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From Static to Dynamic Tag Population Estimation: An Extended Kalman Filter Perspective

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***Abstract*—Tag population estimation has recently attracted significant research attention due to its paramount importance on a variety of radio frequency identification (RFID) applications. However, most, if not all, of existing estimation mechanisms are proposed for the static case where tag population remains constant during the estimation process, thus leaving the more challenging dynamic case unaddressed, despite the fundamental importance of the latter case on both theoretical analysis and practical application. In order to bridge this gap, we devote this paper to designing a generic framework of stable and accurate tag population estimation schemes based on Kalman filter for both static and dynamic RFID systems. Technically, we first model the dynamics of RFID systems as discrete stochastic processes and leverage the techniques in extended Kalman filter (EKF) and cumulative sum control chart (CUSUM) to estimate tag population for both static and dynamic systems. By employing Lyapunov drift analysis, we mathematically characterise the performance of the proposed framework in terms of estimation accuracy and convergence speed by deriving the closed-form conditions on the design parameters under which our scheme can stabilise around the real population size with bounded relative estimation error that tends to zero with exponential convergence rate.**

***Index Terms*—RFID, tag population estimation, extended Kalman filtre, stochastic stability.**

I. Introduction

A. Context and Motivation

Recent years have witnessed an unprecedented development and application of the radio frequency identification (RFID) technology. As a promising low-cost technology, RFID is widely utilized in various applications ranging from inventory control [1][2], supply chain management [3] to tracking/location [4][5]. A standard RFID system has two types of devices: a set of RFID tags and one or multiple RFID readers (simply called *tags* and *readers*). A tag is typically a low-cost microchip labeled with a unique serial number (ID) to identify an object. A reader, on the other hand, is equipped with an antenna and can collect the information of tags within its coverage area.

Tag population estimation and counting is a fundamental functionality for many RFID applications such as warehouse management, inventory control and tag identification. For example, quickly and accurately estimating the number of tagged objects is crucial in establishing inventory reports for large retailers such as Wal-Mart [6]. Due to the paramount practical importance of tag population estimation, a large body of studies [7][8][9][10][11] have been devoted to the design of efficient estimation algorithms. Most of them, as reviewed in Sec. II, are focused on the static scenario where the tag population is constant during the estimation process. However, many practical RFID applications, such as logistic control, are dynamic in the sense that tags may be activated or terminated as specialized in C1G2 standard [12], or the tagged objects may enter and/or leave the reader's covered area frequently, thus resulting in tag population variation. In such dynamic applications, a fundamental research question is how to design efficient algorithms to dynamically trace the tag population quickly and accurately.

B. Summary of Contributions

In this paper, we develop a generic framework of stable and accurate tag population estimation schemes for both static and dynamic RFID systems. By generic, we mean that our framework both supports the real-time monitoring and can estimate the number of tags accurately without any prior knowledge on the tag arrival and departure patterns. Our design is based on the extended Kalman filter (EKF) [13], a powerful tool in optimal estimation and system control. By performing Lyapunov drift analysis, we mathematically prove the efficiency and stability of our framework in terms of the boundedness of estimation error.

The main technical contributions of this paper are articulated as follows. We formulate the system dynamics of the tag population for both static and dynamic RFID systems where the number of tags remains constant and varies during the estimation process. We design an EKF-based population estimation algorithm for static RFID systems and further enhance it to dynamic RFID systems by leveraging the cumulative sum control chart (CUSUM) to detect the population change. By employing Lyapunov drift analysis, we mathematically characterise the performance of the proposed framework in terms of estimation accuracy and convergence speed by deriving the closed-form conditions on the design parameters under which our scheme can stabilise around the real population size with bounded relative estimation error that tends to zero within exponential convergence rate. To the best of our knowledge, our work is the first theoretical framework that dynamically traces the tag population with closed form conditions on the estimation stability and accuracy.

II. Related Work

Due to its fundamental importance, tag population estimation has received significant research attention, which we

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briefly review in this section.

A. Tag Population Estimation for Static RFID systems

Most of existing works are focused on the static scenario where the tag population is constant during the estimation process. The central question there is to design efficient algorithms quickly and accurately estimating the static tag population. Kodialam *et al.* design an estimator called PZE which uses the probabilistic properties of empty and collision slots to estimate the tag population size [14]. The authors then further enhance PZE by taking the average of the probability of idle slots in multiple frames as an estimator in order to eliminate the constant additive bias [7]. Han *et al.* exploit the average number of idle slots before the first non-empty slots to estimate the tag population size [15]. Later, *Qian et al.* develop Lottery-Frame scheme that employs geometrically distributed hash function such that the jth slot is chosen with prob. $\frac{1}{2^{j+1}}$ [9]. As a result, the first idle slot approaches around the logarithm of the tag population and the frame size can be reduced to the logarithm of the tag population, thus reducing the estimation time. Subsequently, a new estimation scheme called ART is proposed in [10] based on the average length of consecutive non-empty slots. The design rational of ART is that the average length of consecutive non-empty slots is correlated to the tag population. ART is shown to have smaller variance than prior schemes. More recently, *Zheng et al.* propose another estimation algorithm, ZOE, where each frame just has a single slot and the random variable indicating whether a slot is idle follows Bernoulli distribution [11]. The average of multiple individual observations is used to estimate the tag population.

We would like to point out that the above research work does not consider the estimation problem for dynamic RFID systems and thus may fail to monitor the system dynamics in real time. Specifically, in typical static tag population estimation schemes, the final estimation result is the average of the outputs of multi-round executions. When applied to dynamic tag population estimation, additional estimation error occurs due to the variation of the tag population size during the estimation process.

B. Tag Population Estimation for Dynamic RFID systems

Only a few propositions have tackled the dynamic scenario. The works in [16] and [17] consider specific tag mobility patterns that the tags move along the conveyor in a constant speed, while tags may move in and out by different workers from different positions, so these two algorithm cannot be applicable to generic dynamic scenarios. Subsequently, Xiao *et al.* develop a differential estimation algorithm, ZDE, in dynamic RFID systems to estimate the number of arriving and removed tags [18]. More recently, they further generalize ZDE by taking into account the snapshots of variable frame sizes [19]. Though the algorithms in [18] and [19] can monitor the dynamic RFID systems, they may fail to estimate the tag population size accurately, because they must use the same hash seed in the whole monitoring process, which cannot reduce the estimation variance. Using the same seed is an effective way in tracing tag departure and arrival. However, it may significantly limit the estimation accuracy, even in the static case.

Besides the limitations above, prior works do not provide formal analysis on the stability and the convergence rate. To fill this vide, we develop a generic framework for tag population estimation in dynamic RFID systems. By generic, we mean that our framework can both support real-time monitoring and estimate the number of tags accurately without the requirement for any prior knowledge on the tag arrival and departure patterns. As another distinguished feature, the efficiency and stability of our framework is mathematically established.

III. Technical Preliminaries

In this section, we briefly introduce the extended Kalman filter and some fundamental concepts and results in stochastic process which are useful in the subsequent analysis.

A. Extended Kalman Filter

The extended Kalman filter is a powerful tool to estimate system state in nonlinear discrete-time systems. Formally, a nonlinear discrete-time system can be described as follows:

$$z_{k+1} = f(z_k, x_k) + w_k^* \tag{1}$$

$$y_k = h(z_k) + u_k^*, \tag{2}$$

where $z_{k+1} \in \mathbb{R}^n$ denotes the state of the system, $x_k \in \mathbb{R}^d$ is the controlled inputs and $y_k \in \mathbb{R}^m$ stands for the measurement observed from the system. The uncorrelated stochastic variables $w_k^* \in \mathbb{R}^n$ and $u_k^* \in \mathbb{R}^m$ denote the process noise and the measurement noise, respectively. The functions f and h are assumed to be the continuously differentiable.

For the above system, we introduce an EKF-based state estimator given in Definition 1.

Definition 1 (Extended Kalman filter [13])**.** *A two-step discrete-time extended Kalman filter consists of state prediction and measurement update, defined as*

1) Time update (prediction)

$$\hat{z}_{k+1|k} = f(\hat{z}_{k|k}, x_k) \tag{3}$$

$$P_{k+1|k} = P_{k|k} + Q_k, \tag{4}$$

2) Measurement update (correction)

$$\hat{z}_{k+1|k+1} = f(\hat{z}_{k+1|k}, x_k) + K_{k+1}v_{k+1} \tag{5}$$

$$P_{k+1|k+1} = P_{k+1|k}\left(1 - K_{k+1}C_{k+1}\right) \tag{6}$$

$$K_{k+1} = \frac{P_{k+1|k}C_{k+1}}{P_{k+1|k}{C_{k+1}}^2 + R_{k+1}}, \tag{7}$$

where $v_{k+1} = y_{k+1} - h(\hat{z}_{k+1|k})$ (8)

$$C_{k+1} = \left.\frac{\partial h(z_{k+1})}{\partial z_{k+1}}\right|_{z_{k+1}=\hat{z}_{k+1|k}}. \tag{9}$$

Remark. *In the above definition of extended Kalman filter, the parameters can be interpreted in our context as follows:*

- $\hat{z}_{k+1|k}$ *is the prediction of* z_{k+1} *at the beginning of frame* $k+1$ *given by the previous state estimate, while* $\hat{z}_{k+1|k+1}$ *is the estimate of* z_{k+1} *after the adjustment based on the measure at the end of frame* $k+1$.

- v_{k+1}, *referred to as innovation, is the measurement residual in frame* $k+1$. *It represents the estimated error of the measure.*
- K_{k+1} *is the Kalman gain. With reference to equation* (5), *it weighs the innovation* v_{k+1} *w.r.t.* $f(\hat{z}_{k+1|k}, x_k)$.
- $P_{k+1|k}$ *and* $P_{k+1|k+1}$, *in contrast to the linear case, are not equal to the covariance of estimation error of the system state. Here, we will refer to them as pseudo-covariance.*
- Q_k *and* R_k *are two tunable parameters which play the role as that of the covariance of the process and measurement noises in linear stochastic systems to achieve optimal filtering in the maximum likelihood sense. We will show later that* Q_k *and* R_k *also play an important role in improving the stability and convergence of our EKF-based estimators.*

B. Boundedness of Stochastic Process

In order to analyse the stability of an estimation algorithm, we need to check the boundedness of the estimation error defined as follows:

$$e_{k|k-1} \triangleq z_k - \hat{z}_{k|k-1}. \tag{10}$$

Due to probabilistic nature of the estimation algorithm, the estimation process is a stochastic process. Thus, we further introduce the following two mathematical definitions [20] [21] on the boundedness of stochastic process.

Definition 2 (Boundedness of Random Variable)**.** *The stochastic process of the estimation error* $e_{k|k-1}$ *is said to be bounded with probability one (w.p.o.), if there exists* $X > 0$ *such that*

$$\lim_{k\to\infty} \sup_{k\geq 1} \mathbb{P}\{|e_{k|k-1}| > X\} = 0. \tag{11}$$

Definition 3 (Boundedness in Mean Square)**.** *The stochastic process* $e_{k|k-1}$ *is said to be exponentially bounded in the mean square with exponent* ζ, *if there exist real numbers* ψ_1, $\psi_2 > 0$ *and* $0 < \zeta < 1$ *such that*

$$E[e^2_{k|k-1}] \leq \psi_1 e^2_{1|0}\zeta^{k-1} + \psi_2. \tag{12}$$

To investigate the boundedness defined in Definition 2 and 3, we introduce the following lemma [22].

Lemma 1. *Given a stochastic process* $V_k(e_{k|k-1})$ *and constants* $\underline{\beta}$, $\overline{\beta}$, $\tau{>}0$ *and* $0{<}\alpha{\leq}1$ *with the following properties:*

$$\underline{\beta} e^2_{k|k-1} \leq V_k(e_{k|k-1}) \leq \overline{\beta} e^2_{k|k-1}, \tag{13}$$

$$E[V_{k+1}(e_{k+1|k})|e_{k|k-1}] - V_k(e_{k|k-1}) \leq -\alpha V_k(e_{k|k-1}) + \tau, \tag{14}$$

then for any $k \geq 1$ *it holds that*

- *the stochastic process* $e_{k|k-1}$ *is exponentially bounded in the mean square, i.e.,*

$$\begin{aligned} E[e^2_{k|k-1}] &\leq \frac{\overline{\beta}}{\underline{\beta}} E[e^2_{1|0}](1-\alpha)^{k-1} + \frac{\tau}{\underline{\beta}} \sum_{j=1}^{k-2} (1-\alpha)^j \\ &\leq \frac{\overline{\beta}}{\underline{\beta}} E[e^2_{1|0}](1-\alpha)^{k-1} + \frac{\tau}{\underline{\beta}\alpha}, \end{aligned} \tag{15}$$

- *the stochastic process* $e_{k|k-1}$ *is bounded w.p.o..*

From Lemma 1, if we can construct $V_k(e_{k|k-1})$, a function of $e_{k|k-1}$, such that both its drift and $\frac{V_k(e_{k|k-1})}{e^2_{k|k-1}}$ are bounded, i.e, (14) and (13) hold, then $e_{k|k-1}$ is also bounded and the convergence rate depends on constant α mostly. Besides, it can be noted that Lemma 1 can only be implemented offline. To address this limit, we adjust Lemma 1 to an online version with time-varying parameters, which can be proven by the same method as in [21] and [23].

Lemma 2. *If there exist a stochastic process* $V_k(e_{k|k-1})$ *and real numbers* β^*, β_k, $\tau_k{>}0$ *and* $0{<}\alpha^*_k{\leq}1$ *with the following properties:*

$$V_1(e_{1|0}) \leq \beta^* e^2_{1|0}, \tag{16}$$

$$\beta_k e^2_{k|k-1} \leq V_k(e_{k|k-1}), \tag{17}$$

$$E[V_{k+1}(e_{k+1|k})|e_{k|k-1}] - V_k(e_{k|k-1}) \leq -\alpha^*_k V_k(e_{k|k-1}) + \tau_k; \tag{18}$$

then for any $k \geq 1$ *it holds that*

- *the stochastic process* $e_{k|k-1}$ *is exponentially bounded in the mean square, i.e.,*

$$\begin{aligned} E[e^2_{k|k-1}] \leq{}& \frac{\beta^*}{\beta_k} E[{e_{1|0}}^2] \prod_{i=1}^{k-1} (1-\alpha^*_i) \\ &+ \frac{1}{\beta_k} \sum_{i=1}^{k-2} \tau_{k-i-1} \prod_{j=1}^{i} (1-\alpha^*_{k-j}), \end{aligned} \tag{19}$$

- *the stochastic process* $e_{k|k-1}$ *is bounded w.p.o..*

Remark. *The conditions in Lemma 2 can be interpreted as follows: To prove the boundedness of* $e_{k|k-1}$, *it is sufficient by constructing a function* $V_k(e_{k|k-1})$ *such that both its drift, i.e,* (18), *and* $\frac{V_k(e_{k|k-1})}{e^2_{k|k-1}}$, *i.e,* (16), (17), *are bounded.*

IV. System Model and Problem Formulation

A. System Model

Consider a RFID system consisting of a reader and a mass of tags operating on one frequency channel. The number of tags is unknown *a priori* and can be constant or dynamic (time-varying), which we refer to as *static* and *dynamic* systems, respectively throughout the paper. The MAC protocol for the RFID system is the standard framed-slotted ALOHA protocol, where the standard *Listen-before-Talk* mechanism is employed by the tags to respond the reader's interrogation [24].

The reader initiates a series of frames indexed by an integer $k \in \mathbb{Z}_+$. Each individual frame, referred to as a round, consists of a number of slots. The reader starts frame k by broadcasting a `begin-round` command with frame size L_k, persistence probability r_k and a random seed Rs_k. When a tag receives a `begin-round` command, it uses a hash function $h(\cdot)$, L_k, Rs_k, and its ID to generate a random number i in the range $[0, L_k - 1]$ and reply in slot i of frame k with probability r_k.[1]

Since every tag picks its own response slot individually, there may be zero, one, or more than one tags transmitting in a slot, which are referred to as *idle*, *singleton*, and *collision* slots, respectively. The reader is not assumed to be able to distinguish between a singleton or a collision slot, but it can detect an idle slot. We term both singleton and collision slots as *occupied* slots throughout the paper. By collecting all replies

[1]The outputs of the hash function have a uniform distribution such that the tag can choose any slot within the round with the equal probability.

in a frame, the reader can generate a bit-string B_k illustrated as $B_k = \{\cdots|0|0|1|0|1|1|\cdots\}$, where '0' indicates an idle slot, and '1' stands for an occupied one.

Subsequently, the reader finalizes the current frame by sending an `end round` command. Based on the number of idle slots, i.e., the number of '0' in B_k, the reader runs the estimation algorithm, detailed in the next section, to trace the tag population.

B. Tag Population Estimation Problem

Our objective is to design a stable and accurate tag population estimation algorithm for both static and dynamic systems. By stable and accurate we mean that

- the estimation error of our algorithm is bounded in the sense of Definition 2 and 3 and the relative estimation error tends to zero;
- the estimated population size converges to the real value with exponential rate.

Mathematically, we consider a large-scale RFID system of a reader and a set of tags with the unknown size z_k in frame k which can be static or dynamic. Denote by $\hat{z}_{k|k-1}$ the prior estimate of z_k in the beginning of frame k. At the end of frame k, the reader updates the estimate $\hat{z}_{k|k-1}$ to $\hat{z}_{k|k}$ by running the estimation algorithm. Our designed estimation scheme need to guarantee the following properties:

- $\lim\limits_{z_k\to\infty}\left|\dfrac{\hat{z}_{k|k-1}-z_k}{z_k}\right| = 0$;
- the converges rate is exponential.

V. Tag Population Estimation: Static Systems

In this section, we focus on the baseline scenario of static systems where the tag population is constant during the estimation process. We first establish the discrete-time model for the system dynamics and the measurement model using the bit string B_k observed during frame k. We then present our EKF-based estimation algorithm.

A. System Dynamics and Measurement Model

Consider the static RFID systems where the tag population stays constant, the system state evolves as

$$z_{k+1} = z_k, \tag{20}$$

meaning that the number of tags z_{k+1} in the system in frame $k+1$ equals that in frame k.

In order to estimate z_k, we leverage the measurement on the number of idle slots during a frame. To start, we study the stochastic characteristics of the number of idle slots.

Assume that the initial tag population z_0 falls in the interval $z_0 \in [\underline{z}_0, \overline{z}_0]$, yet the exact value of z_0 is unknown and should be estimated. The range $[\underline{z}_0, \overline{z}_0]$ can be a very coarse estimation that can be obtained by any existing population estimation method. Recall the system model that in frame k, the reader probes the tags with the frame size L_k. Denote by variable N_k the number of idle slots in frame k, that is, the number of '0's in B_k, we have the following results on N_k according to [14], [25].

Lemma 3. *If each tag replies in a random slot among the L_k slots with probability r_k, then it holds that $N_k \sim \mathcal{N}[\mu, \sigma^2]$ for large L_k and z_k, where $\mu = L_k(1-\frac{r_k}{L_k})^{z_k}$ and $\sigma^2 = L_k(L_k-1)(1-\frac{2r_k}{L_k})^{z_k} + L_k(1-\frac{r_k}{L_k})^{z_k} - {L_k}^2(1-\frac{r_k}{L_k})^{2z_k}$.*

Lemma 4. *For any $\epsilon^* > 0$, there exists some $M > 0$, such that if $z_k \geq M$ or $L_k = \hat{z}_{k|k-1} \geq M$, then it holds that*

$$\left|\mu - L_k e^{-r_k\rho}\right| \leq \epsilon^*, \tag{21}$$

$$\left|\sigma^2 - L_k(e^{-r_k\rho} - (1+r_k^2\rho)e^{-2r_k\rho})\right| \leq \epsilon^*, \tag{22}$$

where $\rho = \frac{z_k}{L_k}$ is referred to as the reader load factor.

Lemmas 3 and 4 imply that in large-scale RFID systems, we can use $L_k e^{-r_k\rho}$ and $L_k(e^{-r_k\rho} - (1+r_k^2\rho)e^{-2r_k\rho})$ to approximate μ and σ^2.

At the end of each frame k, the reader gets a measure y_k of the idle slot frequency defined as

$$y_k = \frac{N_k}{L_k}. \tag{23}$$

Recall Lemma 3, it holds that y_k is a Normal distributed random variable specified as follows: $E[y_k] = e^{-r_k\rho}$ and $Var[y_k] = \frac{1}{L_k}(e^{-r_k\rho} - (1+r_k^2\rho)e^{-2r_k\rho})$. Since there are z_k tags reply in frame k with probability r_k, the probability that a slot is idle, denoted as $p(z_k)$, can be calculated as

$$p(z_k) = (1-\frac{r_k}{L_k})^{z_k} \approx e^{-\frac{r_k z_k}{L_k}}. \tag{24}$$

Notice that for large z_k, $p(z_k)$ can be regarded as a continuously differentiable function of z_k.

Using the language in the Kalman filter, we can write y_k as follows:

$$y_k = p(z_k) + u_k, \tag{25}$$

where, based on the statistic characteristics of y_k, u_k is a Gaussian random variable with zero mean and variance

$$Var[u_k] = \frac{1}{L_k}(e^{-r_k\rho} - (1+r_k^2\rho)e^{-2r_k\rho}). \tag{26}$$

We note that u_k measures the uncertainty of y_k.

To summarise, the discrete-time model for static RFID systems is characterized by (20) and (25).

B. Tag Population Estimation Algorithm

Noticing that the system state characterised by (20) and (25) is a discrete-time nonlinear system, we thus leverage the two-step EKF described in Definition 1 to estimate the system state. In (7), the Kalman gain K_k increases with Q_k while decreases with R_k. As a result, Q_k and R_k can be used to tune the EKF such that increasing Q_k and/or decreasing R_k accelerates the convergence rate but leads to larger estimation error. In our design, we set Q_k to a constant $q > 0$ and introduce a parameter ϕ_k as follows to replace R_k to facilitate our demonstration:

$$R_k = \phi_k P_{k|k-1} {C_k}^2. \tag{27}$$

It can be noted from (7) and (27) that K_k is monotonously decreasing in ϕ_k, i.e., a small ϕ_k leads to quick convergence with the price of relatively high estimation error. Hence, choosing the appropriate value for ϕ_k consists of striking a balance between the convergence rate and the estimation error. In our work, we take a dynamic approach by setting ϕ_k to a small value $\underline{\phi}$ but satisfying (62) at the first few rounds (J rounds) of estimation to allow the system to act quickly since

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the estimation in the beginning phase can be very coarse. After that we set ϕ_k to a relatively high value $\overline{\phi}$ to achieve high estimation accuracy.

Now, we present our tag population estimation algorithm in Algorithm 1 where $P_{0|0}$, q can be set to some constants straightforward since the performance mostly depends on ϕ_k and k_{max} is the time horizon during which the system needs to be monitored, e.g., if the RFID system needs to be monitored from time T_1 to T_2, then at T_1, k_{max} is simply set as $T_2 - T_1$. The major procedures can be summarised as:

1) *In the beginning of frame k: prediction (line 3).* The reader first predicts the number of tags based on the estimation at the end of frame $k-1$. The predicted value is defined as $\hat{z}_{k|k-1}$. Then the reader sets the persistence probability r_k following Lemma 8 and z_k is set to $\hat{z}_{k|k-1}$.
2) *Line 4-5*. The reader launches the *Listen-before-talk* protocol as introduced in IV-A in order to receive the feedbacks from tags.
3) *At the end of frame k: correction (line 6-14)*. The reader computes N_k based on B_k and further calculates y_k and v_k from N_k. It then updates the prediction with the corrected estimate $\hat{z}_{k|k}$ following (5).

We will theoretically establish the stability and accuracy of the algorithm in Sec. VII.

Algorithm 1 Tag population estimation (static cases): executed by the reader

Input: $\underline{z}_0$, $P_{0|0}$, q, J, L, $\underline{\phi}$, $\overline{\phi}$, maximum number of rounds k_{max}
Output: Estimated tag population set $S_z = \{\hat{z}_{k|k} : k \in [0, k_{max}]\}$
1: **Initialisation:** $\hat{z}_{0|0} \leftarrow \underline{z}_0$, $Q_0 \leftarrow q$, $S_z = \{\hat{z}_{0|0}\}$
2: **for** $k = 1$ to k_{max} **do**
3: $\quad \hat{z}_{k|k-1} \leftarrow \hat{z}_{k-1|k-1}$, $L_k \leftarrow L$, $r_k \leftarrow 1.59 L_k / \hat{z}_{k|k-1}$, $P_{k|k-1} \leftarrow P_{k-1|k-1} + Q_{k-1}$
4: $\quad$ Generate a new random seed Rs_k and broadcast (L_k, r_k, Rs_k)
5: $\quad$ Run *Listen-before-Talk* protocol
6: $\quad$ Obtain the number of idle slots N_k, and compute y_k and v_k using (23) and (8)
7: $\quad Q_k \leftarrow q$
8: $\quad$ **if** $k \leq J$ **then**
9: $\quad\quad \phi_k \leftarrow \underline{\phi}$
10: $\quad$ **else**
11: $\quad\quad \phi_k \leftarrow \overline{\phi}$
12: $\quad$ **end if**
13: $\quad$ Calculate R_k and K_k using (27) and (7)
14: $\quad$ Update $\hat{z}_{k|k}$ and $P_{k|k}$ using (5) and (6)
15: $\quad S_z \leftarrow S_z \cup \{\hat{z}_{k|k}\}$
16: **end for**

VI. Tag Population Estimation: Dynamic Systems

In this section, we further tackle the dynamic case where the tag population may vary during the estimation process. The objective for the dynamic systems is to promptly detect the global tag papulation change and accurately estimate the quantity of this change. To that end, we first establish the system model and then present our estimation algorithm.

A. System Dynamics and Measurement Model

In dynamic RFID systems, we can formulate the system dynamics as

$$z_{k+1} = z_k + w_k, \tag{28}$$

where the tag population z_{k+1} in frame $k+1$ consists of two parts: i) the tag population in frame k and ii) a random variable w_k which accounts for the stochastic variation of tag population resulting from the tag arrival/departure during frame k. Notice that w_k is referred to as process noise in Kalman filters and the appropriate characterisation of w_k is crucial in the design of stable Kalman filters, which will be investigated in detail later. Besides, the measurement model is the same as the static case. Hence, the discrete-time model for dynamic RFID systems can be characterized by (28) and (25).

B. Tag Population Estimation Algorithm

In the dynamic case, we leverage the two-step EKF to estimate the system state combined with the CUSUM test to further trace the tag population fluctuation.

Algorithm 2 Tag population estimation (unified framework): executed by the reader

Input: $\underline{z}_0$, $P_{0|0}$, q, J, L, $\underline{\phi}$, $\overline{\phi}$, maximum number of rounds k_{max}
Output: Estimation set $S_z = \{\hat{z}_{k|k} : k \in [0, k_{max}]\}$
Initialisation: $\hat{z}_{0|0} \leftarrow \underline{z}_0$, $Q_0 \leftarrow q$, $S_z = \{\hat{z}_{0|0}\}$
for $k = 1$ to k_{max} **do**
$\quad \hat{z}_{k|k-1} \leftarrow \hat{z}_{k-1|k-1}$, $L_k \leftarrow L$, $P_{k|k-1} \leftarrow P_{k-1|k-1} + Q_{k-1}$, $r_k \leftarrow \min(1, 1.59 L_k / \hat{z}_{k|k-1})$,
$\quad$ Generate a new seed Rs_k and broadcast (L_k, Rs_k) and run *Listen-before-Talk* protocol
$\quad$ Obtain the number of idle slots N_k, and compute y_k and v_k using (23) and (8)
$\quad Q_k \leftarrow q$
$\quad$ **if** $k \leq J$ **then**
$\quad\quad \phi_k \leftarrow \underline{\phi}$
$\quad$ **else**
$\quad\quad$ Execute Algorithm 3
$\quad\quad \phi_k \leftarrow$ *output of Algorithm 3*
$\quad$ **end if**
$\quad$ Calculate R_k and K_k using (27) and (7), and update $\hat{z}_{k|k}$ and $P_{k|k}$ using (5) and (6)
$\quad S_z \leftarrow S_z \cup \{\hat{z}_{k|k}\}$
end for

Our main estimation algorithm is illustrated in Algorithm 2. The difference compared to the static scenario is that tag population variation needs to be detected by the CUSUM test presented in Algorithm 3 in the next subsection and the output of Algorithm 3 acts as a feedback to ϕ_k, meaning ϕ_k is no more a constant after the first J rounds as the static case due to the tag population variation. Specifically, if a change on the tag population is detected in frame k, ϕ_k is set to $\underline{\phi}$ to quickly

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react to the change, otherwise ϕ_k sticks to $\overline{\phi}$ to stabilize the estimation. We note that in the case where z_k is constant, Algorithm 2 degenerates to Algorithm 1.

C. Detecting Tag Population Change: CUSUM Test

The CUSUM Detection Framework. We leverage the CUSUM test to detect the change of tag population and further adjust ϕ_k. CUSUM test is a sequential analysis technique typically used for change detection [26]. It is shown to be asymptotically optimal in the sense of the minimum detection time subject to a fixed worst-case expected false alarm rate [27].

In the context of dynamic tag population detection, the reader monitors the innovation process $v_k = y_k - p(\hat{z}_{k|k-1})$. If the number of the tags population is constant, v_k equals to u_k which is a Gaussian process with zero mean. In contrast, upon the system state changes, i.e., tag population changes, v_k drifts away from the zero mean. In our design, we use Φ_k as a normalised input to the CUSUM test by normalising v_k with its estimated standard variance, specified as follows:

$$\Phi_k = \frac{v_k}{\sqrt{(P_{k|k-1} + Q_{k-1}){C_k}^2 + Var[u_k]\big|_{z_k=\hat{z}_{k|k-1}}}}. \quad (29)$$

The reader updates the CUSUM statistics g_k^+ and g_k^- as follows:

$$g_k^+ = \max\{0, g_{k-1}^+ + \Phi_k - \Upsilon\}, \quad (30)$$
$$g_k^- = \min\{0, g_{k-1}^- + \Phi_k + \Upsilon\}, \quad (31)$$
$$g_k^+ = g_k^- = 0, \text{if } \delta = 1, \quad (32)$$

where g_0^+=0 and $g_0^- = 0$. And Υ≥0, referred to as reference value, is a filter design parameter indicating the sensitivity of the CUSUM test to the fluctuation of Φ_k, Moreover, by δ we define an indicator flag indicating tag population change:

$$\delta = \begin{cases} 1 & \text{if } g_k^+ > \theta \text{ or } g_k^- < -\theta, \\ 0 & \text{otherwise}, \end{cases} \quad (33)$$

where $\theta > 0$ is a pre-specified CUSUM threshold.

The detailed procedure of the change detection is illustrated in Algorithm 3 where $\varphi_1(\delta)$ shown in (37)is used to assign the value to ϕ_k according to whether the system state changes.

Algorithm 3 CUSUM test: executed by the reader in frame k

Input: Υ, θ
Output: ϕ_k
Initialisation: $g_0^+ \leftarrow 0$, $g_0^- \leftarrow 0$
Compute Φ_k using equation (29)
$g_k^+ \leftarrow$ (30), $g_k^- \leftarrow$ (31)
if $g_k^+ > \theta$ or $g_k^- < -\theta$ **then**
  $\delta \leftarrow 1$, $\phi_k \leftarrow \varphi_1(\delta)$, $g_k^+ \leftarrow 0$, $g_k^- \leftarrow 0$
else
  $\delta \leftarrow 0$, $\phi_k \leftarrow \varphi_1(\delta)$
end if
Return ϕ_k

Parameter tuning in CUSUM test. The choice of the threshold θ and the drift parameter Υ has a directly impact on the performance of the CUSUM test in terms of detection delay and false alarm rate. Formally, the average running length (ARL) $L(\mu^*)$ is used to denote the duration between two actions [28]. For a large θ, $L(\mu^*)$ is approximated as

$$L(\mu^*) = \begin{cases} \Theta(\theta), & \text{if } \mu^* \neq 0, \\ \Theta(\theta^2), & \text{if } \mu^* = 0, \end{cases} \quad (34)$$

where μ^* denotes the mean of the process Φ_k.

In our context, ARL corresponds to the mean time between two false alarms in the static case and the mean detection delay of the tag population change in the dynamic case. It is easy to see from (34) that a higher value of θ leads to lower false alarm rate at the price of longer detection delay. Therefore, the choices of θ and Υ consists of a tradeoff between the false alarm rate and the detection delay.

Recall that Φ_k can be approximated to a white noise process, i.e, $\Phi_k \sim \mathcal{N}[\mu^*, {\sigma^*}^2]$ with $\mu^* = 0$, $\sigma^* = 1$ if the system state does not change. Generically, as recommended in [29], setting θ and Υ as follows achieves good ARL from the engineering perspective.

$$\theta = 4\sigma^*, \quad (35)$$
$$\Upsilon = \mu^* + 0.5\sigma^*. \quad (36)$$

In the CUSUM framework, we set ϕ_k by $\varphi_1(\delta)$ as follows:

$$\varphi_1(\delta) = \begin{cases} \underline{\phi}, & \text{if } \delta = 1, \\ \overline{\phi}, & \text{if } \delta = 0. \end{cases} \quad (37)$$

The rationale is that once a change on the tag population is detected in frame k, ϕ_k is set to $\underline{\phi}$ to quickly react to the change, while ϕ_k sticks to $\overline{\phi}$ when no system change is detected.

Finally, we conclude this section by summarizing the algorithm parameter settings.

TABLE I
ALGORITHM PARAMETER SETTINGS

Parameters	Settings
$P_{0\|0}, J, q$	constants as shown in Sec. IX
k_{max}	required monitoring time as shown in Sec. IX
ϕ_k	following Algorithm 3 while satisfying (62)
L_k, r_k	$L_k = L, r_k = \min(1, 1.59L/\hat{z}_{k\|k-1})$ with constant L
θ, Υ	based on (35) and (36) following [29]

VII. PERFORMANCE ANALYSIS

In this section, we establish the stability and the accuracy of our estimation algorithms for both static and dynamic cases.

A. Static Case

Our analysis is composed of two steps. We first derive the estimation error and then establish the stability and the accuracy of Algorithm 1 in terms of the boundedness of estimation error.

Computing Estimation Error. We first approximate the non-linear discrete system by a linear one. To that end, as the function $p(z_k)$ is continuously differentiable at $z_k = \hat{z}_{k|k-1}$, using the Taylor expansion, we have

$$p(z_k) = p(\hat{z}_{k|k-1}) + C_k(z_k - \hat{z}_{k|k-1}) + \chi(z_k, \hat{z}_{k|k-1}), \quad (38)$$

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where $C_k = -\frac{r_k \rho}{e^{r_k \rho} \hat{z}_{k|k-1}}$, (39)

$$\chi(z_k, \hat{z}_{k|k-1}) = \sum_{j=2}^{\infty} \frac{1}{e^{r_k \rho} j!} (r_k \rho - \frac{r_k \rho z_k}{\hat{z}_{k|k-1}})^j. \quad (40)$$

Regarding the convergence of $\chi(z_k, \hat{z}_{k|k-1})$ in (40), assume that

$$z_k = a'_k \hat{z}_{k|k-1}, \quad (41)$$

we can obtain the following boundedness of the residual for the case $|a'_k - 1| < \frac{1}{r_k \rho}$:

$$|\chi(z_k, \hat{z}_{k|k-1})| = \sum_{j=0}^{\infty} \frac{(\hat{z}_{k|k-1} - z_k)^2 (r_k \rho)^{j+2}}{e^{r_k \rho} \hat{z}_{k|k-1}^2 (j+2)!} \Big|1 - \frac{z_k}{\hat{z}_{k|k-1}}\Big|^j$$
$$\leq \frac{(r_k \rho)^2 (\hat{z}_{k|k-1} - z_k)^2}{2 e^{(r_k \rho)} a_k \hat{z}_{k|k-1}^2}, \quad (42)$$

where $a_k = 1 - (r_k \rho)|1 - a'_k|$. (43)

Recall the definition of the estimation error in (10) and using (20), (3) and (5), we can derive the estimation error $e_{k+1|k}$ as follows:

$$e_{k+1|k} = z_{k+1} - \hat{z}_{k+1|k} = z_k - \hat{z}_{k|k}$$
$$= z_k - \hat{z}_{k|k-1} - K_k \left[C_k (z_k - \hat{z}_{k|k-1}) + \chi(z_k, \hat{z}_{k|k-1}) + u_k \right]$$
$$= (1 - K_k C_k) e_{k|k-1} + s_k + m_k, \quad (44)$$

where s_k and m_k are defined as

$$s_k = -K_k u_k, \quad (45)$$
$$m_k = -K_k \chi(z_k, \hat{z}_{k|k-1}). \quad (46)$$

Boundedness of Estimation Error. Having derived the dynamics of the estimation error, we now state the main result on the stochastic stability and accuracy of Algorithm 1.

Theorem 1. *Consider the discrete-time stochastic system given by* (20) *and* (25) *and Algorithm 1, the estimation error* $e_{k|k-1}$ *defined by* (10) *is exponentially bounded in mean square and bounded w.p.o., if the following conditions hold:*

1) *there are positive numbers* $\underline{q}$, $\overline{q}$, $\underline{\phi}$ *and* $\overline{\phi}$ *such that the bounds on* Q_k *and* ϕ_k *are satisfied for every* $k \geq 0$*, as in*
$$\underline{q} \leq Q_k \leq \overline{q}, \quad (47)$$
$$\underline{\phi} \leq \phi_k \leq \overline{\phi}, \quad (48)$$
2) *The initialization must follow the rules*
$$P_{0|0} > 0, \quad (49)$$
$$|e_{1|0}| \leq \epsilon \quad (50)$$
with positive real number $\epsilon > 0$.

Remark. *By referring to the design objective posed in Section IV, Theorem 1 prove the following properties of our estimation algorithm:*

- *the estimation error of our algorithm is bounded in mean square and the relative estimation error tends to zero;*
- *the estimated population size converges to the real value with exponential rate.*

Moreover, the conditions in Theorem 1 can be interpreted as follows:

1) *The inequalities* (47) *and* (48) *can be satisfied by the configuring the correspondent parameters in Algorithm 1, which guarantees the boundedness of the pseudo-covariance* $P_{k|k-1}$ *as shown later.*
2) *The inequality* (49) *consists of establishing positive* $P_{k|k-1}$ *for every* $k \geq 1$.
3) *As a sufficient condition for stability, the upper bound* ϵ *may be too stringent. As shown in the simulation results, stability is still ensured even with a relatively large* ϵ.

Before the proof of Theorem 1, we first state several auxiliary lemmas to streamline the proof and show how to apply these lemmas to prove Theorem 1 subsequently.

Lemma 5. *Under the conditions of Theorem 1, if* $P_{0|0} > 0$*, there exist* $\underline{p}_k, \overline{p}_k > 0$ *such that the pseudo-covariance* $P_{k|k-1}$ *is bounded for every* $k \geq 1$*, i.e.,*

$$\underline{p}_k \leq P_{k|k-1} \leq \overline{p}_k. \quad (51)$$

Proof: Recall (4) and (6), we have $P_{k|k-1} \geq Q_{k-1}$, and

$$P_{k|k-1} = P_{k-1|k-2}(1 - K_{k-1} C_{k-1}) + Q_{k-1}$$
$$= P_{k-1|k-2} - \frac{P_{k-1|k-2}^2 C_{k-1}^2}{P_{k-1|k-2} {C_{k-1}}^2 + R_{k-1}} + Q_{k-1}. \quad (52)$$

Following the design of R_k in (27) and by iteration, we further get

$$P_{k|k-1} = P_{k-1|k-2} \left(1 - \frac{1}{1 + \phi_{k-1}}\right) + Q_{k-1} = P_{1|0}$$
$$\cdot \prod_{i=1}^{k-1} \left(1 - \frac{1}{1 + \phi_i}\right) + \sum_{i=0}^{k-2} Q_i \prod_{j=i}^{k-2} \left(1 - \frac{1}{1 + \phi_{j+1}}\right) + Q_{k-1}.$$

Since ϕ_k and Q_k are controllable parameters, we can set $\underline{\phi}_k \leq \overline{\phi}$ and $Q_k \leq \overline{q}$ for every $k \geq 0$ in Algorithm 1, where $\overline{\phi}, \overline{q} > 0$. Consequently, we have

$$P_{k|k-1} \leq P_{1|0} \left(1 - \frac{1}{1 + \overline{\phi}}\right)^{k-1} + \overline{q} \sum_{j=1}^{k-1} \left(1 - \frac{1}{1 + \overline{\phi}}\right)^j +$$
$$Q_{k-1} \leq (P_{0|0} + Q_0) \left(1 - \frac{1}{1 + \overline{\phi}}\right)^{k-1} + \overline{q}\overline{\phi} + Q_{k-1} \quad (53)$$

Let $\overline{p}_k = ((P_{0|0} + Q_0) \left(1 - \frac{1}{1 + \overline{\phi}}\right)^{k-1} + \overline{q}\overline{\phi} + Q_{k-1}$ and $\underline{p}_k = Q_{k-1}$, we have $\underline{p}_k \leq P_{k|k-1} \leq \overline{p}_k$. ■

Lemma 6. *Let* $\alpha_k \triangleq \frac{1}{1 + \phi_k}$*, it holds that*

$$\frac{(1 - K_k C_k)^2}{P_{k+1|k}} e_{k|k-1}^2 \leq (1 - \alpha_k) \frac{e_{k|k-1}^2}{P_{k|k-1}}, \ \forall k \geq 1. \quad (54)$$

Proof: From (52), we have

$$P_{k+1|k} = P_{k|k-1} \left(1 - K_k C_k\right) + Q_k$$
$$\geq P_{k|k-1} \left(1 - K_k C_k\right). \quad (55)$$

By substituting it into the left-hand side of (54) and using the fact that $R_k = \phi_k P_{k|k-1} {C_k}^2$ for every $k \geq 1$, we get

$$\frac{(1 - K_k C_k)^2}{P_{k+1|k}} e_{k|k-1}^2 \leq \frac{(1 - K_k C_k)^2}{P_{k|k-1} \left(1 - K_k C_k\right)} e_{k|k-1}^2$$
$$\leq (1 - K_k C_k) \frac{{e_{k|k-1}}^2}{P_{k|k-1}} \leq \left(1 - \frac{1}{1 + \phi_k}\right) \frac{e_{k|k-1}^2}{P_{k|k-1}}.$$

We are thus able to prove (54). ■

Lemma 7. *Let* $b_k \triangleq \frac{r_k \rho (4 a_k \phi_k + 1 - a_k)}{4 a_k^2 \phi_k (1 + \phi_k) \hat{z}_{k|k-1} P_{k|k-1}}$*, it holds that*

$$\frac{m_k [2(1 - K_k C_k) e_{k|k-1} + m_k]}{P_{k+1|k}} \leq b_k |\hat{z}_{k|k-1} - z_k|^3. \quad (56)$$

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Proof: From (46), we get the following expansion

$$\frac{m_k[2(1-K_kC_k)e_{k|k-1}+m_k]}{P_{k+1|k}} = \frac{-P_{k|k-1}C_k\chi(z_k,\hat{z}_{k|k-1})}{P_{k+1|k}(P_{k|k-1}{C_k}^2+R_k)}\cdot\left[2\left(1-\frac{P_{k|k-1}{C_k}^2}{P_{k|k-1}{C_k}^2+R_k}\right)\cdot e_{k|k-1}-\frac{P_{k|k-1}C_k}{P_{k|k-1}{C_k}^2+R_k}\chi(z_k,\hat{z}_{k|k-1})\right].$$

It then follows from (39), (41) and (55) that

$$\frac{m_k[2(1-K_kC_k)e_{k|k-1}+m_k]}{P_{k+1|k}} \le \frac{-P_{k|k-1}C_k(r_k\rho)^2}{2e^{r_k\rho}a_k\hat{z}_{k|k-1}^2P_{k|k-1}} \frac{(\hat{z}_{k|k-1}-z_k)^2}{(1-K_kC_k)(P_{k|k-1}{C_k}^2+R_k)}\left[2-\frac{2P_{k|k-1}{C_k}^2}{P_{k|k-1}{C_k}^2+R_k}\cdot|\hat{z}_{k|k-1}-z_k|-\frac{P_{k|k-1}C_k}{P_{k|k-1}{C_k}^2+R_k}\frac{(r_k\rho)^2(\hat{z}_{k|k-1}-z_k)^2}{2e^{1.59}a_k\hat{z}_{k|k-1}^2}\right] \le \frac{r_k\rho(4a_k\phi_k+1-a_k)}{4a_k^2\phi_k(1+\phi_k)\hat{z}_{k|k-1}P_{k|k-1}}|\hat{z}_{k|k-1}-z_k|^3.$$

We are thus able to prove (56). ■

Lemma 8. $E\left[\frac{{s_k}^2}{P_{k+1|k}}|e_{k|k-1}\right] \le \frac{2.46\hat{z}_{k|k-1}}{\phi_k(1+\phi_k)r_kP_{k|k-1}}$ *when* $r_k\rho = 1.59$.

Proof: From (45), we have $E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right] = \frac{K_k^2E[u_k^2]}{P_{k+1|k}}$. With (7), (26) and (55), we have $E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right] \le \frac{e^{2r_k\rho}\hat{z}_{k|k-1}(e^{-r_k\rho}-(1+r_k^2\rho)e^{-2r_k\rho})}{\phi_k(1+\phi_k)P_{k|k-1}\rho r_k^2}$.

Since item $E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right]$ influences the estimation accuracy, we set the optimal persistence probability to minimize this item. Denote $\Lambda(r_k) = \frac{e^{2r_k\rho}}{r_k^2}(e^{-r_k\rho}-(1+r_k^2\rho)e^{-2r_k\rho})$, we have

$$\frac{\mathrm{d}\Lambda}{\mathrm{d}r_k} = \frac{(r_k\rho-2)e^{r_k\rho}+2}{r_k^3}.$$

Since $r_k\rho > 0$ and $\frac{\mathrm{d}((r_k\rho-2)e^{r_k\rho}+2)}{\mathrm{d}r_k\rho} = (r_k\rho-1)e^{r_k\rho}$ which is greater zero if $r_k\rho > 1$ and is smaller than zero if $r_k\rho < 1$, and 1) if $r_k\rho = 1$, $\frac{\mathrm{d}\Lambda}{\mathrm{d}r_k} < 0$; 2) if $r_k\rho = 0$, $\frac{\mathrm{d}\Lambda}{\mathrm{d}r_k} = 0$; 3) if $r_k\rho = 2$, $\frac{\mathrm{d}\Lambda}{\mathrm{d}r_k} > 0$, there exists a unique solution $r_k\rho \in (1,2)$ for $\frac{\mathrm{d}\Lambda}{\mathrm{d}r_k} = 0$ such that $\Lambda(r_k)$ is minimized. Searching in $(1,2)$, we find the optimal $r_k\rho = 1.59$. Therefore, we can obtain that

$$E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right] \le \frac{2.46\hat{z}_{k|k-1}}{\phi_k(1+\phi_k)r_kP_{k|k-1}} \triangleq \xi_k, \quad (57)$$

which completes the proof. ■

Armed with the above auxiliary lemmas, we next prove Theorem 1.

Proof of Theorem 1: First, we construct the following Lyapunov function to define the stochastic process:

$$V_k(e_{k|k-1}) = \frac{e_{k|k-1}^2}{P_{k|k-1}},$$

which satisfies (4) and (49) as $P_{k|k-1} > 0$.

Next, we use Lemma 2 to develop the proof. Because it follows from Lemma 5 that the properties (16) and (17) in Lemma 2 are satisfied, the main task left is to prove (18).

From (44), expanding $V_{k+1}(e_{k+1|k})$ leads to

$$V_{k+1}(e_{k+1|k}) = \frac{e_{k+1|k}^2}{P_{k+1|k}} = \frac{[(1-K_kC_k)e_{k|k-1}+s_k+m_k]^2}{P_{k+1|k}} = \frac{(1-K_kC_k)^2}{P_{k+1|k}}e_{k|k-1}^2+\frac{m_k[2(1-K_kC_k)e_{k|k-1}+m_k]}{P_{k+1|k}} + \frac{2s_k[(1-K_kC_k)e_{k|k-1}+m_k]}{P_{k+1|k}}+\frac{s_k^2}{P_{k+1|k}}.$$

Furthermore, by Lemmas 6, 7 and 8 and some algebraic operations, we have

$$E\left[V_{k+1}(e_{k+1|k})|e_{k|k-1}\right]-V_k(e_{k|k-1}) \le -\alpha_kV_k(e_{k|k-1})+b_k|e_{k|k-1}|^3+\xi_k. \quad (58)$$

To obtain the same formation with (18), we further proceed to bound the second term in b_k in (58) as follows:

$$b_k|e_{k|k-1}|^3 \le \varsigma\alpha_kV_k(e_{k|k-1}), \quad (59)$$

where $0 < \varsigma < 1$ is preset controllable parameter. To prove the above inequality, we need to prove $|e_{k|k-1}| \le \frac{4\varsigma a_k^2\phi_k\hat{z}_{k|k-1}}{1.59(4a_k\phi_k+1.59|a_k'-1|)}$. Since $|e_{k|k-1}| = |a_k'-1|\hat{z}_{k|k-1}$, it suffices to show

$$|a_k'-1| \le \frac{4\varsigma a_k^2\phi_k}{1.59(4a_k\phi_k+1.59|a_k'-1|)}, \quad (60)$$

which is equivalent to $(1-4\phi_k-4\phi_k\varsigma){a_k}^2+(4\phi_k-2)a_k+1 \le 0$ because of (43). With some algebraic operations, we obtain 1) $\frac{1-2\phi_k-2\sqrt{\phi_k(\phi_k+\varsigma)}}{1-4\phi_k(1+\varsigma)} < a_k \le 1$, if $\phi_k < \frac{1}{4(1+\varsigma)}$; and 2) $\frac{2\phi_k-1+2\sqrt{\phi_k(\phi_k+\varsigma)}}{4\phi_k(1+\varsigma)-1} \le a_k \le 1$, if $\phi_k > \frac{1}{4(1+\varsigma)}$; and 3) $\frac{1+\varsigma}{1+2\varsigma} \le a_k \le 1$, if $\phi_k = \frac{1}{4(1+\varsigma)}$. Since it holds that $\frac{2\phi_k-1+2\sqrt{\phi_k(\phi_k+\varsigma)}}{4\phi_k(1+\varsigma)-1} < \frac{1+\varsigma}{1+2\varsigma}$ for every ς and $\frac{2\phi_k-1+2\sqrt{\phi_k(\phi_k+\varsigma)}}{4\phi_k(1+\varsigma)-1}$ will decrease monotonically to $\frac{1}{1+\varsigma}$ for a large ϕ_k, we have in the worst case for $\phi_k \ge \frac{1}{4(1+\varsigma)}$,

$$\frac{1+\varsigma}{1+2\varsigma} \le a_k \le 1. \quad (61)$$

It follows from the analysis that if we set

$$\phi_k \ge \frac{1}{4(1+\varsigma)}, \quad (62)$$

(60) can be satisfied. Moreover, it holds that

$$|a_k'-1| \le \frac{0.63\varsigma}{1+2\varsigma}. \quad (63)$$

That is,

$$|e_{k|k-1}| \le \epsilon_k, \quad (64)$$

where $\epsilon_k \triangleq \frac{0.63\varsigma}{1+2\varsigma}\hat{z}_{k|k-1}$. By setting ϕ_k in (62), for $|e_{k|k-1}| \le \epsilon_k$, we thus have

$$E\left[V_{k+1}(e_{k+1|k})|e_{k|k-1}\right]-V_k(e_{k|k-1}) \le -(1-\varsigma)\alpha_kV_k(e_{k|k-1})+\xi_k. \quad (65)$$

Therefore, we are able to apply Lemma 2 to prove Theorem 1 by setting $\epsilon = \frac{0.63\varsigma}{1+2\varsigma}\hat{z}_{1|0}$, $\beta^* = \frac{1}{Q_0}$, $\alpha_k^* = (1-\varsigma)\alpha_k$, $\beta_k = \frac{1}{\overline{p}_k}$ and $\tau_k = \xi_k$. ■

Remark. *Theorem 1 also holds in the sense of Lemma 1 (the off-line version of Lemma 2) by setting the parameters in* (15) *as* $\overline{\beta} = \frac{1}{Q_0}$, $\alpha = \frac{1-\varsigma}{1+\overline{\phi}} \le \alpha_k^*$, $\underline{\beta} = (P_{0|0}+Q_0+\overline{q}(\overline{\phi}+1) \ge \overline{p}_k$, *and* $\tau = \frac{Q_0\hat{z}_{max}}{\underline{\phi}(1+\underline{\phi})} \ge \xi_k$, *where* $\hat{z}_{max}$ *is the maximum estimate.*

We conclude the analysis on the performance of our estimation algorithm for the static case with a more profound

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investigation on the evolution of the estimation error $|e_{k|k-1}|$. More specifically, we can distinguish three regions:

- *Region 1:* $\sqrt{\frac{2.46M\hat{z}_{k|k-1}}{\phi_k(M-1)r_k(1-\varsigma)}} \leq |e_{k|k-1}| \leq \epsilon_k$. By substituting the condition into the right hand side of (65), we obtain: $-(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + \xi_k \leq -\frac{(1-\varsigma)\alpha_k}{M} V_k(e_{k|k-1})$, where $M > 1$ is a positive constant and can be set beforehand. It then follows that $E[V_{k+1}(e_{k+1|k})|e_{k|k-1}] \leq \left(\frac{M-(1-\varsigma)\alpha_k}{M}\right) V_k(e_{k|k-1})$. Consequently, we can bound $E[e_{k|k-1}^2]$ as:

$$E[e_{k|k-1}^2] \leq \frac{\overline{p}_k}{Q_0} E[{e_{1|0}}^2] \prod_{i=1}^{k-1} (1-\alpha_i^*) \tag{66}$$

  with $\alpha_k^* = \frac{(1-\varsigma)\alpha_k}{M}$. It can then be noted that $E[e_{k|k-1}^2] \to 0$ at an exponential rate as $k \to \infty$.
- *Region 2:* $\sqrt{\frac{2.46\hat{z}_{k|k-1}}{\phi_k r_k(1-\varsigma)}} \leq |e_{k|k-1}| < \sqrt{\frac{2.46M\hat{z}_{k|k-1}}{\phi_k(M-1)r_k(1-\varsigma)}}$. In this case, we have $-\frac{(1-\varsigma)\alpha_k}{M}V_k(e_{k|k-1}) < -(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + \xi_k \leq 0$. It then follows from Lemma 2 that

$$E[e_{k|k-1}^2] \leq \frac{\overline{p}_k}{Q_0} E[{e_{1|0}}^2] \prod_{i=1}^{k-1} (1-\alpha_i^*) + \overline{p}_k \sum_{i=1}^{k-2} \xi_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*).$$

  Hence, when $k \to \infty$, $E[e_{k|k-1}^2]$ converges at exponential rate to $\overline{p}_k \sum_{i=1}^{k-2} \xi_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*) \sim \Theta(\hat{z}_{k|k-1})$, which is decoupled with the initial estimation error and it thus holds $\frac{E[e_{k|k-1}]}{z_k} = \Theta\Big(\frac{1}{\sqrt{z_k}}\Big) \to 0$ when $z_k \to \infty$.
- *Region 3:* $0 \leq |e_{k|k-1}| < \sqrt{\frac{2.46\hat{z}_{k|k-1}}{\phi_k r_k(1-\varsigma)}}$. In this case, we can show that the right hand side of (65) is positive, i.e., $-(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + \xi_k > 0$. It also follows from Lemma 2 that

$$E[e_{k|k-1}^2] \leq \frac{\overline{p}_k}{Q_0} E[{e_{1|0}}^2] \prod_{i=1}^{k-1} (1-\alpha_i^*) + \overline{p}_k \sum_{i=1}^{k-2} \xi_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*).$$

  Hence, when $k \to \infty$, $E[e_{k|k-1}^2]$ converges exponentially to $\overline{p}_k \sum_{i=1}^{k-2} \xi_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*) \sim \Theta(\hat{z}_{k|k-1})$, which is decoupled with the initial estimation error and it thus holds $\frac{E[e_{k|k-1}]}{z_k} \leq \Theta\Big(\frac{1}{\sqrt{z_k}}\Big) \to 0$ when $z_k \to \infty$.

Combining the above three regions, we get the following results on the convergence of the expected estimation error $E[e_{k|k-1}]$: (1) if the estimation error is small (Region 3), it will converge to a value smaller than $\Theta(\sqrt{\hat{z}_{k|k-1}})$ as analysed in Region 3; (2) if the estimation error is larger (Region 1), it will decrease as analysed in Region 1 and fall into either Region 2 or Region 3 where $E[e_{k|k-1}] \leq \Theta(\sqrt{\hat{z}_{k|k-1}})$ such that the relative estimation error $\frac{E[e_{k|k-1}]}{z_k} \to 0$ when $z_k \to \infty$.

B. Dynamic Case

Our analysis on the stability of Algorithm 2 for the dynamic case is also composed of two steps. First, we derive the estimation error. Second, we establish the stability and the accuracy of Algorithm 2 in terms of the boundedness of estimation error.

We first derive the dynamics of the estimation error as follows:

$$e_{k+1|k} = (1-K_kC_k)e_{k|k-1} + s_k + m_k, \tag{67}$$

which differs from the static case (44) in s_k. In the dynamic case, we have

$$s_k = w_k - K_k u_k \tag{68}$$

Next, we show the boundedness of the estimation error in Theorem 2.

Theorem 2. *Under the conditions of Theorem 1, consider the discrete-time stochastic system given by* (28) *and* (25) *and Algorithm 2, if there exist time-varying positive real number* λ_k, $\sigma_k > 0$ *such that*

$$E[w_k] \leq \lambda_k, \tag{69}$$

$$E[{w_k}^2] \leq \sigma_k, \tag{70}$$

then the estimation error $e_{k|k-1}$ *defined by* (10) *is exponentially bounded in mean square and bounded w.p.o..*

Remark. *Note that the condition* $E[w_k] \leq \lambda_k$ *always holds for* $E[w_k] < 0$, *we thus focus on the case that* $E[w_k] \geq 0$. *In the proof, the explicit formulas of* λ_k *and* σ_k *are derived. As in the static case, the conditions may be too stringent such that the results still hold even if the conditions are not satisfied, as illustrated in the simulations.*

The proof of Theorem 2 is also based on Lemmas 6, 7 and 8, but due to the introduction of w_k into s_k, we need another two auxiliary lemmas on $E[s_k]$ and $E[s_k^2]$.

Lemma 9. *If* $E[w_k] \geq 0$, *then there exists a time-varying real number* $d_k > 0$ *such that*

$$E\Big[\frac{2s_k[(1-K_kC_k)e_{k|k-1} + m_k]}{P_{k+1|k}}\Big|e_{k|k-1}\Big] \leq d_k|e_{k|k-1}|E[w_k].$$

Proof: When $E[w_k] \geq 0$, from $E[v_k] = 0$, (41), (55) and the independence between w_k and $e_{k|k-1}$, we can derive

$$\begin{aligned} &E\left[\frac{2s_k[(1-K_kC_k)e_{k|k-1} + m_k]}{P_{k+1|k}}\Big|e_{k|k-1}\right] \\ &\leq 2E[w_k]\frac{1+\phi_k}{\phi_k P_{k|k-1}}\left[\frac{\phi_k|e_{k|k-1}|}{1+\phi_k} + \frac{1.59|e_{k|k-1}|^2}{2a_k(1+\phi_k)\hat{z}_{k|k-1}}\right] \\ &\leq E[w_k]\frac{2a_k\phi_k + (1-a_k)}{a_k\phi_k P_{k|k-1}}|e_{k|k-1}|. \end{aligned}$$

By setting

$$d_k = \frac{2a_k\phi_k + (1-a_k)}{a_k\phi_k P_{k|k-1}}, \tag{71}$$

we thus complete the proof. ■

Lemma 10. *There exists a time-varying parameter* $\xi_k^* > 0$ *such that* $E\left[\frac{{s_k}^2}{P_{k+1|k}}\Big|e_{k|k-1}\right] \leq \xi_k^*$.

Proof: By (68), we have $s_k^2 = w_k^2 - 2K_k w_k u_k + K_k^2 u_k^2$. Since w_k and u_k are uncorrelated and $e_{k|k-1}$ does not depend on either w_k or u_k, we have

$$E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right] = \frac{E[w_k^2]}{P_{k+1|k}} + \frac{K_k^2 E[u_k^2]}{P_{k+1|k}}. \quad (72)$$

Substituting (7), (55) and using Lemma 8, noticing that $E[u_k] = 0$, we get $E\left[\frac{s_k^2}{P_{k+1|k}}|e_{k|k-1}\right] \leq \frac{(1+\phi_k)E[w_k^2]}{\phi_k P_{k|k-1}} + \frac{2.46\hat{z}_{k|k-1}}{\phi_k(1+\phi_k)r_k P_{k|k-1}}$. Finally, by setting ξ_k^* as

$$\xi_k^* = \frac{1+\phi_k}{\phi_k P_{k|k-1}} E[{w_k}^2] + \frac{2.46\hat{z}_{k|k-1}}{\phi_k(1+\phi_k)r_k P_{k|k-1}}, \quad (73)$$

we complete the proof. ■

Armed with the above lemmas, we next prove Theorem 2 by utilizing the same method with the proof of Theorem 1.

Proof of Theorem 2: Recall (45) and (68), we notice that the only difference between the estimation errors of Algorithms 2 and 1 is s_k. Therefore, it suffices to study the impact of w_k on $V_k(e_{k|k-1})$.

It follows from Lemmas 6, 7, 8, 9 and 10 that

$$\begin{aligned} &E\left[V_{k+1}(e_{k+1|k})|e_{k|k-1}\right] - V_k(e_{k|k-1}) \\ &\leq -\alpha_k V_k(e_{k|k-1}) + b_k|e_{k|k-1}|^3 + d_k|e_{k|k-1}|E[w_k] + \xi_k^*. \end{aligned}$$

Furthermore, bounding the second item in b_k as (59) and given ϕ_k in (62), yields

$$\begin{aligned} &E\left[V_{k+1}(e_{k+1|k})|e_{k|k-1}\right] - V_k(e_{k|k-1}) \\ &\qquad \leq -(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + d_k|e_{k|k-1}|E[w_k] + \xi_k^* \end{aligned}$$

for $|e_{k|k-1}| \leq \epsilon_k$.

And we can thus prove Theorem 2 by setting $\epsilon = \frac{0.63\varsigma}{1+2\varsigma}\hat{z}_{1|0}$, $\beta^* = \frac{1}{Q_0}$, $\alpha_k^* = (1-\varsigma)\alpha_k$, $\tau_k = \xi_k^* + d_k|e_{k|k-1}|\lambda_k$ and $\beta_k = \frac{1}{\overline{p}_k}$. ■

We conclude the analysis on the performance of our estimation algorithm for the dynamic case with a more profound investigation on the evolution of the estimation error $|e_{k|k-1}|$ and derive the explicit formulas for λ_k and σ_k. More specifically, we can distinguish three regions:

- *Region 1*: $\sqrt{\frac{9.84M\hat{z}_{k|k-1}}{\phi_k(M-1)r_k(1-\varsigma)}} \leq |e_{k|k-1}| \leq \epsilon_k$. In this case, the objective is to achieve

$$\begin{aligned} &E\left[V_{k+1}(e_{k+1|k})|e_{k|k-1}\right] - V_k(e_{k|k-1}) \\ &\qquad \leq -\frac{1}{M}(1-\varsigma)\alpha_k V_k(e_{k|k-1}) \end{aligned}$$

so that $E[e_{k|k-1}^2]$ is bounded as

$$E[e_{k|k-1}^2] \leq \frac{\overline{p}_k}{Q_0} E[{e_{1|0}}^2] \prod_{i=1}^{k-1}(1-\alpha_i^*). \quad (74)$$

That is, it should hold that $d_k|e_{k|k-1}|E[w_k] + \xi_k^* \leq \frac{M-1}{M}(1-\varsigma)\alpha_k V_k(e_{k|k-1})$. To that end, we firstly let the following inequalities hold

$$\begin{cases} d_k|e_{k|k-1}|E[w_k] \leq \frac{M-1}{2M}(1-\varsigma)\alpha_k V_k(e_{k|k-1}), \\ \xi_k^* \leq \frac{M-1}{2M}(1-\varsigma)\alpha_k V_k(e_{k|k-1}). \end{cases} \quad (75)$$

Secondly, substituting (71), (73) into (75) leads to

$$E[w_k] \leq \frac{a_k\phi_k(1-\varsigma)|e_{k|k-1}|}{(1+\phi_k)(2a_k\phi_k+1-a_k)}, \quad (76)$$

$$E[{w_k}^2] \leq \frac{\phi_k(M-1)r_k(1-\varsigma)|e_{k|k-1}|^2}{2M(1+\phi_k)^2} - \frac{2.46\hat{z}_{k|k-1}}{(1+\phi_k)^2}. \quad (77)$$

Thirdly, let

$$\frac{\phi_k(M-1)r_k(1-\varsigma)|e_{k|k-1}|^2}{2M(1+\phi_k)^2} \geq \frac{4.92\hat{z}_{k|k-1}}{(1+\phi_k)^2}, \quad (78)$$

and we thus have

$$|e_{k|k-1}| \geq \sqrt{\frac{9.84M\hat{z}_{k|k-1}}{\phi_k(M-1)r_k(1-\varsigma)}} \triangleq \tilde{\epsilon}, \quad (79)$$

$$E[{w_k}^2] \leq \frac{2.46\hat{z}_{k|k-1}}{(1+\phi_k)^2} \triangleq \sigma_k. \quad (80)$$

The rational behind can be interpreted as follows: i) the right term of (77) cannot be less than zero and ii) there always exists the measurement uncertainty in the system. Consequently, the impact of tag population change plus the measurement uncertainty should equal in order of magnitude that of only measurement uncertainty, which can be achieved by establishing $E[{w_k}^2] \leq K_k^2 E[{u_k}^2]$ and (78) with reference to (72) and (73).

However, since a_k' and a_k are unknown a priori, we thus need to transform the right hand side of (76) to a computable form. From (61), we get $\frac{1}{a_k} - 1 \leq \frac{\varsigma}{1+\varsigma}$ such that it holds for the right hand side of (76) that $\frac{a_k\phi_k(1-\varsigma)|e_{k|k-1}|}{3(1+\phi_k)(2a_k\phi_k+1-a_k)} \geq \frac{\phi_k(1-\varsigma)\tilde{\epsilon}}{3(1+\phi_k)\left(2\phi_k+\frac{\varsigma}{1+\varsigma}\right)}$. Finally, let

$$E[w_k] \leq \frac{\phi_k(1-\varsigma)\tilde{\epsilon}}{3(1+\phi_k)\left(2\phi_k + \frac{\varsigma}{1+\varsigma}\right)} \triangleq \lambda_k, \quad (81)$$

we can establish (74) and thus get that $E[e_{k|k-1}^2] \to 0$ at an exponential rate when $k \to \infty$.

- *Region 2:* $\sqrt{\frac{9.84\hat{z}_{k|k-1}}{\phi_k r_k(1-\varsigma)}} \leq |e_{k|k-1}| < \sqrt{\frac{9.84M\hat{z}_{k|k-1}}{\phi_k(M-1)r_k(1-\varsigma)}}$. Given $\tilde{\epsilon}$, λ_k and σ_k as in *Region 1*, in this case, we have $-(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + d_k|e_{k|k-1}|E[w_k] + \xi_k^* \leq 0$. It then follows from Lemma 2 that

$$\begin{aligned} E[e_{k|k-1}^2] \leq{}& \frac{\overline{p}_k}{Q_0}E[{e_{1|0}}^2]\prod_{i=1}^{k-1}(1-\alpha_i^*) \\ &+ \overline{p}_k\sum_{i=1}^{k-2}\tau_{k-i-1}\prod_{j=1}^{i}(1-\alpha_{k-j}^*). \end{aligned}$$

Hence, when $k \to \infty$, $E[e_{k|k-1}^2]$ converges exponentially to $\overline{p}_k\sum_{i=1}^{k-2}\tau_{k-i-1}\prod_{j=1}^{i}(1-\alpha_{k-j}^*) \sim \Theta(\hat{z}_{k|k-1})$ and it thus holds that $\frac{E[e_{k|k-1}]}{z_k} = \Theta(\frac{1}{\sqrt{z_k}}) \to 0$ for $z_k \to \infty$.

- *Region 3:* $0 \leq |e_{k|k-1}| < \sqrt{\frac{9.84\hat{z}_{k|k-1}}{\phi_k r_k(1-\varsigma)}}$. The circumstances in this region are very complicated due to $E[w_k]$ and $E[{w_k}^2]$, we here thus just consider the worst case that $E[w_k] = \lambda_k$ and $E[{w_k}^2] = \sigma_k$. Consequently, we have $-(1-\varsigma)\alpha_k V_k(e_{k|k-1}) + d_k|e_{k|k-1}|E[w_k] + \xi_k^* > 0$, and it then follows from Lemma 2 that

$$E[e_{k|k-1}^2] \leq \frac{\overline{p}_k}{Q_0} E[e_{1|0}{}^2] \prod_{i=1}^{k-1} (1-\alpha_i^*) + \overline{p}_k \sum_{i=1}^{k-2} \tau_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*).$$

Hence, when $k \to \infty$, $E[e_{k|k-1}^2]$ converges at exponential rate to $\overline{p}_k \sum_{i=1}^{k-2} \tau_{k-i-1} \prod_{j=1}^{i} (1-\alpha_{k-j}^*) \sim \Theta(\hat{z}_{k|k-1})$, and thus $\frac{E[e_{k|k-1}]}{z_k} \leq \Theta(\frac{1}{\sqrt{z_k}}) \to 0$ for $z_k \to \infty$.

Note that for the case that $E[w_k] < \lambda_k$ and $E[w_k{}^2] < \sigma_k$, the range of *Region 3* will shrink and the range of *Region 2* will largen.

Integrating the above three regions, we can get the similar results on the convergence of the expected estimation error $E[e_{k|k-1}]$ as in the static case.

VIII. Discussion

This section discusses the application of the proposed algorithm to the unreliable channel and multi-reader scenario.

Error-prone channel. The unreliable channel may corrupt an idle slot into a busy slot and vice versa. We consider the random error model as [30]. Let t_0 and t_1 $(0 \leq t_1, t_2 < 0.5)$ be the false positive rate that an empty slot turns into a busy slot and the false negative rate, respectively. For notation convenience, we mark parameters in the perfect channel model with a superscript $*$ to define their counterparts under the imperfect channel model. With some straightforward algebraic operations, we have

$$p^*(Z_k) = t_1 + (1-t_0-t_1)p(Z_k), \tag{82}$$

$$Var^*[u_k] = (1-t_0-t_1)^2 Var[u_k]. \tag{83}$$

We can compute the new Kalman gain K_k^* as

$$K_k^* = \frac{1}{(1-t_0-t_1)} K_k. \tag{84}$$

The ideal channel case is equivalent to the case where $t_0 = t_1 = 0$. It is then straightforward to derive the stability in the case of imperfect channel by using $p^*(Z_k)$, $Var^*[u_k]$ and K_k^*. In this regard, we find that Theorem 1 and 2 still hold under the same conditions, meaning that our approach is robust against channel errors.

Multi-reader case. In multi-reader scenarios, we leverage the same approach as [31]. The main idea is that a back-end server can be used to synchronize all readers such that the RFID system with multiple readers operates as the single-reader case. Specially, the back-end server calculates all the parameters and sends them to all readers such that they broadcast the same parameters to the tags. Subsequently, each reader sends its bitmap to the back-end server. Then the back-end server applies OR operator on all bitmaps, which eliminates the impact of the duplicate readings of tags in the overlapped interrogation region.

IX. Numerical Analysis

In this section, we conduct extensive simulations to evaluate the performance of the proposed tag population estimation algorithms by focusing on the relative estimation error denoted as $REE_k = \left|\frac{z_k - \hat{z}_{k|k-1}}{z_k}\right|$. Specifically, unless otherwise specified, we simulate in sequence both static and dynamic RFID systems where the initial tag population are $z_0 = 10^4$ with the following default parameters: $q = 0.1$, $P_{0|0} = 1$, $J = 3$, $\theta = 4$ and $\Upsilon = 0.5$ with reference to (35) and (36), $L = 1500$, $\underline{\phi} = 0.25$ and $\overline{\phi} = 100$ such that (62) always holds. Since the proposed algorithms do not require collision detection, we set a slot to 0.4ms as in the EPCglobal C1G2 standard [12]. We will discuss the effect of $\underline{\phi}$ and $\overline{\phi}$ on the performance in next section.

A. Algorithm Verification

In the subsection, we show the impact of $\underline{\phi}$ and $\overline{\phi}$ on the system performance. To that end, with REE_0=0.5, we keep z_k=10^4 in static scenario while the tag population varies in order of magnitude from $\sqrt{\hat{z}_{k|k-1}}$ to $0.4\hat{z}_{k|k-1}$ in different patterns in dynamic scenario. Specifically, we set $\overline{\phi}$=100 while varying $\underline{\phi}$=0.25, 0.5, 1 in Fig. 1, 2, and fix $\underline{\phi}$=0.25 with varying $\overline{\phi}$=1, 10, 100 in Fig. 3, 4. As shown in the figures, a smaller $\underline{\phi}$ leads to rapider convergence rate while the bigger $\overline{\phi}$, the smaller the deviation. Thus, we choose $\underline{\phi}$=0.25 and $\overline{\phi}$=100 in the rest of the simulation.

Moreover, we make the following observations. First, as derived in Theorem 2, the estimation is stable and accurate facing to a relative small population change, i.e., around the order of magnitude $\sqrt{\hat{z}_{k|k-1}}$. Second, the proposed scheme also functions nicely even when the estimation error is as high as $0.4\hat{z}_{k|k-1}$ tags as shown in Fig. 2 and 4. This is due to the CUSUM-based change detection which detects state changes promptly such that a small value is set for ϕ_k, leading to rapid convergence rate.

B. Algorithm Performance

In this section, we evaluate the performance of the proposed EKF-based estimator, referred to as EEKF here, in comparison with [7] in static scenario and with [19] in dynamic scenario.

1) Static System ($z_k = 10^4$): We evaluate the performance by varying initial relative error as

- REE_0=$\frac{z_0 - \hat{z}_{0|0}}{z_0}$=0.2 implies a small initial estimation error and satisfies (64) with $0.5 \leq \varsigma < 1$.
- $REE_0 = 0.5$ means a medium initial estimation error.
- $REE_0 = 0.8$ means a large initial estimation error.

In this three case, we simulate EZB with the optimal parameters as specified in [7] with the accuracy 0.95, specifically, when REE_0=0.2, frame size $L = 35$, the number of trails $n = 70$ and persistence probability $p' = 0.0057$; when REE_0=0.5, then L=66, n=43 and p'=0.0113; when REE_0=0.8, then L=215, n=20 and p'=0.0449. For legible presentation, we set the simulation time here to $215 * 30$ based on the case REE_0=0.8, then $k_{max} = 185, 97, 30$, respectively. Note that we use the same frame size with each case of EZB.

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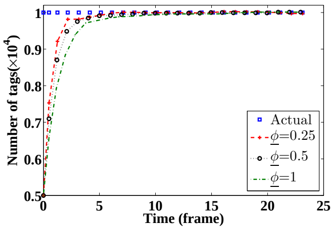


Fig. 1. Static:$\overline{\phi}$=100.

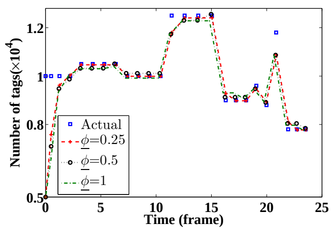


Fig. 2. Dynamic:$\overline{\phi}$=100.

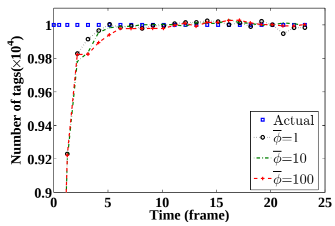


Fig. 3. Static:$\underline{\phi}$=0.25.

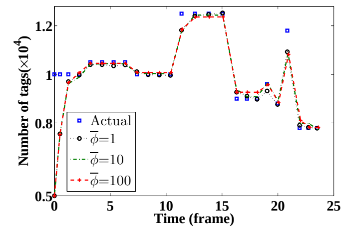
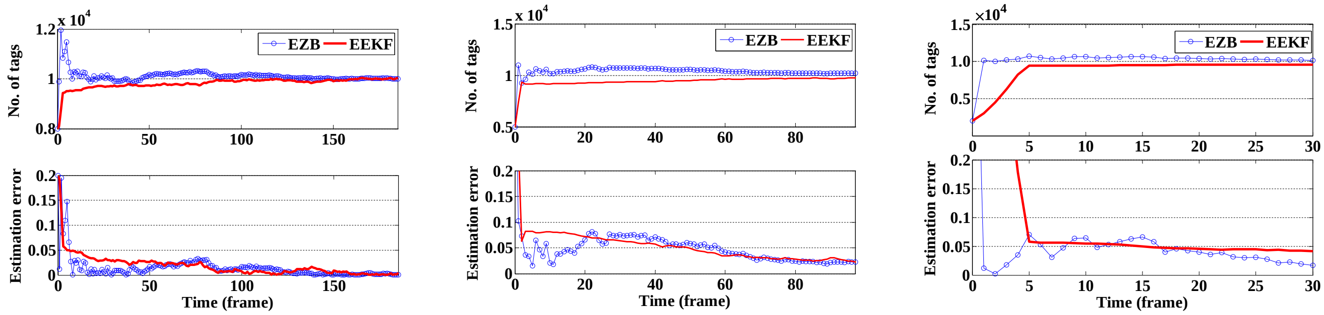


Fig. 4. Dynamic:$\underline{\phi}$=0.25.



(a) $REE_0 = 0.2$ (b) $REE_0 = 0.5$ (c) $REE_0 = 0.8$

Fig. 5. Algorithm performance under different initial estimation errors.

Fig. 5 illustrates the estimation processes with different initial estimation errors. As shown in the figures, the estimation $\hat{z}_{k|k-1}$ converges towards the actual number of tags within very short time in all cases, despite the initial estimation error. Though EZB is faster than EEKF to stabilize around the actual size with the error less than 0.05, EEKF achieves smaller deviation.

TABLE II
EXECUTION TIME

Algorithm	**Variation of tag population** ($\times 10^3$)				
	10→12.5	→6.737	→9.364	→7.049	→8.616
JREP	3.2768	3.2768	3.2768	3.2768	3.2768
EEKF	1.2	1.2	1.2	0.6	0.6

2) Dynamic system: In this subsection, we evaluate the performance of EEKF for dynamic systems by comparing with the start-of-the-art solution JREP [19] in terms of execution time to achieve the required accuracy. To that end, we refer to the simulation setting in [19]. Specifically, the initial estimation error is 10%. The tag population size changes by following the normal distribution with the mean of 10000 and the variance of 2000^2 and the accuracy requirement is 95%. By taking 5 samplings, we obtain the results as listed in Table II. As shown in Table II, EEKF is more time-efficient than JREP. This is because the persistence probability in JREP is set to optimise the power-of-two frame size, which increases the variance of the number of empty slots and leads to the performance degradation. In contrast, EEKF can minimise this variance while promptly detecting the tag population changes.

X. Conclusion

In this paper, we have addressed the problem of tag estimation in dynamic RFID systems and designed a generic framework of stable and accurate tag population estimation schemes based on Kalman filter. Technically, we leveraged the techniques in extended Kalman filter (EKF) and cumulative sum control chart (CUSUM) to estimate tag population for both static and dynamic systems. By employing Lyapunov drift analysis, we mathematically characterised the performance of the proposed framework in terms of estimation accuracy and convergence speed by deriving the closed-form conditions on the design parameters under which our scheme can stabilise around the real population size with bounded relative estimation error that tends to zero within exponential convergence rate.

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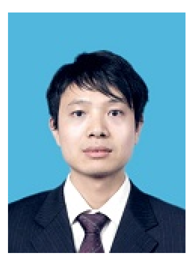

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