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Stability Analysis of Frame Slotted Aloha Protocol

Jihong Yu, Lin Chen

***Abstract*—Frame Slotted Aloha (FSA) protocol has been widely applied in Radio Frequency Identification (RFID) systems as the *de facto* standard in tag identification. However, very limited work has been done on the stability of FSA despite its fundamental importance both on the theoretical characterisation of FSA performance and its effective operation in practical systems. In order to bridge this gap, we devote this paper to investigating the stability properties of p-persistent FSA by focusing on two physical layer models of practical importance, the models with single packet reception and multipacket reception capabilities.**

Technically, we model the FSA system backlog as a Markov chain with its states being backlog size at the beginning of each frame. The objective is to analyze the ergodicity of the Markov chain and demonstrate its properties in different regions, particularly the instability region. By employing drift analysis, we obtain the closed-form conditions for the stability of FSA and show that the stability region is maximised when the frame length equals the number of packets to be sent in the single packet reception model and the upper bound of stability region is maximised when the ratio of the number of packets to be sent to frame length equals in an order of magnitude the maximum multipacket reception capacity in the multipacket reception model. Furthermore, to characterise system behavior in the instability region, we mathematically demonstrate the existence of transience of the backlog Markov chain. Finally, the analytical results are validated by the numerical experiments.

***Index Terms*—Frame slotted Aloha, stability, multipacket reception.**

I. Introduction

A. Context and Motivation

Since the introduction of Aloha protocol in 1970 [1], a variety of such protocols have been proposed to improve its performance, such as Slotted Aloha (SA) [23] and Frame Slotted Aloha (FSA) [19]. SA is a well known random access scheme where the time of the channel is divided into identical slots of duration equal to the packet transmission time and the users contend to access the server with a predefined slot-access probability. As a variant of SA, FSA divides time-slots into *frames* and a user is allowed to transmit only a single packet per frame in a randomly chosen time-slot.

Due to their effectiveness to tackle collisions in wireless networks, SA-and-FSA-based protocols have been applied extensively to various networked systems ranging from the satellite networks [18], wireless LANs [28], [34] to the emerging Machine-to-Machine (M2M) networks [32], [29]. Specifically, in radio frequency identification (RFID) systems, FSA plays a fundamental role in the identification of tags [35], [14] and is standardized in the EPCGlobal Class-1 Generation-2 (C1G2) RFID standard [4]. In FSA-based protocols, all users with packets transmit in the selected slot of the frame respectively, but only packets experiencing no collisions are successful while the other packets referred to as backlogged packets (or simply backlogs), are retransmitted in the subsequent frames.

Given the paramount importance of the stability for systems operating on top of Aloha-based protocols, a large body of studies have been devoted to stability analysis in a slotted collision channel [2], [13], [5] where a transmission is successful if and only if just a single user transmits in the selected slot, referred to as single packet reception (SPR). Differently with SPR, the emerging multipacket reception (MPR) technologies in wireless networks, such as Code Division Multiple Access (CDMA) and Multiple-Input and Multiple-Output (MIMO), make it possible to receive multiple packets in a time-slot simultaneously, which remarkably boosts system performance at the cost of the system complexity.

More recently, the application of FSA in RFID systems and M2M networks has received considerable research attention. However, very limited work has been done on the stability of FSA despite its fundamental importance both on the theoretical characterisation of FSA performance and its effective operation in practical systems. Motivated by the above observation, we argue that a systematic study on the stability properties of FSA incorporating the MPR capability is called for in order to lay the theoretical foundations for the design and optimization of FSA-based communication systems.

B. Summary of Contributions

In this paper, we investigate the stability properties of p-persistent FSA with SPR and MPR capabilities. The main contributions of this paper are articulated as follows. We model the packet transmission process in a frame as the bins and balls problem [11] and derive the number of successfully received packets under both SPR and MPR models. We formulate a homogeneous Markov chain to characterize the number of the backlogged packets and derive the one-step transition probability with the persistence probability p. By employing drift analysis, we obtain the closed-form conditions for the stability of p-persistent FSA and derive conditions maximising the stability regions for both SPR and MPR models. To characterise system behavior in the instability region with the persistence probability p, we mathematically demonstrate the existence of transience of the backlog Markov chain. Besides, we investigate how to achieve the stability condition and give the control algorithm for updating the frame size.

Our work demonstrates that the stability region is maximised when the frame length equals the number of sent packets in the SPR model and the upper bound of stability region is maximised when the ratio of the number of sent packets to frame length equals in an order of magnitude the maximum multipacket reception capacity in the MPR model. In addition, it is also shown that FSA-MPR outperforms FSA-SPR remarkably in terms of the stability region size.

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C. *Paper Organisation*

The remainder of the paper is organised as follows. Sec. II gives a brief overview of related work and compares our results with existing results. In Sec. III, we present the system model, including random access model, traffic model and packet success probability. In Sec. IV, we summary the main result of this paper and the detailed proofs on the stability properties of FSA-SPR and FSA-MPR are given in Sec. V and Sec. VI, respectively. In Sec. VII, we study the frame size control in practice. In Sec. VIII, we conduct the numerical analysis. Finally, we conclude our paper in Sec. IX.

II. Related Work

Aloha-based protocols are basic schemes for random medium access and are applied extensively in many communication systems. As a central property, the stability of Aloha protocols has received lots of attention, which we briefly review here.

Stability of slotted Aloha. Tsybakov and Mikhailov [27] initiated the stability analysis of finite-user slotted Aloha. They found sufficient conditions for stability of the queues in the system using the principle of stochastic dominance and derived the stability region for two users explicitly. For the case of more than two users, the inner bounds to the stability region were shown in [22]. Subsequently, Szpankowski [26] found necessary and sufficient conditions for the stability under a fixed transmission probability vector for three-user case. However, the derived conditions are not closed-form, meaning the difficulty on verifying them. In [2] an approximate stability region was derived for an arbitrary number of users based on the mean-field asymptotics. It was claimed that this approximate stability region is exact under large user population and it is accurate for small-sized networks. The sufficient condition for the stability was further derived to be linear in arrival rates without the requirement on the knowledge of the stationary joint statistics of queue lengths in [13]. Recently, the stability region of SA with K-exponential backoff was derived in [5] by modeling the network as inter-related quasi-birth-death processes. We would like to point out that all the above stability analysis results were derived for the SPR model.

Stability of slotted Aloha with MPR. The first attempt at analyzing stability properties of SA with MPR was made by Ghez *et al.* in [7], [8] in an infinite-user single-buffer model. They drew a conclusion that the system could be stabilized under the symmetrical MPR model with a non-zero probability that all packets were transmitted successfully. Afterwards, Sant and Sharma [24] studied a special case of the symmetrical MPR model for finite-user with an infinite buffer. They derived sufficient conditions on arrival rate for stability of the system under the stationary ergodic arrival process. Subsequently, the effect of MPR on stability and delay was investigated in [17] and it was shown that stability region undergoes a phase transition and then reaches the maximization. Besides, in [9] necessary and sufficient conditions were obtained for a Nash equilibrium strategy for wireless networks with MPR based on noncooperation game theory. More recently, Jeon and Ephremides [10] characterised the exact stability region of SA with stochastic energy harvesting and MPR for a pair of bursty users. Although the work aforementioned analyzed the stability of system without MPR or/and with MPR, they are mostly, if not all, focused on SA protocol, while our focus is FSA with both SPR and MPR.

Performance analysis of FSA. There exist several studies on the performance of FSA. Wieselthier and Anthony [31] introduced a combinatorial technique to analyse performance of FSA-MPR for the case of finite users. Schoute [25] investigated dynamic FSA and obtained the expected number of time-slots needed until the backlog becomes zero. Consider the application of FSA to RFID identification problem, the asymptotic sum of all frame sizes for optimal identification efficiency is proved to be $ne - 1.09\ln(n)$ in [21] where n is the RFID tag cardinality. However, these works did not address the stability of FSA, which is of fundamental importance.

In summary, only very limited work has been done on the stability of FSA despite its fundamental importance both on the theoretical characterisation of FSA performance and its effective operation in practical systems. In order to bridge this gap, we devote this paper to investigating the stability properties of FSA under both SPR and MPR models.

III. System Model

A. *Random access model in FSA*

We consider a system of infinite identical users with buffer capacity of one packet and operating on one frequency channel. In one slot, a node can complete a packet transmission.

The random access process operates as follows: FSA organises time-slots with each frame containing a number of consecutive time-slots. Each user is allowed to randomly and independently choose a time-slot to send his packet at most once per frame. More specifically, suppose the length of frame t is equal to L_t, then in the beginning of frame t each user generates a random number $\mathfrak{R}$ and selects the $(\mathfrak{R} \mod L_t)$-th time-slot in frame t to transmit his packet. Note that unsuccessful packets in the current frame are retransmitted in the next frame with the constant persistence probability p while newly generated packets are transmitted in the next frame following their arrivals with probability one.

Moreover, we investigate two physical layer models of practical importance, the models with single packet reception (SPR) and multipacket reception (MPR) capabilities:

- Under the SPR model, a packet suffers a collision if more than one packet is transmitted in the same time-slot. SPR is a classical and baseline physical layer model.
- Under the MPR model, up to $\overline{M}$ ($\overline{M} > 1$) concurrently transmitted packets can be received successfully with non-zero probabilities as specified by a stochastic matrix Ξ defined as follows:

$$\Xi \triangleq \begin{pmatrix} \hat{\xi}_{10} & \hat{\xi}_{11} & & & & \\ \vdots & \vdots & \vdots & \ddots & & \\ \hat{\xi}_{x_0 0} & \hat{\xi}_{x_0 1} & \cdots & \cdots & \hat{\xi}_{x_0 x_0} & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ \hat{\xi}_{\overline{M}0} & \hat{\xi}_{\overline{M}1} & \cdots & \cdots & \hat{\xi}_{\overline{M}\,\overline{M}} & \\ \mathbf{1} & \mathbf{0} & \cdots & \cdots & \cdots & \mathbf{0} \end{pmatrix} \quad (1)$$

where $\hat{\xi}_{x_0k_0}$ ($k_0 \leq x_0 \leq \overline{M}$) is the probability of having k_0 successful packets among x_0 transmitted packets in one slot. Ξ is referred to as the reception matrix. The last two decades have witnessed an increasing prevalence of MPR technologies such as CDMA and MIMO. Mathematically, the SPR model can be regarded as a degenerated MPR model with $\overline{M} = 1$ and

$$\Xi = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ \vdots & \vdots & \mathbf{0} \\ 1 & 0 & \end{pmatrix}.$$

For notational convenience, we use FSA-SPR and FSA-MPR to denote the FSA system operating on the SPR and MPR models, respectively.

B. Traffic model

Let random variable N_t denote the total number of new arrivals during frame t and denote by A_{tl} the number of new arrivals in time-slot l in frame t where $l = 1, 2, \cdots, L_t$. Assume that $(A_{tl})_{1\leq l\leq L_t}$ are independent and identically Poisson distributed random variables with probability distribution:

$$P\{A_{tl} = u\} = \Lambda_u (u \geq 0) \quad (2)$$

such that the expected number of arrivals per time-slot $\Lambda = \sum_1^\infty u\Lambda_u$ is finite.

Then as $N_t = \sum_{l=1}^{L_t} A_{tl}$, the distribution of N_t, defined as $\{\lambda_t(n)\}_{n\geq 0}$, also follows Poisson distribution with the expectation $\mathfrak{N}_t = L_t\Lambda$.

C. Packet success probability

The process of randomly and independently choosing a time-slot in a frame to transmit packets can be cast into a class of problems that are known as occupancy problems, or bins and balls problem [11]. Specifically, consider the setting where a number of balls are randomly and independently placed into a number of bins, the classical occupance problem studies the maximum load of an individual bin.

In our context, time-slots and packets to be transmitted in a frame can be cast into bins and balls, respectively. Denote by Y_t the random variable for the number of packets to be transmitted in frame t. Given $Y_t = \hat{h}$ in frame t and the frame length L_t, the number x_0 of packets sent in one time-slot, referred as to occupancy number, is binomially distributed with parameters $\hat{h}$ and $\frac{1}{L_t}$:

$$B_{\hat{h},\frac{1}{L_t}}(x_0) = \binom{\hat{h}}{x_0}(\frac{1}{L_t})^{x_0}(1-\frac{1}{L_t})^{\hat{h}-x_0}. \quad (3)$$

Applying the distribution of equation (3) to all L slots in the frame, we can get the expected value $b(x_0)$ of the number of time-slots with occupance number x_0 in a frame as follows:

$$b(x_0) = L_t B_{\hat{h},\frac{1}{L_t}}(x_0) = L_t\binom{\hat{h}}{x_0}(\frac{1}{L_t})^{x_0}(1-\frac{1}{L_t})^{\hat{h}-x_0}. \quad (4)$$

We further derive the probability that a packet is transmitted successfully under both SPR and MPR.

Packet success probability of FSA-SPR

In FSA-SPR, the number of successfully received packets equals that of time-slots with occupance number $x_0 = 1$. Following the result of [30], we can obtain the probability that under SPR there exist exactly k successful packets among $\hat{h}$ transmitted packets in the frame, denoted by $\xi^{SPR}_{\hat{h}k}$, as follows:

$$\xi^{SPR}_{\hat{h}k} = \begin{cases} \frac{\binom{L_t}{k}\binom{\hat{h}}{k}k!G(L_t-k,\hat{h}-k)}{L_t^{\hat{h}}}, & 0 < k < \min(\hat{h}, L_t) \\ \frac{\binom{L_t}{\hat{h}}\hat{h}!}{L_t^{\hat{h}}}, & k = \hat{h} \leq L_t \\ 0, & k > \min(\hat{h}, L_t) \\ 0, & k = L_t < \hat{h} \end{cases} \quad (5)$$

where

$$G(V, w) = V^{\hat{w}} + \sum_{t=1}^{\hat{w}}(-1)^t\prod_{j=0}^{t-1}[(\hat{w}-j)(V-j)](V-t)^{\hat{w}-t}\frac{1}{t!}$$

with $V \triangleq L_t - k$ and $\hat{w} \triangleq \hat{h} - k$.

Consequently, the expected number of successfully received packets in one frame in FSA-SPR, denoted as $r^{SPR}_{\hat{h}}$, is

$$r^{SPR}_{\hat{h}} = \sum_{k=1}^{\min(\hat{h},L_t)} k\xi^{SPR}_{\hat{h}k} = b(1). \quad (6)$$

Packet success probability of FSA-MPR

Let occupancy numbers x_l and k_l be the number of transmitted packets and successful packets in the l-th time-slot, respectively, where $l = 1, 2, \cdots, L_t$. The probability that k packets are received successfully among $\hat{h}$ transmitted packets in the frame, denoted by $\xi^{MPR}_{\hat{h}k}$, can be expressed as

$$\xi^{MPR}_{\hat{h}k} = \sum_{\sum_l x_l=\hat{h}}\sum_{\sum_l k_l=k}\prod\hat{\xi}_{x_lk_l} \quad (7)$$

We can further derive the expected number of successfully received packets in one frame as

$$r^{MPR}_{\hat{h}} = \sum_{k=1}^{\hat{h}} k\xi^{MPR}_{\hat{h}k} = L_t\sum_{x_0=1}^{\hat{h}}\sum_{k_0=1}^{x_0} B_{\hat{h},\frac{1}{L_t}}(x_0)k_0\hat{\xi}_{x_0k_0}. \quad (8)$$

In the subsequent analysis, to make the presentation concise without introducing ambiguity, we use $\xi_{\hat{h}k}$ to denote $\xi^{SPR}_{\hat{h}k}$ in FSA-SPR and $\xi^{MPR}_{\hat{h}k}$ in FSA-MPR. The notations used throughout the paper are summarized in Table I.

TABLE I
MAIN NOTATIONS

Symbols	Descriptions
p	persistence probability
$\overline{M}$	maximum MPR capacity
Λ	expected arrival rate per slot
$\mathfrak{N}_t$	expected arrival rate in frame t
$\lambda_t(n)$	prob. of n new arrivals in frame t
L_t	the length of frame t
X_t	random variable: No. of backlogs in frame t
i	the value of backlogs in frame t, i.e., $X_t = i$
Y_t	random variable: No. of transmitted packet in frame t
$\hat{h}$	the value of packets sent in frame t, i.e., $Y_t = \hat{h}$
Z_t	random variable: No. of retransmitted packet in frame t
h	the value of retransmitted packets in frame t, i.e., $Z_t = h$
α	the ratio of $\hat{h}$ to L_t
$\hat{\xi}_{x_0k_0}$	prob. of having k_0 out of x_0 successful packets in a slot
$\xi_{\hat{h}k}$	prob. of having k out of $\hat{h}$ successful packets in frame t
P_{is}	one-step transition probability
D_i	drift in frame t

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IV. Main results

To streamline the presentation, we summarize the main results in this section and give the detailed proof and analysis in the subsequent sections that follow.

Aiming at studying the stability of FSA, we decompose our global objective into the following three questions, all of which are of fundamental importance both on the theoretical characterisation of FSA performance and its effective operation in practical systems:

- **Q1**: Under what condition(s) is FSA stable?
- **Q2**: When is the stability region maximised?
- **Q3**: How does FSA behave in the instability region?

Before answering the questions, we first introduce the formal definition of stability employed by Ghez *et al.* in [7].

Define by random variable X_t the number of backlogged packets in the system at the start of frame t. The discrete-time process $(X_t)_{t\geq 0}$ can be seen as a homogeneous Markov chain.

Definition 1. *An FSA system is stable if $(X_t)_{t\geq 0}$ is ergodic and unstable otherwise.*

By Definition 1, we can transform the study of stability of FSA into investigating the ergodicity of the backlog Markov chain. The rationality of this transformation is two-fold. One interpretation is the property of ergodicity that there exists a unique stationary distribution of a Markov chain if it is ergodic. The other can be interpreted from the nature of ergodicity that each state of the Markov chain can recur in finite time with probability 1.

From an engineering perspective, if FSA is stable, then the number of backlogs in the system will reduce overall; otherwise, it will increase as the system operates.

We then establish the following results characterizing the stability region and demonstrating the behavior of the Markov chain in nonergodicity regions under both SPR and MPR.

A. Results for FSA-SPR

Denote by i and $\hat{h}$ the value of the number of backlogs and sent packets in frame t and $\alpha \triangleq \frac{\hat{h}}{L_t}$. Recall the definitions of X_t and Y_t, we can suppose that $X_t = i$ and $Y_t = \hat{h}$.

Theorem 1. *Under FSA-SPR, consider an irreducible and aperiodic backlog Markov chain $(X_t)_{t\geq 0}$ with nonnegative integers. When $i \to \infty$, we have* [1]

1) *The system is always stable if $\Lambda < \alpha e^{-\alpha}$ and $L_t = \Theta(\hat{h})$. Specially, $\alpha = 1$ maximizes the stability region*[2] *and also the stable throughput.*
2) *The system is unstable under each of the following three conditions: (1) $L_t = o(\hat{h})$; (2) $L_t = O(\hat{h})$; (3) $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha e^{-\alpha}$.*

[1]For two variables X, Y, we use the following asymptotic notations:

- $X = o(Y)^*$ if $0 \leq \frac{X}{Y} \leq \theta_0$, as $Y \to \infty$, where constant $\theta_0 \geq 0$;
- $X = o(Y)$ if $\frac{X}{Y} = 0$, as $Y \to \infty$;
- $X = O(Y)$ if $\frac{X}{Y} = \infty$, as $Y \to \infty$;
- $X = \Theta(Y)$ if $\theta_1 \leq \frac{X}{Y} \leq \theta_2$, as $Y \to \infty$, where constants $\theta_2 \geq \theta_1 > 0$.

[2]The ergodicity region of a Markov chain in this paper is referred to as stability region.

Remark 1. *Theorem 1 answers the first two questions and can be interpreted as follows:*

- *When $L_t = o(\hat{h})$, i.e., the number of sent packets $\hat{h}$ is far larger than the frame length L_t, a packet experiences collision with high probability (w.h.p.), thus increasing the backlog size and destabilising the system;*
- *When $L_t = O(\hat{h})$, i.e., the number of sent packets $\hat{h}$ is far smaller than the frame length, a packet is transmitted successfully w.h.p.. However, the expected number of successful packets is still significantly less than that of new arrivals in the frame. The system is thus unstable.*
- *When $L_t = \Theta(\hat{h})$, i.e., $\hat{h}$ has the same order of magnitude with the frame length, the system is stable when the backlog can be reduced gradually, i.e., when the expected arrival rate is less than the successful rate.*

It is well known that an irreducible and aperiodic Markov chain falls into one of three mutually exclusive classes: positive recurrent, null recurrent and transient. So, our next step after deriving the stability conditions is to show whether the backlog Markov chain in the instability region is transient or recurrent, which answers the third question.

Theorem 2. *With the same notations as in Theorem 1, $(X_t)_{t\geq 0}$ is always transient in the instability region, i.e., under each of the following three conditions: (1) $L_t = o(\hat{h})$; (2) $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha e^{-\alpha}$; (3) $L_t = O(\hat{h})$.*

Remark 2. *If a state of a Markov chain is transient, then the probability of returning to itself for the first time in finite time is less than 1. Hence, Theorem 2 implies that once out of the stability region, the system is not guaranteed to return to stable state in finite time, that is, the number of backlogs will increase persistently.*

B. Results for FSA-MPR

Theorem 3. *Under FSA-MPR, using the same notations as in Theorem 1, we have*

1) *The system is always stable if $L_t = \Theta(\hat{h})$ and $\Lambda < \sum_{x_0=1}^{\overline{M}} e^{-\alpha} \frac{\alpha^{x_0}}{x_0!} \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0}$. Specially, let α^* denote the value of α that maximises the upper bound of stability region, it holds that $\alpha^* = \Theta(\overline{M})$.*
2) *The system is unstable under each of the following conditions: (1) $L_t = o(\hat{h}^{1-\epsilon_1})$ where $0 < \epsilon_1 \leq 1$; (2) $L_t = O(\hat{h})$; (3) $\Lambda > \alpha$ and $L_t = \Theta(\hat{h})$.*

Remark 3. *Comparing the results of Theorem 3 to Theorem 1, we can quantify the performance gap between FSA-SPR and FSA-MPR in terms of stability. For example, when $\alpha = 1$, the stability region is maximised in FSA-SPR with $\Lambda < e^{-1}$, while the upper bound of the stability region in FSA-MPR is $e^{-1} \sum_{x_0=1}^{\overline{M}} \frac{1}{(x_0-1)!}$. Note that for $\overline{M} > 2$, it holds that*

$$1 + 1 + \frac{1}{2} < \sum_{x_0=1}^{\overline{M}} \frac{1}{(x_0-1)!} < 1 + 1 + \sum_{x_0=1}^{\overline{M}} \frac{1}{x_0(x_0+1)}$$
$$< 2 + \left(\sum_{x_0=1}^{\overline{M}} \frac{1}{x_0} - \frac{1}{x_0+1} \right) = 3 - \frac{1}{\overline{M}+1}.$$

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The upper bound of the stability region of FSA-MPR when $\alpha = 1$ is thus between 2.5 and 3 times the maximum stability region of FSA-SPR. And hence the maximum upper bound of the stability region of FSA-MPR achieved when $\alpha^* = \Theta(\overline{M})$ is far larger than that of FSA-SPR.

Theorem 4. *With the same notations as in Theorem 3, $(X_t)_{t\geq 0}$ is transient under each of the following three conditions: (1) $L_t = o(\hat{h}^{1-\epsilon_1})$; (2) $L_t = O(\hat{h})$; (3) $\Lambda > \alpha$ and $L_t = \Theta(\hat{h})$.*

Remark 4. *Theorem 4 demonstrates that despite the gain on the stability region size of FSA-MPR over FSA-SPR, their behaviors in the unstable region are essentially the same.*

V. Stability Analysis of FSA-SPR

In this section, we will analyse the stability of FSA-SPR and prove Theorem 1 and 2.

A. Characterising backlog Markov chain

As mentioned in Sec. IV, we characterize the number of the backlogged packets in the system at the beginning of frame t as a homogeneous Markov chain $(X_t)_{t\geq 0}$. We assume that $X_t = i$ and $Y_t = \hat{h}$. Denote by Z_t the random variable for the number of retransmitted packets in frame t. Since the transmitted packets in frame t consists of the new arrivals during frame $t-1$ and the retransmitted packets in frame t, we have

$$Y_t = Z_t + N_{t-1}. \tag{9}$$

Suppose w new packets arrive in frame $t-1$ and h out of $i-w$ backlogs are retransmitted in frame t of which the probability is as follows:

$$B_{i-w}(h) \triangleq \binom{i-w}{h} p^h (1-p)^{i-w-h}.$$

As a consequence, the number of packets transmitted in frame t is $\hat{h} = w + h$.

We now calculate the one-step transition probability as a function of $\xi_{\hat{h}k}$, retransmission probability p and $\{\lambda_t(n)\}_{n\geq 0}$. Denote by $P_{is} = P\{X_{t+1} = s | X_t = i\}$ the one-step transition probability, we can derive the following results:

1) For $i = 0$:

$$P_{00} = \lambda_t(0), \quad P_{0s} = \lambda_t(s), \quad s \geq 1.$$

2) For $i \geq 1$:

$$\begin{cases} P_{i,i-s} = \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=\{s-w\}^+}^{i-w} B_{i-w}(h) \cdot \\ \qquad \sum_{n=0}^{\min(L,\hat{h})-s} \lambda_t(n) \xi_{\hat{h},n+s}, \qquad 1 \leq s \leq i, \\ P_{i,i} = \lambda_t(0)\Big(\lambda_{t-1}(0) B_i(0) + \sum_{w=0}^{i} \lambda_{t-1}(w) \cdot \\ \qquad \sum_{h=0}^{i-w} B_{i-w}(h) \xi_{\hat{h},0}\Big) + \sum_{w=0}^{i} \lambda_{t-1}(w) \cdot \\ \qquad \sum_{h=0}^{i-w} B_{i-w}(h) \sum_{n=1}^{\min(\hat{h},L)} \lambda_t(n) \xi_{\hat{h}n}, \\ P_{i,i+s} = \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) \cdot \\ \qquad \sum_{n=0}^{\min(\hat{h},L)} \lambda_t(n+s) \xi_{\hat{h}n}, \qquad s \geq 1, \end{cases} \tag{10}$$

where $\{s-w\}^+ = \max\{s-w, 0\}$.

The rationale for the calculation of the transition probability is explained as follows:

- When $i = 0$, i.e., there are no backlogs in the frame, the backlog size remains zero if no new packets arrive and increases by s if s new packets arrive in the frame.
- When $i > 0$, we have three possibilities, corresponding to the cases where the backlog size decreases, remains unchanged and increases, respectively:
  - The state $1 \leq s \leq \min(\hat{h}, L_t)$ corresponds to the case where the backlog size decreases by s when $n \leq \min(\hat{h}, L_t) - s$ new packets arrive but $n+s$ backlogged packets are received successfully.
  - The backlog size remains unchanged if either of two following events happens: (a) no new packets are generated and either no backlogged packets are transmitted or all the transmitted backlogged packets fail; (b) $n \leq \min(\hat{h}, L_t)$ new packets arrive but n backlogged packets are successfully received.
  - The backlog size increases when the number of successful packets is less than that of new arrivals.

In order to establish the ergodicity of the backlog Markov chain $(X_t)_{t\geq 0}$, it is necessary to ensure $(X_t)_{t\geq 0}$ is irreducible and aperiodic. To this end, we conclude this subsection by providing the sufficient conditions on $\{\lambda_t(n)\}$ for the irreducibility and the aperiodicity of $(X_t)_{t\geq 0}$ as

$$0 < \lambda_t(n) < 1, \ \forall \ n \geq 0. \tag{11}$$

We would like to point out that most of traffic models can satisfy (11). Throughout the paper, it is assumed that (11) holds and hence $(X_t)_{t\geq 0}$ is irreducible and aperiodic.

B. Stability analysis

Recall Definition 1, to study the stability of FSA, we need to analyse the ergodicity of the backlog Markov chain $(X_t)_{t\geq 0}$. We first define the drift and then introduce two auxiliary lemmas which will be useful in the ergodicity demonstration.

Definition 2. *The drift D_i of the backlog Markov chain $(X_t)_{t\geq 0}$ at state $X_t = i$ where $i \geq 0$ is defined as*

$$D_i = E[X_{t+1} - X_t | X_t = i]. \tag{12}$$

Lemma 1 ([20]). *Given an irreducible and aperiodic Markov chain $(X_t)_{i\geq 0}$ having nonnegative integers as state space with the transition probability matrix $\mathbf{P} = \{P_{is}\}$, $(X_t)_{t\geq 0}$ is ergodic if for some integer $Q \geq 0$ and constant $\epsilon_0 > 0$, it holds that*

1) $|D_i| < \infty$, for $i \leq Q$,
2) $D_i < -\epsilon_0$, for $i > Q$.

Lemma 2 ([12]). *Under the assumptions of Lemma 1, $(X_t)_{t\geq 0}$ is not ergodic, if there exist some integer $Q \geq 0$ and some constants $B \geq 0$, $c \in [0,1]$ such that*

1) $D_i > 0$ for all $i \geq Q$,
2) $\phi^i - \sum_s P_{is} \phi^i \geq -B(1-\phi)$ for all $i \geq Q$, $\phi \in [c, 1]$.

Armed with Lemma 1 and 2, we start to prove Theorem 1.

Proof of Theorem 1: In the proof, we first explicitly formulate the drift defined by (12) and then study the ergodicity of Markov chain based on drift analysis.

Denote by random variable C_t the number of successful transmissions in frame t, we have

$$X_{t+1} - X_t = N_t - C_t.$$

Recall (12), it then follows that

$$D_i = E[N_t - C_t | X_t = i] = \mathfrak{N}_t - E[C_t | X_t]. \tag{13}$$

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Since all new arrivals and unsuccessful packets in frame $t-1$ are transmitted in frame t with probability one and p, respectively, we have

$$P\{C_t = k | X_t = i, N_{t-1} = w, Z_t = h\} = \xi_{\hat{h}k}^{SPR},$$

for $0 \leq k \leq \min(\hat{h}, L)$. Recall (6), we have

$$\begin{aligned} E[C_t|X_t = i] &= \sum_{h=0}^{i-w} B_{i-w}(h) E[C_t = k | X_t = i, N_{t-1} = w] \\ &= \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{SPR}. \end{aligned} \tag{14}$$

Following (13) and (14), we obtain the value of the drift:

$$D_i = \mathfrak{N}_t - \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{SPR}. \tag{15}$$

After formulating the drift, we then proceed by two steps.

Step 1: $L_t = \Theta(\hat{h})$ **and** $\Lambda < \alpha e^{-\alpha}$**.**

In this step, we intend to corroborate that the conditions in Lemma 1 can be satisfied if $L_t = \Theta(\hat{h})$ and $\Lambda < \alpha e^{-\alpha}$. We first show that $|D_i|$ is finite. This is true for $i \leq Q$ since

$$\begin{aligned} |D_i| &< \max\{\mathfrak{N}_t, \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{SPR}\} \\ &< \max\{\mathfrak{N}_t, \min\{L_t, \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} (w+h) B_{i-w}(h)\}\} \\ &< \max\{\mathfrak{N}_t, \min\{L_t, \sum_{w=0}^{i} w\lambda_{t-1}(w) + \sum_{w=0}^{i} (i-w) p \lambda_{t-1}(w)\}\} \\ &< \max\{\mathfrak{N}_t, \min\{L_t, (1-p)\lambda + ip\}\}. \end{aligned} \tag{16}$$

Next, to derive the limit of D_i, we start with the following lemma which is proved in Appendix A.

Lemma 3. *If $r_{\hat{h}}^{SPR}$ has a limit $\hat{r}$, then it holds that* $\lim_{i\to\infty} \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{SPR} = \hat{r}$.

Following Lemma 3, we have

$$\begin{aligned} \lim_{i\to\infty} D_i &= \mathfrak{N}_t - \lim_{\hat{h}\to\infty} r_{\hat{h}}^{SPR} \\ &= \lim_{\hat{h}\to\infty} L_t \left\{ \Lambda - \binom{\hat{h}}{1} \frac{1}{L_t} \cdot \left(1 - \frac{1}{L_t}\right)^{\hat{h}-1} \right\} \\ &= L_t(\Lambda - \alpha e^{-\alpha}), \end{aligned} \tag{17}$$

where $\alpha \triangleq \frac{\hat{h}}{L_t}$. It thus holds that $\lim_{i\to\infty} D_i < -\epsilon_0$ with $\epsilon_0 = \frac{\alpha e^{-\alpha} - \Lambda}{2}$ since both α and Λ are constants when $L_t = \Theta(\hat{h})$ and $\Lambda < \alpha e^{-\alpha}$.

It then follows from Lemma 1 that $(X_t)_{t\geq 0}$ is ergodic. Specially, when $\alpha = 1$, the system stability region is maximized, i.e., $\Lambda < e^{-1}$.

Step 2: $L_t = o(\hat{h})$ **or** $L_t = O(\hat{h})$ **or** $L_t = \Theta(\hat{h})$ **and** $\Lambda > \alpha e^{-\alpha}$**.**

In this step, we prove the instability of $(X_t)_{t\geq 0}$ by applying Lemma 2. Taking into consideration the impact of different relation between L_t and $\hat{h}$ on the limit of D_i. With (17), the following results hold for $\hat{h} \to \infty$:

- $\Lambda - \lim_{\alpha\to\infty} \alpha e^{-\alpha} = \Lambda > 0$, when $L_t = o(\hat{h})$,
- $\Lambda - \lim_{\alpha\to 0} \alpha e^{-\alpha} = \Lambda > 0$, when $L_t = O(\hat{h})$,
- $\Lambda - \alpha e^{-\alpha} > 0$, when $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha e^{-\alpha}$.

Consequently, we have $\lim_{i\to\infty} D_i > 0$, which proves the first condition in Lemma 2.

Next, we will validate the second condition of Lemma 2 in two cases according to the probable relationship between $\hat{h}$ and i, i.e., $\hat{h} = o(i)$ and $\hat{h} = \Theta(i)$.

Note that the second condition apparently holds for ϕ=0 and ϕ=1, we thus focus on the remaining value of ϕ, i.e., $\phi \in (c, 1)$. Moreover, given $\hat{h}$, $P_{i,i-s}$ in (10) can also be expressed as

$$P_{i,i-s} = \sum_{w=0}^{\hat{h}} \lambda_{t-1}(w) B_{i-w}(\hat{h} - w) \sum_{n=0}^{\hat{h}-s} \lambda_t(n) \xi_{\hat{h},n+s}. \tag{18}$$

Now, we start the proof with the above arms.

Case 1: $\hat{h} = o(i)$**.**

Given $\hat{h} = o(i)$, we can derive the result as follows:

$$\begin{aligned} \sum_{s=0}^{\infty} \phi^s P_{is} &= \sum_{s=0}^{i-\hat{h}-1} \phi^s P_{is} + \sum_{s=i-\hat{h}}^{i} \phi^s P_{is} + \sum_{s=i+1}^{\infty} \phi^s P_{is} \\ &\leq \phi^{i+1} + \sum_{s=i-\hat{h}}^{i} \phi^s \sum_{w=0}^{\hat{h}} \lambda_{t-1}(w) B_{i-w}(\hat{h} - w) \\ &\quad \cdot \sum_{n=0}^{\hat{h}+s-i} \lambda_t(n) \xi_{\hat{h},n+i-s} \\ &\leq \phi^{i+1} + \sum_{s=i-\hat{h}}^{i} \phi^s \sum_{w=0}^{\hat{h}} B_{i-w}(\hat{h} - w) \\ &\leq \phi^{i+1} + \hat{h} e^{-\frac{ip}{2}(1-\frac{\hat{h}}{ip})^2} \phi^{i-\hat{h}} \leq \phi^i, \text{ as } i \to \infty, \end{aligned} \tag{19}$$

where we use the Chernoff's inequality to bound the cumulative probability of $B_{i-w}(\hat{h} - w)$. Therefore, the second condition of Lemma 2 holds when $\hat{h} = o(i)$.

Case 2: $\hat{h} = \Theta(i)$**.**

In this case, we need to distinguish the three instability regions. Without loss of generality, we assume that $\hat{h} = \beta i$ where constant $\beta \in (0, 1]$.

(1) $L_t = o(\hat{h})$**.**

When $L_t = o(\hat{h})$, it also holds that $L_t = o(i)$ and that at most $L_t - 1$ packets are successfully received, we thus have

$$\begin{aligned} \sum_{s=0}^{\infty} \phi^s P_{is} &= \sum_{s=0}^{i-L_t-1} \phi^s P_{is} + \sum_{s=i-L_t}^{i} \phi^s P_{is} + \sum_{s=i+1}^{\infty} \phi^s P_{is} \\ &\leq \phi^{i+1} + L_t \phi^{i-L_t} \\ &\leq \phi^i + B(1-\phi), \text{ as } i \to \infty, \end{aligned} \tag{20}$$

for any positive constant B.

(2) $L_t = \Theta(\hat{h})$.

The key steps we need in this case are to obtain upper bounds of $\xi_{\hat{h}k}^{SPR}$ and the arrival rate in a new way. To this end, we first recomputed $\xi_{\hat{h}k}^{SPR}$ when $L_t = \Theta(\hat{h})$ as follows:

$$\begin{cases} \xi_{\hat{h}k}^{SPR} &= \frac{\binom{L}{k} k! (L_t - k)^{\hat{h}-k}}{L_t^{\hat{h}}} - \frac{\binom{L_t}{k+1}(k+1)!(L_t-k-1)^{\hat{h}-k-1}}{L_t^{\hat{h}}} \\ &\leq \frac{\binom{L_t}{k} k!}{L_t^k} \left((1 - \frac{k}{L_t})^{\hat{h}-k} - (1 - \frac{k+1}{L_t})^{\hat{h}-k} \right) \\ &\leq \frac{\binom{L_t}{k} k!}{L_t^k} \leq (1 - \frac{k/2}{L_t})^{k/2}, \quad k < \min(\hat{h}, L_t), \\ \xi_{\hat{h}\hat{h}}^{SPR} &= \frac{\binom{L_t}{\hat{h}} \hat{h}!}{L_t^{\hat{h}}} \leq (1 - \frac{\hat{h}/2}{L_t})^{\hat{h}/2} \\ &\leq (1 - \alpha/2)^{\hat{h}/2}, \quad k = \hat{h} \leq L_t. \end{cases} \tag{21}$$

The rationale behind the above inequalities is as follows: Given $\hat{h}$ transmitted packets, the probability of exactly k successful packets equals the absolute value of the difference between the probability of at least k successful packets and that of at least $k+1$ successful packets.

Next, we introduce an auxiliary lemma to bound the probability distribution of the arrival rate. When the number of new arrivals per slot A_{tl} is Poisson distributed with the mean Λ, the number of new arrivals per frame N_t (A_{tl} and N_t is formally defined in Sec. III.) is also a Poisson random variable with the mean $\mathfrak{N}_t = L_t \Lambda > \hat{h}e^{-\alpha}$.

Lemma 4 ([16]). *Given a Poisson distributed variable X with the mean μ, it holds that*

$$Pr[X \le x] \le \frac{e^{-\mu}(e\mu)^x}{x^x}, \ \forall \ x < \mu, \tag{22}$$

$$Pr[X \ge x] \le \frac{e^{-\mu}(e\mu)^x}{x^x}, \ \forall \ x > \mu. \tag{23}$$

In the case that $L_t = \Theta(i)$, it holds that $\mathfrak{N}_t = L_t\Lambda > L_t^{\frac{2}{3}}$, for the constant Λ and a large i. Consequently, applying (22) in Lemma 4, we have

$$P\{N_t \le L_t^{2/3}\} \le \frac{e^{-\lambda}(e\lambda)^{L_t^{2/3}}}{(L_t^{2/3})^{L_t^{2/3}}} \le e^{-L_t^{2/3}(\frac{L\Lambda}{L_t^{2/3}}-1)} \left(\frac{L_t\Lambda}{L_t^{2/3}}\right)^{L_t^{2/3}}$$
$$\le \left(e^{\frac{L_t\Lambda}{L_t^{2/3}}-1}\frac{L^{2/3}}{L_t\Lambda}\right)^{-L_t^{2/3}} \le \frac{1}{a_1^{L_t^{2/3}}}, \tag{24}$$

where $a_1 \triangleq \frac{e^{\Lambda L_t^{1/3}-1}}{\Lambda L_t^{1/3}} \gg 1$, following the fact that $e^x > 1+x$, for $\forall \ x > 0$.

Armed with (21) and (24) and noticing the fact that at most $\hat{h}$ packets are successfully received, we start developing the proof and obtain the results as follows:

$$\begin{aligned}
\sum_{s=0}^{\infty}\phi^s P_{is} &= \sum_{s=0}^{i-\hat{h}-1}\phi^s P_{is} + \sum_{s=i-\hat{h}}^{i}\phi^s P_{is} + \sum_{s=i+1}^{\infty}\phi^s P_{is} \\
&\le \phi^{i+1} + \sum_{s=i-\hat{h}}^{i}\sum_{w=0}^{\hat{h}}\lambda_{t-1}(w)B_{i-w}(\hat{h}-w)\cdot \\
&\quad \sum_{n=0}^{\hat{h}+s-i}\lambda_t(n)(1-\frac{n+i-s}{2L_t})^{(n+i-s)/2} \\
&\le \phi^{i+1} + \sum_{s=i-\hat{h}}^{i}\phi^s\sum_{n=0}^{\hat{h}}\lambda_t(n)(1-\frac{n}{2L_t})^{\frac{n}{2}} \\
&\le \phi^{i+1} + \sum_{s=i-\hat{h}}^{i}\phi^s\Big(\sum_{n=0}^{L_t^{2/3}}\lambda_t(n) + \sum_{n=L_t^{2/3}}^{\hat{h}}\lambda_t(n)(1-\frac{n}{2L_t})^{\frac{n}{2}}\Big) \\
&\le \phi^{i+1} + \sum_{s=i-\hat{h}}^{i}\phi^s\Big(\frac{1}{a_1^{L_t^{2/3}}} + (1-\frac{1}{2L_t^{1/3}})^{\frac{L_t^{2/3}}{2}}\Big) \\
&\le (\hat{h}+1)\big(\frac{1}{a_1^{L_t^{2/3}}} + e^{-\frac{L_t^{1/3}}{4}}\big)\phi^{i-\hat{h}} + \phi^{i+1} \\
&\le \phi^i + B(1-\phi), \ \text{ as } \ i \to \infty,
\end{aligned} \tag{25}$$

for any positive constant B, where the last inequality holds for $(\hat{h}+1)(\frac{1}{a_1^{L_t^{2/3}}} + e^{-\frac{L_t^{1/3}}{4}}) \sim \Theta(ie^{-i^{1/3}}) \to 0$ as $i \to \infty$, while $B(1-\phi)$ is positive constant.

Consequently, the second condition in Lemma 2 holds for Case 2. Next, we proceed with the proof for the third case.

(3) $L_t = O(\hat{h})$**.**

When $L_t = O(\hat{h})$, it also holds that $L_t = O(i)$ such that the expected number of new arrivals per frame $\mathfrak{N}_t = L_t\Lambda \gg i$. Since N_t is Poisson distributed as mentioned in Case 2 above, recall (24), it also holds that

$$P\{N_t \le i\} \le \frac{1}{a_2^i}, \tag{26}$$

where $a_2 \triangleq \frac{i}{L_t\Lambda}\cdot e^{\frac{L_t\Lambda}{i}-1} \ge \frac{L_t\Lambda}{i}$, following the fact that $e^x > 1+x+\frac{x^2}{2}+\frac{x^3}{6}$, for $\forall \ x > 0$.

Using (26) then yields

$$\begin{aligned}
\sum_{s=0}^{\infty}\phi^s P_{is} &= \sum_{s=0}^{i}\phi^s P_{is} + \sum_{s=i+1}^{\infty}\phi^s P_{is} \\
&\le \sum_{s=0}^{i}\phi^s\sum_{n=0}^{s}\lambda_t(n) + \phi^{i+1} \le \sum_{s=0}^{i}\sum_{n=0}^{s}\lambda_t(n) + \phi^{i+1} \\
&\le \frac{i+1}{(\phi\frac{L_t\Lambda}{i})^i}\phi^i + \phi^{i+1} \le \phi^i, \ \text{ as } i \to \infty,
\end{aligned} \tag{27}$$

since ϕ is constant while $\frac{L_t\Lambda}{i} \to \infty$ as $i \to \infty$.

Combining the analysis above, it follows Lemma 2 that the backlog Markov chain $(X_t)_{t\ge0}$ is unstable when $L_t = o(\hat{h})$ or $L_t = O(\hat{h})$ or $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha e^{-\alpha}$. And the proof of Algorithm 1 is thus completed. ■

C. System behavior in instability region

It follows from Theorem 1 that the system is unstable in the following three conditions: $L_t = o(\hat{h})$; $L_t = O(\hat{h})$; and $L_t = \Theta(\hat{h})$ but $\Lambda > \alpha e^{-\alpha}$. Lemma 2, however, is not sufficient to ensure the transience of a Markov chain, we thus in this section further investigate the system behavior in the instability region, i.e., when $(X_t)_{t\ge0}$ is nonergodic. The key results are given in Theorem 2.

Before proving Theorem 2, we first introduce the following lemma [15] on the conditions for the transience of a Markov chain.

Lemma 5 ([15]). *Let $(X_t)_{t\ge0}$ be an irreducible and aperiodic Markov chain with the nonnegative integers as its state space and one-step transition probability matrix $\mathbf{P} = \{P_{is}\}$. $(X_t)_{t\ge0}$ is transient if and only if there exists a sequence $\{y_i\}_{i\ge0}$ such that*

1) *y_i $(i \ge 0)$ is bounded,*
2) *for some $i \ge N$, $y_i < y_0, \ y_1, \ \cdots, \ y_{N-1}$,*
3) *for some integer $N > 0$, $\sum_{s=0}^{\infty} y_s P_{is} \le y_i, \ \forall \ i \ge N$.*

Armed with Lemma 5, we now prove Theorem 2.

Proof of Theorem 2: The key to prove Theorem 2 is to show the existence of a sequence satisfying the properties listed in Lemma 5, so we first construct the following sequence (28) and then prove that it satisfies the required conditions.

$$y_i = \frac{1}{(i+1)^\theta}, \ \theta \in (0,1). \tag{28}$$

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It can be easily checked that $\{y_i\}$ satisfies the first two properties in Lemma 5.

Noticing that the sequence $\{\phi^i\}$ in Lemma 2 satisfies the first two properties in Lemma 5 for $0 < \phi < 1$, and recall (19) and (27), we can conclude that $(X_t)_{t\geq 0}$ is transient if $\hat{h} = o(i)$ or $L_t = O(i)$. Therefore, we next proceed with $\hat{h} = \Theta(i)$ by distinguish two cases.

Case 1: $L_t = o(\hat{h})$**.**

When $\hat{h} = \Theta(i)$, it also holds that $L_t = o(i)$. To streamline the complicated analysis in this case, we partition the region $L_t{=}o(i)$ into two parts, i.e., 1) $L_t{=}o((\ln i)^4)^*$, and 2) $L_t{=}o(i)$ except part 1), i.e., the region $[O((\ln i)^4), o(i)]$.

- Part 1): $L_t{=}o((\ln i)^4)^*$.
  The result in this part is shown in the following lemma for the third property in Lemma 5. The proof is detailed in Appendix B.

  Lemma 6. *If* $L_t{=}o((\ln i)^4)^*$, $(X_t)_{t\geq 0}$ *is always transient.*
- Part 2): $L_t{=}o(i)$ except part 1).
  In this case, since $a_1 > \ln i$ and $y_i - y_{i+1} = \frac{1}{(i+1)^\theta}(1 - (1-\frac{1}{i+2})^\theta) \geq \frac{\theta}{(i+1)^\theta(i+2)}$ where we use the fact that $(1-\frac{1}{i+2})^\theta \leq 1 - \frac{\theta}{i+2}$ following Taylor's theorem, using (24) and (28) yields

$$\begin{aligned}
&\sum_{s=0}^{\infty} y_s P_{is} = \sum_{s=0}^{i-L_t} y_s P_{is} + \sum_{s=i-L_t+1}^{i} y_s P_{is} + \sum_{s=i+1}^{\infty} y_s P_{is}\\
&\leq y_{i+1} + \sum_{s=i-L_t+1}^{i} y_s \sum_{n=0}^{s} \lambda_t(n)(1-\frac{n}{2L_t})^{\frac{n}{2}}\\
&\leq \sum_{s=i-L_t+1}^{i} y_s \Big(\sum_{n=0}^{L_t^{\frac{2}{3}}} \lambda_n + \sum_{n=L_t^{\frac{2}{3}}+1}^{s} \lambda_t(n)(1-\frac{n}{2L_t})^{\frac{n}{2}} \Big) + y_{i+1}\\
&\leq \sum_{s=i-L_t+1}^{i} y_s \Big(\frac{1}{a_1^{L_t^{2/3}}} + (1-\frac{1}{2L_t^{1/3}})^{\frac{L_t^{2/3}}{2}} \Big) + y_{i+1}\\
&\leq \frac{L}{(i-L_t+2)^\theta} \Big(\frac{1}{a_1^{L_t^{2/3}}} + e^{-\frac{L_t^{1/3}}{4}} \Big) + \frac{1}{(i+2)^\theta}\\
&\leq \frac{L_t}{(i-L_t+2)^\theta} \Big((\ln i)^{-(\ln i)^{8/3}} + i^{-\frac{(\ln i)^{1/3}}{4}} \Big) + \frac{1}{(i+2)^\theta}\\
&\leq i^{-4} + \frac{1}{(i+2)^\theta} \leq \frac{1}{(i+1)^\theta}, \text{ as } i \to \infty.
\end{aligned} \tag{29}$$

Case 2: $L_t = \Theta(\hat{h})$**.**

In this case, the method to prove is similar with that used in (25). Recall (25) , we have

$$\begin{aligned}
&\sum_{s=0}^{\infty} y_s P_{is} = \sum_{s=0}^{i-\hat{h}-1} y_s P_{is} + \sum_{s=i-\hat{h}}^{i} y_s P_{is} + \sum_{s=i+1}^{\infty} y_s s P_{is}\\
&\leq y_{i+1} + \sum_{s=i-\hat{h}}^{i} y_s \Big(\frac{1}{a_1^{L^{2/3}}} + (1-\frac{1}{2L^{1/3}})^{\frac{L^{2/3}}{2}} \Big)\\
&\leq \frac{\hat{h}+1}{(i-\hat{h}+1)^\theta} \big(\frac{1}{a_1^{L^{2/3}}} + e^{-\frac{L^{1/3}}{4}} \big) + \frac{1}{(i+2)^\theta}\\
&\leq \frac{1}{(i+1)^\theta}, \quad \text{as } i \to \infty.
\end{aligned} \tag{30}$$

Consequently, it follows Lemma 5 that the backlog Markov chain $(X_t)_{t\geq 0}$ is transient in the instability region, which completes the proof of Theorem 2. ■

VI. Stability Analysis of FSA-MPR

In this section, we study stability properties of FSA-MPR. Following a similar procedure as the analysis of FSA-SPR, we first establish conditions for the stability of FSA-MPR and further analyse the system behavior in the instability region.

A. Stability analysis

We employ Lemma 1 and Lemma 2 as mathematical base to study the stability properties of FSA-MPR, more specifically, in the proof of Theorem 3.

Proof of Theorem 3: We develop our proof in 3 steps.

Step 1: stability conditions.

In step 1, we prove the conditions for the stability of $(X_t)_{t\geq 0}$, i.e., $\Lambda < \sum_{x_0=1}^{\overline{M}} e^{-\alpha} \frac{\alpha^{x_0}}{x_0!} \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0}$ and $L_t = \Theta(\hat{h})$.

Similar to (15), the drift at state i of $(X_t)_{t\geq 0}$ in FSA-MPR can be written as:

$$D_i = \mathfrak{N}_t - \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{MPR}. \tag{31}$$

According to (16), D_i is finite as shown in the following inequality:

$$|D_i| < \max\{\mathfrak{N}_t, (1-p)\mathfrak{N}_t + ip\},$$

which demonstrates the first conditions in Lemma 1 for the ergodicity of $(X_t)_{t\geq 0}$.

Recall (8) and Lemma 3, we have

$$\begin{aligned}
\lim_{i\to\infty} D_i &= \lim_{i\to\infty} \mathfrak{N}_t - \sum_{w=0}^{i} \lambda_{t-1}(w) \sum_{h=0}^{i-w} B_{i-w}(h) r_{\hat{h}}^{MPR}\\
&= \lim_{\hat{h}\to\infty} L_t \left(\Lambda - \sum_{x_0=1}^{\hat{h}} B_{\hat{h},\frac{1}{L}}(x_0) \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} \right)\\
&= L_t \left(\Lambda - \sum_{x_0=1}^{\overline{M}} e^{-\alpha} \frac{\alpha^{x_0}}{x_0!} \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} \right).
\end{aligned} \tag{32}$$

Therefore, it holds that $\lim_{i\to\infty} D_i < -\epsilon_0$ if $L_t = \Theta(\hat{h})$ and $\Lambda < \sum_{x_0=1}^{\overline{M}} e^{-\alpha} \frac{\alpha^{x_0}}{x_0!} \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} \triangleq \hat{R}_1$. It then follows from Lemma 1 that $(X_t)_{t\geq 0}$ is ergodic with $\epsilon_0 = \frac{\hat{R}_1 - \Lambda}{2}$.

Step 2: $\alpha^* = \Theta(\overline{M})$**.**

In Step 2, we show that $\alpha^* = \Theta(\overline{M})$. Since the proof consists mainly of algebraic operations of function optimization, we state the following lemma proving Step 2 and detail its proof in Appendix C.

Lemma 7. *Let* α^* *denote the value of* α *that maximises the upper bound of the stability region, it holds that* $\alpha^* = \Theta(\overline{M})$.

Step 3: instability region.

In Step 3, we prove the instability region of $(X_i)_{i\geq 0}$ by applying Lemma 2.

When $L_t = o(\hat{h})$, recall (32), we have

$$\sum_{x_0=1}^{\hat{h}} B_{\hat{h},\frac{1}{L}}(x_0) \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} = \sum_{x_0=1}^{\overline{M}} B_{\hat{h},\frac{1}{L}}(x_0) \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} \le \sum_{x_0=1}^{\overline{M}} x_0 B_{\hat{h},\frac{1}{L}}(x_0) \to 0, \text{as } \hat{h} \to \infty,$$

since $\lim_{\hat{h}\to\infty} B_{\hat{h},\frac{1}{L}}(x_0) = 0$ for a finite $\overline{M}$.

Moreover, for $L_t = O(\hat{h})$, it can be derived from (32) that $\sum_{x_0=1}^{\overline{M}} e^{-\alpha} \frac{\alpha^{x_0}}{x_0!} \sum_{k_0=1}^{x_0} k_0 \hat{\xi}_{x_0 k_0} \to 0$ since $\alpha \to 0$ as $\hat{h} \to \infty$.

Furthermore, according to the analysis in the first step, we know that $\lim_{i\to\infty} D_i > 0$, if the conditions in the first step are not satisfied.

Additionally, in the analysis of FSA-SPR system, we have proven that if $\hat{h} = o(i)$ or $L_t{=}o(\hat{h})$ or $L_t{=}O(\hat{h})$, the Markov chain $(X_t)_{t\ge 0}$ is always unstable, independent of $\xi_{\hat{h}k}$. Noticing that $\xi_{\hat{h}k}$ is the only difference between FSA-SPR and FSA-MPR, it thus also holds that $(X_t)_{t\ge 0}$ is unstable under FSA-MPR in the three cases.

We next study the instability of FSA-MPR when $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha$. In this case, it holds that $\mathfrak{N}_t = L_t\Lambda > \hat{h}$ such that

$$P\{N_t \le \hat{h}\} \le \frac{1}{a_3^{\hat{h}}}, \tag{33}$$

where $a_3 \triangleq \frac{\alpha}{\Lambda} e^{\frac{\Lambda}{\alpha}-1} > 1$.

Note that the one-step transition probability P_{is} in FSA-MPR can be obtained by replacing $\min(\hat{h}, L_t)$ with $\hat{h}$ in (10).

Hence, recall (25), we have

$$\begin{aligned}\sum_{s=0}^{\infty} \phi^s P_{is} &= \sum_{s=0}^{i} \phi^s P_{is} + \sum_{s=i}^{\infty} \phi^s P_{is} \\ &\le \sum_{s=0}^{i} \phi^s \sum_{n=0}^{\hat{h}} \lambda_t(n) \xi_{\hat{h},n+i-s} + \phi^{i+1} \\ &\le \frac{1}{a_3^{\beta i}} + \phi^{i+1} \le \phi^i + B(1-\phi), \text{ as } i \to \infty,\end{aligned} \tag{34}$$

which proves the instability of FSA-MPR following Lemma 2 and also completes the proof of Theorem 3. ∎

B. System behavior in instability region

It follows from Theorem 3 that the system is unstable under the following three conditions: $L_t = o(\hat{h})$; $L_t = O(\hat{h})$; $L_t = \Theta(\hat{h})$ and $\Lambda > \alpha$. In this subsection, we further investigate the system behavior in the instability region, i.e., when $(X_t)_{t\ge 0}$ is nonergodic. The key results are given in Theorem 4, whose proof is detailed as follows.

Proof of Theorem 4: In the proof of Theorem 2, we have proven that when $L_t{=}O(\hat{h})$ or $\hat{h}{=}o(i)$, the Markov chain $(X_t)_{t\ge 0}$ is always transient, we thus develop the proof for $\hat{h}{=}\Theta(i)$ by distinguishing two cases.

Case 1: $L_t = o(\hat{h}^{1-\epsilon_1})^*$ **with** $\epsilon_1 \in (0, 1]$.

In this case, it holds that $L_t = o(i^{1-\epsilon_1})^*$ for $\hat{h}{=}\Theta(i)$. As counterparts in FSA-SPR, we also partition the region into two parts, i.e., 1) $L_t{=}o((\ln i)^4)^*$, and 2) $L_t{=}o(i)$ except part 1), i.e., the region $[O((\ln i)^4), o(i)]$.

Recall the proof of Lemma 6, it has been shown that $(X_t)_{t\ge 0}$ is always transient, independent of $\xi_{\hat{h}k}$, meaning $(X_t)_{t\ge 0}$ is also transient in FSA-MPR when $L_t{=}o((\ln i)^4)^*$.

As a consequence, it is sufficient to show the transience of $(X_t)_{t\ge 0}$ in part 2). The key step here is to obtain the upper bound of $\xi_{\hat{h}k}$. To this end, we first introduce the following auxiliary lemma.

Lemma 8 ([6]). *Given $\hat{h}$ packets, each packet is sent in a slot picked randomly among L_t time-slots in frame t. If $\rho_j = L_t \frac{e^{-\hat{h}/L_t}}{j!} (\frac{\hat{h}}{L_t})^j$ remains bounded for $\hat{h}, L_t \to \infty$, then the probability $P(m_j)$ of finding exactly m_j time-slots with j packets can be approximated by the following Poisson distribution with the parameter ρ_j,*

$$P(m_j) = e^{-\rho_j} \frac{{\rho_j}^{m_j}}{m_j!}. \tag{35}$$

We next show that Lemma 8 is applicable to FSA-MPR when $L_t = o(\hat{h}^{1-\epsilon_1})^*$ for a large enough $\hat{h}$. To that end, we verify the boundedness of ρ_j, which is derived as

$$0 \le \rho_j \le \frac{\hat{h}^j}{j! L_t^{j-1} e^{\hat{h}^\epsilon}} \le \frac{\hat{h}^j}{j! L_t^{j-1}} \cdot \frac{(\lceil\frac{1}{\epsilon}\rceil j)!}{(\hat{h}^\epsilon)^{\lceil\frac{1}{\epsilon}\rceil j}} \le \frac{(\lceil\frac{1}{\epsilon}\rceil j)!}{j! L_t^{j-1}}, \tag{36}$$

meaning that ρ_j is bounded if j is finite.

Apparently, when $L_t = o(\hat{h}^{1-\epsilon_1})^*$, the probability of finding exactly m_j time-slots with j packets in FSA-MPR can be approximated by the Poisson distribution with the parameter ρ_j, following from Lemma 8 with $j = 1, 2, \cdots, \overline{M}$.

Consequently, we can derive the probability $\xi_{1\to\overline{M}}^{MPR}$ that there are no slots with $1 \le j \le \overline{M}$ packets as follows:

$$\xi_{1\to\overline{M}}^{MPR} = e^{-(\rho_1+\rho_2+\cdots+\rho_{\overline{M}})}. \tag{37}$$

Furthermore, since the event that all $\hat{h}$ packets fail to be received has two probabilities, i.e., 1) there are no slots with $1 \le j \le \overline{M}$ packets in the whole frame, and 2) there exists slots with $1 \le j \le \overline{M}$ packets, but all of these packets are unsuccessful. As a result, it holds that $\xi_{\hat{h}0}^{MPR} \ge \xi_{1\to\overline{M}}^{MPR}$.

We thus can get the following inequalities:

$$\begin{aligned}\xi_{\hat{h}k}^{MPR} &\le 1 - \xi_{\hat{h}0}^{MPR} \le 1 - e^{-(\rho_1+\rho_2+\cdots+\rho_M)} \\ &\le 1 - e^{-\overline{M}\rho_{\overline{M}}}, k \ge 1,\end{aligned} \tag{38}$$

where we use the fact that the probability of exact $k \ge 1$ successfully received packets among $\hat{h}$ packets is less than that of at least one packet received successfully in the first inequality. And the third inequality above follows from the monotonicity of ρ_j when $L = o(\hat{h}^{1-\epsilon_1})^*$, i.e.,

$$\rho_{\overline{M}} > \rho_{\overline{M}-1} > \cdots > \rho_2 > \rho_1.$$

In addition, we can also derive the following results:

$$\begin{aligned}0 &\le \lim_{\hat{h}\to\infty} \hat{h}^4 (1 - e^{-\overline{M}\rho_{\overline{M}}}) \le \lim_{\hat{h}\to\infty} \frac{e^{\overline{M}\rho_{\overline{M}}} - 1}{(1/\hat{h}^4)} \\ &\le \lim_{\hat{h}\to\infty} \frac{\epsilon_1 \hat{h}^{\overline{M}\epsilon_1+5}}{4\overline{M}! e^{\hat{h}^{\epsilon_1}}} \\ &\le \lim_{\hat{h}\to\infty} \frac{\prod_{x=0}^{\overline{M}-1+\lceil\frac{5}{\epsilon_1}\rceil} (\overline{M} + \frac{5}{\epsilon_1} - x)}{4\overline{M}! e^{\hat{h}^{\epsilon_1}}} \le 0,\end{aligned}$$

which means $1 - e^{-\overline{M}\rho_{\overline{M}}} \leq \frac{1}{\hat{h}^4}$. Using this inequality and recall (39), we have

$$\begin{aligned}
\sum_{s=0}^{\infty} y_s P_{is} =& \sum_{s=0}^{i-L} y_s P_{is} + \sum_{s=i-L+1}^{i} y_s P_{is} + \sum_{s=i+1}^{\infty} y_s P_{is} \\
\leq& y_{i+1} + \sum_{s=i-L+1}^{i} y_s \sum_{n=0}^{s} \lambda_t(n)\xi_{\hat{h},n+i-s} \\
\leq& \sum_{s=i-L+1}^{i} y_s \Big(\sum_{n=0}^{L^{\frac{2}{3}}} \lambda_n + \sum_{n=L^{\frac{2}{3}}+1}^{s} \lambda_t(n)\xi_{\hat{h},n+i-s} \Big) + y_{i+1} \\
\leq& \sum_{s=i-L+1}^{i} y_s \Big(\frac{1}{a_1^{L^{2/3}}} + \frac{1}{\hat{h}^4} \Big) + y_{i+1} \\
\leq& \frac{L}{(i-L+2)^\theta} \Big(\frac{1}{a_1^{L^{2/3}}} + \frac{1}{\hat{h}^4} \Big) + \frac{1}{(i+2)^\theta} \\
\leq& 2(\beta i)^{-3} + \frac{1}{(i+2)^\theta} \leq \frac{1}{(i+1)^\theta}, \text{ as } i \to \infty. \qquad (39)
\end{aligned}$$

Thus, according to Lemma 6, the backlog Markov chain $(X_t)_{t\geq 0}$ is transient when $L_t = o(\hat{h}^{1-\epsilon_1})^*$.

Case 2: $L_t = \Theta(\hat{h})$ **and** $\Lambda > \alpha$**.**

In this case, we have $\mathfrak{N}_t = L_t\Lambda > \hat{h}$. Using similar reasoning as (34), we have

$$\sum_{s=0}^{\infty} y_s P_{is} \leq \frac{\beta i + 1}{a_3^{\beta i}} + \frac{1}{(i+2)^\theta} \leq \frac{1}{(i+1)^\theta}, \text{ as } i \to \infty.$$

Therefore, $(X_t)_{t\geq 0}$ is also transient in this case and the proof of Theorem 4 is completed. ■

VII. Discussion

In previous sections, we prove that the stability of FSA relies on the relationship between the frame size and the number of packets to be transmitted in the frame. In order to set the frame size to stabilize FSA systems, the users need to know the number of transmitted packets in the current frame, which is not always observable. In this section, we discuss how to estimate its approximate value from an engineering perspective.

Recall (9), because N_t follows the Poisson distribution and Z_t follows the binomial distribution which can be approximated as the Poisson distribution, Y_t can also be approximated as a Poisson distributed random variable. According to Lemma 4, the value of Y_t sharply concentrates around its expectation, we thus use the following $E[Y_t]$ to approximate $\hat{h}$:

$$\begin{aligned}
E[Y_t] &= ip + (1-p)E[N_{t-1}] \\
&= ip + (1-p)L_{t-1}\Lambda. \qquad (40)
\end{aligned}$$

As a result, we can set the frame size following the control algorithm as follows:

$$L_t = c_1 \left(ip + (1-p)L_{t-1}\Lambda, \right), \qquad (41)$$

$$L_0 = c_1\vartheta, \qquad (42)$$

where $X_0 = \vartheta$ means the initial number of packets in the system, and $c_1 = 1$ for FSA-SPR and $c_1 = \frac{1}{\alpha^*}$ for FSA-MPR.

By the above control algorithm, the frame size L_t only depends on the value of backlog population size X_t, i.e., i, so the original problem is translated to estimate the number of backlogs X_t, i.e., i. Fortunately, there exist several estimation approaches which exploit the channel feedback, such as the probability of a idle or collision slot, and the number of idle or collision slots. Here, we simply illustrate one of feasible estimation methods for FSA-SPR. Denote by I_{t-1} the number of idle slots and by $\hat{C}_{t-1}$ the actual number of successful packets in frame $t-1$, which can be observed at the end of frame $t-1$. Since it holds on the idle slot probability that $\frac{I_{t-1}}{L_{t-1}} \approx \left(1 - \frac{1}{L_{t-1}}\right)^{Y_{t-1}}$ and $Y_{t-1} \approx X_{t-1}p + (1-p)L_{t-2}\Lambda$ and $X_t \approx X_{t-1} - \hat{C}_{t-1} + L_{t-1}\Lambda$, we can estimate actual value of X_t and configure L_t.

More theoretical analysis are conducted in the following references. Specifically, according to the requirement on the estimation accuracy, a rough estimator or an accurate estimator can be selected. Since $ip \leq L_t \leq i$ in (41), rough value of X_t can be estimated so that the estimate $\tilde{X}_t = \Theta(X_t)$ in very short time, more specifically, in $\log(X_t)$ or $\log\log(X_t)$ slots [3]. While if the accurate result is required, we can use the additive estimator as in [32] and Kalman filter-based estimator as in our another work [33] to estimate the value of X_t and update L_t. To summarize, the frame size can be updated based on these estimation schemes in practical scenarios.

VIII. Numerical Results

In this section, we conduct simulations via MATLAB to verify our theoretical results by illustrating the evolution of the number of backlogs in each frame under different parameters with the following default settings: the initial number of backlogs $X_0 = 10^4$, the simulation duration $t_{\max} = 100$, $o(\hat{h}) \leq 0.01\hat{h}$, $O(\hat{h}) \geq 100\hat{h}$, $\alpha = 1$ in FSA-SPR and $\alpha = \alpha^* \in (\frac{\overline{M}-1}{e}, \overline{M})$ in FSA-MPR when $L_t = \Theta(\hat{h})$. To simulate FSA, each user first generates a random number among $[0, L_t - 1]$ uniformly and responds in the corresponding slot. And all results are obtained by taking the average of 100 trials.

A. Stability properties of FSA

FSA-SPR systems. We start by investigating numerically the stability properties of FSA-SPR. As stated in Theorem 1, if $\Lambda < \frac{1}{e}$, the system is stable and unstable otherwise when $\alpha = 1$, we thus set the expected arrival rate per slot Λ to 0.3 and 0.37 for the analysis of stability and instability for $L_t = \hat{h}$, respectively. Moreover, we set a small $\Lambda = 0.01$ to analyze the instability for the cases $L_t = o(\hat{h})$ and $L_t = O(\hat{h})$.

As shown in Fig. 1(a), for the case $L_t = \hat{h}$, the number of backlogs decreases to zero at a rate in proportion to the retransmission probability if $\Lambda < \frac{1}{e}$, while increasing gradually otherwise. This is due to the nature of FSA that frame size varies with the number of the sent packets to maximize the throughput per slot. Moreover, Fig. 1(b) and Fig. 1(c) illustrate the instability when $L_t = o(\hat{h})$ and $L_t = O(\hat{h})$. The numerical results is in accordance with the analytical results on FSA-SPR in Theorem 1.

FSA-MPR System. We then move to the FSA-MPR exploiting MPR model as in [34]. By varying $\overline{M}$ from 2 to 10, we observe that the maximum stability region monotonously increases from 0.84 to 5.84. In the following, we take an example of $\overline{M} = 10$ when $\alpha^* = 10/1.37$ following from

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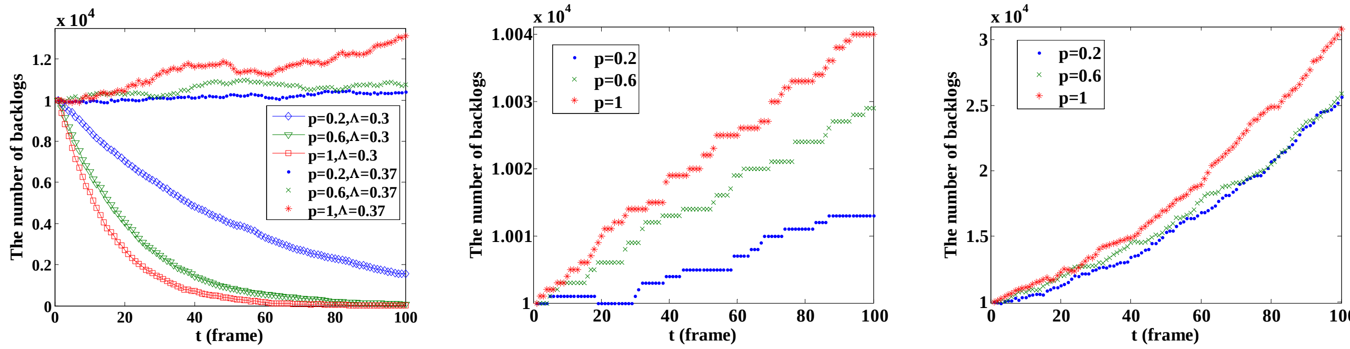
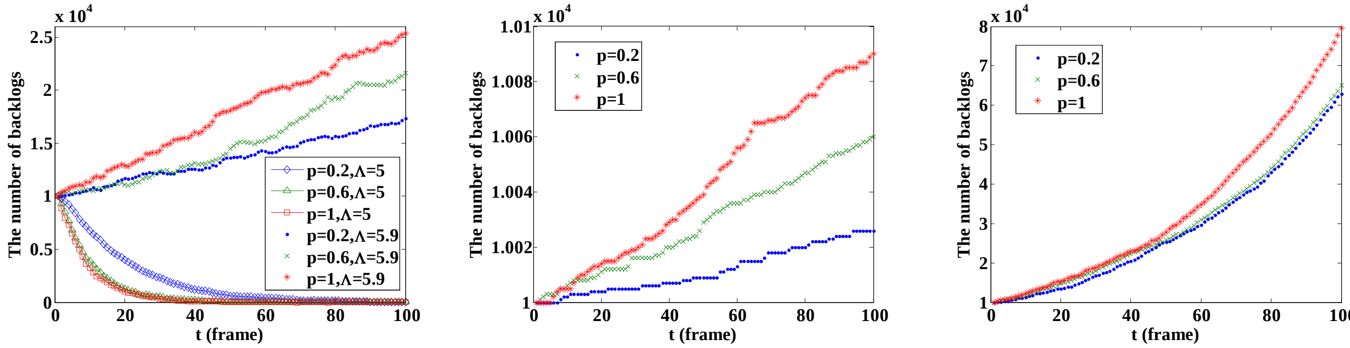


(a) $L = \hat{h}$ (b) $L = o(h)$ and $\Lambda = 0.01$ (c) $L = O(h)$ and $\Lambda = 0.01$

Fig. 1. The evolution of backlog population in FSA-SPR.



(a) $L = \hat{h}/\alpha^*$ (b) $L = o(h)$ and $\Lambda = 0.01$ (c) $L = O(h)$ and $\Lambda = 0.01$

Fig. 2. The evolution of backlog population in FSA-MPR.

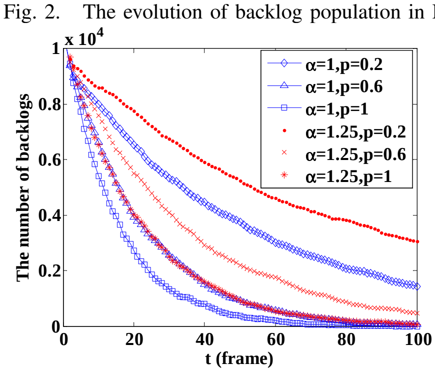


Fig. 3. SPR: No. of backlogs vs. frame size.

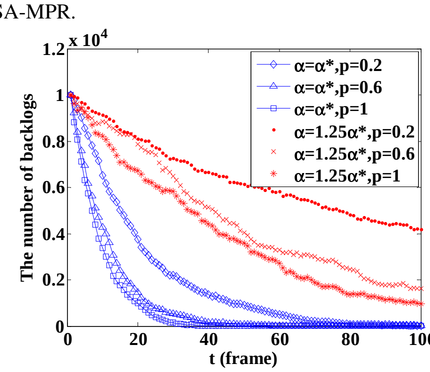


Fig. 4. MPR: No. of backlogs vs. frame size.

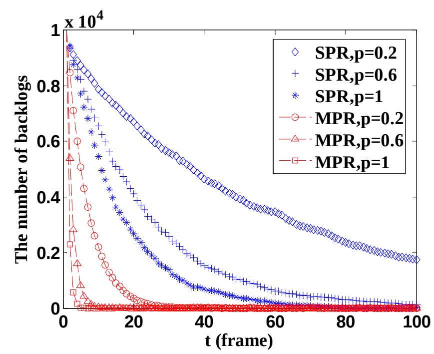


Fig. 5. FSA-SPR vs. FSA-MPR.

Lemma 7. We thus set $\Lambda = 5$ and $\Lambda = 5.9$ for the stability analysis in the case $L_t = \hat{h}/\alpha^*$. As shown in Fig. 2, the numerical results are in accordance with the analytical results on FSA-MPR. Similar trends are observed in another MPR model that only one packet can be successfully captured out of at most $\overline{M}$.

B. Comparison under different frame sizes

We evaluate the performance difference when the frame size deviates from its optimum value that $\alpha = \frac{\hat{h}}{L_t} = 1$ in FSA-SPR and $\alpha = \alpha^*$ in FSA-MPR. To that end, we set $\Lambda = 0.3$ for FSA-SPR and $\Lambda = 5$ for FSA-MPR. As shown in Fig. 3 and Fig. 4, the performance degrades significantly when the frame size is not optimal.

C. Comparison between FSA-SPR and FSA-MPR

We further compare the performance of FSA-SPR and FSA-MPR. To that end, for both FSA-SPR and FSA-MPR, we set $\Lambda = 0.3$ and $L_t = \hat{h}$ where $\hat{h}$ is the number of backlogs in FSA-SPR maximizing the throughput of FSA-SPR. We can also see from Fig. 5 that FSA-MPR, even in the case with non-optimal settings, remarkably outperforms FSA-SPR.

IX. Conclusion

In this paper, we have studied the stability of FSA-SPR and FSA-MPR by modeling the system backlog as a Markov chain. By employing drift analysis, we have obtained the closed-form conditions for the stability of FSA and shown that the stability region is maximised when the frame length equals the number of sent packets in FSA-SPR and the upper bound of stability region is maximised when the ratio of the number of sent packets to frame length equals in an order of magnitude the maximum multipacket reception capacity in FSA-MPR. Furthermore, to characterise system behavior in the instable region, we have mathematically demonstrated the existence of transience of the Markov chain. In addition, we conduct the numerical analysis to verify the theoretical results. Our results provide theoretical guidelines on the design of stable FSA-based protocols in practical applications such as RFID systems and M2M networks.

REFERENCES

[1] N. Abramson. The Aloha system: Another alternative for computer communications. In *Proceedings of Fall 1970 AFIPS fall joint computer conference*, pages 281–285, 1970.

[2] C. Bordenave, D. McDonald, and A. Proutiere. Asymptotic stability region of slotted aloha. *IEEE Transactions on Information Theory*, 58(9):5841–5855, 2012.

[3] B. Chen, Z. Zhou, and H. Yu. Understanding RFID counting protocols. In *ACM MobiHoc*, pages 291–302, 2013.

[4] EPCglobal Inc. Radio-frequency identity protocols class-1 generation-2 UHF RFID protocol for communications at 860 mhz - 960 mhz version 1.0.9. *EPCglobal Inc.*, 17, 2005.

[5] F. Farhadi and F. Ashtiani. Stability region of a slotted aloha network with k-exponential backoff. *arXiv preprint:1406.4448*, 2014.

[6] W. Feller. *An introduction to probability theory and its applications*, volume 2. John Wiley & Sons, 2008.

[7] S. Ghez, S. Verdu, and S. C. Schwartz. Stability properties of slotted Aloha with multipacket reception capability. *IEEE Transactions on Automatic Control*, 33(7):640–649, 1988.

[8] S. Ghez, S. Verdu, and S. C. Schwartz. Optimal decentralized control in the random access multipacket channel. *IEEE Transactions on Automatic Control*, 34(11):1153–1163, 1989.

[9] H. Inaltekin, M. Chiang, H. V. Poor, and S. B. Wicker. Selfish random access over wireless channels with multipacket reception. *IEEE Journal on Selected Areas in Communications*, 30(1):138–152, 2012.

[10] J. Jeon and A. Ephremides. On the stability of random multiple access with stochastic energy harvesting. *IEEE Journal on Selected Areas in Communications*, 33(3):571–584, 2015.

[11] N. Johnson and S. Kotz. *Urn models and their application: an approach to modern discrete probability theory*. Wiley, 1977.

[12] M. Kaplan. A sufficient condition for nonergodicity of a Markov chain. *IEEE Transection on Information Theory*, 25(4):470–471, 1979.

[13] S. C. Kompalli and R. R. Mazumdar. On the stability of finite queue slotted aloha protocol. *IEEE Transactions on Information Theory*, 59(10):6357–6366, 2013.

[14] X. Liu, K. Li, G. Min, K. Lin, B. Xiao, Y. Shen, and W. Qu. Efficient unknown tag identification protocols in large-scale rfid systems. *IEEE Trans. on Parallel and Distributed Systems*, 25(12):3145–3155, 2014.

[15] J. F. Mertens, E. S. Cahn, and S. Zamir. Necessary and sufficient conditions for recurrence and transience of Markov chains, in terms of inequalities. *Journal of Applied Probability*, 15(4):848–851, 1978.

[16] M. Mitzenmacher and E. Upfal. *Probability and computing: Randomized algorithms and probabilistic analysis*. Cambridge University Press, 2005.

[17] V. Naware, G. Mergen, and L. Tong. Stability and delay of finite-user slotted ALOHA with multipacket reception. *IEEE Transactions on Information Theory*, 51(7):2636–2656, 2005.

[18] H.-J. Noh, J.-K. Lee, and J.-S. Lim. Anc-aloha: Analog network coding ALOHA for satellite networks. *IEEE Communications Letters*, 18(6):957–960, 2014.

[19] H. Okada, Y. Igarashi, and Y. Nakanishi. Analysis and application of framed ALOHA channel in satellite packet switching networks-fadra method. *Electronics and Communications in Japan*, 60:72–80, 1977.

[20] A. G. Pakes. Some conditions for ergodicity and recurrence of Markov chains. *Operations Research*, 17(6):1058–1061, 1969.

[21] Z. G. Prodanoff. Optimal frame size analysis for framed slotted Aloha based rfid networks. *Computer Communications*, 33(5):648–653, 2010.

[22] R. R. Rao and A. Ephremides. On the stability of interacting queues in a multiple-access system. *IEEE Transactions on Information Theory*, 34(5):918–930, 1988.

[23] L. G. Robert. ALOHA packet system with and without slots and capture. *ACM SIGCOMM Computer Communication Review*, 5(2):28–42, 1975.

[24] J. Sant and V. Sharma. Performance analysis of a slotted-ALOHA protocol on a capture channel with fading. *Queueing Systems*, 34(1-4):1–35, 2000.

[25] F. C. Schoute. Dynamic frame length ALOHA. *IEEE Transactions on Communications*, 31(4):565–568, 1983.

[26] W. Szpankowski. Stability conditions for some distributed systems: Buffered random access systems. *Advances in Applied Probability*, 26(2):498–515, 1994.

[27] B. S. Tsybakov and V. A. Mikhailov. Ergodicity of a slotted ALOHA system. *Problemy Peredachi Informatsii*, 15(4):73–87, 1979.

[28] S. Vasudevan, D. Towsley, D. Goeckel, and R. Khalili. Neighbor discovery in wireless networks and the coupon collector's problem. In *ACM MobiCom*, pages 181–192, 2009.

[29] F. Vázquez Gallego, J. Alonso-Zarate, and L. Alonso. Energy and delay analysis of contention resolution mechanisms for machine-to-machine networks based on low-power wifi. In *IEEE ICC*, pages 2235–2240, 2013.

[30] H. Vogt. Efficient object identification with passive RFID tags. In *International Conference on Pervasive Computing*, pages 98–113, 2002.

[31] J. E. Wieselthier, A. Ephremides, and A. Larry. An exact analysis and performance evaluation of framed ALOHA with capture. *IEEE Transactions on Communications*, 37(2):125–137, 1989.

[32] H. Wu, C. Zhu, R. J. La, and X. Liu. Fasa: Accelerated S-ALOHA using access history for event-driven M2M communications. *IEEE/ACM Transactions on Networking*, 21(6):1904–1917, 2013.

[33] J. Yu and L. Chen. From static to dynamic tag population estimation: An extended Kalman Filter perspective. *arXiv preprint:1511.08355*, 2015.

[34] W. Zeng, S. Vasudevan, X. Chen, B. Wang, A. Russell, and W. Wei. Neighbor discovery in wireless networks with multipacket reception. In *ACM MobiHoc*, page 3, 2011.

[35] Y. Zhu, W. Jiang, Q. Zhang, and H. Guan. Energy-efficient identification in large-scale rfid systems with handheld reader. *IEEE Transactions on Parallel and Distributed Systems*, 25(5):1211–1222, 2014.

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