be synchronised and their antennas can be heterogeneous in terms of beamwidth. Neighbor discovery protocols should be able to guarantee discovery in this challenging environment in a fully decentralised manner without any prior coordination.

We coin the term *oblivious neighbor discovery* problem to denote the following problem: *How can neighbor nodes with heterogeneous antenna beamwidth and without clock synchronization discover each other within a bounded delay in a fully decentralised manner without any prior coordination?* Particularly, the following requirements should be satisfied:

- Bounded (and minimum) worst-case discovery delay;
- Discovery oblivity, the capability of guaranteeing discovery regardless of the antenna beamwidth and the relative positions of nodes. This requirement is particular in the neighbor discovery with directional antennas.

We emphasize that it is the combination of the above design requirements that makes the oblivious neighbor discovery problem far from trivial and should be handled holistically. As reviewed in Section II, no existing work to our knowledge can satisfy both of them simultaneously. Aiming at providing a comprehensive investigation on the oblivious neighbor discovery problem, we articulate our work as follows:

- *Theoretical framework.* We establish a theoretical framework on oblivious neighbor discovery and establish the performance bound of any oblivious neighbor discovery protocol. Our theoretical results not only shed light on the structure of the problem, but also serve as design guidelines for oblivious neighbor discovery protocols.
- *Protocol design.* Guided by the theoretical results, we further design an oblivious neighbor discovery protocol and prove that it achieves guaranteed oblivious discovery with order-minimal worst-case discovery delay in the asynchronous and heterogeneous environment. We further demonstrate how the protocol can be configured to achieve a desired trade-off between average and worst-case performance.

II. Related Work

As discussed in Section I, designing efficient neighbor discovery protocols for devices with directional antennas is particularly challenging. A natural approach to contour the challenge is to use omni-directional antennas in the neighbor discovery process [1], [2] (cf. [3–7] for major neighbor discovery protocols with omni-directional antennas). The main disadvantages of this approach is two-fold. Firstly, it requires an additional omni-directional antenna; Secondly, the discovered neighbor set using the omni-directional antenna can be significantly different from that using the directional one.

Neighbor discovery protocols using purely directional antennas can be categorised into two classes, *probabilistic* and *deterministic* protocols. In probabilistic approaches [8–14], each node randomly chooses a direction to steer its antenna. Probabilistic protocols have the advantages of being memoryless and stationary and thus are especially robust and suitable in decentralised environments where no prior coordination or synchronisation is available. The main drawback of them is the lack of performance guarantee in terms of discovery delay. This problem is referred to as the long-tail discovery latency problem in which two neighbor nodes may experience extremely long delay before discovering each other. Deterministic protocols [13], [15–18], where each node points its antenna based on a predefined sequence, are proposed to provide guaranteed upper-bound on the worst-case discovery delay. However, the current state-of-the-art deterministic neighbor discovery solutions with directional antennas either fail to achieve bounded discovery delay, or require time synchronisation among nodes, which may be not be practical in many applications or require prior coordination among nodes.

In spite of the existing research in the literature, none of them can solve the oblivious neighbor discovery problem by ensuring nodes with heterogeneous antenna configurations and without clock synchronization to discover each other within a bounded delay in a fully decentralised manner without any prior coordination, which is the focus of this paper.

III. System Model and Problem Formulation

A. System Model

We consider a time-slotted (but not necessarily synchronised) two-dimensional wireless network operating on a single frequency band. The set of nodes in the network is denoted by $\mathcal{S}$ with cardinality $S \triangleq |\mathcal{S}|$. Each node $i \in \mathcal{S}$ is equipped with a directional antenna with beamwidth θ_i ($0 < \theta_i \leq 2\pi$). When $\theta_i = 2\pi$, the antenna of node i degenerates to an omni-directional one. Under such generic antenna model, the communication range of node i can be divided into $N_i \triangleq \frac{2\pi}{\theta_i}$ non-overlapping sectors, indexed from 0 to $N_i - 1$ in clockwise[1].

To discover its neighbors, each node $i \in \mathcal{S}$ lets its antenna scan the communication range which is a disk around itself. As analysed in Section II, any probabilistic antenna scan strategy cannot achieve bounded discovery delay and suffers from the long-tail discovery latency problem in which two neighbor nodes a and b within the communication range of each other may experience extremely long delay before they can discover each other. Motivated by this observation, we consider deterministic neighbor discovery algorithms in which each node switches its antenna in each slot based on a specific pattern so as to discover its neighbors. We term such antenna pattern the *antenna scan pattern*, or *antenna scan sequence* and give its formal definition in the following.

[1]To make our analysis concise, we assume that N_i is an integer. The generation to non-integer N_i is trivial by letting the last sector be partially overlapped with its neighbor sectors.

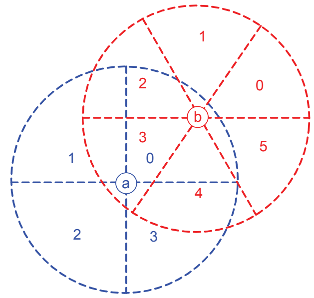

Fig. 1: Example of antenna configuration: the dotted blue and red circles represent the communication range of node a and b; a has $N_a = 4$ antenna sectors, b has $N_b = 6$ sectors; a is situated in sector $h_b = 3$ of b, b is situated in sector $h_a = 0$ of a.

Definition 1 (Antenna Scan Sequence)**.** *The antenna scan sequence is defined as a sequence* $\mathbf{u} \triangleq \{u_t\}_{0 \leq t \leq T_u - 1}$ *where* u_t *is the index of sector at which the antenna is steered,* T_u *is the period of the sequence*[2].

Now consider a pair of neighbor nodes a and b and assume that a is situated in the sector $h_b \in [0, N_b - 1]$ of b and b is situated in the sector $h_a \in [0, N_a - 1]$ of a, they can discover each other if and only if they steer their antennas towards each other. Formally, let $\mathbf{u}$ and $\mathbf{v}$ denote the antenna scan sequences of a and b, with periods T_a and T_b, if there exists $t \in [0, T_aT_b - 1]$ such that $u_t = h_a$ and $v_t = h_b$, a and b can discover each other in slot t. Figure 1 and Example 1 further illustrate the above definition.

Example 1. *Consider the setting of Figure 1 with the following two scenarios:*

- *Scenario 1:* $\mathbf{u} = \{0, 1, 2, 3\}$ *with* $T_a = 4$ *and* $\mathbf{v} = \{5, 4, 3, 2, 1, 0\}$ *with* $T_b = 6$*, i.e., a lets its antenna scan counter-clockwise while b lets its antenna scan clockwise;*
- *Scenario 2:* $\mathbf{u} = \{0, 1, 2, 3\}$ *with* $T_a = 4$ *and* $\mathbf{v} = \{0, 1, 2, 3, 4, 5\}$ *with* $T_b = 6$*, i.e., both of them let their antenna scan counter-clockwise.*

If b is situated in sector 0 of a ($h_a = 0$) and a in sector 3 of b ($h_b = 3$), it can be checked that they can discover each other in slot 8 in Scenario 1, while they cannot discover each other in Scenario 2. The antenna scan sequences and the discovery process are illustrated in Figure 2.

Fig. 2: Example of antenna scan sequences: upper: Scenario 1; lower: Scenario 2.

To model the situation where nodes are not synchronised, we apply the concept of *cyclic rotation* to antenna scan sequences. Specifically, given an antenna scan sequence $\mathbf{w}$, we

[2]A probabilistic neighbor discovery strategy can be regarded as a special case where $T_u \to \infty$.

denote $\mathbf{w}(k)$ a cyclic rotation of $\mathbf{w}$ by k where k is referred to as the *cyclic rotation phase*. Consider an example where $\mathbf{u} = \{0, 1, 2, 3\}$ with $T_u = 4$, we have $\mathbf{u}(2) = \{2, 3, 0, 1\}$. The situation where k is fractional, corresponding to the case where time slots of different nodes are not aligned, is analysed in Section V-D.

B. Problem Formulation

From Example 1, we can see that the antenna scan sequences should be carefully devised to guarantee discovery between any pair of neighbor nodes. To evaluate the performance of a neighbor discovery protocol, we introduce the following two performance metrics:

- *Discovery oblivity.* The first metric, specific for the problem of neighbor discovery with directional antennas, is the discovery oblivity, which characterizes the capability of a neighbor discovery protocol of discovering neighbors regardless of their antenna beamwidth, relative positions and clock drift. A neighbor discovery protocol is oblivious if it can guarantee discovery between any pair of neighbors a and b for any combination $(N_a, N_b) \in \mathbb{N}^2$, $(h_a, h_b) \in [0, N_a - 1] \times [0, N_b - 1]$, and any initial clock offset combination $(t_a^0, t_b^0) \in [0, T_a - 1] \times [0, T_b - 1]$.
- *Worst-case discovery delay.* Given two nodes a and b, the worst-case discovery delay between them is defined as the upper-bound of the latency (in number of slots) before successful discovery for all possible clock drfits.

We emphasize that discovery oblivity is particularly important in neighbor discovery with directional antennas due to the following two reasons.

- The antennas of nodes may have heterogeneous beamwidth (i.e., arbitrary N_a and N_b);
- The relative positions of nodes are usually not known beforehand and may change if nodes are mobile (i.e., h_a and h_b are arbitrary and not known).

Armed with the above definitions and related mathematic notations introduced in this section, we can formulate the oblivious neighbor discovery problem.

Problem 1 (Oblivious neighbor discovery problem)**.** *The oblivious neighbor discovery problem is defined as follows:*

$$\begin{aligned} &\textit{minimise} && T, \\ &\textit{subject to} && \forall t_a^0 \in [0, T_a - 1], t_b^0 \in [0, T_b - 1], \forall N_a, N_b \in \mathbb{N}, \\ &&& \textit{and } \forall h_a \in [0, N_a - 1], h_b \in [0, N_b - 1], \\ &&& \exists t \le T \textit{ such that } u_t(t_a^0) = h_a, v_t(t_b^0) = h_b. \end{aligned}$$

That is, devising antenna scan sequences to minimize T, the worst-case discovery delay, while guaranteeing discovery between any pair of neighbor nodes a and b for any combination of (N_a, N_b), any combination of $(h_a, h_b) \in [0, N_a - 1] \times [0, N_b - 1]$, and any combination of (t_a^0, t_b^0).

IV. Theoretical Performance Bound

In this section, we establish the worst-case neighbor discovery delay bound for any oblivious neighbor discovery protocol. We also analyse the structure of the antenna scan sequence to guarantee oblivious discovery between any pair of nodes a and b. The results derived in this section serve as design guidelines for the oblivious neighbor discovery protocol devised later in Section V that approaches the performance bound.

We start by showing a structural property of the antenna scan sequence of any oblivious neighbor discovery protocol.

Lemma 1. *If two nodes a and b can achieve oblivious discovery with the worst-case discovery delay D by using the antenna scan sequences $\mathbf{u}$ and $\mathbf{v}$, then for any combination of cyclic rotation phases (t_a^0, t_b^0) and any combination (h_a, h_b) where $0 \le h_a \le N_a - 1$ and $0 \le h_b \le N_b - 1$, there exists $t < D$ such that $u_t(t_a^0) = h_a$ and $v_t(t_b^0) = h_b$.*

Proof: We prove the lemma by contradiction. Assume that there exits a combination (h_a^0, h_b^0) such that there does not exist $t < D$ such that $u_t(t_a^0) = h_a^0$ and $v_t(t_b^0) = h_b^0$. Then consider the case where a is situated in the sector h_b^0 of b and b is situated in the sector h_a^0 of a, it can be noted that a and b cannot discover each other within D slots in this case, which contradicts the condition that they can achieve oblivious discovery within D slots.

Remark. *Lemma 1 shows that given any cyclic rotation phases t_a^0 and t_b^0, to ensure discovery within D slots, the pair $(u_t(t_a^0), v_t(t_b^0))$ $(0 \le t < D)$ must cover all combinations of (h_a, h_b) where $0 \le h_a \le N_a - 1$ and $0 \le h_b \le N_b - 1$, i.e., all combinations in $[0, N_a - 1] \times [0, N_b - 1]$.*

We then investigate the period of the antenna scan sequences of any oblivious neighbor discovery protocol. For any node a whose antenna scan sequence is denoted by $\mathbf{u}$, the sequence period T_u is a function of N_a. In the following theorem, we prove that $T_u \ge N_a^2$.

Theorem 1 (Lower-bound of T_u)**.** *For any oblivious neighbor discovery protocol, it holds that $T_u \ge N_a^2$ for each node a.*

Proof: Assume, by contradiction, that $T_u < N_a^2$. We prove the theorem by considering a symmetrical setting between a pair of neighbor nodes a and b where $N_b = N_a$, which leads to $T_v = T_u$ where T_v is the period of the antenna scan sequence of b, denoted by $\mathbf{v}$.

Let $n_{u,g}$ $(n_{v,h})$ denote the number of slots in sequence $\mathbf{u}$ $(\mathbf{v})$ in which a (b) points its antenna in direction $g \in [0, N_a - 1]$ $(h \in [0, N_b - 1])$. Recall that $N_b = N_a$, we can express the period length of $\mathbf{u}$ and $\mathbf{v}$ as follows:

$$\begin{aligned} T_u = T_v &= \sum_{h=0}^{N_a-1} n_{u,g} = \sum_{g=0}^{N_b-1} n_{v,h} \\ &= \sum_{h=0}^{N_b-1} \sum_{g=0}^{N_a-1} \frac{n_{u,g} + n_{v,h}}{2N_a}. \end{aligned} \tag{1}$$

Moreover, since a and b use the antenna scan sequences of the same period T_u, the discovery must occur within T_u slots under an oblivious neighbor discovery protocol.

We now fix $\mathbf{u}$ and cyclically rotate $\mathbf{v}$ by k slots from $k = 0$ to $T_u - 1$. Recall Lemma 1, for any k, there exists at least one slot t such that $u_t = g$ and $v_t(k) = h$ for any pair of $(g, h) \in [0, N_a - 1] \times [0, N_b - 1]$. It follows that the total accumulated number of slots in which $u_t = g$ and $v_t(k) = h$, as k is incremented from 0 to $T_u - 1$, is at least T_u. On the other hand, we can count the total accumulated number of slots in which $u_t = g$ and $v_t(k) = h$, as k is incremented from 0 to $T_u - 1$, as $n_{u,g} \cdot n_{v,h}$. Hence it holds that

$$n_{u,g} n_{v,h} \geq T_u.$$

Recall (1), we have

$$\begin{aligned} T_u &= \sum_{h=0}^{N_b-1} \sum_{g=0}^{N_a-1} \frac{n_{u,g} + n_{v,h}}{2N_a} \\ &\geq \sum_{h=0}^{N_b-1} \sum_{g=0}^{N_a-1} \frac{\sqrt{n_{u,g} n_{v,h}}}{N_a} \geq N_a \sqrt{T_u}. \end{aligned}$$

It then follows that $T_u \geq N_a^2$, which contradicts with the assumption $T_u < N_a^2$ and completes the proof.

We next establish the worst-case discovery delay bound for any oblivious neighbor discovery protocol.

Theorem 2 (Worst-case Discovery Delay Bound)**.** *For any oblivious neighbor discovery protocol, the worst-case discovery delay between any pair of neighbor nodes a and b cannot be lower than $N_a N_b$.*

Proof: We prove the theorem by contradiction. Assume that there exist a pair of antenna scan sequences, $\mathbf{u}$ for a and $\mathbf{v}$ for b, with which the worst-case discovery delay is less than $N_a N_b$. It follows from Lemma 1 that for any cyclic rotation phases t_a^0 and t_b^0 and any combination $(h_a, h_b) \in [0, N_a - 1] \times [0, N_b - 1]$, there exists $t < D$ such that $u_t(t_a^0) = h_a$ and $v_t(t_b^0) = h_b$. That is, the pair $(u_t(t_a^0), v_t(t_b^0))$ $(0 \leq t < D)$ must cover all the possible combinations (h_a, h_b), which is impossible with $D < N_a N_b$.

Theorem 2 derives the performance limit of any oblivious neighbor discovery protocol. We can further generalise Theorem 2 on the pair-wise neighbor discovery to the network-wise neighbor discovery, as stated in the following corollary.

Corollary 1. *For any network where the largest two antenna sector numbers of neighbor nodes are N_1 and N_2, the worst-case discovery delay for any pair of neighbor nodes in the network to discover each other, denoted by D_n, is lower-bounded by $N_1 N_2$ for any oblivious neighbor discovery protocol. Asymptotically, when $N_1 \simeq N_2 \simeq O(N)$, $D_n \simeq O(N^2)$.*

We conclude this section by summarising the derived results on any oblivious neighbor discovery protocol:

- *Discovery delay bound (Theorem 2)*: The worst-case discovery delay is lower-bounded by $N_a N_b$, or $O(N^2)$ if $N_a \simeq N_b \simeq N$;
- *Antenna scan sequence structure (Lemma 1, Theorem 1)*: Given two antenna scan sequences $\mathbf{u}$ and $\mathbf{v}$, for any cyclic rotation phases t_a^0 and t_b^0, the pair $(u_t(t_a^0), v_t(t_b^0))$ $(0 \leq t < D)$ must cover all combinations in $[0, N_a - 1] \times [0, N_b - 1]$; The period of the antenna scan sequence of node i cannot be shorter than N_i^2.

V. An Oblivious Neighbor Discovery Protocol with Directional Antennas

In this section, we devise an oblivious neighbor discovery protocol with directional antennas, which (1) achieves oblivious discovery between any pair of neighbors a and b, (2) approaches the performance bound derived in Section IV without any prior knowledge or coordination.

Our design is composed of two steps. In the first step, each node independently constructs a binary sequence such that the sequences of any two distinct nodes are cyclic rotationally distinct to each other. In the second step, each node generates its antenna scan sequence based on the sequence constructed in the first step.

A. Constructing Cyclic Rotationally Distinct Sequence

In our approach, the antenna scan sequence for each node is constructed based on its globally unique ID (e.g., address), which can be mathematically expressed as a binary sequence of length l. Using globally unique IDs is a typical method to break the symmetry of any pair of nodes.

In the first step, each node independently generates a binary sequence based on its ID such that the binary sequences of any two nodes are *cyclic rotationally distinct* one to the other. Note that the sequences resulting from cyclic rotations of a sequence are not considered to be cyclic rotationally distinct to each other and the original sequence. We term the sequences generated from the ID sequences the *extended ID sequences*.

A simple way of constructing cyclic rotationally distinct extended ID sequences has been proposed in [19] as summarised as follows: let $\mathbf{i}$ denote the ID of node i, which is an l-bit binary sequence; let $\mathbf{1}(k)$ $(\mathbf{0}(k))$ denote a binary sequence of 1 (0) of length k; construct the following binary sequence $\mathbf{I} \triangleq \mathbf{i}||\mathbf{1}(l)||\mathbf{0}(l)$. It is proved in [19] that sequences constructed in this way are cyclic rotationally distinct to each other.

We now generalise the above algorithm to the following algorithm that constructs cyclic rotationally distinct sequences. The reason of developing a new algorithm is two-fold.

- The establishment of bounded discovery delay in our problem requires the length of the extended ID sequence to be odd, (cf. proof of Theorem 3) which cannot be achieved by the algorithm in [19];
- The length of the extended ID sequence generated by our algorithm is shorter than that in [19], which leads to shorter discovery delay.

Lemma 2. *Given any two extended ID sequences $\mathbf{a}$ and $\mathbf{b}$ generated from two ID sequences α and β as follows:*

$$\mathbf{a} \triangleq \mathbf{0}(l_1)||\alpha||\mathbf{1}(l_2) \text{ and } \mathbf{b} \triangleq \mathbf{0}(l_1)||\beta||\mathbf{1}(l_2),$$

under the condition that $l_1 + l_2 \geq l + 1$, it holds that $\mathbf{a}$ and $\mathbf{b}$ are cyclic rotationally distinct to each other, i.e., it holds that

$$\alpha \neq \beta \quad \Longrightarrow \quad \mathbf{a} \neq \mathbf{b}(k), \ \forall k \in [0, l + l_1 + l_2),$$

where $\mathbf{b}(k)$ is $\mathbf{b}$ with a cyclic rotation of k bits.

Proof: We prove the lemma by considering the three possible scenarios illustrated in Figure 3, and showing, in each scenario, that a bit in $\mathbf{a}$ and another bit in $\mathbf{b}(k)$ have different values although the two bits are in the same position within the respective extended ID sequences. This is sufficient to prove that the two extended ID sequences $\mathbf{a}$ and $\mathbf{b}$ are cyclic rotationally distinct one to the other.

Case 1: $k \in (0, l_1)$. As indicated by the arrow in Figure 3, it holds that $a_{L-1} = 1$ and $b_{L-1}(k) = 0$ where $L = l + l_1 + l_2$.

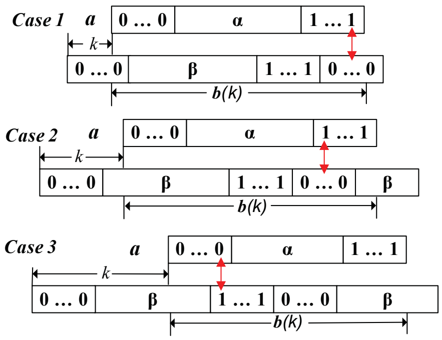

Fig. 3: Illustration of the three cases in the proof of Lemma 2.

Case 2: $k \in [l_1, l + l_2]$. Note that $l_1 + l_2 \geq l + 1$, as indicated by the arrow in Figure 3, it holds that $a_{l+l_1} = 1$ and $b_{l+l_1}(k) = 0$.

Case 3: $k \in (l + l_2, l + l_1 + l_2)$. As indicated by the arrow in Figure 3, it holds that $a_{l_1-1} = 0$ and $b_{l_1-1}(k) = 1$.

Noticing that $\alpha \neq \beta \implies \mathbf{a} \neq \mathbf{b}$, we thus conclude that $\mathbf{a} \neq \mathbf{b}(k),\ \forall k \in [0, l + l_1 + l_2)$.

In our design, we set l_1 and l_2 such that the total length of the extended ID sequence $L \triangleq l + l_1 + l_2$ is odd, e.g., $l_1 + l_2 = l + 1$, leading to $L = 2l + 1$.

B. Constructing Antenna Scan Sequence

In the second step, each node i constructs its antenna scan sequence $\mathbf{u}$ based on the extended ID sequence, denoted as $\mathbf{e}^i$, generated in the first step by choosing l_1 and l_2 such that the resulting sequence length $L = l+l_1+l_2$ is odd. Specifically, let p_i denote the smallest odd prime number not smaller than N_i and co-prime to L; let b_i denote the smallest integer satisfying $2^{b_i} \geq N_i$ and set $q_i = 2^{b_i}$; the antenna scan sequence of node i, $\mathbf{u}$, is constructed as follows:

$$u_t = \begin{cases} t \bmod p_i & e^i_t = 0 \text{ and } t \bmod p_i < N_i, \\ t \bmod q_i & e^i_t = 1 \text{ and } t \bmod q_i < N_i, \\ \text{rand}(N_i - 1) & \text{otherwise,} \end{cases} \quad (2)$$

where $\text{rand}(N_i - 1)$ denotes a random integer in $[0, N_i - 1]$. It can be noted that the period of the antenna scan sequence $\mathbf{u}$ is Lp_iq_i without taking into account the random part. Figure 4 provides an example of the antenna scan sequences for two nodes a and b and their discovery process.

C. Discovery Delay Analysis

In the following theorem, we prove the correctness of our protocol in achieving oblivious discovery and establish the worst-case discovery delay bound.

Theorem 3 (Correctness and Worst-case Discovery Delay Bound)**.** *Our neighbor discovery protocol can ensure oblivious discovery between any pair of neighbors a and b. The worst-case discovery delay between them is upper-bounded by $L \max\{p_aq_b, p_bq_a\}$, asymptotically $O(N_aN_b)$.*

Proof: Given any system parameter combination (t^0_a, t^0_b), (N_a, N_b) and (h_a, h_b), by Lemma 2, there exit $0 \leq l_0 < L$ such that $e^a_{l_0}(t^0_a) \neq e^b_{l_0}(t^0_b)$. Without loss of generality, assume that $e^a_{l_0}(t^0_a) = 0$ while $e^b_{l_0}(t^0_b) = 1$.

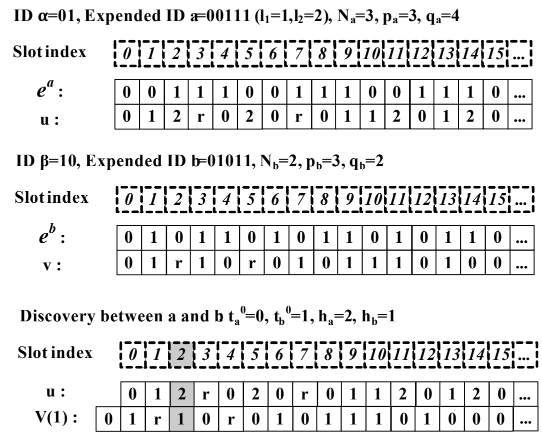

Fig. 4: Example of antenna scan sequences for node a and b (r denotes a randomly number in $[0, N_i - 1]$): they can discover each other in slot 2.

Since p_a is an odd prime and q_b is a power-multiple of 2, it holds that p_a is co-prime with q_b. Let $\mathbf{u}$ and $\mathbf{v}$ denote the antenna scan sequences of a and b. We examine the slots $t_k = l_0 + kL$ where $k \in \mathbb{N}$. More specifically, we consider the subsequences of $\mathbf{u}(t^0_a)$ and $\mathbf{v}(t^0_b)$ in these slots, i.e., $\{u_{t_k}(t^0_a)\}$ and $\{v_{t_k}(t^0_b)\}$. Recall (2), we can write $u_{t_k}(t^0_a)$ and $v_{t_k}(t^0_b)$ as follows:

$$\begin{cases} u_{t_k}(t^0_a) = t^0_a + l_0 + kL \bmod p_a, \\ v_{t_k}(t^0_b) = t^0_b + l_0 + kL \bmod q_b. \end{cases}$$

Recall that (1) L is odd, (2) p_a is an odd prime and co-prime to L, (3) q_b is a power-multiple of 2, it holds that L, p_a and q_b are co-prime one to another. It then follows from the Chinese Remainder Theorem [20] that for any parameter settings (t^0_a, t^0_b), (N_a, N_b) and (h_a, h_b), there exists $k_0 < p_aq_b$ such that

$$\begin{cases} k_0L \bmod p_a = h_a - t^0_a - l_0 \bmod p_a, \\ k_0L \bmod q_b = h_b - t^0_b - l_0 \bmod q_b. \end{cases}$$

It then follows that

$$\begin{cases} u_{t_k}(t^0_a) = t^0_a + l_0 + kL \bmod p_a = h_a, \\ v_{t_k}(t^0_b) = t^0_b + l_0 + kL \bmod q_b = h_b. \end{cases}$$

Hence, a and b can discover each other in slot t_{k_0} with the worst-case discovery delay bounded by Lp_aq_b.

Similarly, when $e^a_{l_0}(t^0_a) = 1$ while $e^b_{l_0}(t^0_b) = 0$, we can prove that the worst-case discovery delay is upper-bounded by Lq_ap_b. Therefore, it holds that the worst-case discovery delay of our protocol is upper-bounded by $L \max\{p_aq_b, q_ap_b\}$. In the asymptotical case, we have $p_a \simeq q_a \simeq N_a$ and $p_b \simeq q_b \simeq N_b$ and hence the delay upper-bound is $O(N_aN_b)$.

We end this subsection with the following two remarks:

- *Tightness of worst-case discovery delay.* Theorem 3 establishes the worst-case discovery delay bound as $L \max\{p_aq_b, p_bq_a\}$. We illustrate via an example in Figure 5 that this bound is actually very tight. In the example where the initial clock drift is $t^0_a = 10$ and $t^0_b = 0$, a and b discover each other only at slot 58, which corresponds to the discovery delay of 59 slots. The worst-case discovery delay bound derived by Theorem 3 in this example is 60.
- *Upper-bound of average discovery delay.* We can derive

the upper-bound of the average discovery delay by using the same technique as the proof of Theorem 3. Specifically, using the same notation, given a random pair of t_a^0 and t_b^0, the expectation of k_0 is bounded by $\frac{p_a q_b - 1}{2}$. Assume that node IDs can be regarded as random binary sequences, the expectation of l_0 is bounded by $\frac{L}{2}$. The average discovery delay is thus upper-bounded by $\frac{L p_a q_b}{2}$, and asymptotically when $p_i \simeq q_i \simeq N_i \simeq N$, it can be bounded by $\frac{LN^2}{2}$. Note that this is a very conservative and thus loose bound, as illustrated in the simulations.

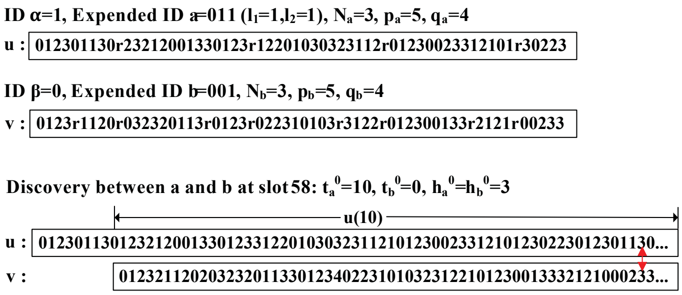

Fig. 5: Tightness of worst-case discovery delay.

D. Discovery Analysis with Non-aligned Slots

Our previous results implicitly assume slots are aligned. In this subsection, we relax this assumption to study the situation where slots are non-aligned. In this context to ensure discovery, it is required that a neighbor discovery protocol should be able to ensure that any pair of neighbor nodes can discover each other with an overlap of α slot where $\alpha \in (0, 1]$ is a system-dependent parameter[3]. A typical condition widely imposed in the literature is to require a discovery to last at least half of the slot duration, i.e., $\alpha = 0.5$.

We next demonstrate that our protocol can achieve the above practical objective. To show this, consider two nodes a and b whose extended ID sequences are denoted as e^a and e^b. Given any parameter setting (t_a^0, t_b^0), (N_a, N_b) and (h_a, h_b) with non-aligned slots, it holds that either $u_t(t_0^a)$ and $v_t(t_0^b)$ overlap for at least half slot duration for any $t \geq 0$ or $u_t(t_0^a + 1)$ and $v_t(t_0^b)$ overlap for at least half slot duration for any $t \geq 0$. We thus investigate these two cases:

- *Case 1:* $u_t(t_0^a)$ and $v_t(t_0^b)$ overlap for at least half slot duration for any $t \geq 0$. In this case, the previous analysis can be directly applied. The only difference is that instead of an entire overlap, a discovery in this case is a partial overlap of at least half slot duration.
- *Case 2:* $u_t(t_0^a + 1)$ and $v_t(t_0^b)$ overlap for at least half slot duration for any $t \geq 0$. In this case, since $\mathbf{u}$ and $\mathbf{v}$ are cyclic rotationally distinct to each other, we can prove in the same way as Theorem 3 that within the same delay bound, there exists t^* such that $u_{t^*}(t_0^a + 1) = h_a$ and $v_{t^*}(t_0^b) = h_b$. Hence a and b can discovery each other in slot t^* with an overlap of at least half slot duration.

[3]A practical example is that switching antenna from one direction to another incurs non-negligible delay.

Figure 6 illustrates the two cases of the neighbor discovery with non-aligned slots with the scan sequences of the example in Figure 4. As proved in this subsection as well as illustrated in Figure 6, in both cases, a and b can discover each other within the worst-case delay derived in Theorem 3 with an overlap of more than half slot.

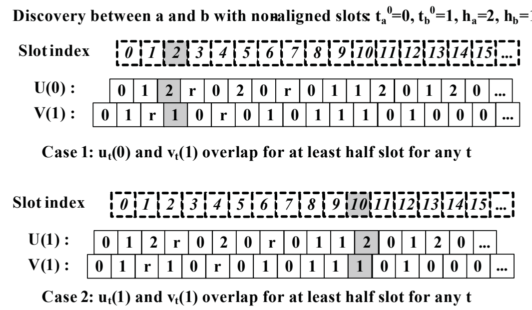

Fig. 6: Neighbor discovery with non-aligned slots.

E. Discovery Beacon Scheduling

Our theoretical analysis hinges on the fact that two neighbor nodes are able to discover each other once they steer their antennas to each other at the same slot for at least half of a slot during which they exchange discovery beacons. This assumption is also largely made in the literature. In this subsection, we design discovery beacon scheduling to achieve discovery once an overlap of at least half slot occurs. By overlap, we mean that in the overlapping slot, a and b steer their antennas toward each other.

Before motivating and discussing our design, we present a beacon scheduling mechanism initially proposed in [3] and improved in [5]. In this approach, each node sends two beacons each active slot, one at the beginning of the slot, the other at the end. The node remains in listening mode in the intermediate period. Under the condition that the slots of two nodes are not perfectly aligned, they can receive a beacon from the other node in each overlapping active slots. To handle perfect slot alignment, the slot overflowing scheme is developed in [5], where each active slot overflows by δ, a small amount that is sufficient to receive a beacon from another node. However, their approach cannot be applied in our context as it requires that active slots are separated by inactive slots to allow slot overflow, but in our context a node remains active in each slot, making slot overflow impossible.

Motivated by the above argument, we devise the following beacon scheduling scheme.

- Consider node i in slot t, we call slot t a p-slot if $e_t^i = 0$ and $t \bmod p_i < N_i$, i.e., the condition of first line of (2) is satisfied; in the same way we define the q-slot. If the condition of the third line of (2) holds, the node randomly chooses between a p-slot and a q-slot. Recall the proof of Theorem 3, given any pair of neighbors a and b, there must exists an overlap between a p-slot of a and a q-slot of b and between a q-slot of a and a p-slot of b.
- At each p-slot, node i sends two beacons, one beacon scheduled δ_p after the beginning of the slot and the other

scheduled δ_p before the end of the slot, as illustrated in Figure 7 (upper left). The beacon schedule in the q-slots proceeds in the same way. The parameters δ_p and δ_q are set such that $\delta_p + \delta_q < \frac{1}{2}$, where we normalise slot duration to 1 to simplify notation.

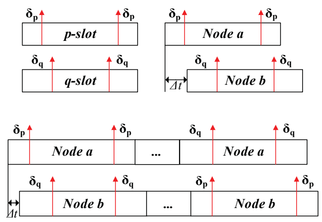

Fig. 7: Illustration of our beacon scheduling and the resulting mutual discovery.

The following theorem formally proves that the proposed beacon scheduling mechanism can guarantee mutual discovery.

Theorem 4. *Our beacon scheduling mechanism can guarantee mutual discovery.*

Proof: We prove the theorem by distinguishing two cases: (1) slots are perfectly aligned, (2) slots are not aligned. To make the notation concise, we assume each slot last unit time.

In the first case with aligned slots, recall the proof of Theorem 3, given any pair of neighbors a and b, there must exist an entire overlap between a p-slot of a and a q-slot of b. Since $\delta_p \neq \delta_q$ and $\delta_p + \delta_q < \frac{1}{2}$, each node can successfully receive two beacons from the other, as illustrated in Figure 7 (upper left).

In the second case with non-aligned slots, recall the analysis in Section V-D and the proof of Theorem 3, given any pair of neighbors a and b, there must exists an overlap of at least half slot between a p-slot of a and a q-slot of b and between a p-slot of b and a q-slot of a. Consider the overlap between a p-slot of a and a q-slot of b and assume, without loss of generality, that $\delta_p < \delta_q$. We first prove that a can receive the first q-beacon of b in the overlapping slot (cf. Figure 7 upper right). To that end, we need to show that the first q-beacon of b does not collide with the second p-beacon of a. Assume, by contradiction, that they collide. It then holds that

$$1 - \delta_p = \Delta t + \delta_q, \tag{3}$$

where Δt denotes the time offset between the overlapped slots. It follows from the analysis in Section V-D that $\Delta t \leq \frac{1}{2}$. It follows from (3) that

$$\delta_p + \delta_q = 1 - \Delta t \geq \frac{1}{2},$$

which contradicts to the setting that $\delta_p + \delta_q < \frac{1}{2}$. Hence, a can receive the first q-beacon of b without collision.

We then prove that b can receive a discovery beacon from a by distinguishing the following two subcases:

- If $\Delta t \neq \delta_q - \delta_p$, b can receive the second p-beacon of a because by applying similar analysis, we can show that the transmission time of the second p-beacon of a differs that of the second q-beacon of b (cf. Figure 7 upper right).
- If $\Delta t = \delta_q - \delta_p$, b cannot receive the second p-beacon of a because the transmission time of the second p-beacon of a coincides with that of the second q-beacon of b. In this case, we consider the overlap between a q-slot of a and a p-slot of b. It can be easily checked that the transmission time of the second q-beacon of a in this slot differs that of the second p-beacon of b (cf. Fig. 7 lower). b can thus receive the second q-beacon of a in this slot.

We have thus proved that both a and b can receive at least a beacon of each other. The mutual discovery is achieved.

The devised beacon scheduling mechanism can be further adapted in collision-prone environments when the network size is large. In this context, more than one simultaneously transmitted beacons lead to collision and thus cannot be recovered at the receiver. To limit collision, we can desynchronize p-beacons and q-beacons by adding a small random time drift to δ_p and δ_q. Note that in such context, discovery delay cannot be bounded due to collision. The utility of our neighbor discovery protocol is to ensure that any pair of neighbors will eventually steer their antennas toward each other, without which discovery can never be achieved.

VI. Trading off Worst-case and Average Discovery Delay

In the previous section, we have shown that the worst-case discovery delay of our protocol is bounded by $L \max\{p_a q_b, p_b q_a\}$ and this bound is very tight, i.e., there are extremely unlucky cases where discovery cannot be achieved before the worst-case delay. On the other hand, it is easy to see that a purely random strategy where each node points its antenna to a random direction each slot leads to an average delay of $N_a N_b$ even in such extremely unlucky cases. However, the worst-case discovery delay of any random discovery strategy cannot be bounded. Generally, random or probabilistic neighbor discovery protocols usually perform well in the average case by limiting the expected discovery delay, with the main drawback being the lack of performance guarantee in terms of worst-case discovery delay. The following question naturally arises: *How to improve the average performance of our protocol in those extremely unlucky cases while still ensuring a bounded discovery delay*.

In this section, we investigate how a desired trade-off between the worst-case and the average discovery delay can be achieved by adapting our neighbor discovery protocol, more specifically by properly choosing the parameters p_i and q_i. To make our analysis tractable, we focus on a synchronised case where $N_a = N_b = N$ and a and b choose the same parameters, i.e., $p_a = p_b$ and $q_a = q_b$. However, the idea presented via this example also holds in the general cases. Recall the antenna scan sequence in our approach (equation (2)), we note that for slots $N_i \leq t \bmod p_i$ (when $e_t^i = 0$) and $N_i \leq t \bmod q_i$ (when $e_t^i = 1$), each node i randomly points its antenna. We can configure the number of such "random slots" via p_i and q_i so as to improve the average performance while still ensuring the bounded discovery delay by the operations in the remaining "deterministic slots". Specifically, choosing larger p_i and q_i results in more "random slots", thus improving the average performance at the price of increasing the worst-case delay. By choosing proper p_i and q_i, we can trade off the worst-case and the average discovery delay.

We next provide an approximative quantitative analysis on the above trade-off. Consider the case where N is sufficiently large and $p_i \simeq q_i \simeq p$. Approximatively, within each p slots, there are $p-N$ “random slots” where player i randomly points its antenna. We call such $p-N$ “random slots” a random frame. The probability that discovery can be achieved within one random frame can be calculated as

$$q = 1 - \left(1 - \frac{1}{N^2}\right)^{p-N}.$$

Recall that the worst-case delay is bounded by approximatively Lp^2 rounds, i.e., Lp random frames, we can then calculate the upper-bound of the average discovery delay $\overline{d}$ as follows:

$$\overline{d} \leq q \cdot p + (1-q)q \cdot 2p + \cdots + (1-q)^{Lp-1} \cdot Lp^2.$$

Given a target expected delay bound $\bar{d}$, p can be chosen based on the above inequality. To get more insight, we consider the case where we set p sufficiently larger than N but linear to N, i.e., $p = (1+\lambda)N$ with sufficiently large λ. We have $q \simeq \frac{\lambda}{N}$. After some algebraic operations, the average delay is bounded by

$$\overline{d} < \sum_{k=0}^{\infty}(1-q^k)q \cdot (k+1)p = \left(1 + \frac{1}{\lambda}\right) N^2,$$

with the worst-case discovery delay being $L(1+\lambda)^2N^2$. We can thus trade off the worst-case and the average discovery delay by choosing proper λ.

VII. Numerical Analysis

In this section, we conduct a suite of simulations to illustrate the theoretical results established in previous analysis and to evaluate the performance of the developed neighbor discovery protocol in several typical application scenarios.

A. Pair-wise Neighbor Discovery

We start by simulating the baseline scenario of pair-wise discovery between a pair of neighbor nodes a and b. Specifically, we trace the discovery delay for different antenna configurations of a and b, i.e., different combinations of (N_a, N_b). The relative positions of a and b, represented by (h_a, h_b) is also randomly generated. The clock drift between a and b is randomly generated from $[0, 1000]$ slots. Both a and b have an ID of 8 bits randomly attributed to them. Throughout our simulations, each point represents the worst-case or average value of a number of independent simulation runs, with the required number of simulation runs calculated using “independent replications” [21].

Figure 8 traces the worst-case and the average discovery delay of our protocol. For comparison, we also trace the average discovery delay of the random strategy where each node steers its antenna at a random direction each slot. We cannot trace the worst-case delay of the random strategy because we observe that in some cases, discovery cannot be achieved within the simulation duration which is set to 10^5 slots. From the results, we make the following observations: (1) For given N_a, the worst-case delay increases linearly w.r.t. N_b, which is in accordance of our theoretical result established in Theorem 2. (2) The worst-case and average discovery delay trade-off between the random strategy and the deterministic one as ours is clearly demonstrated. Our simulation results seem to favor a well-designed deterministic strategy as we see a limited performance loss in terms of average delay with the advantage of having strict worst-case delay bound.

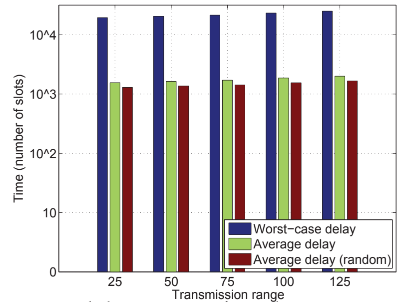

Fig. 10: Discovery delay comparison between our protocol and the random discovery strategy in a network.

We then investigate trading off worst-case and average discovery delay by incorporating the mechanism proposed in Section VI. To that end, we pick settings where a and b can only discover each other almost with the worst-case discovery delay. We implement the mechanism proposed in Section VI and trace the resulting trade-off between the worst-case and average discovery delay in Figure 9. Specifically, we implement two settings: (1) Small p and q, in this setting, p and q are chosen as the smallest eligible values larger than $2N$; (2) Large p and q, in this setting, p and q are chosen as the smallest eligible values larger than $10N$. The simulation results clearly demonstrate the trade-off between worst-case and average discovery delay: with larger p and q, the worst-case discovery delay is more important, while the average delay is less. The trade-off can thus be parameterised to satisfy specific application requirement by tuning p and q.

B. Network-wide Neighbor Discovery

We further complete our simulation study by investigating a more complex scenario of a randomly deployed wireless network. To that end, we simulate in a network with 100 nodes randomly deployed in a $200m \times 200m$ square. We vary the transmission range of nodes from $25m$ to $125m$ such that the average number of neighbors of a node varies from around 3 to more than 50, which we believe can cover a wide range of practical scenarios. For each node i, its antenna parameter N_i is randomly chosen from $\{6, 12, 18, 24, 30, 36\}$. Other parameters are the same as previous simulations. We use the standard beacon format as in the literature [5], [6] and our beacon scheduling mechanism with $\delta_p = 0.1$ and $\delta_q = 0.2$.

We trace the worst-case and the average discovery delay of our protocol and the average discovery delay of the random strategy. Again, we observe that in some cases, discovery cannot be achieved under random strategy within the simulation duration which is set to 10^5 slots. As illustrated in Figure 10, the worst-case discovery delay of our protocol is bounded and only increases slightly w.r.t. the number of neighbors. This result reflects the fact that in the simulated cases, collisions among beacons only have limited impact on the discovery performance. This is because beacons are very short, especially compared to normal data packets, thus

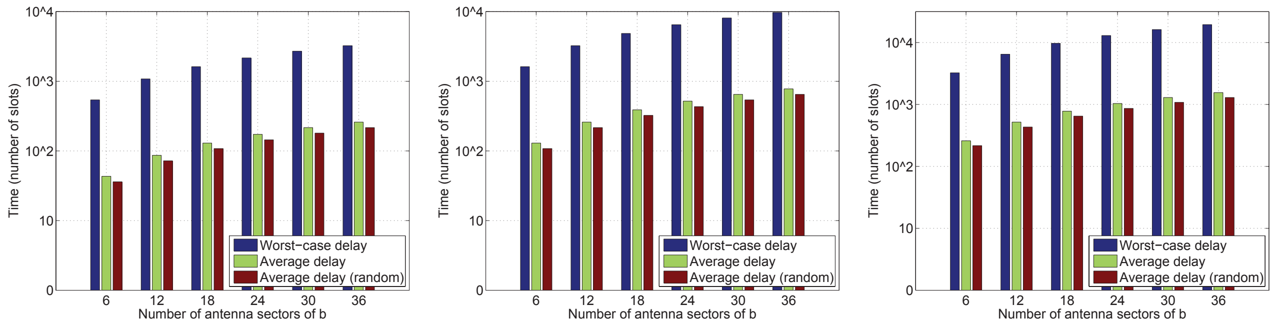

Fig. 8: Discovery delay comparison between our protocol and the random discovery strategy under fixed N_a and varying N_b: left $N_a = 6$, middle $N_a = 18$, right $N_a = 36$.

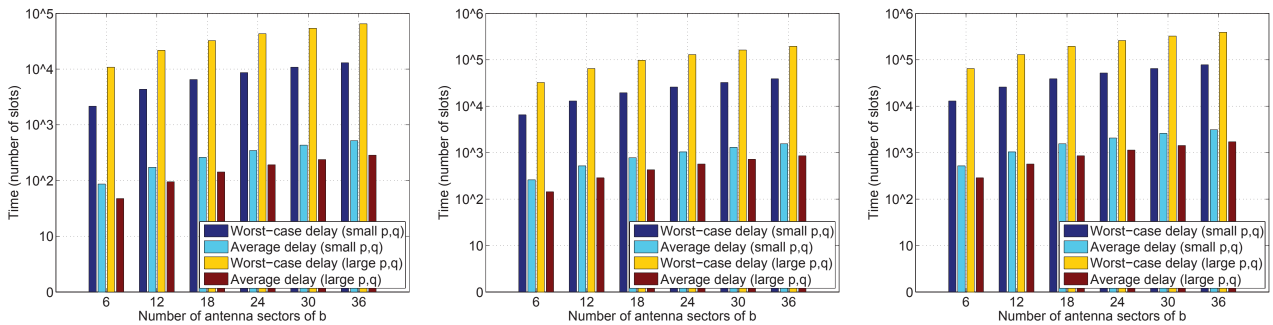

Fig. 9: Trading off worst-case and average discovery delay under fixed N_a and varying N_b: left $N_a = 6$, middle $N_a = 18$, right $N_a = 36$.

limiting the collision probability, the fact that the antennas are directional also limits the collision probability. Consequently, the probability of having consecutive collisions on the discovering slots is even more rare. In terms of average discovery delay, we observe that our protocol is only slightly outperformed by the random strategy.

VIII. Conclusion

We have formulated and studied the oblivious neighbor discovery problem. We have established the performance bound of any neighbor discovery protocol achieving oblivious discovery. Guided by the theoretical results, we have designed an oblivious discovery protocol and proved that it achieves guaranteed oblivious discovery with order-minimal worst-case discovery delay in the asynchronous and heterogeneous environment. In the future research, we plan to investigate the energy-constraint case where nodes stay in the dormant state most of the time while only wakes up periodically.

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