Imitation-based Spectrum Access Policy for CSMA/CA-based Cognitive Radio Networks

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Abstract—In this paper, we tackle the problem of opportunistic spectrum access in cognitive radio networks where a number of unlicensed Secondary Users (SU) operating on the standard CSMA/CA protocol access a number of frequency channels partially occupied by licensed Primary Users (PU). We apply evolutionary game theory to model the spectrum access problem and derive distributed mechanisms to converge to the Nash equilibrium. To this end, we combine a payoff computation methodology, relying on the estimation on the number of SUs on the same channel, with the channel access policy derived by the evolutionary game model. The conducted numerical analysis shows that a fast convergence is achieved and the proposed mechanisms are robust against errors in payoff computation.

I. Introduction

Cognitive radio [1], with its capability to flexibly configure its transmission parameters, has emerged in recent years as a promising paradigm to enable more efficient spectrum utilization. Spectrum access models in cognitive radio networks can be classified into three categories, namely exclusive use (or operator sharing), commons and shared use of primary licensed spectrum [2]. In the last model, unlicensed secondary users (SU) are allowed to access the spectrum of licensed primary users (PU) in an opportunistic way. In this case, a well-designed spectrum access mechanism is crucial to achieve efficient spectrum usage.

In this paper, we focus on the generic model of cognitive networks consisting of multiple frequency channels, each characterized by a channel availability probability determined by the activity of PUs on it. In such model, from the individual SU's perspective, a challenging problem is to compete (or coordinate) with other SUs in order to opportunistically access the unused spectrum of PUs to maximize its own payoff; at the system level, a crucial research issue is to design efficient spectrum access protocols achieving optimal spectrum usage.

We tackle the problem of opportunistic spectrum access in CSMA/CA-based Cognitive Radio Networks from an evolutionary game theoretic angle. The motivation of applying evolutionary game theory in the study of the spectrum access problem is twofold. First, it is a powerful tool to study the interaction among players and the system dynamic in terms of population. Stemmed from classic game theory and Darwins evolution theory, it can explicitly capture the fundamental relationship among *competition*, *cooperation* and *communication*, three crucial elements in the design of any spectrum access

protocols in cognitive radio networks. Second, evolutionary game theory provides a theoretic tool for the design of distributed channel access protocols based on local information which is particularly suited in decentralized environments as cognitive radio networks.

We formulate the spectrum access game as a population game where the Secondary Users are aimed at load-balancing the achieved throughput. To this end, we apply the Proportional Imitation-based Spectrum Access Policy (PISAP) proposed in [3], which can be implemented distributedly based on solely local interactions and thus is especially suited in decentralized adaptive learning environments as cognitive radio networks.

The current work represents a natural extension of the theoretical work in [3], that assumes an abstract MAC layer with which each SU gets a fair portion of the channel capacity. Given that the most commonly used distributed MAC layer protocol in autonomous wireless networks is CSMA/CA, this paper aims at filling the gap between theoretic and practical implementation of the proposed imitation dynamic. To this end, we provide a systematical study based on IEEE 802.11 DCF and we demonstrate the convergence to an efficient and stable system equilibrium, i.e. to a fair throughput allocation. The difficulty lies in the fact that CSMA/CA presents the well-known problem of *short-term unfairness*, i.e., a SU can experience large variations on the throughput if the measure time is not long enough. Hence implementing directly the imitation based on throughput may provide very poor system performance. To overcome this difficulty, we propose to implement the imitation protocol by estimating the number of SUs on the same channel. We show that this operation is equivalent to estimating the expected throughput (on the long term) and thus will lead to the same optimal operating point of the system.

II. Related Work

Due to the success of applying evolutionary game theory in the study of biological and economic problems, a handful of recent studies have applied evolutionary game theory as a tool to study resource allocation problems arisen from wired and wireless networks, among which Shakkottai *et al.* addressed the problem of non-cooperative multi-homing of users to access points in IEEE 802.11 WLANs by modeling it as a population game and studied the equilibrium properties of the

game [4]; Berenbrik *et al.* studied the convergence speed for achieving fair resource allocation by means of the Proportional Imitation Rule [5]; Niyato *et al.* studied the dynamics of network selection in a heterogeneous wireless network using the theory of evolutionary game and the replicator dynamic and proposed two network selection algorithm to reach the evolutionary equilibrium [6]; Ackermann *et al.* investigated the concurrent imitation dynamics in the context of symmetric congestion games by focusing on the convergence properties [7]; Niyato *et al.* studied the multiple-seller and multiple-buyer spectrum trading game in cognitive radio networks using the replicator dynamic and provided a theoretic analysis for the two-seller two-group-buyer case [8]. Coucheney *et al.* studied the user-network association problem in wireless networks with multi-technology and proposed an algorithm to achieve the fair and efficient solution [9].

On the other hand, there are several works in the literature addressing the problem of estimating the number of competing stations using CSMA/CA as random access protocol. In [10] for example, Maskery *et al.* propose a method based on the probability of channel capture. In [11] Heusse *et al.* propose a novel technique consisting in dynamically changing the contention window depending on the amount of sensed idle slots. Doing so, it is shown that the contention window of each user converge to a unique optimal value, which maximize the throughput and which can be used to estimate the number of competing hosts on channel. Bianchi *et al.* define in [12] a method for estimating the number of active hosts by means of a Kalman filter.

The rest of the paper is structured as follows. Section III presents the system model followed by the formulation of the spectrum access game. Section IV describes and analyzes distributed techniques for payoff calculation. Section V discusses a natural enhancement to our work. Section VI presents simulations results to evaluate the performance of the proposed techniques. Section VII concludes the paper.

III. Network Model and Game Theory Formulation

In this section, we present the system model of our work, followed by the game formulation of the spectrum access problem, which serves as the basis of the analysis presented in subsequent sections.

A. Network Model

We consider a primary network consisting of a set $\mathcal{C}$ of C frequency channels, each with bandwidth B[1]. The users in the primary network are operated in a synchronous time-slotted fashion. A set $\mathcal{N}$ of N SUs, each operating in saturation condition, tries to opportunistically access the channels when they are left free by PUs. Let $Z_i(k)$ be the random variable equal to 1 when of channel i is unoccupied by any PU at slot k and 0 otherwise. We assume that the process $\{Z_i(k)\}$ is stationary and independent for each i and k. We also assume that at each time slot, channel i is free with probability μ_i,

i.e., $\mathbb{E}[Z_i(k)] = \mu_i$. The channel availability probabilities $\mu \triangleq \{\mu_i\}$ are *a priori* not known by SUs. Channel conditions are ideal: no packet corruption and no hidden terminals are considered. We assume perfect sensing at the SUs, i.e., any transmission of any PU on a channel is perfectly sensed by SUs sensing that channel and thus no collision occurs between PUs and SUs.

The duration of one PU-slots is fixed. A small initial part of it, of fixed length as well, is used by the SUs for sensing the presence of the PU. It is assumed that if a packet is partially transmitted at the end of the PU-slot, the transmission can be reactivated, without loss of information, at the first PU-slot which is senses non-occupied by the PU.

We define an iteration t as a block of PU-slots of fixed duration T during which the SUs don't change their strategy. At the end of each iteration, SUs obtain a payoff which corresponds to the achieved throughput. We assume that such information is sent in a specific field of the packet header such that it can be seized by any SU in the system.

We let the SUs access the selected channels at each iteration through the Distributed Coordination Function (DCF), which is a carrier sense multiple access with collision avoidance (CSMA/CA) scheme and binary slotted exponential back-off[2]. The related random backoff algorithm (defined in the IEEE 802.11 DCF) is designed to give each host a fair chance of obtaining the channel under contention.

For our study, we simplify the DCF model as in [14], i.e. by using a non uniform discrete time scale, where a generic slot, corresponding either to an empty slot or a busy slot, will be referred to as a *coarse slot*. Thus, while idle slots have fixed duration σ, busy slots consist of:

- Packet time (from 34 up to 2346 Bytes)
- SIFS (Short Interframe Space)
- ACK time (usually 14 Bytes)
- DIFS (Distributed Interframe Space)

and can host either a transmission or a collision. This approach allows to derive results regardless of the considered access mode (Basic, RTS/CTS or a combination of the two), since this only affects the duration of the busy slot.

B. Spectrum Access Game Formulation

In our work, each SU j is modeled as a rational decision maker, aiming at load-balancing the total system throughput. The *instantaneous* throughput it can achieve in terms of packets per second, denoted as T_j, can be expressed as a function of μ_{s_j} and n_{s_j}, where s_j denotes the channel which j chooses, n_{s_j} denotes the number of *neighbors* on the same channel s_j. The expected value of T_j, which has to be intended as the *long-term* throughput when T is very large, can be written as:

$$\mathbb{E}[T_j] = f(\mu_{s_j}, n_{s_j}).$$

In this paper, SUs implement DCF as a random access protocol to avoid collisions. This yields:

$$f(\mu_i, n_i) = B\mu_i p(n_i), \tag{1}$$

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[1]Our analysis can be extended to study the heterogeneous case with different channel capacities.

[2]For a detailed presentation about DCF, refer to the 802.11 standard [13]. For a complete analysis of the performance instead, refer to [14].
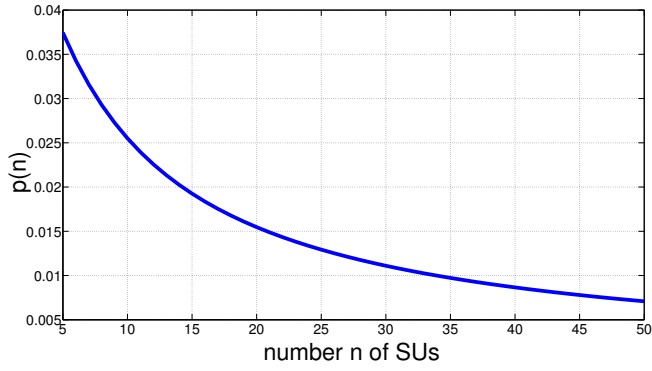
Fig. 1. Probability of successful transmission per SU per coarse slot as a function of the number of users on channel. The contention window has been set according to the DSSS schema parameters [13]

where $p(n_i)$ denotes the successful transmission probability with n_i SUs on the same channel i. Note that the channel is here assumed perfectly fair. That means, by fixing n_i in (1), $\mathbb{E}[p(n_i)]$ takes the same value for all individuals on channel i.

The game is defined formally as follows:

Definition 1. *The spectrum access game G is a 3-tuple ($\mathcal{N}$, $\mathcal{C}$, $\{U_j\}$), where $\mathcal{N}$ is the player set, $\mathcal{C}$ is the strategy set of each player. Each player j chooses its strategy $s_j \in \mathcal{C}$ where its normalized utility function U_j is defined as*

$$U_j = \mathbb{E}[T_j]/B = \mu_{s_j}p(n_{s_j}).$$

We can recognize that G is a congestion game, which is also a potential game with the following potential function:

$$P(n_1,...,n_C) = \sum_{i=1}^{C}\sum_{k=1}^{n_i}\mu_i p(k) \text{ , where } n_1 + ... + n_C = N \tag{2}$$

Theorem 1. *There exists at least one Nash Equilibrium for the described Spectrum Access Game.*

Proof: The Nash Equilibria are the maximizers of the potential function defined by (2). There is at least one maximizer since P can only take a finite set of values. ∎

C. Computation of $p(n)$

Looking at (1), it can be noticed that the payoff U_i can be expressed as a function of two factors:

- the channel availability μ_i.
- the probability of successful transmission per SU per slot $p(n_i)$. Such an event requires a transmission attempt by a single SU and the absence of all the others.

As the channel availabilities are assumed to be constant and hence can be easily derived by learning, the problem reduces to the evaluation of $p(n_i)$. The latter, drawn in Fig. 1 as a function of n_i, can be expressed as the joint probability that a single SU transmits and all the other users contending the channel do not transmit:

$$p(n) = (1 - P(Tx))^{N-1}P(Tx) \tag{3}$$

where $P(Tx)$ is the probability of transmission in a randomly chosen coarse slot. Note that $P(Tx)$ can be derived by solving

a non-linear system generated by (7) and (9) in [14] by using numerical methods ($P(Tx)$ is denoted τ in [14]).

IV. DISTRIBUTED APPROACHES TO CONVERGE TO NASH EQUILIBRIUM

In this section we propose distributed approaches to reach the NE.

A. Imitation

Our distributed approach is based on imitation and more specifically on a recently proposed algorithm [3] called PISAP. We first give details on PISAP, standing for Proportional Imitation-based Spectrum Access Policies. The core idea behind PISAP is the following: at each iteration, each SU randomly selects another SU; if the payoff (i.e. the throughput) of the selected SU is higher than its own payoff, the SU imitates the strategy of the selected SU at the next iteration with a probability proportional to the payoff difference multiplicated by the switching rate.

We propose two throughput computation methodologies:

1) Greedy Payoff Computation Technique (GPCT): the SUs simply associate their payoff to their *measured* instantaneous normalized throughput, i.e. they divide at each iteration the number of packets being successfully transmitted by the iteration time T.
2) Neighbors Estimate-based Payoff Computation Technique (NEPCT): the SUs exchange information about *estimated* average throughput. This can be accomplished if at each iteration t each SU j settling in channel i computes his own payoff in the following manner:
   - get $\hat{n}_i$, i.e. estimation of the number of users on the same channel.
   - substitute $\hat{n}_i$ in (3) and gets $p(\hat{n}_i)$.
   - calculate $U_j = \mu_i p(\hat{n}_i)$.

The performance of NEPCT-PISAP and GPCT-PISAP, standing for NEPCT and GPCT in conjunction with PISAP, will be displayed and analyzed in Section VI.

B. Neighbors Estimation

μ being easily achievable, the problem can be reconducted to the estimate of the number of neighbors at each iteration. There are several works in the literature addressing this problem (e.g. [10] [11]), but we will focus on the solution proposed in [12]. Such solution builds on the existence of a mathematical relationship between the number of competing stations and the packet collision probability encountered on the shared medium. This can be accomplished by monitoring all single coarse slots, regardless of the fact that a transmission attempt has been performed or not. Following [12], the number of competing terminals accessing a channel via-DCF can be expressed as follows:

$$n = 1 + \frac{log(1-p_c)}{log\left(1 - \frac{2(1-2p_c)}{(1-2p_c)(CW_{min}+1)+p_c \cdot CW_{min}(1-(2p_c)^m)}\right)}$$
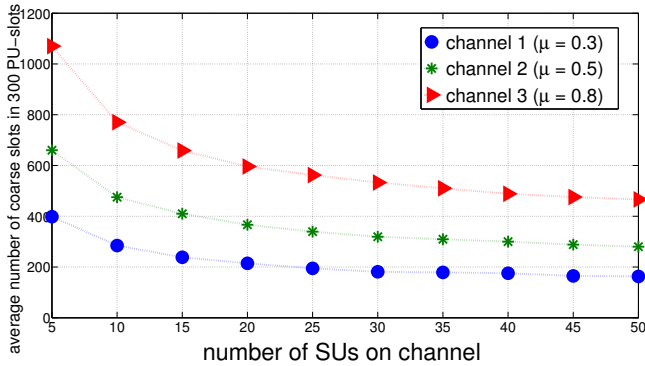
Fig. 2. Average number of coarse slots as a function of the number of the Secondary Users on the considered channel. The iteration block size is fixed to 300 PU-slots.

packet payload	6912 bits
MAC header	272 bits
ACK length	112 bits + PHY header
PHY header	128 bits
Channels bit rate	54 Mb/s
Propagation delay	1 μs
RxTx Turnaround Time	20 μs
Busy Detect Time	20 μs
ACK Timeout	300 μs
slot time σ	20 μs
CW_{min}	32
CW_{max}	1024
SIFS	10 μs
DIFS	50 μs
Switching rate	0.5

TABLE I
SIMULATION SETTINGS.

where p_c is the probability that a packet being transmitted on the packet collides. Such value can be obtained as a function of the number of experienced collisions C_{coll}, of the observed busy slots C_{busy} (a SU cannot distinguish whether an observed busy slot hosts a successful transmission or a collision) and of the total number S of observed slots: $p_c = (C_{busy} + C_{coll})/S$.

C. Estimate error

Errors in estimating the number of competing users on channel affect the computation of obtained payoffs. Such estimate error varies depending on the number of coarse slots per iteration, which in turns depends on two factors:

1) channel availabilities: higher channel availabilities result in a higher amount of coarse slots and hence in better estimations (see Fig. 2 and Fig. 3).
2) number of users on the considered channel: if the packet has a fixed length, busy slots containing packets being successfully transmitted have fixed duration t_{busy}^s [s]. Similarly, busy slots hosting collided packets last t_{busy}^c [s]. As the number n of SUs increases, this provokes an increase of the number of busy slots to the detriment of the number of idle slots. If n increases further, the busy slots will get more and more filled with collisions. As $t_{idle} < t_{busy}^c < t_{busy}^s$, where t_{idle} denotes the standard duration of an idle slot, the number of coarse slots as a function of n decreases, as shown in Fig. 2. As a consequence, the average absolute relative error, drawn as a function of n in Fig 3, increases while n gets larger.

V. DISCUSSION

One might argue that a better estimate in terms of speed and accuracy can be obtained by using an extended Kalman filter coupled with a change estimation mechanism [12]. Such methodology takes into account several additional information, such as state update laws and variance of p_c, in order to fast track variations in the network occupancy status. Nevertheless these advantages are the privilege only of the SUs who select the same channel in adjacent iterations. That means, in the system there would be in the one hand SUs experiencing very precise payoff estimations, and on the other hand SUs experiencing very inaccurate estimations. Such a big difference can cause convergence problems and its study is a major subject of our future investigations.

VI. PERFORMANCE EVALUATION

In this section we conduct extensive simulations to evaluate the performance of NEPCT-PISAP and GPCT-PISAP in terms of fairness, convergence speed and switching cost. In addition, the underlying techniques will be compared to the ideal case, referred to in the following as ideal-PISAP, where the payoffs are perfectly known by the SUs at each iteration.

A. Simulation Model

We simulate a cognitive radio network of $N = 50$ SUs and $C = 3$ channels, on which a generic PU has different activity rates on different channels, leading to different channel availability probabilities characterized by $\mu = [0.3, 0.5, 0.8]$. We let SUs employ DCF-*basic access* mechanism to access the channels. Slot time σ and contention window values are calculated according to the Direct Sequence Spread Spectrum (DSSS) access schema, as specified by the 802.11 standard. The duration of one PU-slots is fixed to 500 μs, of which 50 μs are needed by the SUs for sensing. Hence the SUs have 450 μs per unoccupied PU-slot to access the channel via-DCF. The packet format and the parameters adopted in the simulations are displayed in Table I.

In the *basic access* mode, t_{busy}^s and t_{busy}^c can be derived as follows:

$$\begin{cases} t_{busy}^s &= P + SIFS + \delta + ACK + DIFS + \delta \\ t_{busy}^c &= P + DIFS + \delta \end{cases}$$

where P is the fixed packet length (made up of the physical header, the MAC header and the payload) and δ is the propagation delay.

B. Fairness and Convergence

We want to firstly analyze the fairness of NEPCT-PISAP, GPCT-PISAP and ideal-PISAP. To this end, we adopt the Jain's fairness index [15], which varies in $[0, 1]$ and reaches its maximum when the resource (the throughput in our case) is equally shared amongst users.

Fig. 4 and Fig. 5 shows NEPCT-PISAP and GPCT-PISAP fairness convergence trends for different iteration block sizes.
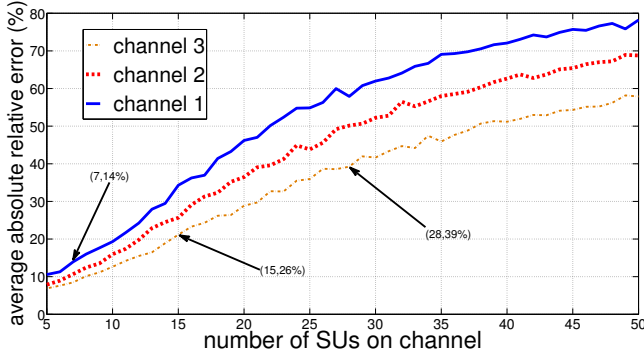
Fig. 3. Neighbors estimation accuracy as a function of the users on channel. Each point represents an average over 100 independent realizations.
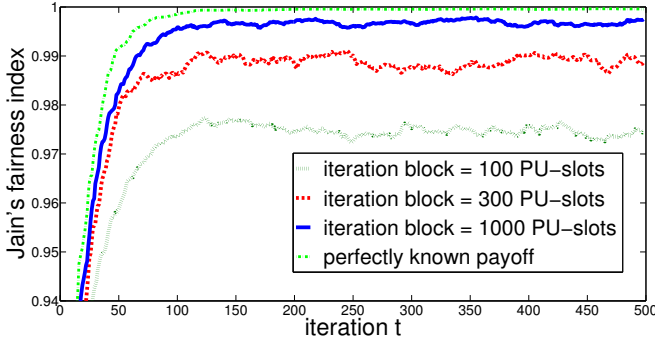

Fig. 4. Fairness convergence of ideal-PISAP and NEPCT-PISAP, the latter plotted for different iteration block sizes. Each curve represents an average over 100 independent realizations

We observe that NEPCT-PISAP clearly outperforms GPCT-PISAP for average fairness at the equilibrium in all analyzed cases. If we consider iteration blocks of 300 PU-slots for example, the achieved fairness indexes for NEPCT-PISAP and GPCT-PISAP are approximately 0.99 and 0.72 respectively. We also notice from the same plots that boosting T, i.e. the iteration block length, is equivalent in both cases to increasing the fairness. This is a natural consequence of the fact that $\hat{n}_i \to n_i$ and $T_j \to \mathbb{E}[T_j]$ as T gets larger.

From Fig. 4 and Fig. 5 one can further infer that a greater estimate error does not result in a slower convergence. Regardless of the iteration block length in fact, NEPCT-PISAP, GPCT-PISAP and ideal-PISAP always converge within 125, 40 and 100 iterations respectively. With respect to the time however, that means that iteration block size is in direct ratio
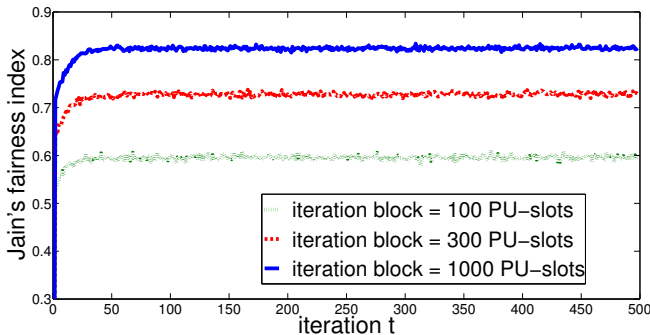

Fig. 5. Fairness convergence of PISAP based on GPCT for different iteration block sizes. Each curve represents an average over 100 independent realizations

to convergence speed. Hence GPCT-PISAP convergence time can be calculated as $40*PUslot\_length*block\_length$, which in our case is $2s$, $6s$ and $20s$ for iteration blocks of 100, 300 and 1000 PU-slots respectively. With a similar procedure we can calculate NEPCT-PISAP convergence time, which takes the values of $6.25s$, $18.75s$ and $62.5s$ for iteration blocks of 100, 300 and 1000 PU-slots respectively.

These results are confirmed by Fig. 6, which displays one realization of ideal-PISAP, NEPCT-PISAP and GPCT-PISAP. In particular, the convergence trends in terms of number of SUs per channel and average utility per user per channel are depicted. Looking at the three plots, it is easy to notice that the one induced by ideal-PISAP is less noisy than the ones induced by NEPCT-PISAP and GPCT-PISAP. Not enough, in the latter case the curves even intersect several times. This is due to the fact that larger estimate errors (GPCT-PISAP high short-time throughput variations can be regarded as errors around the average throughput) produce greater amounts of revision opportunities[3] both in the convergence and the stability phase.

C. Reaching the NE

After solving the discrete optimization problem defined by (2) with numerical methods, it is possible to find out that 1) there exists a *unique* NE for our system 2) at the NE there are 7 SUs on channel 1, 15 SUs on channel 2 and 28 SUs on channel 3.

Looking at Fig. 6, we notice that while this solution is clearly searched out by ideal-PISAP, the same doesn't happen for NEPCT-PISAP, where a sub-optimal solution is selected. This is due to the unbalance of the average neighbors estimate error on the three channel at the NE, as shown in Fig. 3 for $T = 300$ PU-slots. If we fix on this figure the values $7, 15, 28$ on the x-axis in fact, it can be noticed that the average absolute estimate error is of nearly 14%, 26%, 39% on channel 1, channel 2 and channel 3 respectively. As NEPCT tends more to overestimate the number of neighbors rather than underestimate, this means that the SUs settling in channel 3 (which is the channel hosting at the NE the greater number of users) will experience at the true NE a lower payoff (in average) than the ones settling in channel 1 and channel 2. This provokes a modest migration especially towards channel 1, the latter being the channel where the estimate error is smaller. Despite this, fairness values close to the unity are always achieved even for very small iteration block sizes, as shown in Fig. 4.

D. Switching cost

We now turn to the analysis of the switching cost, i.e., the global number of channel switches. Due to the drastic cost of changing frequencies in current wireless devices in terms of delay, packet loss and protocol overhead, an efficient channel access policy should avoid frequently channel switching, unless necessarily.

Fig. 7 shows the NEPCT-PISAP and GPCT-PISAP switching cost trend as a function of the iteration t for

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[3]A revision opportunity is defined in evolutionary game theory as the chance to modify the current adopted strategy with non-zero probability.
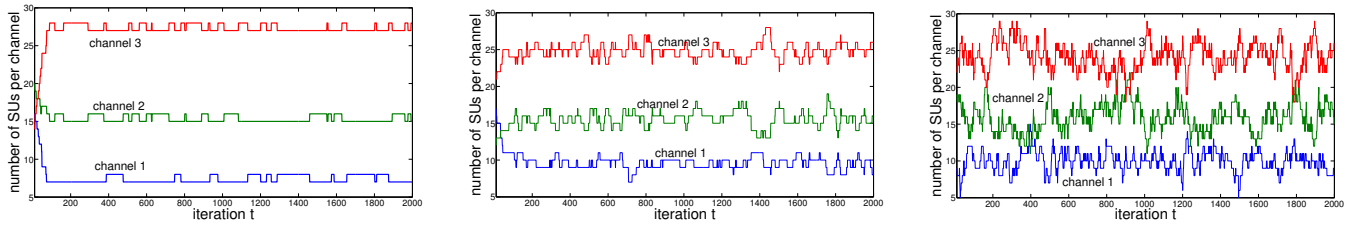
Fig. 6. Convergence of PISAP when, at each iteration, n is perfectly known (left), is estimated by means of NEPCT (center) and by means of GPCT (right). The iteration block size is fixed to 300 PU-slots.
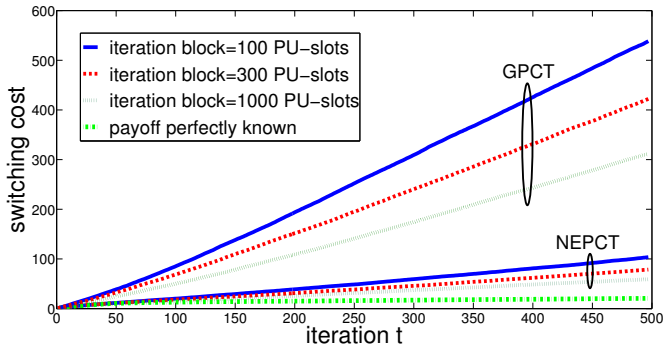

Fig. 7. Switching cost performance comparison between NEPCT-PISAP, GPCT-PISAP and ideal-PISAP. Each curve represents an average over 100 independent realizations.

different iteration block lengths. Ideal-PISAP trend is also plotted by for performance comparison. We observe that NEPCT-PISAP clearly outperforms GPCT-PISAP for any iteration block size. The slope of the curved induced by NEPCT-PISAP are in fact noticeably less steep than the ones induced by GPCT-PISAP. As a consequence, if for instance we fix the iteration block length to 100 PU-slots, after 500 iterations the charged switching cost with NEPCT-PISAP and GPCT-PISAP is approximately 50 and 300 respectively.

VII. Conclusion

In this paper we have discussed the problem of opportunistic spectrum access in CSMA/CA-based cognitive radio networks where the Secondary Users aim at load-balancing the achieved throughput. We have demonstrated that this is possible if each SU adopts the Proportional Imitation-based Spectrum Access Policy, which is totally distributed and relies on solely local interactions amongst users.

We have shown that computing payoffs (i.e., throughputs) can be problematic, and we have analyzed the performance of two payoff computation methodologies relying on CSMA/CA, namely the Neighbors Estimate-based Payoff Computation Technique (NEPCT), relying on the estimate of the number of SUs settling in the same channel at each iteration, and the Greedy Payoff Computation Technique (GPCT), relying on the measured instantaneous throughput. We have simulated a scenario where the underlying techniques are used in conjunction with PISAP and we found out that, although estimate inaccuracy affects negatively the Nash Equilibrium exactness reached by means of NEPCT-PISAP, the latter achieves high fairness values even for very small iteration block sizes and clearly outperforms GPCT-PISAP in terms of both fairness and switching cost.

Following the path traced by this paper, we plan to further explore payoff computation techniques and to apply them to distributed scenarios such as the one offered by imitation-based Cognitive Radios. Especially, at the actual stage we are attempting to study the spectrum access game with estimate errors, in order to analytically show the convergence of the derived dynamics.

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