Efficient Medium Access Control Design – A Game Theoretical Approach

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***Abstract*—In this paper, we address the question that how to design efficient MAC protocols in selfish and non-cooperative networks, which is crucial in nowadays open environments. We model the medium access control problem as a non-cooperative game in which the MAC protocol can be regarded as distributed strategy update scheme approaching the equilibrium point. Under such game theoretical framework, three MAC protocols, the aggressive, conservative and cheat-proof MAC protocol, are then proposed with tunable parameters allowing them to converge to desired social optimal point. The first two MAC protocols require network participants to follow the rules, while the cheat-proof MAC protocol can survive the selfish environments where nodes are pure self-interested.**

Based on our game theoretical analysis, we provide a general methodology for designing efficient MAC protocols in non-cooperative and selfish environments. We believe that the proposed methodology not only provides a general way of designing stable and controllable MAC protocols achieving high performance even in selfish environments, but also provides a general framework that can be extended to design efficient protocols in other non-cooperative and selfish environments.

I. Introduction

Medium Access Control (MAC) is crucial for networks where the communication medium is shared by network participants competing for the channel access. Designing efficient MAC protocols is a challenging task, especially in wireless environments where channel sensing is much less effective than in wired medium. IEEE 802.11 DCF (Distributed Coordination Function), the most popular MAC protocol for WLANs, uses the exponential backoff (EB) mechanism where each node doubles its contention window (CW) upon a collision until CW_{max} and sets it to the basic value CW_{min} upon a successful transmission. This MAC protocol results in too many collisions and thus leads to network sub-optimality when the network scales. Moreover, it has short-term fairness problem due to the EB mechanism applied after the collision.

This motivates us to address the fundamental question that how to design efficient MAC protocols, what are the methodology and guidelines to follow? An efficient MAC protocol should satisfy the following properties: 1) *Convergence and stability*: the protocol should converge to a stable equilibrium; 2) *Social optimality and fairness*: the converged equilibrium should be network-wide optimal or at least quasi-optimal and each participant should get a fair share of payoff at this point.

Besides the above requirements on performance, we pose another requirement: the *survivability* of the MAC protocol in selfish environments. Nowadays, networks become more and more open. Hence network participants may behave selfishly rather than cooperatively, i.e., they adapt the strategy that maximizes their own utility, regardless of others. Thus we can not implicitly assume all participants act cooperatively by following the designed protocols. Under such circumstance, we require an efficient MAC protocol to be survivable such that it can guide the individual nodes to operate on the designed equilibrium point even they are purely self-interested and non-cooperative. In other words, the MAC protocol consists of a strategy that each selfish individual node has no incentive to deviate.

We conduct our work using game theory, a powerful tool to study the interaction among decision makers with conflicting objectives. Our motivation of using game theoretical approach rather than global optimization approach is two-fold: 1) Game theory is a powerful tool to model selfish behaviors and their impact on the system performance in distributed environments with self-interested players; 2) Game theory can model the features or constraints such as lack of coordination and network feedback in distributed environments. In fact in such environments, selfish behavior is often much more robust and scalable than any centralized cooperative control, which is very expensive or even impossible to implement.

We begin by modelling the medium access control problem as a non-cooperative game G_{MAC}, where each player chooses its strategy, the channel access probability, to maximize its utility function, defined as the difference between its throughput and transmission cost. In such non-cooperatives games, a Nash equilibrium (NE) is the strategy profile where no player has incentive to deviate unilaterally. The MAC protocol can be viewed as the distributed strategy update mechanism approaching the NE.

We then conduct an in-depth study on the NE of G_{MAC}. We find that there always exists a biased NE and that under certain condition, there also exists a unique non-biased NE, in which we are specially interested since at the non-biased NE, the fairness is ensured among players. However, by studying the network-wide utility on the non-biased NE and the social optimal point, we find that the non-biased NE is inefficient. We then propose two pricing schemes to improve the efficiency of the non-biased NE. We show that by wisely choosing the pricing factors, the non-biased NE can be approached to the social optimal point. We then seek the MAC protocol that can lead the network to converge to the social optimal point. However, the convergence is not guaranteed under the best response and subgradient control, the two basic dynamic control mechanisms in game theory.

We thus turn to more sophisticated control mechanisms under which the game is provably convergent to the social optimal point. We propose three MAC protocols: aggressive, conservative and cheat-proof MAC protocol. We show that the proposed MAC protocols have the following desirable properties: 1) They consist of nature access probability update schemes for rational players; 2) They provide tunable parameters with which one can control the convergent point of the network; 3) They lead the network to a stable state where both the convergence and the social optimality are ensured. The aggressive and the conservative MAC protocol require network participants to respect their rules. In contrast, the cheat-proof MAC protocol with the correspondent pricing function based on observable information is designed such that the channel access probability update scheme is the best response for the rational selfish players when maximizing their utility. Hence no player has incentive to deviate from the cheat-proof MAC protocol. As a result, the cheat-proof MAC protocol can survive the selfish environments where players are purely self-interested and may break any protocol rules if they can get more payoff than obeying the rules.

Based on our game theoretical analysis, we answer the posed question by providing the following methodology in designing efficient MAC protocols in non-cooperative and selfish environments: 1) choosing a natural channel access update scheme; 2) configuring the parameters in the chosen update scheme to ensure that the network converges to the designed optimal equilibrium under the update scheme; 3) deriving appropriate pricing functions based on observable information such that no player has incentive to deviate from the MAC protocol.

Our main contributions can be summarized as follows:

- We formulate the medium access control as a non-cooperative game and perform an in-depth analysis on the game, including the existence, uniqueness, convergence and efficiency of the NE;
- Three MAC protocols, the aggressive, conservative and cheat-proof MAC protocol, are proposed with tunable parameters allowing them to converge to desired social optimal point. The first two MAC protocols require network participants to follow the rules, while the cheat-proof MAC protocol can survive the non-cooperative and selfish environments.
- Based on our game theoretical analysis, we provide a general methodology for designing efficient MAC protocols in non-cooperative and selfish environments.

We believe these contributions are very relevant for the medium access control design achieving high efficiency and survivability in selfish and non-cooperative environments.

II. Related Work

Game theory has been employed widely to study the non-cooperative behaviors at MAC layer. [6] studies the non-cooperative equilibria of Aloha for heterogeneous users. [5] studies the stability of multi-packet slotted Aloha with selfish users and perfect information. [7] shows that the 802.11 MAC protocol leads to inefficient equilibria if users configure their packet size and data rate to maximize their own throughput. [8] shows that the existence of small population of selfish nodes leads to network collapse. The authors thus propose a penalizing scheme to prevent the network from being paralyzed. [2] reverse-engineers binary exponential backoff algorithm in game theory framework.

In the field of MAC protocol design, much recent work [10] [11] applies the network utility maximization (NUM) framework by viewing the network as an optimization solver and the MAC protocols as distributed algorithms solving some global network utility maximization problem. Our work, however, is based on a game theoretical framework under which the medium access control problem is modeled as a non-cooperative game and the MAC protocol is regarded as the distributed strategy update scheme approaching the equilibrium. We argue that our work is more suited in selfish non-cooperative environments such as nowadays open accessed networks where participants are purely self-interested. [1] also studies the MAC design from a game theoretical angle. It is focused on modelling a large class of system-wide quality of services models via utility functions and deriving distributed contention resolution algorithm based on continuous feedback signal rather than binary contention signal to approach the NE (may not be social optimal). In contrast, our work focuses on providing a methodology on how to design efficient MAC protocols in selfish environments that can guide the network to a stable equilibrium which is network-wide optimal and at which each participant gets a fair share of payoff.

III. System Model

We consider a LAN consisting of a set $\mathcal{N} = \{1, 2, \cdots, n\}$ of nodes sharing the common medium. We base our study on a general and basic MAC layer model: Time is divided into synchronized slots. Each node can send one packet in a slot. If a node i has a new packet to send, it attempts transmission during the next slot with probability p_i called channel access probability. The channel access probability p_i can be realized via contention window in the case where a backoff mechanism is implemented such as CSMA. In the above simple model, we do not assume any collision avoidance or detection mechanism although such mechanisms may facilitate the MAC protocol design. Built on a basic MAC layer model without any added functionalities, our proposed MAC protocols can be implemented in almost all nowadays network systems, from slotted Aloha to CSMA.

IV. Non-cooperative Medium Access Control Game

In game theory, the utility function describes the satisfaction level of the player as the result of their strategies. In our study, we consider a utility function as follows

$$U_i = p_i \prod_{j \in \mathcal{N}, j \neq i} (1 - p_j) - c_i p_i$$

In the above defined utility function, node i gets payoff 1 for a successful frame transmission and no payoff if the transmission fails due to a collision. On the other hand, the transmission of a frame also incurs the transmission cost c_i (c_i is normalized), e.g., in terms of energy. In this paper, for the reason of simplicity, we assume that $c_i = c$. The utility function U_i thus represents the net benefit of a node i when operating on p_i.

We now formulate the medium access control problem as a non-cooperative game G_{MAC}.

Definition 1: The non-cooperative medium access game G_{MAC} is a triple $(\mathcal{N}, \{A_i\}, \{U_i\})$, where $\mathcal{N}$ is the player set, A_i is the strategy set of player i, U_i is the utility function of player i defined previously. Each player i selects its channel access probability $p_i \in A_i = [0, 1]$ to maximize its utility U_i. Formally, G_{MAC} is expressed as:

$$G_{MAC}: \quad \max_{p_i \in A_i} U_i(p_i, p_{-i}), \quad i \in \mathcal{N}$$

A. Nash Equilibrium Analysis

For non-cooperative games as G_{MAC}, the most important concept is the Nash equilibrium (NE), where no player has incentive to deviate from its current strategy. The NE can be seen as optimal "agreements" between the opponents of the game. In the case of the G_{MAC}, we have the following definition of NE.

Definition 2: A channel access probability vector $\mathbf{p}^* = (p_1^*, \cdots, p_n^*)$ is a NE of G_{MAC} if no player can improve its utility by unilaterally deviating from $\mathbf{p}^*$:

$$U_i(p_i^*, p_{-i}^*) \geq U_i(p_i', p_{-i}^*), \quad 0 \leq p_i' \leq 1, \quad \forall i \in \mathcal{N}$$

We use the concept of Pareto-optimality and social welfare optimality to characterize the efficiency of different strategy profiles.

Definition 3: The strategy profile $\mathbf{s}$ is Pareto-optimal if there does not exist another strategy profile $\mathbf{s}'$ such that for each player i, it holds that $U_i(\mathbf{s}') > U_i(\mathbf{s})$.

Definition 4: The strategy profile $\mathbf{s}$ is social welfare optimal if it maximizes the aggregated payoff $\sum_{i \in \mathcal{N}} U_i$.

Theorem 1 studies the NE of G_{MAC}.

Theorem 1: G_{NPC} admits at least one NE.

Proof: It can be verified that the strategy set of each player i $A_i = [0, 1]$ is a nonempty compact convex subset of Euclidian space. The utility function U_i is continuous and concave w.r.t. p_i on A_i. Hence, by Theorem 1 in [3], there exists at least one NE. ■

Since the utility function U_i is concave, $\mathbf{p}^*$ is either on the border of the strategy space or satisfies $\frac{\partial U_i}{\partial p_i} = 0$. We call a NE $\mathbf{p}^*$ a non-biased equilibrium if, for all nodes i, $\mathbf{p}^*$ satisfies $\frac{\partial U_i}{\partial p_i} = 0$, and biased equilibrium otherwise. Theorem 2 provides a more in-depth insight on the NE of G_{MAC}. The proof is straightforward and is omitted here.

Theorem 2: If $c \geq 1$, then G_{MAC} has only one biased NE $\{p_i^* = 0\}$ and no non-biased NE; If $c < 1$, then G_{MAC} has n biased NE $NE_i = \{p_i^* = 1, p_j^* = 0 (j \neq i)\} (i = 1, \cdots, n)$ and one non-biased NE $\{p_i^* = 1 - c^{\frac{1}{n-1}}\}$.

Remark 1: $c \geq 1$ is the trivial case where the transmission cost is so expensive that all players choose to keep silent. If $c < 1$, the biased NE corresponds to the situation that one player captures the channel and others always defer their transmission. The non-biased NE is the case where each player gets a fair share of the channel. We are mainly interested in the non-biased NE in our study and any efficient MAC protocol should not lead the network to the biased NE. In the rest of the paper, we focus on the non-trivial case where $c < 1$.

Remark 2: We can analyze the non-biased NE from another angle: consider each player has two pure strategies: transmit or wait. The non-biased NE is thus the mixed strategy NE of G_{MAC} and such NE is guaranteed to exist.

Remark 3: From an economic point of view, c can be regarded as the price for player i operating on p_i. The NE is thus the point where the marginal gain $\frac{\partial p_i \prod_{j \in \mathcal{N}, j \neq i}(1 - p_j)}{\partial p_i}$ equals to the price c. From the players's point of view, operating at higher p_i increases the gain at the expense of paying more in terms of price. Hence, to search the NE is actually to seek a compromised point between the gain and the cost.

V. Inefficiency of the Non-biased NE of G_{MAC}

The non-biased NE discussed in last section provides a solution where no player can increase its utility any further through individual effort. A natural question we pose is that whether the non-biased NE is efficient, i.e., Pareto-optimal and social welfare optimal. In this section, we answer this question by comparing the utility at above non-biased NE and the social welfare optimal point.

Let $\widehat{\mathbf{P}} = \{\widehat{p_i}\}$ denote the social welfare optimal point of G_{MAC} maximizing the global network utility $\sum_{i \in \mathcal{N}} U_i$, we investigate the fair social welfare optimal point where $\widehat{p_i} = \widehat{p}$ for all $i \in \mathcal{N}$ in the following lemma.

Lemma 1: Under the condition that $n \geq 2$ and $0 < c < 1$, there is a unique fair social welfare optimal point where $\widehat{p_i} = \widehat{p}$ for all $i \in \mathcal{N}$. Moreover, it holds that

1. $\widehat{p}$ is the root of $n(1-p)^{n-1} - (n-1)(1-p)^{n-2} - c = 0$.
2. $0 < \widehat{p} < 1/n$
3. $\widehat{p} < p_i^*$

Proof: It is easy to verify the case where $n = 2$. We consider the case where $n \geq 3$. Let $Q(p) = \frac{\partial \sum_{i \in \mathcal{N}} U_i(\{p_i = p\})}{\partial p}$, by imposing $Q(p) = 0$, we get $Q(p) = n(1-p)^{n-1} - (n-1)(1-p)^{n-2} - c = 0$. It follows

$$Q'(p) = -n(n-1)(1-p)^{n-2} + (n-1)(n-2)(1-p)^{n-3}$$

Hence $Q(p)$ is monotonously decreasing w.r.t. p in $(0, \frac{2}{n})$ and monotonously increasing in $(\frac{2}{n}, 1)$. Noticing that $Q(1) = -c < 0$, $Q(\frac{1}{n}) = -c < 0$ and $Q(0) = 1 - c > 0$, we obtain that $Q(p) = 0$ admits a unique solution $\widehat{p} \in (0, \frac{1}{n})$. Moreover, $Q(p) < 0$ when $p \in (\widehat{p}, 1)$ and $Q(p) > 0$ when $p \in (0, \widehat{p})$. Hence, $\widehat{p}$ is the unique maximizer of $\sum_{i \in \mathcal{N}} U_i$.

On the other hand, following $0 < c < 1$, we have $Q(p_i^*) = (n-1)c(1 - c^{-\frac{1}{n-1}}) < 0$. This leads to $p_i^* > \widehat{p}$. ■

Consider the utility function at the non-biased NE $\mathbf{p}^*$, we have $U_i(\mathbf{p}^*) = 0$ for all player i. Since all players are self-interested and rational and would never accept a negative payoff, operating at $\mathbf{p}^*$ actually minimizes both the individual and network-wide utility. On the other hand, at $\widehat{\mathbf{P}}$, we have $\sum_{i\in\mathcal{N}} U_i = n\left(\widehat{p}(1-\widehat{p})^{n-1} - c\widehat{p}\right)$. By expressing c by $\widehat{\mathbf{p}}$, after some mathematic operations, we get $\sum_{i\in\mathcal{N}} U_i = n(n-1)(1-\widehat{p})^{n-2}\widehat{p}^2 > 0$[1].

The non-biased NE is not Pareto optimal either. If all players switch from the non-biased NE to the social optimal point, both the individual and the network utility increase. This is due to the fact of lack of cooperation and the incentive to operate at social optimal point. The following theorem summarizes our result of this section.

Theorem 3: The non-biased NE of G_{MAC} is inefficient, i.e., neither Pareto-optimal nor social welfare optimal.

Figure 1 and Figure 2 show the non-biased NE and the social optimal point $\widehat{p}$ as a function of c.

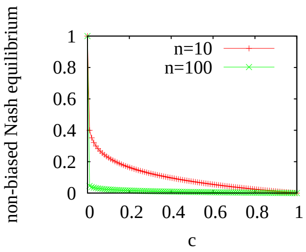

Fig. 1. p_i^* as a function of c

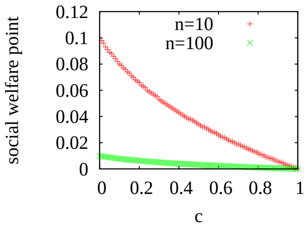

Fig. 2. $\widehat{p}$ as a function of c

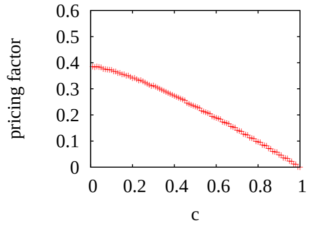

Fig. 3. b_i as a function of c

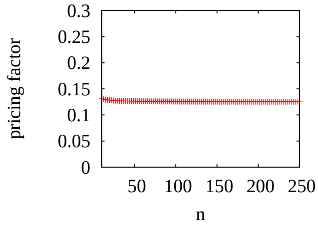

Fig. 4. b_i as a function of n

VI. Non-cooperative Medium Control Access Game With Pricing

Pricing is a powerful technique in game theory to motivate selfish players to adopt desirable behaviors. In our context, we turn to pricing to let the network converge to the social optimal point. From Lemma 1, at the NE, players tend to operate at higher p_i than the optimal point $\widehat{p}$, we encourage the players to decrease their p_i via pricing to approach the social optimal point. In this new context, we develop a non-cooperative game with pricing denoted by G_{MACP}=$(\mathcal{N}, \{A_i\}, \{U_i'\})$, where the utility function $U_i'(\cdot)$ is defined as $U_i' = U_i + \tau_i(p_i)$, where $\tau_i : A_i \to \mathbb{R}$ is the general form of the pricing function. In this paper, we investigate the following two pricing schemes.

A. Pricing Scheme 1

Motivated by the fact that, at the non-biased NE of G_{MAC}, players operate at higher p_i than the social optimal case, we impose a linear pricing function $\tau_i = -b_i p_i$ which is monotonously decreasing w.r.t. p_i by setting $b_i > 0$ to encourage the players to decrease their p_i. $b_i p_i$ can be regarded as extra price to players imposed by the pricing policy. The non-cooperative medium access game with this pricing scheme G^1_{MACP} is thus formally expressed as

$$G^1_{MACP} : \max_{0\le p_i\le 1} U_i^1(p_i, p_{-i}) = U_i(p_i, p_{-i}) - b_i p_i, i \in \mathcal{N}$$

We rewrite the utility function as $U_i^1 = p_i \prod_{j\in\mathcal{N}, j\neq i}(1 - p_j) - c' p_i$, where $c' = c + b_i$. It can be shown that if $c' < 1$, G^1_{MACP} admits n biased NEs and a unique non-biased NE $\{p_i = p_1^* = 1 - (c')^{\frac{1}{n-1}}\}$. By imposing $p_1^* = \widehat{p}$, the non-biased NE coincides to the social optimal point.

Theorem 4: By setting the pricing factor $b_i = (1 - \widehat{p})^{n-1} - c$, G^1_{MACP} admits a unique efficient non-biased NE which is also the social optimal point.

In the analysis of G_{MAC}, we can interpret c as the price for player i operating on p_i. Here in G^1_{MACP}, the above price becomes $c + b_i = (1 - \widehat{p})^{n-1}$. As the price increases, each player i tends to decrease its p_i at the non-biased NE.

Figure 3 and Figure 4 show b_i as a function of c ($n = 10$) and n (c=0.1).

B. Pricing Scheme 2

In this pricing scheme, we impose a gain discount on the utility function. Under this circumstance, $U_i^2 = d_i p_i \prod_{j\in\mathcal{N}, j\neq i}(1 - p_j) - c p_i$, where $d_i < 1$ is the discounting factor applied to discourage players to increase their p_i. The non-cooperative medium access game with this pricing scheme G^2_{MACP} is formally expressed as

$$G^2_{MACP} : \max_{0\le p_i\le 1} U_i^2(p_i, p_{-i}) = d_i p_i \prod_{j\in\mathcal{N}, j\neq i}(1 - p_j) - c p_i, i \in \mathcal{N}$$

For G^2_{MACP}, n biased NEs and a unique non-biased NE $\{p_i = p_2^* = 1 - (\frac{c}{d_i})^{\frac{1}{n-1}}\}$ exist if $\frac{c}{d_i} < 1$. By imposing $p_2^* = \widehat{p}$, the non-biased NE coincides to the social optimal point.

Theorem 5: By setting the pricing factor $d_i = \frac{c}{(1-\widehat{p})^{n-1}}$, G^2_{MACP} admits a unique efficient non-biased NE which is also the social optimal point.

Figure 5 and Figure 6 show d_i as a function of c ($n = 10$) and n (c=0.2).

Different from the first pricing scheme achieving the goal by increasing the cost of the transmission from c to c', the second pricing scheme attains the same goal by decreasing the gain of the successful transmission from $p_i \prod_{j\in\mathcal{N}, j\neq i}(1 - p_j)$ to $d_i p_i \prod_{j\in\mathcal{N}, j\neq i}(1 - p_j)$.

[1] Under our model, when $c = 0$, we get $\widehat{p} = 1/n$, the aggregated utility becomes the network throughput S. From Lemma 1 we have $S = \sum_{i\in\mathcal{N}} U_i = (1 - 1/n)^{n-1} < 1/e$ and $\lim_{n\to\infty} S = 1/e$. The result is coherent to the traditional performance bound of slotted-Aloha.

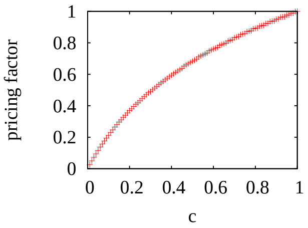

Fig. 5. d_i as a function of c

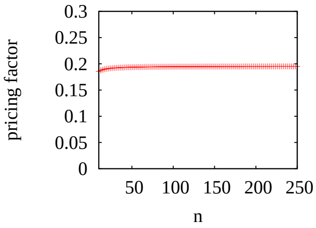

Fig. 6. d_i as a function of n

VII. Approaching the Non-biased NE

Until now, we have studied the NE of G_{MAC} and two pricing schemes to improve the efficiency of the non-biased NE. The MAC protocol can be viewed as the distributed strategy update mechanism to converge to the NE. To study such game dynamics, we consider the repeated play of G_{MAC}, and look for update mechanism in which players repeatedly adjust strategies in response to observations of other player actions so as to approach the non-biased NE. In this section, we study the convergence of two basic strategy update mechanisms widely used in game theory: the best response update and the subgradient update.

A. Best Response Update

In game theory, the simplest strategy update mechanism is best response update: at each iteration, every node chooses the best response to the actions of all the other nodes in previous iteration. Mathematically, at iteration $t+1$, player i updates its channel access probability as

$$p_i^{t+1} = r(\mathbf{p^t}) := \operatorname*{argmax}_{0\le p_i\le 1} U_i(p_i, p_{-i}^t)$$

Clearly, if the above dynamic reaches a stable state, this state is a NE. The convergence to the NE under best response update is also guaranteed.

For our medium access game without or with pricing, the convergence under the best response update is not guaranteed. Take G_{MAC} as an example, the best response is

$$p_i^{t+1} = r(\mathbf{p^t}) = \begin{cases} 1 & \prod_{j\in\mathcal{N},j\neq i}(1-p_j) > c \\ 0 & \prod_{j\in\mathcal{N},j\neq i}(1-p_j) < c \\ \forall p_i \in [0,1] & \prod_{j\in\mathcal{N},j\neq i}(1-p_j) = c \end{cases}$$

Staring by $p_i^0 = 0, \forall i \in \mathcal{N}$, we have $p_i^t = 1$ if t is even, $p_i^t = 1$ if t is odd. Hence, the best response update of the medium access games may not converge to the NE.

B. Subgradient Update

An alternative strategy is the subgradient update. Compared to the best response update, subgradient update can be viewed as the better response update in which every player adjusts its channel access probability in the gradient direction suggested by observations of other player actions. Mathematically, player i updates its strategy according to

$$p_i^{t+1} = p_i^t + \lambda_i^t \frac{\partial U_i}{\partial p_i}\Big|_{\mathbf{p}=\mathbf{p^t}}$$

where $\lambda_i^t > 0$ is the stepsize of player i at iteration t. The subgradient update scheme can be interpreted from an economic point of view. If the marginal gain is greater than contention price c, i.e., $\frac{\partial U_i}{\partial p_i}\Big|_{\mathbf{p}=\mathbf{p^t}} > 0$, i increases the access probability p_i, otherwise, i decreases p_i. Since at each iteration, players update channel access probabilities by a small amount, they usually experience smooth trajectories.

We take G_{MAC} as an example to study the convergence to the NE under the above subgradient update scheme. For G_{MAC}, the subgradient update can be derived as

$$p_i^{t+1} = p_i^t + \lambda_i^t \left(\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) - c \right)$$

where the fixed point of the subgraidient update is $p_i = 1 - c^{\frac{1}{n-1}}$ which is also the non-biased NE. Consider a simple case where $n=2$, starting from $\mathbf{p^0} = (p_1,p_2) = (1-c+\epsilon, 1-c-\epsilon)$ where ϵ is a small positive value, we have $\mathbf{p^t} \to (1,0)$ as $t\to+\infty$; starting from $\mathbf{p^0} = (1-c-\epsilon, 1-c+\epsilon)$, $\mathbf{p^t} \to (0,1)$ as $t \to +\infty$. Hence, the subgradient update may not converge to the non-biased NE.

VIII. Medium Access Control Design

Given the fact that the two basic update schemes studied above do not guarantee the convergence to the non-biased NE in the medium access game with or without pricing, we investigate more sophisticated medium access control mechanisms and propose the following three medium access control schemes with provable convergence to the desired equilibrium.

A. Medium Access Control Scheme 1: Aggressive Control

The first medium access control scheme is defined as:

$$p_i^{t+1} = p_{max} \prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) + \beta p_i^t \left(1 - \prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) \right) \quad (1)$$

where $0 < p_{max} < 1$, $\beta < 1$. One interpretation of the scheme is that at each iteration player i sets its channel access probability p_i to the maximum value p_{max} with probability $\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ depending on the channel access probability of other players in the last iteration, while reduces p_i by a factor β otherwise. If one iteration corresponds to one slot, p_i^{t+1} is the expected channel access probability of the following update scheme based on channel condition: if the channel is not occupied by other players during the last slot, then player i sets p_i to p_{max} for the coming slot; otherwise it reduces p_i by β. We refer this scheme as the aggressive medium access control as players set their channel access probability to the maximum value once the channel is not occupied by others.

The following theorem studies the dynamics under the aggressive medium access control scheme.

Theorem 6: If $\max\left\{(n-1)p_{max}, \beta + (1-p_{max})^{n-1}\left(\frac{(n-1)p_{max}}{1-p_{max}} - \beta\right)\right\} < 1$, the update scheme defined in (1) admits a unique fixed point $\mathbf{p^{f1}} = \{p_i^{f1}\}$ and it holds that:1) $0 < p_i^{f1} < p_{max}$; 2) Starting from any initial point $\mathbf{p^0} = \{p_i^0\}$ where $0 < p_i^0 < p_{max}$, the iteration defined by (1) converges to $\mathbf{p^{f1}}$.

Proof: We first show that with any initial point $0 < p_i^0 < p_{max}$, it holds that $0 < p_i^t < p_{max}, \forall i \in \mathcal{N}, t > 0$. We prove it by showing that if $0 < p_i^t < p_{max}, t \geq 0$, then $0 < p_i^{t+1} < p_{max}, \forall i \in \mathcal{N}$. Noticing that $0 < \beta p_i^t < p_i^t < p_{max}$, we get $0 < p_i^{t+1} < p_{max}\left(\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) + 1 - \prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)\right) = p_{max}$.

We then show that (1) admits a unique fixed point by using the following lemma concerning the fixed point of a contraction [4]:

Lemma 2: If the update scheme defined in (1) is a contraction, then it admits a unique fixed point; Moreover, starting from any initial point, the iteration under it converges to the unique fixed point.

The contraction is defined in [4] as follows: let (X, d) be a metric space, f: $X \to X$ is a contraction if there exists a constant k with $0 \leq k < 1$ such that

$$d(f(x), f(y)) \leq kd(x, y) \quad \forall x, y \in X$$

where $d(x, y) = ||x - y|| = \max_i ||x_i - y_i||$.

The key point to establish the uniqueness of the fixed point is thus to show the update scheme defined in (1) is a contraction.

We have

$$\begin{aligned} d(f(x), f(y)) &= ||f(x) - f(y)|| \\ &\leq ||\frac{\partial f}{\partial x}||||x - y|| = ||\frac{\partial f}{\partial x}||d(x, y) \end{aligned}$$

If the Jacobian $||\frac{\partial f}{\partial x}|| \leq k < 1$, f is a contraction. In our context, we show that the update scheme of (1) is a contraction by proving $||J||_\infty \leq k$, where $J = \{J_{ij}\}$ is the Jacobian of the update scheme of (1) defined by $J_{ij} = \frac{\partial p_i^{t+1}}{\partial p_j^t}$.

At iteration t ($t \geq 0$), we have

$$J_{ij} = \begin{cases} (\beta p_i^t - p_{max})\prod_{l\in\mathcal{N},l\neq i,l\neq j}(1-p_l^t) & i \neq j \\ \beta(1 - \prod_{l\in\mathcal{N},l\neq i}(1-p_l^t)) & i = j \end{cases}$$

Noticing that $0 < \beta p_i^t < p_i^t < p_{max}$, we have

$$\begin{aligned} ||J||_\infty &= \max_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}|J_{ij}| = \beta(1 - \prod_{l\in\mathcal{N},l\neq i}(1-p_l^t)) \\ &\quad - (\beta p_i^t - p_{max})\sum_{j\in\mathcal{N},j\neq i}\prod_{l\in\mathcal{N},l\neq i,l\neq j}(1-p_l^t) \\ &< \beta(1 - \prod_{l\in\mathcal{N},l\neq i}(1-p_l^t)) + p_{max}\sum_{j\in\mathcal{N},j\neq i}\prod_{l\in\mathcal{N},l\neq i,l\neq j}(1-p_l^t) \\ &= \beta + \prod_{l\in\mathcal{N},l\neq i}(1-p_l^t)\left(p_{max}\sum_{l\in\mathcal{N},l\neq i}\frac{1}{1-p_l^t} - \beta\right) \end{aligned}$$

Let $\mathbf{p^t} = \{p_i^t\}$, $Q(\mathbf{p^t}) = \beta + \prod_{l\in\mathcal{N},l\neq i}(1 - p_l^t)\left(p_{max}\sum_{l\in\mathcal{N},l\neq i}\frac{1}{1-p_l^t} - \beta\right)$, we bound $Q(\mathbf{p^t})$ by Q_{max}. To this end, we rewrite Q as

$$\begin{aligned} Q(p_j^t) = \beta + &\prod_{l\in\mathcal{N},l\neq i,l\neq j}(1-p_l^t) * \\ &\left(\left(p_{max}\sum_{l\in\mathcal{N},l\neq i,l\neq j}\frac{1}{1-p_l^t} - \beta\right)(1-p_j^t) + p_{max}\right) \end{aligned}$$

It follows that Q attains Q_{max} when $p_j^t = 0$ or $p_j^t = p_{max}$. Performing the same analysis for all $j \in \mathcal{N}$, we show that Q attains Q_{max} at the border of the strategy space. Let $N(0 \leq N \leq n-1)$ be the number of players with the access probability p_{max} at Q_{max}, it follows that

$$Q(N) = \beta + (1-p_{max})^N\left(p_{max}\left(\frac{N}{1-p_{max}} + n - 1 - N\right) - \beta\right)$$

Imposing $\frac{\partial Q}{\partial N} = 0$, we obtain

$$\frac{p_{max}}{1-p_{max}}N^2 + \left(n - \frac{\beta}{p_{max}}\right)N - (1-p_{max}) = 0$$

We can further verify that $\frac{\partial Q}{\partial N} = 0$ at $N_0 = \frac{-\left(n - \frac{\beta}{p_{max}}\right) + \sqrt{\left(\frac{\beta}{p_{max}}\right)^2 + 4p_{max}}}{\frac{2p_{max}}{(1-p_{max})}}$, If $0 \leq N_0 \leq n-1$, $\frac{\partial Q}{\partial N} < 0$ in $[0, N_0)$, $\frac{\partial Q}{\partial N} > 0$ in $(N_0, n-1]$. Hence Q is minimized at $N = N_0$ and maximized at $N = 0$ or $N = n-1$. If $N_0 < 0$ or $N_0 > n-1$, Q has no local maximizer in $(0, n-1)$ and attains its maximum at border. In both cases, we have $Q_{max} = \max\left\{Q(0), Q(n-1)\right\} = \max\left\{(n-1)p_{max}, \beta + (1-p_{max})^{n-1}\left(\frac{(n-1)p_{max}}{1-p_{max}} - \beta\right)\right\}$

Let $k = \max\left\{(n-1)p_{max},\ \beta + (1-p_{max})^{n-1}\left(\frac{(n-1)p_{max}}{1-p_{max}} - \beta\right)\right\}$, if the condition in the theorem holds, i.e., $k < 1$, we have $||J||_\infty \leq k < 1$. The update defined in (1) is a contraction. It admits a unique fixed point and the update converges to the fixed point, i.e., $\lim_{t\to\infty} p_i^t = p_i^{f1}$. Since we have shown that $0 < p_i^t < p_{max}$, we have $0 < p_i^{f1} < p_{max}$. This concludes our proof. ■

Recall that our goal of the medium access control design is to encourage the players to operate stably at the social optimal point, to this end, we impose $\widehat{p} = p_i^{f1}$. The following theorem is immediate.

Theorem 7: Under the condition of Theorem 6, by tuning β and p_{max} such that $\widehat{p} = p_i^{f1}$, or $\widehat{p} = p_{max}(1-\widehat{p})^{n-1} + \beta\widehat{p}\left(1 - (1-\widehat{p})^{n-1}\right)$, the proposed aggressive medium control scheme is convergent to the social optimal point, which is also the non-biased NE of the medium access game with pricing.

Theorem 6 and Theorem 7 provide guidelines for choosing parameters for aggressive MAC scheme. From Theorem 6, we can see that small p_{max} and large β help the network operate at a stable point. Theorem 7 further quantifies p_{max} and β to approach the stable convergent point to the social optimal point. As an example, if n is large ($(1-\frac{1}{n})^{n-1} \sim \frac{1}{e}$), $c \to 0$, then $p_{max} = \frac{1}{n} + \epsilon$ where ϵ is a positive small number, e.g. $\epsilon = O(n^{-2})$, $\beta = 1 - \frac{n\epsilon}{(e-1)}$ is a possible setting.

B. Medium Access Control Scheme 2:Conservative Control

In this section, we propose a more conservative medium access control scheme defined as:

$$p_i^{t+1} = p_i^t f(p_i^t) \prod_{j\in\mathcal{N}, j\neq i} (1-p_j^t) + p_{min}\left(1 - \prod_{j\in\mathcal{N}, j\neq i}(1-p_j^t)\right) \quad (2)$$

where $f(p_i^t) = 1 + \frac{p_{max}-p_i^t}{p_{max}-p_{min}}\delta$, $0 < p_{min} < p_{max} < 1$, $0 < \delta \leq \frac{p_{max}-p_{min}}{p_{max}}$. We pose the above constraint of δ to ensure that with any $p_{min} \leq p_i^t \leq p_{max}$, it holds that $p_{min} \leq p_i^t f(p_i^t) \leq p_{max}$. One interpretation of the above update scheme is that at each iteration player i sets p_i to the minimum value p_{min} with probability $1 - \prod_{j\in\mathcal{N}, j\neq i}(1-p_j^t)$, while increases p_i by a factor $f(p_i^t)$ otherwise. $f(p_i^t)$ is specially designed such that $f(p_i^t) = 1+\delta$ at p_{min}, $f(p_i^t) = 1$ at p_{max}, $1 < f(p_i^t) < 1+\delta$ and is linearly decreasing w.r.t. p_i^t in (p_{min},p_{max}). The increasing factor is thus adaptable based on the current channel access probability p_i^t. If one iteration consists of one slot, p_i^{t+1} becomes the expected channel access probability of the following update scheme: if the channel is occupied by other players during the last slot, then i sets p_i to p_{min} for the coming slot; otherwise it increase p_i by $1+\delta$. We refer this scheme as the conservative medium access control as players set the channel access probability to the minimum value once the channel is sensed occupied by others.

We next investigate the dynamics under the above conservative medium access control scheme.

Theorem 8: If $0 < \delta \leq \frac{p_{max}-p_{min}}{p_{max}}$ and $(n-1)(p_{max} - p_{min})(1-p_{min})^{n-2} + \left(1 - \frac{p_{max}\delta}{p_{max}-p_{min}}\right)(1-p_{min})^{n-1} < 1$, the update scheme defined in (2) admits a unique fixed point $\mathbf{p^{f2}} = \{p_i^{f2}\}$ and it holds that:1) $p_{min} < p_i^{f2} < p_{max}$; 2) Starting from any initial point $\mathbf{p^0}$ where $p_{min} < p_i^0 < p_{max}$, the iteration defined by (2) converges to $\mathbf{p^{f2}}$.

Proof: We follow the same way as the proof of Theorem 6 by showing that the Jacobian for (2) $||J'||_\infty \leq k' < 1$. To this end, we compute J'_{ij} as

$$J_{ij} = \begin{cases} \left(p_{min} - p_i^t\left(1 + \delta\frac{p_{max}-p_i^t}{p_{max}-p_{min}}\right)\right) \\ \times \prod_{l\in\mathcal{N}, l\neq i, l\neq j}(1-p_l^t) & i \neq j \\ \left(1 + \delta\frac{p_{max}-2p_i^t}{p_{max}-p_{min}}\right)\prod_{l\in\mathcal{N}, l\neq i}(1-p_l^t) & i = j \end{cases}$$

Noticing that $0 < \delta \leq \frac{p_{max}-p_{min}}{p_{max}}$, we can show that $||J'||_\infty = \max_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} J_{ij}$ is maximized at $p_i^t = p_{max}$, $p_j^t = p_{min}$ for $j \neq i$, we thus have

$$||J'||_\infty \leq (n-1)(p_{max}-p_{min})(1-p_{min})^{n-2} + \left(1 - \frac{p_{max}\delta}{p_{max}-p_{min}}\right)(1-p_{min})^{n-1}$$

If the condition in the theorem holds, i.e., $k' = (n-1)(p_{max} - p_{min})(1-p_{min})^{n-2} + \left(1 - \frac{p_{max}\delta}{p_{max}-p_{min}}\right)(1-p_{min})^{n-1} < 1$, $||J'||_\infty \leq k' < 1$. (2) is a contraction. Both the uniqueness of the fixed point and the convergence are guaranteed. It is further easy to show that the fixed point $p_{min} < p_i^{f2} < p_{max}$. ■

We then study the convergence of the conservative medium access control to the social optimal point.

Theorem 9: Under the condition of Theorem 8, by tuning δ, p_{max} and p_{min} such that $\widehat{p} = p_i^{f2}$, or $\widehat{p} = \widehat{p}f(\widehat{p})(1-\widehat{p})^{n-1} + p_{min}\left(1-(1-\widehat{p})^{n-1}\right)$, the conservative medium control scheme is convergent to the social optimal point.

C. Medium Access Control Scheme 3: Cheat-proof Control

The above MAC protocols have following desirable properties: 1) they consist of natural access probability update schemes for rational players; 2) they provide tunable parameters with which one can control the convergent point of the network; 3) they lead the network to a stable state where both fairness and social optimality are ensured;

The above MAC protocols meet the requirements for efficient MAC protocols in terms of performance. However, they both require network participants to respect the rules. Hence, they cannot survive selfish and non-cooperative environments because in such environments, players only adopt strategies that bring the most benefits to them, regardless of the fact that the adopted strategy leads to social optimality or not. In such environments, we can not implicitly assume that all participants act cooperatively by following the designed MAC protocols. Under such circumstance, we require an efficient MAC protocol to be survivable such that it can guide the individual nodes to operate on the designed optimal equilibrium even they are purely self-interested and non-cooperative. In other words, the MAC protocol consists of a set of strategies that each selfish individual node has no incentive to deviate.

To this end, we apply the pricing technique again. On one hand, the pricing scheme approaches the non-biased NE to the social optimal point; on the other hand, the pricing scheme encourages the individual selfish players to follow the MAC protocol. In the following part of this section, we propose the cheat-proof MAC protocol. The above desirable properties are maintained in the cheat-proof MAC protocol. Moreover, the imposed pricing function can encourage the players to follow the proposed protocol.

The channel access probability update scheme in the cheat-proof MAC protocol is defined as

$$p_i^{t+1} = p_{max}\prod_{j\in\mathcal{N}, j\neq i}(1-p_j^t) + p_{min}\left(1 - \prod_{j\in\mathcal{N}, j\neq i}(1-p_j^t)\right) \quad (3)$$

One interpretation of the above update scheme is that at each iteration player i sets its access probability p_i to p_{max} with probability $\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ and p_{min} otherwise. If one iteration corresponds to one slot, p_i^t becomes the expected channel access probability of the following update scheme: if the channel is occupied by other players during the last slot, then player i sets p_i to p_{min} for the coming slot; otherwise it sets p_i to p_{max}. In this scheme, p_i^{t+1} is decoupled with p_i^t, which is a necessary condition for the following demonstration.

(3) can not only be regarded as the update scheme for the channel access probability, but also be viewed as the strategy update that implicitly maximizes some utility function, as studied in the following theorem.

Theorem 10: Regard (3) as the best response function for each player $i\in\mathcal{N}$ at each iteration, the underlying utility function that each player tries to maximize is

$$U_i^C = -\Big(p_i - p_{min} - (p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j)\Big)^2 + C$$

where $C > \max\Big\{\Big(p_{max} - p_{min} - (p_{max} - p_{min})(1-p_{max})^{n-1}\Big)^2, \Big((p_{max}-p_{min})(1-p_{min})^{n-1}\Big)^2\Big\}$ is a constant large enough to avoid the negative utility value.

Proof: The proof is straightforward noticing (3) can be written as $p_i^{t+1} = p_{min} + (p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ ■

Recall that in G_{MAC}, the utility function U_i is

$$U_i = p_i \prod_{j\in\mathcal{N},j\neq i}(1-p_j) - cp_i$$

We impose the following pricing function

$$\begin{aligned}\tau_i(p_i) &= U_i^C - U_i\\ &= -(p_i - p_{min} - (p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j))^2\\ &+ C - p_i\prod_{j\in\mathcal{N},j\neq i}(1-p_j) - cp_i\end{aligned}$$

Next we define the non-cooperative game with the above pricing function as

$$G_{MACP}^C: \quad \max_{0\le p_i\le 1} U_i^C(p_i, p_{-i}), \quad i\in\mathcal{N}$$

The above proposed MAC scheme with pricing is cheat-proof in that the access probability update scheme corresponds to the best response strategy of G_{MAC}^C, thus a rational player will follow (3) to maximize its payoff.

The following theorem establishes the existence, uniqueness of the NE in G_{MACP}^C and the convergence to the unique NE under the cheat-proof MAC scheme.

Theorem 11: Under the condition that $(n-1)(p_{max} - p_{min})(1-p_{min})^{n-1} < 1$, G_{MACP}^C admits a unique NE. Starting from any initial point $\mathbf{p_0}$, the cheat-proof MAC scheme is convergent to the unique NE.

Proof: We use the following theorem in game theory concerning the uniqueness of NE [4]:

Lemma 3: If the best response function is a contraction, then the game admits a unique NE; Moreover, starting from any initial point, the iteration under the best response converges to the unique NE.

The above lemma shows that actually the NE consists of the fixed point of the best response function.

We now prove that the update scheme (3) is a contraction. This can be shown by noticing that

$$J_{ij}^C = \begin{cases} -(p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) & i\neq j\\ 0 & i=j\end{cases}$$

$$\begin{aligned}||J||_\infty^C &= (n-1)(p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)\\ &\le (n-1)(p_{max}-p_{min})(1-p_{min})^{n-1} < 1\end{aligned}$$

Thus (3) is a contraction. The theorem is proven ■

Furthermore, if the condition in Theorem 11 is satisfied, let $\mathbf{p^C} = \{p_i^C\}$ be the unique NE, we can show that $p_{min} < p_i^C < p_{max}$. The following theorem studies the efficiency of the unique NE of G_{MAC}^C.

Theorem 12: If $\widehat{p} = p_{min} + (p_{max}-p_{min})(1-\widehat{p})^{n-1}$, the unique NE is efficient, i.e., $p_i^C = \widehat{p}$.

Theorem 11 and 12 provide sufficient condition on the convergence to the NE under (3), which can be regarded as the best response update. One draw back is that the best response update often leads to large fluctuations that may cause temporary system instability. We address this issue by studying the subgradient update in G_{MACP}^C. By setting the step size sufficiently small, the subgradient update scheme experiences a smooth trajectory. Theorem 13 gives the sufficient condition on the convergence of the subgradient update to the NE of G_{MACP}^C. The proof follows the similar way as that of Theorem 6 and is omitted here.

Theorem 13: Consider the subgradient update for cheat-proof MAC scheme defined as

$$\begin{aligned}p_i^{t+1} &= p_i^t + \lambda\frac{\partial U_i^C}{\partial p_i}\\ &= p_i^t - 2\lambda\left(p_i^t - p_{min} - (p_{max}-p_{min})\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)\right)\end{aligned}$$

under the same condition as Theorem 12, the subgradient update scheme converges to the unique NE.

The above subgradient update scheme is actually a mild version of the cheat-proof MAC scheme in (3). By controlling the step size λ, players experience less variation in their strategies than the best response update (3). The system is thus more stable. As price, the convergence delay increases.

D. Implementation Issues

In the practical implementation of the proposed MAC protocols, players usually do not have access to the access probability of others, so they can not directly calculate $\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ which is needed to update p_i^t.

To solve this problem, we apply the *Idle Sense* approach proposed in [9] allowing a player to estimate the channel condition by observing the average number of consecutive idle slots between two transmission attempts. Let $P_{idle}^t = \prod_{j\in\mathcal{N}}(1-p_j^t)$ be the probability of an idle slot and n_{idle}^t be the number of average consecutive idle slots between two transmission attempts during iteration t, it holds that $n_{idle}^t = \dfrac{P_{idle}^t}{1-P_{idle}^t}$. It follows that $\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t) = \dfrac{n_{idle}^t}{n_{idle}^t+1}\cdot\dfrac{1}{1-p_i^t}$. Thus each player i can update p_i^t by observing n_{idle}^t. Another desirable feature of using *Idle Sense* approach is that our MAC protocols decouple access control from collision perception, thus are immune to problems incurred by packet collision perception.

Algorithm 1 shows the derived cheat-proof MAC protocol. The aggressive and conservative MAC protocols can be derived similarly. In the protocol, a transmission corresponds to an occupied channel slot when only one player transmits (a successful transmission) or multiple players transmit simultaneously (a collision).

Algorithm 1 Cheat proof MAC Protocol

After each transmission **do**
  $sum \leftarrow sum + n,\ \ ntrans \leftarrow ntrans + 1$ /* `Idle sense: the player observes` n `idle slots before the transmission` */
  if $ntrans \geq ntrans_{max}$ **then**
    $n_{idle}^t \leftarrow \dfrac{sum}{ntrans}$ /* `estimate` n_{idle}^t */
    $p_i^{t+1} \leftarrow p_{min} + (p_{max}-p_{min})\dfrac{n_{idle}^t}{n_{idle}^t+1}\cdot\dfrac{1}{1-p_i^t}$ /* `update` p_i */
    $sum \leftarrow 0,\ ntrans \leftarrow 0$ /* `reset variables` */
  end if
end

In our work, we do not address how to realize the pricing, which is not trivial at all. An appropriate pricing scheme in our context should be distributed and cheat-proof in case where players may provide forged information to get extra gain. In previous part of this section, we have shown how to estimate $\prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ based on n_{idle}^t, which is observable to all players. Next we provide a mechanism to estimate p_i^t based on only observable information. This is a crucial issue to implement any pricing scheme.

The mechanism is extended from the *Idle Sense* approach. Let $P_{idle,i}^t = \prod_{j\in\mathcal{N},j\neq i}(1-p_j^t)$ and $n_{idle,i}^t$ be the number of average consecutive idle slots between two "i-transmissions" during iteration t, where "i-transmission" corresponds to an occupied channel slot when only one player except i transmits or multiple players transmit simultaneously. $P_{idle,i}^t$ is in fact the probability of the slot with no "i-transmission". It holds that $n_{idle,i}^t = \dfrac{P_{idle,i}}{1-P_{idle,i}}$. It follows that $p_i^t = 1 - \dfrac{P_{idle}}{P_{idle,i}} = \dfrac{n_{idle,i}^t - n_{idle}^t}{n_{idle,i}^t(1+n_{idle}^t)}$. Hence, p_i^t can be estimated based on n_{idle}^t which is observable to all players, and $n_{idle,i}^t$ which is observable to all players except i (i knows p_i^t). By employing the above mechanism, the pricing scheme can be realized in a distributed and cheat-proof way based on only observable information.

E. Methodology for Efficient MAC Protocol Design

Based on the analysis on the medium access game and the three proposed MAC protocols, we introduce the following methodology for designing efficient MAC protocols for non-cooperative and selfish environments:

1. Choosing a natural channel access update scheme;
2. Configuring the parameters in the chosen update scheme to ensure that the network converges to the global optimal point under the update scheme;
3. Deriving appropriate pricing functions based on observable information such that no player has incentive to deviate from the designed MAC protocol.

We believe that the proposed methodology not only provides a general way of designing stable and controllable MAC protocols achieving high performance even in selfish environments, but also provides a general framework that can be extended to design efficient protocols in other non-cooperative and selfish environments.

IX. Numerical Results

In this section, we provide numerical results on the performance of the proposed MAC protocols. First, we consider a network of 10 nodes. We set $c = 0.25$, $ntrans_{max} = 10$. We calculate the social optimal point to be $\widehat{p} = 0.058$. Based on Theorem 6 and Theorem 7, we set $p_{max} = 0.08$, $\beta = 0.33$. The correspondent aggressive MAC protocol leads the network to the social optimal point. Similarly, for the conservative MAC protocol, we set $\delta = 0.133$, $p_{max} = 0.08$, $p_{min} = 0.05$. For the cheat-proof MAC protocol, we set $p_{min} = 0.001$ and $p_{max} = 0.105$. Under these parameter settings, based on our analytical model, the network converges to the social optimal point under the proposed three MAC protocols. This is confirmed by the numerical result shown in Figure 7-9, which plot the channel access probability trajectories of each player. Figure 10 plots the trajectories of the access probability under subgradient update, a mild version of the cheat-proof MAC protocol. The trajectory converges in a smoother way with longer convergence delay.

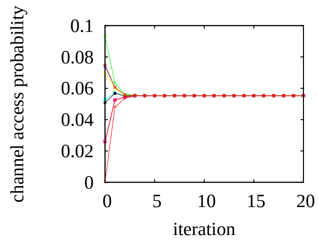

Fig. 7. Aggressive MAC

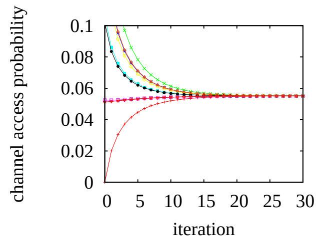

Fig. 8. Conservative MAC

We then focus on the cheat-proof MAC protocol. In the protocol implementation, the number of network participants n is required to configure the protocol parameters.

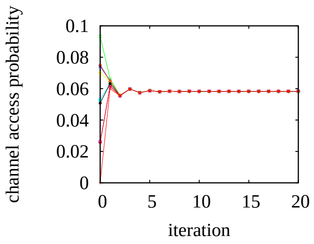

Fig. 9. Cheat-proof MAC

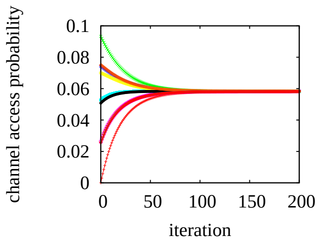

Fig. 10. Cheat-proof MAC: mild version

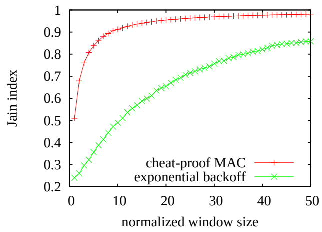

Fig. 13. Fairness comparison

We now study the impact of the estimation error of n on the protocol performance by allowing certain estimation error on n and studying the performance under such estimation error. Figure 11 plots U_{act}/U_{opt} under estimation error $10\%-50\%$, where U_{act} is the actual global utility with estimation error, U_{opt} is the optimal global utility without error. We can see that our protocol is quite robust in that the global utility does not degrade dramatically w.r.t. the estimation error of n, even when the estimation error reaches 50%. This is a disable feature when running the protocol in dynamical environments.

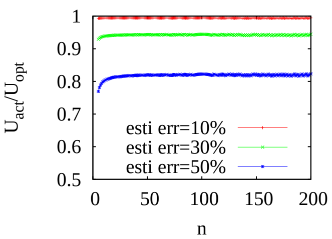

Fig. 11. Cheat-proof MAC: performance with estimation error of n

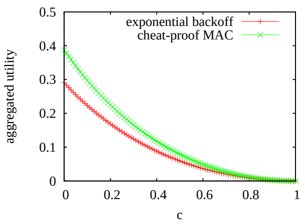

Fig. 12. $\sum_{i\in\mathcal{N}} U_i$ as function of c

Finally, we compare the performance of our protocol with the EB based MAC protocol widely employed in nowadays WLANs. We set $n = 50$, $c = 0.01$. Since the performance of the EB protocol highly depends on p_{max} and p_{min}, we simulate the EB protocol with different p_{max} and p_{min} values and plot the maximum aggregated utility with the aggregated utility achieved by the cheat-proof protocol in Figure 12. We also compares the fairness of the two protocols by plotting the normalized Jain fairness index [12] in Figure 13. We can see that our protocol achieves higher utility with better short-term fairness. The result is due to the fact that the EB protocol relies on an inefficient collision resolution mechanism which causes both network sub-optimality on performance and the short-term fairness problem. However, the cheat-proof MAC protocol decouples the access control from collision perception and players have much less variation in channel access probability around the social optimal point. Hence it is not surprising that the proposed MAC protocol outperforms the EB protocol in both performance and fairness.

X. Conclusion and Future Work

In this paper, we address the question that how to design efficient MAC protocols in selfish and non-cooperative networks, which is crucial in nowadays open environments. To this end, we conduct an in-depth study on the medium access control under game theoretical framework. Three MAC protocols, the aggressive, the conservative and the cheat-proof MAC protocols, are then proposed with tunable parameters allowing them to converge to desired optimal point. The first two MAC protocols require network participants to follow the rules, while the cheat-proof MAC protocol can survive the selfish environments where nodes are purely self-interested.

Based on our game theoretical analysis, we provide a general methodology for designing efficient MAC protocols for non-cooperative and selfish environments. We believe that the proposed methodology not only provides a general way of designing stable and controllable MAC protocols achieving high performance even in selfish environments, but also provides a general framework that can be extended to design efficient protocols in other non-cooperative and selfish environments.

As future work, we plan to develop a general practical pricing scheme to guide selfish players act cooperatively. Another direction is to apply the methodology proposed in this paper to the network and transport layers.

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