Lexicographic Relay Selection and Channel Allocation for Multichannel Cooperative Multicast

Yitu Wang* Wei Wang* Lin Chen† Zhaoyang Zhang*
*Department of Information Science and Electronic Engineering
Zhejiang Key Lab. of Information Network Technology
Zhejiang University, Hangzhou 310027, P.R. China
Email: wangw@zju.edu.cn
†Lab. de Recherche en Informatique (LRI)
University of Paris-Sud 11
Orsay, France, 91405 Cedex
Email: Lin.Chen@lri.fr

***Abstract*—Cooperative multicast has been demonstrated to achieve significant performance gain over the classic source-destination transmission paradigm by exploiting spatial diversity through the participation of multiple relay nodes. As a major technical challenge, the selection of relays for a multicast session has significant impact on the multicast performance. The challenge is even more pronounced when the number of channels are limited as the relay selection is in this context coupled with channel allocation. We establish an analytical framework for joint relay selection and channel allocation problem and develop a lexicographic max-min multicast relay selection scheme. Our design consists of two technical steps. 1) We consider the maximization of the minimal data rate. By decoupling relay selection and channel allocation, the problem is transformed to a max-min-max problem, which is difficult to solve. To make this problem tractable, we reformulate it as a convex optimization problem via relaxation and smoothing, and prove the asymptotic equivalence from a geometrical perspective. 2) We propose an adjustment algorithm based on the initial max-min solution, and prove that the proposed scheme achieves lexicographic optimality.**

I. Introduction

Multicast is a spectrum-efficient method for one-to-many transmissions over wireless channels by enabling service providers to send multimedia data to multiple users simultaneously [1]. Scalable video coding (SVC) divides a multimedia stream into multiple layers. An SVC stream has one base layer and multiple enhancement layers, where the base layer provides a minimum quality of the multimedia while the enhancement layers gradually increase the quality. To further proliferate multimedia applications over wireless networks, multicast with SVC plays an important role to improve the wireless resource utilization and provide differentiated QoS [2].

Cooperative multicast takes the merit of spatial diversity and efficiently combats the influence of path loss and channel fading to further enhance the multicast capacity [3][4]. In two-hop cooperative multicast systems, the source node first transmits data to relay nodes, then the users requesting the same data can be logically grouped as multicast groups and served by designated relay nodes respectively. Although cooperative multicast has the potential to increase the capacity, an improper relay selection scheme will result in an even lower data rate.

This work is supported in part by National Natural Science Foundation of China (No. 61571396), Zhejiang Provincial Natural Science Foundation of China (No. R17F010006), and National Hi-Tech H&D Program (No. 2014AA01A702).

To use in multicast with SVC scenarios, most publications assume that the number of orthogonal channels are many enough to avoid co-channel interference among relays [5][6]. However, few publication studies the multicast relay selection problem with considering the limitation of the number of channels. In some practical networks, e.g., IEEE 802.11 [7], the number of available channels is small. Such limitation brings fundamental challenge and complexity to the multicast relay selection problem, because different relay nodes are coupled with each other, leading to complicated relay activation, relay selection and channel allocation problems. Moreover, the effects of the limitation to the system capacity is not straightforward. It is nontrivial to obtain a multicast relay selection scheme taking the limitation into consideration.

In this paper, we develop an analytical framework for lexicographic max-min multicast relay selection scheme with a limited number of channels for cooperative multicast. Lexicographic optimization is a well-recognized fair optimility criterion [8][9] for multi-objective optimization problems. Lexicographic max-min optimization provides a unique solution and outperforms all possible classic max-min solutions [10]. Specifically, we design the scheme in two steps. 1) We consider the maximization of the minimal data rate first. By decoupling relay selection and channel allocation, the problem is transformed to a max-min-max problem, which is difficult to solve. To make this problem tractable, we reformulate it as a convex optimization problem via relaxation and smoothing, and prove the asymptotic equivalence from a geometrical perspective. 2) Towards lexicographic optimal solution, besides the minimal rate, it is necessary to further consider the capacities of other nodes. We propose an adjustment approach based on the initial max-min solution, and prove that the proposed scheme achieves lexicographic optimality. Finally, our proposed algorithm is demonstrated by simulation results.

The rest of this paper is organized as follows. Section II presents the system model. Section III proposes the details in designing the relay selection algorithm. The performance of the proposed algorithm is evaluated by simulation in Section IV. Finally, this paper is concluded in Section V.

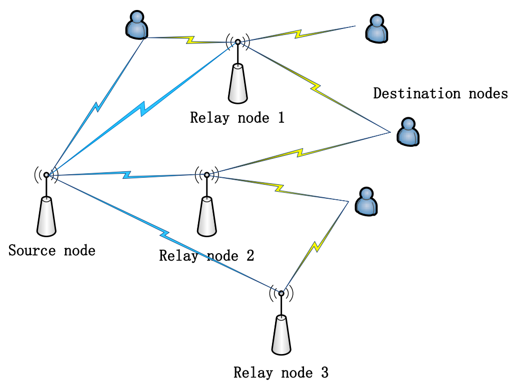

Fig. 1. Cooperative Multicast System

II. System model

A. Cooperative Multicast Model

Consider a wireless cooperative multicast network, consisting of a source node $\mathcal{S} = \{s\}$, M relay nodes $\mathcal{R} = \{r_1, r_2, \cdots, r_M\}$ and N destination nodes $\mathcal{D} = \{d_1, d_2, \cdots, d_N\}$. Relay nodes multicast with SVC to improve the wireless resource utilization and provide differentiated QoS according to the weakest channel conditions in their corresponding multicast group. The time is slotted and it takes two time slots to accomplish the cooperative multicast. In the first time slot, as the blue links in Fig. 1, the source node s broadcasts the data to all relay nodes according to the weakest source-relay channel condition γ. In the second time slot, the relay nodes multicast the received data to all destination nodes simultaneously, as the yellow links. As in [4], we assume that K orthogonal channels are available in the network (e.g., using OFDMA), denoted as $\mathcal{C} = \{c_1, c_2, \cdots, c_K\}$.

Let $\mathcal{G} = (V, E)$ denote the conflict graph, where each relay node $r_i \in V, \forall r_i$ is a vertex of the conflict graph, and $(r_i, r_j) \in E$ implies that r_i and r_j cannot transmit on the same channel simultaneously since their transmissions interfere with each other. If any destination node can receive the signal from both the relay nodes r_i and r_j, then $(r_i, r_j) \in E$. Taking Fig. 1 as an example, we obtain that the relay nodes r_1 and r_2 are conflict, r_2 and r_3 are conflict while r_1 and r_3 are not conflict.

Decode-and-forward transmission mode is adopted in the cooperative multicast systems. Each destination node either receives data directly from source node s or uses cooperative multicast with the help of only one relay node. The relay node r_i decodes and estimates the signal received from the source node s and then transmits the estimated data to the destination node d_j. The capacity from source node s to destination node d_j with the assistance of relay node r_i is

$$C_{ij} = \frac{W}{2} \min\{\log_2(1+\gamma), \log_2(1+\gamma_{ij})\}. \tag{1}$$

where γ_{ij} represents the SINR between i and j.

B. Problem Formulation

Define $\mu : \mathcal{D} \to \mathcal{R}$ as a relay selection scheme, where $\mu(d_j) = r_i$ indicates the relay node r_i is selected to help the transmission to d_j via cooperative multicast, and $\mu(d_j) = \phi$ means that the source node s transmits to the destination node d_j directly. Note that it is possible that $\mu(d_i) = \mu(d_j), d_i \neq d_j$, which is different from the models in [4][5], where a relay node can be assigned to assist only one destination node. The channel allocation matrix of relay nodes is denoted as $\boldsymbol{\tau} = \{\tau_{ik}\}_{M\times N}$, where $\tau_{ik} = 1$ represents that relay node r_i is activated using channel c_k.

Considering the multicast nature of wireless communication systems, a relay node r_i multicasts data to destinations $\mathcal{D}_i = \{d_j | \mu(d_j) = r_i, \forall d_j \in \mathcal{D}\}$ with a maximal rate of

$$R_i = \sum_{c_k \in \mathcal{C}} \tau_{ik} \min_{d_j \in \mathcal{D}_i} \{C_{ij}\}, \tag{2}$$

such that each destination node in $\mathcal{D}_i$ can successfully receive and decode the data.

Our goal is to design a lexicographic max-min relay selection scheme, in which the lexicographic optimal rate vector is no lexicographically less than that of any other scheme.

Definition 1 (Lexicographic Optimality)**.** *Let* $\mathbf{R} = (\nu_1, \nu_2, ..., \nu_N)$ *be an achievable rate vector which is sorted in non-descending order, where* ν *represents the received data rate of destination nodes. Any two relay selection schemes* μ *and* μ' *resulting in two such vectors* $\mathbf{R}$ *and* $\mathbf{R}'$ *have the following relationships:*

- *If* $\nu_i = \nu'_i$ *for any* $i = 1, 2, ..., N$*, then* $\mathbf{R}$ *is lexicographically equal to* $\mathbf{R}'$*.*
- *If there exist a prefix* $(\nu_1, \nu_2, ...\nu_i)$ *of* $\mathbf{R}$ *and a prefix* $(\nu'_1, \nu'_2, ...\nu'_i)$ *of* $\mathbf{R}'$ *such that* $\nu_i > \nu'_i$*, and* $\nu_j = \nu'_j$ *for* $1 \leq j \leq i-1$*. Then* $\mathbf{R}$ *is lexicographically greater than* $\mathbf{R}'$*.*

$\mathbf{R}$ *is lexicographically optimal if it is no lexicographically less than all other feasible rate vectors.* ■

The received data rates for destination nodes are lexicographically optimized by determining which relay nodes should be activated and which destination nodes these relay nodes forward data to. Based on the definition of lexicographic optimality in Definition 1, we can formulate the lexicographic max-min problem as follows.

$$\operatorname{lex}\max_{\boldsymbol{\tau},\mu} \ \mathbf{R}, \tag{3}$$

$$s.t. \ \sum_{c_k \in \mathcal{C}} \tau_{ik} \leq 1, \tag{4}$$

$$\tau_{ik} + \tau_{lk} \leq 1, \forall (r_i, r_l) \in E, \tag{5}$$

$$\tau_{ik} \in \{0, 1\}, \tag{6}$$

where lex max indicates the operation of lexicographic maximization. Constraint (4) indicates that a relay node can use at most one channel which depends on the fact that usually only one radio is deployed in a device. Constraint (5) represents that if two relay nodes r_i and r_l interfere with each other, they must engage different channels to avoid cross-channel interference. The optimization problem in Eq. (3) lexicographically maximizes the rate vector by determining the relay selection scheme μ and channel allocation scheme $\boldsymbol{\tau}$.

III. Algorithm Design

In this section, we propose an analytical framework for lexicographic max-min multicast relay selection scheme for cooperative multicast with a limited number of channels. To overcome the coupling between relay selection and channel allocation, we address the problem in two steps:

- Consider only the minimal data rate and solve the max-min problem to obtain an initial relay selection solution.
- Optimize the rates of other nodes by further adjusting the relay selection to achieve lexicographic optimality.

A. Decoupling in Max-Min Subproblem

Maximizing the minimal data rate among the destination nodes is equivalent to maximizing the minimal multicast rate among the relay nodes according to Eq. (2), we can formulate the max-min subproblem from Eq. (3)-(6) as follows:

$$\begin{aligned} &\max_{\boldsymbol{\tau},\mu} \min_{i} \ R_i \\ &\quad s.t. \ (4)(5)(6). \end{aligned} \tag{7}$$

This max-min problem involves coupled relay selection scheme μ and channel allocation scheme $\boldsymbol{\tau}$. To decouple these two aspects, we exploit the property of the max-min problem and suggest a capacity-based relay selection scheme which is independent to channel allocation in the following lemma.

Lemma 1 (Capacity-Based Relay Selection)**.** *The max-min solution of the cooperative multicast system is achievable only by adjusting channel allocation if each destination node d_j joins the multicast group of the relay r_i which has the largest channel capacity C_{ij}, i.e.,*

$$\max_{\boldsymbol{\tau},\mu} \min_{i} \ R_i = \max_{\boldsymbol{\tau}} \min_{i} \ R_i(\hat{\mu}), \tag{8}$$

where

$$\hat{\mu}(d_j) = \arg\max_{r_i \in \mathcal{R}} \{C_{ij}\}. \tag{9}$$

To decouple relay selection and channel allocation in Eq. (7), we adopt the relay selection scheme $\hat{\mu}$ in Lemma 1 and transform the max-min problem in Eq. (7) to a max-min-max problem which is a channel allocation problem only.

$$\begin{aligned} &\max_{\boldsymbol{\tau}} \min_{j} \max_{i} \ \sum_{c_k \in \mathcal{C}} \tau_{ik} C_{ij} \\ &\quad s.t. \ (4)(5)(6). \end{aligned} \tag{10}$$

B. Tractable Reformulation for Channel Allocation

Due to the non-smooth structure of max-min-max function, the min-max-min problem in Eq. (10) is very difficult to solve both in theoretical analysis and in numerical calculation [11]. To solve it directly, the result comes close to the exhausting method, which faces the curse of dimensionality[12]. Therefore, we transform the min-max-min problem into a convex one by relaxation and smoothing techniques, and further prove that the relaxation and smoothing are tight.

The max-min-max problem is a combinational optimization problem, which is not differentiable due to constraint (6), which should be relaxed to obtain the optimal solution,

$$\tau_{ik} \in [0, 1], \tag{11}$$

so that the objective in Eq. (10) is a continuous function of $\boldsymbol{\tau}$.

With the above relaxation, the max-min-max problem is continuous but still undifferentiated, we further adopt a smoothing technique to approximate the original max-min-max optimization problem, such that the transformed approximation problem is differentiable about $\boldsymbol{\tau}$.

Adopting the smoothing technique in [13], the objective of the max-min-max problem in Eq. (10) can be approximated by

$$\frac{1}{\epsilon} \ln \left(\sum_{i=1}^{N} \frac{1}{\sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}}} \right) + \frac{U}{\epsilon}, \tag{12}$$

where ϵ is the approximation parameter and U is a constant.

For given U and $\epsilon > 0$, consider the exponential of objective function in Eq. (12), the optimality is preserved according to the monotone property of exponential function [14]. The optimization problem in Eq. (10) can be transformed to

$$\begin{aligned} &\min_{\boldsymbol{\tau}} \ \sum_{i=1}^{N} \frac{1}{\sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}}} \\ &\quad s.t. \ (4)(5)(11). \end{aligned} \tag{13}$$

Theorem 1 (Convexity of Problem (13))**.** *The optimization problem in Eq.* (13) *is convex.*

Proof: Please refer to Appendix A. ∎

C. Geometrical Analysis

To obtain some critical insight of the convex problem, we consider the relay selection optimization problem in Eq. (13) as a geometrical problem which investigates the relationship of positions of a line and a few points. On one hand, such a method decreases the computational complexity. On the other hand, we are managed to prove that the relaxation is tight by adopting geometrical analysis.

We first consider the optimality condition of the problem in Eq. (13). Adopting the Karush-Kuhn-Tucker (KKT) condition [14], the problem can be transformed to

$$\begin{aligned} \min P = &\sum_{i=1}^{N} \frac{1}{\sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{L} \tau_{ik} C_{ij}}} + \sum_{i=1}^{N} \lambda_i \left(\sum_{k=1}^{K} \tau_{ik} - 1 \right) \\ &+ \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j \in \mathcal{E}_i} \beta_{ijk} (\tau_{ik} + \tau_{jk} - 1) \\ s.t. \ \ &\lambda_i \left(\sum_{k=1}^{K} \tau_{ik} - 1 \right) = 0 \\ &\beta_{ijk} (\tau_{ik} + \tau_{jk} - 1) = 0, \end{aligned} \tag{14}$$

where $\mathcal{E}_i = \{r_j | (r_i, r_j) \in E, \forall r_j \in \mathcal{R}\}$ represents the set of the conflict relay nodes of r_i, λ_i is the Lagrangian multiplier for constraint Eq. (4) and β_{ijk} is the Lagrangian multiplier for constraint Eq. (5).

The first derivative of P with respect to $\boldsymbol{\tau}$ can be transformed into

$$\begin{aligned} P'^{*} =& \frac{\partial P}{\partial \tau_{in}} \sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}} \\ =& -\sum_{j=1}^{M} \epsilon C_{ij} + \left(\lambda_i + \sum_{j \in \mathcal{E}_i} \beta_{ijn} \right) \sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}}. \end{aligned} \tag{15}$$

Since $\sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}} > 0$, the optimality conditions does not change for P'^{*}.

To analyze the first derivative in Eq. (15), we rewrite the optimality condition for the problem in Eq. (15) as an expression of a line in two-dimensional space [15],

$$y_{in} = A_i x_{in}, \tag{16}$$

where $x_{in} = \lambda_i + \sum_{j \in \mathcal{E}_i} \beta_{ijn}$, $A_i = \sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{K} \tau_{ik} C_{ij}}$ and $y_{in} = \sum_{j=1}^{M} \epsilon C_{ij}$.

From a geometrical perspective, each relay r_i using channel c_n has a corresponding point $S_{in} = (x_{in}, y_{in})$ in the two dimensional space. Define $\mathcal{S}_i$ as the set of all points $S_{in}, \forall n$. For given λ, β and relay r_i, the coordinates x_{in} and y_{in} are determinate. In such a case, the problem is transformed to a geometrical one to find the slope of the line $Y_i = A_i X_i$ by adjusting $\boldsymbol{\tau}$ to let some of the points S_{in} on the line and all the other points under the line. Accordingly, the following rule can be determined

$$\begin{aligned} \tau_{ik} = 1, \exists A_i = \frac{y_{ik}}{x_{ik}} \\ \tau_{ik} = 0, \forall A_i > \frac{y_{ik}}{x_{ik}}, \end{aligned} \tag{17}$$

where $k = \arg\max_j x_{ij}$. To determine the tangency of the line, as the example in Fig. 2, the blue points belong to relay node r_i, the blue region represents the feasible region of the tangency A_i, and there exists a tangency A_i that passes through the right-most point (x_{i3}, y_{i3}) in the region. Thus, relay node r_i is activated to transmit with a channel 3. While there does not exist a tangency A_j that passes through the left-most point (x_{j1}, y_{j1}). As a result, relay node r_j is deactivated.

In this way, we obtain the optimal conditions for the reformulated convex optimization problem in Eq. (13). Besides providing the optimal conditions, we further prove that the relaxation and smoothing in the tractable reformulation step are asymptotically tight in the following theorem.

Theorem 2 (Asymptotic Equivalence). *With sufficiently large smoothing parameter ϵ, the solution of the convex optimization problem in Eq.* (13) *is asymptotically optimal for the original max-min-max problem in Eq.* (10).

Proof: For optimality, for each relay node r_i, only at most one channel it can use to transmit, leading to the relaxation being tight. A similar proof can be found in [15] that a line has probability 0 to go through all the 3 points. Therefore, the relaxation of $\boldsymbol{\tau}$ in Eq. (11) is tight.

According to [13], with sufficiently large smoothing parameter ϵ, the value of the approximating function converges to a stationary point, such that the smoothing in Eq. (12) is tight.

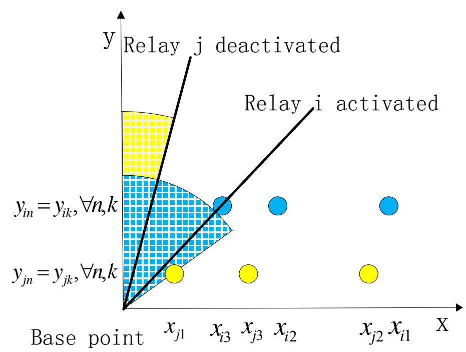

Fig. 2. Relationship between lines and points

Since the relaxation of $\boldsymbol{\tau}(t)$ in Eq. (11) and the smoothing in Eq. (12) are tight, the optimization problem in Eq. (12) and the one in Eq. (10) are asymptotically equivalent. ■

D. Proposed Algorithm

Towards lexicographic optimality, we proposed a relay selection scheme μ^* based on the initial relay selection scheme $\hat{\mu}$ in Lemma 1 and the channel allocation $\boldsymbol{\tau}$. Even though $\hat{\mu}$ achieves max-min optimal, a destination node may actually receives a higher data rate from another relay node.

We first divide the destination nodes into two categories, i.e., the node with the weakest channel quality in each multicast group and other destination nodes. Denote the set of the weakest node in each multicast group as $\mathcal{J}$,

$$\mathcal{J} = \{d(r_i) | d(r_i) = \arg\min_{d_j \in \mathcal{D}_i} \{C_{ij}\}, \forall r_i \in \mathcal{R}\}, \tag{18}$$

where $\mathcal{D}_i = \{d_j | \mu(d_j) = r_i, \forall d_j \in \mathcal{D}\}$ and $d(r_i)$ is the destination node which has the weakest channel quality in the multicast group of relay r_i.

For the destination node $d_j \in \mathcal{J}$,

$$\mu^*(d_j) = \hat{\mu}(d_j) = \arg\max_{r_i \in \mathcal{R}} \{C_{ij}\}. \tag{19}$$

For the destination node $d_j \in \mathcal{D}/\mathcal{J}$,

$$\mu^*(d_j) = \arg\max_{r_i \in \mathcal{R}} \{\min\{C_{ij}, R_i\}\}, \tag{20}$$

where R_i is the multicast rate of relay r_i with the initial relay selection scheme $\hat{\mu}$ and the channel allocation $\boldsymbol{\tau}$ obtained in the first step.

Remark 1 (Interpretation of Relay Selection Scheme). *The destination nodes in $\mathcal{J}$ suffer from bad channel conditions and limit the multicast rate of each relay node. As for the nodes in $\mathcal{J}$, their relay selection scheme stay the same in μ^* as in $\hat{\mu}$. For other destination nodes, they select a relay that reaches their maximal possible data rate they can receive.*

Theorem 3 (Lexicographic Optimality). *The proposed relay selection scheme μ^* is lexicographically optimal.*

Proof: Please refer to Appendix B. ■

In the pseudo-codes, Lines 1 initializes the system parameters, Line 3 calculates the potential transmission rate, Line

TABLE I
RATE VECTOR $\mathbf{R}$

	1	2	3	4	5	6	7	8	9	10
Proposed	12.7496	12.7496	12.7496	14.7067	14.7067	15.8894	15.8894	15.8894	17.2415	17.2415
Random	12.2972	12.2972	12.2972	14.2155	14.2155	14.2155	15.0306	15.0306	18.2415	18.2415
EPSA	11.5488	11.5488	11.5488	13.4623	18.0838	18.0838	18.0838	20.2415	20.2415	20.2415
Max-Min	12.7496	12.7496	12.7496	12.7496	14.7067	14.7067	14.7067	15.8894	15.8894	17.2415

Algorithm 1 Lexicographic Max-Min Relay Selection

1: Initialize parameters $M_J, \epsilon, \mathcal{D}, \mathcal{S}, \mathcal{R}, \mathcal{C}$.
2: **loop**
3: Calculate $C_{ij}, \forall i, j$ according to Eq. (1)
4: Obtain the neighborhood graph $\mathcal{G}, \mathcal{E}_i, \forall i$
5: Each destination node selects a relay node according to Eq. (9)
6: Adjust λ and β by augmented lagrange method
7: **for** $i = 1 : N$ **do**
8: **for** $k = 1 : K$ **do**
9: **if** $\exists A_i = \frac{y_{ik}}{x_{ik}}$ and relay r_i does not have a channel **then**
10: $\tau_{ik} = 1$
11: **end if**
12: **end for**
13: **end for**
14: **end loop** until the approximation gap is within a given threshold by updating the approximation parameter ϵ
15: Each destination node selects a relay node according to Eq. (19) and Eq. (20)
16: The multicast rate of each relay node is determined according to Eq. (2)

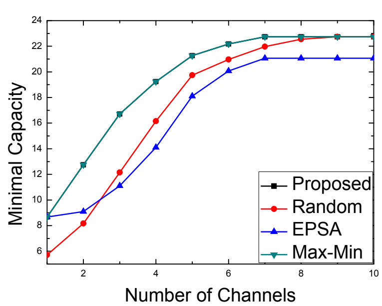

Fig. 3. Minimal capacity varying number of channels

4 obtains the topology, Line 5 adopts the max-min relay selection scheme, Lines 6-14 determine the channel allocation according to the proposed algorithm, Line 15 deploys the lexicographic optimal relay selection scheme, Line 16 determines the scheduled transmission rate.

IV. SIMULATION

In the simulation, destination nodes are randomly distributed in a circular field with a radius of 50 and relay nodes are randomly distributed in a circular field with a radius of 10, and the source node is located at the center. Following the simulation parameter settings in [5], we set the bandwidth W as 22 MHz for all channels. The transmission power is the same for each node, i.e., $P_s = P_{r_i} = 1$ for source node and relay nodes. For the transmission model, we assume that the path loss exponent $\alpha = 4$ and the noise power $N_0 = 10^{-10}$.

For performance comparison with the proposed lexicographic max-min scheme μ^*, three baseline schemes are adopted.

- **Random**: Relay nodes are activated randomly, destination nodes choose the relay with the largest channel quality.
- **EPSA** [17]: The resources are allocated to the multicast groups with large potential to maximize total capacity.
- **Max-Min**: The max-min scheme $\hat{\mu}$ in Lemma 1.

For each setting, we randomly generated 10 instances and obtain the average results.

In Fig. 3, there are 10 relay nodes and 30 destination nodes. The performance of the proposed scheme and Max-Min scheme is the same. The proposed scheme outperforms EPSA and random schemes. It is observed that the performance increment is smaller when the number of the channels is larger than 7, because when 7 channels are available, through proper channel allocation, nearly all relay nodes are all activated for transmission.

Besides the minimal capacity, we further analyze the data rates of other destination nodes. Table I provides the rate vector $\mathbf{R}$ for these three schemes in a scenario with 10 destination nodes and 10 relay nodes with 3 available channels. It can be found that the rate vector of the proposed scheme is lexicographically greater than those of three baseline schemes, which provides relatively homogeneous service quality to all users in the cooperative multicast system.

V. CONCLUSION

In this paper, we construct an analytical framework for lexicographic max-min multicast relay selection for cooperative multicast with a limited number of channels. Specifically, by decoupling relay selection and channel allocation, the problem is transformed to a max-min-max problem. To make this problem tractable, we reformulate it via relaxation and smoothing, and prove the asymptotic equivalence from a geometrical perspective. We propose an adjustment algorithm based on the initial max-min solution, and prove that the proposed scheme achieves lexicographic optimality.

APPENDIX A
PROOF OF THEOREM 1

Denote the objective function in Eq. (13) as $f(\boldsymbol{\tau})$. To analyze the convexity of $f(\boldsymbol{\tau})$, we take the first derivative of $f(\boldsymbol{\tau})$ with respect to τ_{ik} as

$$\frac{\partial f(\boldsymbol{\tau})}{\partial \tau_{ln}} = -\frac{1}{\sum_{j=1}^{M} e^{\epsilon \sum_{k=1}^{L} \tau_{lk} v_{lj}}} \sum_{j=1}^{M} \epsilon v_{lj} < 0, \tag{21}$$

which is a strictly decreasing function of $\boldsymbol{\tau}$.

We derive the hessian of $f(\boldsymbol{\tau})$ as follows

$$\begin{pmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & A_N \end{pmatrix}. \tag{22}$$

where

$$A_i = \begin{pmatrix} a_i & \dots & a_i \\ \vdots & \ddots & \vdots \\ a_i & \dots & a_i \end{pmatrix}. \tag{23}$$

and $a_l = \left(\frac{1}{\sum_{j=1}^{M} e^{\epsilon(\sum_{k=1}^{L} \tau_{lk} v_{lj})}}\right)^2$.

Note that the hessian of $f(\boldsymbol{\tau})$ is a symmetric matrix. Each sub-block of the hessian of $f(\boldsymbol{\tau})$ is also a symmetric matrix. It can be derived according to [16] that

$$\mathbf{E} = (M, 0, \cdots, 0), \tag{24}$$

where $\mathbf{E}$ is the eigenvector of the matrix in Eq. (23).

Therefore, the eigenvalue of the hessian of $f(\boldsymbol{\tau})$ is non-negative. According to [14] [16], the hessian is semi-positive definite, and $f(\boldsymbol{\tau})$ is convex with respect to $\boldsymbol{\tau}$. Since all the constraints in Eq. (13) are linear, we obtain the conclusion that the problem in Eq. (13) is a convex optimization problem.

Appendix B
Proof of Theorem 3

Reasoning by contradiction, suppose there exist a relay selection scheme μ' that lexicographically greater than μ^*. According to Definition 1, $\exists i, \nu'_i > \nu^*_i$ and $\nu'_k = \nu^*_k, \forall k < i$.

First let us check whether $\mu'(d_j) = \mu^*(d_j), \forall d_j \in \mathcal{J}$. If not, then $\exists d_j \in \mathcal{J}, \nu'_j < \nu^*_j$, because according to relay selection scheme μ^*, if $\exists d_j \in \mathcal{J}, \mu'(d_j) \neq r_l = \mu^*(d_j)$, then destination node d_j must receive a lower data rate $\nu'_j < \nu^*_j$, since relay node r_l holds the best channel condition and provides the maximal data rate for destination node d_j.

- Since the weakest link of r_l is removed, the multicast rate of r_l can be larger $R'_l > R^*_l$.
- As for users $d_q \in \mathcal{D}/\mathcal{J}$, their received data rate could change as follows

$$\begin{aligned} &\nu'_q = \nu^*_q, if \nu^*_q < R^*_l \\ &\nu'_q \geq \nu^*_q, if R^*_l \leq \nu^*_q \leq R'_l \\ &\nu'_q = \nu^*_q, if \nu^*_q > R'_l. \end{aligned} \tag{25}$$

The first line of Eq. (25) means that, since destination nodes in $\mathcal{D}/\mathcal{J}$ already select the relay node which provides the maximal possible data rate that one can receive, the increment of the multicast rate of r_l will not affect their choice. The second line of Eq. (25) means that destination nodes which select r_l or could receive higher data rate from r_l might benefit from μ'. The third line of Eq. (25) means that destination nodes who receive higher data rate is not affected by μ'. Assume that the position of d_j in $\mathbf{R}'$ is p, satisfying $\nu'_{p-1} \leq \nu'_p < \nu'_{p+1}$. According to Eq. (25), the top $p-1$ items of both $\mathbf{R}^*$ and $\mathbf{R}'$ are the same. Therefore, $\nu^*_l = \nu'_l, \forall l \leq p-1$. Because d_j is inserted to the position p in $\mathbf{R}'$, the item of the position $p+1$ in $\mathbf{R}'$ equals to the item of the position p in $\mathbf{R}^*$, $\nu^*_p = \nu'_{p+1}$. Therefore, $\nu^*_p > \nu'_p$, which is a contradiction.

If it is positive, then $\nu'_j \leq \nu^*_j, \forall d_j \in \mathcal{D}$, because the proposed relay selection scheme μ^* selects the relay node which provides the maximal possible data rate that one can receive for the rest of the destination nodes. Thus, it is not possible when $\mu'(d_j) = \mu^*(d_j), \forall d_j \in \mathcal{J}$, that $\nu'_j > \nu^*_j, \exists d_j \in \mathcal{D}/\mathcal{J}$.

Therefore, there does not exist a relay selection scheme μ' that lexicographically greater than μ^*, which proves μ^* achieves lexicographic optimality.

References

[1] O. Alay, P. Liu, and Y. Wang, "Cooperative layered video multicast using randomized distributed space time codes," *IEEE Transactions on Multimedia*, vol. 13, no. 5, pp. 1127-1140, Nov. 2011.

[2] P. Li, H. Zhang, and B. Zhao, "Scalable video multicast with adaptive modulation and coding in broadband wireless data systems, *IEEE Transactions on Networking*, vol. 20, no. 1, pp. 57-68, Aug. 2012.

[3] J. Garg, M. Priyanka, and G. Kapil, "A Review on Cooperative Communication Protocols in Wireless World," *International Journal of Wireless and Mobile Networks*, vol. 5, no. 2, Apr. 2013.

[4] Y. Shi, S. Sharma, and Y. T. Hou, "Optimal relay assignment for cooperative communications," *Proc. of ACM ISMNC 2008*, pp. 3-12, Feb. 2008.

[5] D. Yang, X. Fang, and G. Xue, "OPRA: Optimal relay assignment for capacity maximization in cooperative networks," *Proc. of IEEE ICC*, Jun. 2011.

[6] I. Maric and R. D. Yates, "Cooperative multihop broadcast for wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1080-1088, Aug. 2004.

[7] P. Kyasanur and N H. Vaidya, "Capacity of multi-channel wireless networks: impact of number of channels and interfaces," *Proc. of IEEE ICMCN 2005*, pp. 43-57, Aug. 2005.

[8] Y. Li, Q. Peng and X. Wang, "Multicast capacity with max-min fairness for heterogeneous networks," *IEEE Transactions on Networking*, pp. 622-635, Jul. 2014.

[9] P. Li, S. Guo, W. Zhuang, et al, "On efficient resource allocation for cognitive and cooperative communications", *IEEE Journal on Selected Areas in Communications*, pp. 264-273, Oct. 2014.

[10] Radunovic, Bozidar, and Jean-Yves Le Boudec, "A unified framework for max-min and min-max fairness with applications," *IEEE Transactions on Networking*, pp. 1073-1083, Apr. 2007.

[11] D. Cao, M. Chen and H. Wang, "Interval method for global solutions of a class of min-max-min problems," *Applied Mathematics and Computation*, vol. 196, no. 2, pp. 594-602, Oct. 2008.

[12] E. Polak and J. O. Royset, "Algorithms for finite and semi-infinite min-max-min problems using adaptive smoothing techniques," *Journal of optimization theory and applications*, vol. 119, no. 3, pp. 421-457, Dec. 2003.

[13] A .Tsoukalas, P. Parpas, and B.Rustem, "A smoothing algorithm for finite min-max-min problems," *Optimization Letters*, vol. 3, no. 1, pp. 49-62, Jul. 2009.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge university press, 2004.

[15] W. Wang, W. Wang, Q. Lu, K. G. Shin, and T. Peng, "Geometry-based optimal power control of fading multiple access channels for maximum sum-rate in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 9, no. 6, pp. 1843-1848, Jun. 2010.

[16] B. Noble and J W. Daniel, *Applied Linear Algebra*, New Jersey: Prentice-Hall, 1988.

[17] J. Liu, W. Chen, Z. Cao and K. Letaief, "Dynamic power and sub-carrier allocation for OFDMA-based wireless multicast systems," *Proc. of IEEE ICC*, May. 2008.