Wireless Netw (2011) 17:1355–1371
DOI 10.1007/s11276-011-0353-8

Spectrum auction with interference constraint for cognitive radio networks with multiple primary and secondary users

Lin Chen · Stefano Iellamo · Marceau Coupechoux · Philippe Godlewski

Published online: 20 May 2011

Abstract Extensive research in recent years has shown the benefits of *cognitive radio* technologies to improve the flexibility and efficiency of spectrum utilization. This new communication paradigm, however, requires a well-designed spectrum allocation mechanism. In this paper, we propose an auction framework for cognitive radio networks to allow unlicensed secondary users (SUs) to share the available spectrum of licensed primary users (PUs) fairly and efficiently, subject to the interference temperature constraint at each PU. To study the competition among SUs, we formulate a non-cooperative multiple-PU multiple-SU auction game and study the structure of the resulting equilibrium by solving a non-continuous two-dimensional optimization problem, including the existence, uniqueness of the equilibrium and the convergence to the equilibrium in the two auctions. A distributed algorithm is developed in which each SU updates its strategy based on local information to converge to the equilibrium. We also analyze the revenue allocation among PUs and propose an algorithm to set the prices under the guideline that the revenue of each PU should be proportional to its resource. We then extend the proposed auction framework to the more challenging scenario with free spectrum bands. We develop an algorithm based on the no-regret learning to reach a correlated equilibrium of the auction game. The proposed algorithm, which can be implemented distributedly based on local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks. Finally, through numerical experiments, we demonstrate the effectiveness of the proposed auction framework in achieving high efficiency and fairness in spectrum allocation.

Keywords Cognitive radio networks · Spectrum auction · No-regret learning · Distributed algorithm · Game theory

Part of this work is done when Stefano Iellamo was a master student in Department of Electronics and Information, Politecnico di Milano, Piazza L. da Vinci 32, Milan, Italy.

L. Chen (✉)
Laboratoire de Recherche en Informatique (LRI), CNRS, University of Paris-Sud XI and INRIA, 91405 Orsay, France
e-mail: Lin.Chen@lri.fr

S. Iellamo · M. Coupechoux · P. Godlewski
Department of Computer Science and Networking, TELECOM ParisTech, LTCI CNRS 5141, 46 Rue Barrault, 75013 Paris, France
e-mail: dajelvento@gmail.com

M. Coupechoux
e-mail: coupecho@telecom-paristech.fr

P. Godlewski
e-mail: godlewsk@telecom-paristech.fr

1 Introduction

Cognitive radio [1, 2] has emerged in recent years as a promising paradigm to enable more efficient and spectrum utilization. Apart from the conventional command and control model, three more flexible spectrum management models are presented in [3], namely, exclusive use (or operator sharing), commons and shared use of primary licensed spectrum. In the last model, unlicensed secondary users (SUs) are allowed to access the spectrum of licensed primary users (PUs) in an opportunistic way. In such a model, a well-designed spectrum allocation mechanism is crucial to achieve efficient spectrum usage and harmonious coexistence of PUs and SUs. On one hand, the radio resource allocation mechanism should ensure that the spectrum resource (unused by PUs) is allocated efficiently and fairly among SUs. On the other hand, the communication of PUs should not be disturbed by the SUs.

In this paper, we tackle the challenging research problem of designing efficient spectrum allocation mechanism for cognitive radio networks. We consider a generic network scenario in which multiple PUs and SUs coexist. To use the spectrum resource efficiently, the SUs share the available spectrum of the PUs under the condition that the *interference temperature* constraint [4] is always satisfied at each PU, i.e., the total received power of the SUs at each PU should be kept under some threshold in order to protect the PU's traffic. The considered scenario can represent various network scenarios, e.g. the PUs are the access points of a mesh network and the SUs are the mobile devices.

In our work, we develop an auction framework to allow SUs to share the available spectrum of PUs. Under the proposed framework, each PU acts as a resource provider by (1) announcing a price and a reserve bid (2) allocating the received power as a function of the bids submitted by SUs. Each SU acts as a customer by (1) submitting a two-dimensional bid indicating which PU to bid for resource and how much to bid (2) paying the chosen PU an amount of payment proportional to the allocated resource and the announced price. To study the competition among SUs, we formulate a non-cooperative auction game and study the structure of the Nash equilibrium (NE) by solving a non-continuous two-dimensional optimization problem. A distributed algorithm is developed in which each SU updates its strategy based on local information to converge to the NE. Our analysis can serve as a decision and control framework for SUs to exploit the underutilized spectrum resource.

After investigating the competition among resource demanders, i.e., SUs, we extend our study on the natural yet crucial issue that how PUs (resource providers) interact with each other by setting their prices to maximize their revenue. We propose an algorithm to set the prices of PUs under the guideline that the revenue of each PU should be proportional to its resource: i.e., the interference temperature threshold.

We then extend the proposed auction framework to the more challenging scenario with free spectrum bands. In this context, a SU should strike a balance between accessing a free spectrum band with more interference if the competitors take the same strategy, and paying more for communication gains by staying with a licensed band. We show that the *ping-pong effect* may occur under the best-response update, i.e., a SU keeps switching between the free band and a licensed band. To eliminate the ping-pong effect, we develop an algorithm based on the no-regret learning [5] to reach a correlated equilibrium (CE) [6] of the auction game. The proposed algorithm, which can be implemented distributedly and requires only local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks.

Due to their perceived fairness and allocation efficiency [7], auctions are among the best-known market-based mechanisms to allocate spectrum [8–15]. In most proposed auctions, the spectrum resource is treated as goods in traditional auctions studied by economists, i.e., one licensed band (or a collection of multiple bands) is awarded to one SU. However, spectrum auction differs from conventional auctions in that it has to address radio interference. Spectrum auction is essentially a problem of interference-constrained resource allocation. Only a few papers have discussed spectrum auctions under interference constraint, among which [12] and [13] studied conflict-free spectrum allocation with high spectrum efficiency. Huang et al. [11] developed an auction-based spectrum sharing framework to allow a single spectrum manager to share its spectrum with a group of users, subject to the interference temperature constraint at the measurement point, a requirement proposed by FCC in [4]. Based on the same model as [11], our work is among the relative few that investigate the interference-constraint radio resource allocation problem under the auction framework. Compared with previous work, we make the following key contributions:

- Existing auction mechanisms mainly focus on single-PU scenario with very limited analytical and numerical studies on multiple-PU case. Our work, however, conduct an in-depth analysis on the spectrum auction for multiple PUs to allocate their spectrum to multiple SUs efficiently and fairly. As a distinctive feature of the proposed auction framework, the SUs' strategy (bid) is two-dimensional and non-continuous, leading to a competition scenario with more complex interactions among players and requiring an original study of the resulting equilibrium.
- We investigate the spectrum auction with free spectrum bands and develop a distributed adaptive algorithm based on no-regret learning to converge to a CE of the auction game. To the best of our knowledge, our work is the first to adapt the auction framework to address the spectrum sharing problem in heterogeneous environments with both licensed and free bands.

The rest of this paper is structured as follows. Section 2 presents our system model and auction framework followed by the formulation of the non-cooperative auction game. Section 3 solves the auction game and analyzes the structural properties of the resulting NE. Section 4 investigates the revenue allocation among PUs. Section 5 extends our auction framework to the more challenging scenario with free spectrum bands. Simulation results are presented in Sect. 6. Section 7 concludes the paper.

2 System model and spectrum auctions

This section introduces the notation and the system model of our work, followed by the presentation of the proposed spectrum auction framework and the formulation of the auction game under the framework.

2.1 Cognitive radio network model

We consider a cognitive radio network consisting of a set of primary users referred to as PUs and a set of secondary transmitter-receiver pairs referred to as secondary users or SUs. We use $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$ to denote the PU set and the SU set, respectively. We use S_i and D_i to denote the transmitter and the receiver of SU $i \in \mathcal{M}$. Each PU $n \in \mathcal{N}$ operates on a spectrum band n with bandwidth B_n that is non-overlapped with the spectrum bands of other PUs, i.e., $n_1 \bigcap n_2 = \Phi, \forall n_1, n_2 \in \mathcal{N}$.

SU i's valuation of the spectrum is defined by a function $U_i(\gamma_i)$, where γ_i is the received signal-to-interference-plus-noise ratio (SINR) at SU i's receiver D_i. $U_i(\gamma_i)$ characterizes the application payoff (e.g. satisfaction level) of SU i from SINR γ_i. We assume $U_i(\gamma_i)$ is continuously differentiable, strictly increasing and concave in γ_i with $U_i(0) = 0$. For each SU i, the received SINR using PU n's band is given by

$$\gamma_i = \frac{p_i h_{ii}}{n_0 B_n + \sum_{j \neq i} p_j h_{ji}}, \tag{1}$$

where p_i denotes SU i's transmission power, h_{ji} denotes the channel gain from SU j's transmitter S_j to SU i's receiver D_i,n_0 denotes the background noise power spectral density.

In the considered scenario, to ensure that the transmissions of PUs are not significantly degraded by SUs, an interference temperature constraint is imposed such that the total received power of SUs at PU n must satisfy

$$\sum_{i=1}^{M} p_i g_{in} \le P_n, \quad \forall n \in \mathcal{N},$$

where g_{in} is the channel gain from S_i to PU n, P_n is the tolerable interference threshold at PU n.

2.2 Spectrum auction framework

We apply auction mechanisms to tackle the spectrum allocation problem. By definition, an auction is a decentralized market mechanism for allocating resources and can be formulated as a non-cooperative game, where players are bidders, strategies are bids, both allocations and payments are functions of bids. A well-known auction is the Vickrey–Clarke–Groves (VCG) auction [7], which is shown to have social optimal outcome. However, it requires global information to perform centralized computations. To overcome this limitation, two one-dimensional share auction mechanisms, namely the SINR and the power auction are proposed in [11] to study the spectrum allocation problem in single-PU networks. In the following, we extend the work of [11] to the multiple-PU scenario by proposing the two-dimensional SINR and power auction, as shown in Algorithm 1.[1]

Algorithm 1 Two-dimensional spectrum auction algorithm

Price announcing: Each PU n announces a reserve bid β_n and a price $\pi_n > 0$.

Bidding: Based on β_n and π_n, each SU i submits a bid (a_i, b_i) where $a_i \in \mathcal{N}$ and $b_i \ge 0$.

Spectrum allocation: Each SU i is allocated a transmission power p_i from PU a_i as follows:

$$p_i = \frac{P_{a_i}}{g_{ia_i}} \frac{b_i}{\sum_{j \in \mathcal{M}, a_j = a_i} b_j + \beta_{a_i}}. \tag{2}$$

Payment collection: Each SU i pays PU a_i a payment $C_i = \pi_{a_i} \gamma_i g_{ia_i}$ in the SINR auction and $C_i = \pi_{a_i} p_i g_{ia_i}$ in the power auction.

In the proposed two-dimensional spectrum auction, each PU n first announces a reserved bid β_n[2] and a price $\pi_n > 0$. Based on β_n and π_n, each SU i submits a bid (a_i, b_i) where $a_i \in \mathcal{N}$ indicates from which PU the SU wants to buy spectrum and $b_i \ge 0$ further indicates how much spectrum resource the SU wants to bid. After the collection of all the bids from the SUs, each SU i is allocated a transmission power p_i from PU a_i which is proportional to its submitted bid b_i, as shown in (2). From (1) and (2), we can derive the received SINR of SU i as:

$$\gamma_i = \frac{P_{a_i} \frac{h_{ii}}{g_{ia_i}} b_i}{n_0 B_{a_i} \left(\sum_{j \in \mathcal{M}, a_j = a_i} b_j + \beta_{a_i} \right) + \sum_{j \in \mathcal{M}, a_j = a_i, j \neq i} P_{a_i} \frac{h_{ji}}{g_{ja_i}} b_j}. \tag{3}$$

In contrast to [11] where SUs are charged the same price per unit SINR, we apply the economic concept of *price discrimination* in the proposed SINR auction by imposing g_{ia_i} as a user-dependent pricing factor on SU i. The design rationale is that for two SUs choosing the same PU, the SU causing more interference at the PU should be charged more per unit SINR than the SU causing less interference. As we will show via numerical experiments, this feature is especially suited in multi-PU case by resulting a more

[1] In our study, we assume that SUs are honest, and indeed make the payments. We do not consider the issue of *payment enforcement*, which may require a separate mechanism and is beyond the scope of the paper.

[2] From the perspective of auction theory, the reserved bid β_n set by PU n can be seen as a bid made by PU n. By bidding β_n, PU n has a way of declaring a reservation value for its spectrum resource and prevents the possibility of the SUs colluding to purchase the resource for an arbitrarily small amount of money.

balanced equilibrium. For the power auction, noticing that the received power of SU i at PU a_i is $p_i g_{ia_i}$, the auction scheme actually implements a pricing policy under which a price π_n per unit received power is imposed by PU n to the SUs connecting to it.

To end this subsection, we would like to give some insight on the motivation of choosing the price to be proportional to the utility of γ_i in the SINR auction and $g_{ia_i}p_i$ in the power auction: the former is the benefits of the SU when enjoying a SINR γ_i and the latter corresponds to the interference that a SU generates at the receiver of PU (naturally the PUs should be compensated by a payment proportional to the interference). In other words, in the former case, the price is proportional to the benefits of SUs to some extent, while in the latter case, the price is proportional to the "damage" caused by SUs to PUs. Both of these two approaches are natural choices of PU when performing a spectrum auction.

2.3 Non-cooperative spectrum auction game formulation

Under the proposed auction framework, we model the interaction among SUs as a non-cooperative spectrum auction game, denoted as G_{NSA} and G_{NPA} for the SINR and power auction, respectively. Let $s_i = (a_i, b_i)$ denote the strategy of SU i and s_{-i} denote the strategy of the SUs except i, given the price vector $\pi = (\pi_n, n \in \mathcal{N})$, each SU i chooses its strategy s_i to maximize his *utility function* defined as follows:

$$S_i(s_i, s_{-i}) = U_i(\gamma_i(s_i, s_{-i})) - C_i(s_i, s_{-i}).$$

The solution of the game is characterized by *Nash Equilibrium (NE)*, a strategy profile $\mathbf{s}^* = (s_i^*, s_{-i}^*)$ from which no player has incentive to deviate unilaterally [16], i.e.,

$$S_i(s_i^*, s_{-i}^*) \geq S_i(s_i, s_{-i}^*), \quad \forall i \in \mathcal{M}, \forall a_i \in \mathcal{N}, \forall b_i \geq 0.$$

As a distinguished feature from the single-PU auction, the auction framework proposed in our work is two-dimensional and involves both PU selection and bid adjustment, which leads to a competition scenario with more complex interactions among players. Consequently, characterizing structural properties of the auction game in our context requires an original study of the game equilibria that cannot draw on existing well-known results, as will be shown in later analysis.

3 Solving the auction game: NE analysis

In this section, we solve the auction game by deriving the NE of the game and study the structure properties of the NE. To this end, we focus on the following optimization problem faced by each SU i in the spectrum auction game, given the price of PUs $\pi = \{\pi_n, n \in \mathcal{N}\}$ and strategies of others s_{-i}:

$$s_i^* = (a_i^*, b_i^*) = \operatorname*{argmax}_{s_i} S_i(s_i, s_{-i}), \tag{4}$$

which, according to the following lemma, can be written as

$$s_i^* = (a_i^*, b_i^*) = \operatorname*{argmax}_{a_i \in \mathcal{N}} \operatorname{argmax}_{b_i \geq 0} S_i(s_i, s_{-i}).$$

Lemma 1 $\max_{(a_i,b_i)} S_i(s_i, s_{-i}) = \max_{a_i \in \mathcal{N}} \max_{b_i \geq 0} S_i(s_i, s_{-i})$.

Proof On one hand, it follows from (4) that[3]

$$S_i((a_i^*, b_i^*), s_{-i}) \geq \max_{a_i \in \mathcal{N}} \max_{b_i \geq 0} S_i((a_i, b_i), s_{-i}).$$

On the other hand, we have

$$\max_{a_i \in \mathcal{N}} \max_{b_i \geq 0} S_i((a_i, b_i), s_{-i}) \geq \max_{b_i \geq 0} S_i((a_i^*, b_i), s_{-i}) \geq S_i((a_i^*, b_i^*), s_{-i}).$$

Combining the above results completes our proof. □

3.1 SINR auction

We start with the SINR auction game. Unlike the single-PU auction studied in [11], where each SU maximizes its utility function over its bid only, the SU optimization problem in the multiple-PU case is a joint two-dimensional problem over the submitted bid and the PU to whom the SU bids for spectrum. To solve the SUs' optimization problem, a straightforward way to find (a_i^*, b_i^*) is to search over all possible PU settings and perform optimization over bid for every setting, which is computationally intensive and makes the resulting NE intractable. In our analysis, we overcome this technical difficulty by decomposing the two-dimensional optimization problem based on the structural properties of the utility function, detailed in Lemma 2.

Lemma 2 *For each SU i, given* π *and* s_{-i}, *it holds that*

$$a_i^* = \operatorname*{argmax}_{n \in \mathcal{N}} S_i(\gamma_{in}^*) = \operatorname*{argmax}_{n \in \mathcal{N}} U_i(\gamma_{in}^*) - \pi_n g_{in} \gamma_{in}^*,$$

where $\gamma_{in}^* = \min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}, \forall n \in \mathcal{N}$.

Proof Let γ_{in} denote the SINR of SU i when connecting to PU n, recall (3), we can show that

1. γ_{in} is upper-bounded by $P_n h_{ii}/(n_0 B_n g_{in})$;
2. For $\gamma_{in} \leq P_n h_{ii}/(n_0 B_n g_{in})$, there is an one-to-one mapping between γ_{in} and b_i.

From Lemma 1, the optimization problem of SU i is thus equivalent to the following one:

[3] For the sake of simplicity, in case of non-ambiguity, we note $S_i((a_i^*, b_i^*), s_{-i})$ as a function of s_i, i.e., $S_i(s_i)$ or $S_i(a_i^*, b_i^*)$.

$$\max_{n\in\mathcal{N}} \max_{\gamma_{in}\le\frac{P_n h_{ii}}{n_0 B_n g_{in}}} S_i(n,\gamma_{in}).$$

Moreover, when choosing PU n, S_i can be written as a function of γ_{in} as

$$S_i(\gamma_{in}) = U_i(\gamma_{in}) - \pi_n g_{in}\gamma_{in},$$

whose derivative is

$$\frac{\partial S_i}{\partial \gamma_{in}} = U_i'(\gamma_{in}) - \pi_n g_{in}.$$

Following the concavity of U_i, U_i' is monotonously decreasing in γ_{in}. Hence S_i is a quasi-concave function of γ_{in}, thus has a unique global maximizer

$$\gamma_{in}^* = \min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}.$$

The maximum of S_i under PU n is given by $S_i(\gamma_{in}^*)$. It then follows that

$$a_i^* = \operatorname*{argmax}_{n\in\mathcal{N}} S_i(\gamma_{in}^*) = \operatorname*{argmax}_{n\in\mathcal{N}} U_i(\gamma_{in}^*) - \pi_n g_{in}\gamma_{in}^*,$$

where $\gamma_{in}^* = \min\{U_i'^{-1}(\pi_n g_{in}), P_n h_{ii}/(n_0 B_n g_{in})\}$.. □

Specifically, when π_n is significantly large, more precisely,

$$\pi_n g_{in} \ge U_i'(P_n h_{ii}/n_0 B_n g_{in}), \forall n\in\mathcal{N}, \quad \forall i\in\mathcal{M},$$

Lemma 2 can be simplified to Corollary 1.

Corollary 1 *If* $\pi_n g_{in} \ge U_i'(P_n h_{ii}/n_0 B_n g_{in}), \forall n\in\mathcal{N}, \forall i\in\mathcal{M}$, *it holds that*

$$a_i^* = \operatorname*{argmin}_{n\in\mathcal{N}} \pi_n g_{in}.$$

Proof Recall that $U_i(\gamma_i)$ is concave in γ_i, $\pi_n g_{in} \ge U_i'(P_n h_{ii}/n_0 B_n g_{in})$ leads to

$$U_i'^{-1}(\pi_n g_{in}) \le P_n h_{ii}/(n_0 B_n g_{in}).$$

It then follows from Lemma 2 that $\gamma_{in}^* = U_i'^{-1}(\pi_n g_{in})$ and

$$\begin{aligned} a_i^* &= \operatorname*{argmax}_{n\in\mathcal{N}} S_i(U_i'^{-1}(\pi_n g_{in})) \\ &= \operatorname*{argmax}_{n\in\mathcal{N}} U_i(U_i'^{-1}(\pi_n g_{in})) - \pi_n g_{in} U_i'^{-1}(\pi_n g_{in}). \end{aligned}$$

Let $x = \pi_n g_{in}$, regard $S_i = U_i(U_i'^{-1}(x)) - xU_i'^{-1}(x)$ as a function of x, after some mathematical operations, we have

$$\frac{\partial S_i}{\partial x} = -U_i'^{-1}(x),$$

which, following the concavity of U_i, is non-positive. $S_i(x)$ is thus non-increasing in x. Hence

$$a_i^* = \operatorname*{argmax}_{n\in\mathcal{N}} S_i(U_i'^{-1}(\pi_n g_{in})) = \operatorname*{argmin}_{n\in\mathcal{N}} \pi_n g_{in},$$

which concludes our proof. □

If we denote $\pi_n g_{in}$ as the effective price for SU i when choosing PU n, Corollary 1 states that SU i always chooses the PU with the minimum effective price.

As the key results of this subsection, we have demonstrated that in the SINR auction game, the choice of PU only depends on the effective price set by PUs. Consequently, the optimization problem of each SU i can be decomposed into two sub-problems, which can be performed sequentially:

1. i chooses PU a_i^* based on the effective price of PUs and stay with PU a_i^*;
2. i performs bid optimization by adjusting its bid submitted to PU a_i^*, which degenerates into single-PU case.

The following theorem on the NE of the SINR auction game is then immediate whose proof follows straightforwardly from that of Theorem 1 and Proposition 6 in [11].

Theorem 1 *For the SINR auction with* $\beta_n > 0, \forall n\in\mathcal{N}$, *there exists a threshold price vector* $\pi_{th}^s = \{\pi_{th,n}^s, n\in\mathcal{N}\}$ *such that if the price vector* $\pi > \pi^s{}_{th}$,[4] *a NE exists to which the best response update converges. The NE is unique if* a_i^* *is singleton for every SU i. On the other hand, if there exists some* $n_0\in\mathcal{N}$ *such that* $\pi n_0 \ge \pi_{th,n_0}^s$, *there is no NE.*

3.2 Power auction

In this subsection, we turn to the power auction game. As the payment function C_i in the power auction has a different structure to that in the SINR auction (i.e., C_i is a function of p_i instead of γ_i), the decomposition in the previous analysis on the SINR auction is no more applicable here. To characterize the equilibrium of the power auction game, we make the following approximation in the subsequent analysis:

$$\sum_{a_j=a_i, j\ne i} b_j \gg b_i, \\ \forall i\in\mathcal{M}, \text{or equivalently}, \sum_{s_j=a_i, j\ne i} b_j \sim \sum_{s_j=a_i} b_j. \quad (5)$$

The approximation (5) is accurate in large systems where the bid variation of any individual player has neglectable influence on the system state. More specifically, under (5), the impact of b_i on the interference at the receiver D_i, denoted as I_i, can be neglected, in other words, I_i can be regarded independent w.r.t. b_i. The utility function of SU i can then be written as:

[4] Throughout the paper, the inequality between two vectors is defined as the inequality in all components of the vectors.

$$S_i = U_i(\gamma_i) - \frac{\pi_{a_i} g_{ia_i} I_i}{h_{ii}} \gamma_i.$$

where $I_i = n_0 B_{a_i} + \sum_{j \in \mathcal{M}, j \neq i, a_j = a_i} p_j h_{ji}$.

To solve the power auction game, we transform the original game G_{NPA} into another game G_{NPA}' in which the strategy of SU i is (a_i, γ_i) instead of (a_i, b_i) in G_{NPA}. Under the approximation (5), γ_i can be regarded as a linear function of b_i. As I_i is independent w.r.t. b_i, any unilateral change in b_i can be transformed into related change in γ_i without any influence on γ_{-i}. Thus the original game G_{NPA} is equivalent to the transformed game G_{NPA}', formally expressed as:

$$G'_{NPA}: \quad \max_{s_i=(a_i,\gamma_i)} S_i(s_i, s_{-i}), i \in \mathcal{M}.$$

We now concentrate on the new game G_{NPA}'. Performing the same analysis as Lemma 1 and Corollary 1 by noticing that $I_i \geq n_0 B_{a\_i}$, we have the following result that decouples the PU selection and the adjustment of γ_i in G_{NPA}'.

Lemma 3 *If* $\pi_n g_{in} n_0 B_n / h_{ii} \geq U_i'(P_n h_{ii}/n_0 g_{in}), \forall n \in \mathcal{N}, \forall i \in \mathcal{M}$, *it holds that* $a_i^* = \mathrm{argmin}_{n \in \mathcal{N}} \pi_n g_{in} I_i / h_{ii}$.

Compared with the SINR auction game where the effective price imposed by PU n to SU i is $\pi_n g_{in}$, in the power auction game, the corresponding effective price becomes $\pi_n g_{in} I_i / h_{ii}$. Lemma 3 states that SU i always chooses the PU with the minimum effective price. Armed with Lemma 3, we can then establish the existence of NE in G_{NPA}' under the condition that the prices set by PUs are sufficiently high.

Theorem 2 *Under the approximation* (5) *and the condition in Lemma 3,* G_{NPA}' *admits a NE.*

Proof For any SU i, under the strategy of others $s_{-i} = (a_{-i}, \gamma_{-i})$, it follows from Lemma 3 that i chooses PU $a_i^* = \min_{n \in \mathcal{N}} \pi_n g_{in} I_i / h_{ii}$, i.e., for any $a_i' \neq a_i^*$, it holds that

$$\frac{\pi_{a_i^*} h_{ia_i^*} I_i(a_i^*)}{h_{ii}} \leq \frac{\pi_{a_i'} h_{ia_i'} I_i(a_i')}{h_{ii}}.$$

It then follows that for any $\gamma_i \geq 0$

$$\begin{aligned} S_i(a_i^*, \gamma_i) &= U_i(\gamma_i) - \frac{\pi_{a_i^*} h_{ia_i^*} I_i(a_i^*)}{h_{ii}} \gamma_i \geq U_i(\gamma_i) \\ &\quad - \frac{\pi_{a_i'} h_{ia_i'} I_i(a_i')}{h_{ii}} \gamma_i \\ &= S_i(a_i', \gamma_i), \end{aligned}$$

which implies that given the opponents' strategy, choosing PU a_i^* is always the dominating strategy for any γ_i.

On the other hand, performing the same analysis as Lemma 1, we can show that in G_{NPA}',

$$\max_{(a_i,\gamma_i)} S_i(s_i, s_{-i}) = \max_{\gamma_i} \max_{a_i} S_i(s_i, s_{-i}).$$

The optimization problem for SU i thus becomes

$$\max_{(a_i,\gamma_i)} S_i(s_i, s_{-i}) = \max_{\gamma_i} S_i(a_i^*, \gamma_i),$$

in which the utility function of SU i is $S_i(a_i^*, \gamma_i)$, which is concave in γ_i. Furthermore, it follows from $I_i \geq n_0 B_n$ and $p_i \leq P_n / g_{ia_i}$ when SU i chooses PU n that $\gamma_i \leq \max_{n \in \mathcal{N}} h_{ii} P_n / (g_{ia_i} n_0 B_n)$. Thus the strategy space $\gamma = (\gamma_i, i \in \mathcal{M})$ is a nonempty, convex, and compact set. It then follows from Theorem 1 in [17] that G_{NPA}' admits a NE. □

In the rest part of this subsection, we focus on a particular scenario to investigate the dynamics of the power auction game. In the studied scenario, there is only one PU processing N spectrum bands which can be accessed opportunistically by the SUs with logarithmic utilities (i.e., $U_i(\gamma_i) = \theta_i ln(\gamma_i)$ with θ_i being a user-dependant parameter) and the receivers of SUs are collocated at the PU (i.e. $h_{ji} = g_{jn}, \forall i, j \in \mathcal{M}, \forall n \in \mathcal{N}$). This can be viewed as a special case of our model where there are N virtual PUs corresponding to one physical PU. In this scenario, consider that G_{NPA}' is played repeatedly and we base our analysis on the following best response update in which each SU i updates its strategy for the next iteration $t+1$ to maximize its utility based on the strategy of the opponents in the current iteration t:

$$a_i(t+1) = \operatorname*{argmin}_{n \in \mathcal{N}} \frac{\pi_n g_{in} I_i(t)}{h_{ii}} \tag{6}$$

$$\gamma_i(t+1) = \operatorname*{argmax}_{\gamma_i \in \left[0, \frac{h_{ii} P_{a_i(t+1)}}{n_0 B_{a_i(t+1)}}\right]} S_i(s_i = (a_i(t+1), \gamma_i), s_{-i}(t)). \tag{7}$$

Theorem 3 studies the uniqueness of NE and the convergence to the NE under best response update.

Theorem 3 *In the considered scenario, if* $\pi_n > \max\{M/(\theta_i n_0 B_n), \theta_i / P_n\}, \forall n \in \mathcal{N}, \forall i \in \mathcal{M}, G'_{NPA}$ *admits a unique NE. Starting from any initial point, the iteration under the best response update converges to the unique NE.*

Proof In the considered scenario with logarithmic utility, if $\pi_n > \theta_i / P_n$, the condition in Lemma 3 is satisfied. We can write the best response update as

$$\gamma_i(t+1) = U_i'^{-1}\left(\min_{n \in \mathcal{N}} \frac{\pi_n g_{in} I_i}{h_{ii}}\right) = \max_{n \in \mathcal{N}} U_i'^{-1}\left(\frac{\pi_n g_{in} I_i}{h_{ii}}\right). \tag{8}$$

Let $\Delta_i \triangleq \sum_{j \in \mathcal{M}, a_j = a_i, j \neq i} b_j + \beta_{a_i}$, under the approximation (5), we can express I_i and γ_i as:

$$I_i = n_0B_{a_i} + \sum_{j\in\mathcal{M}, a_j=a_i, j\neq i} p_j h_{ji} = n_0 B_{a_i} + P_{a_i} \sum_{j\in\mathcal{M}, a_j=a_i, j\neq i} \frac{b_j}{\sum_{k\in\mathcal{M}, a_k=a_i} b_k + \beta_{a_i}} \simeq n_0B_{a_i} + P_{a_i}\left(1-\frac{b_i+\beta_{a_i}}{\Delta_i}\right) \simeq n_0B_{a_i} + P_{a_i}\left(1-\frac{\beta_{a_i}}{\Delta_i}\right), \tag{9}$$

$$\gamma_i = \frac{h_{ii}p_i}{I_i} \simeq \frac{P_{a_i}\frac{b_i}{\Delta_i}}{n_0B_{a_i} + P_{a_i}\left(1-\frac{\beta_{a_i}}{\Delta_i}\right)} = \frac{b_i}{\left(1+\frac{n_0B_{a_i}}{P_{a_i}}\right)\Delta_i - \beta_{a_i}} \quad \forall i \in \mathcal{M}.$$

It follows that

$$\sum_{\forall j\in\mathcal{M}, a_j=a_i, j\neq i} \gamma_j = \frac{\Delta_i - \beta_{a_i}}{\left(1+\frac{n_0B_{a_i}}{P_{a_i}}\right)\Delta_i - \beta_{a_i}},$$

and

$$1-\frac{\beta_{a_i}}{\Delta_i} = \frac{n_0B_{a_i}}{P_{a_i}} \frac{\sum_{j\in\mathcal{M}, a_j=a_i, j\neq i}\gamma_j}{1-\sum_{j\in\mathcal{M}, a_j=a_i, j\neq i}\gamma_j}. \tag{10}$$

Injecting (10) into (9) with logarithmic utility, (8) becomes

$$\gamma_i(t+1) \triangleq \mathbf{T_i}(\gamma(\mathbf{t})) = \max_{n\in\mathcal{N}} \left\{ \frac{1-\sum_{j\in\mathcal{M}, a_j=a_i, j\neq i}\gamma_j(t)}{\pi_n\theta_i n_0 B_n} \right\}.$$

We then establish the uniqueness of the NE and the convergence to the unique NE under best response update $\mathbf{T} = (\mathbf{T_i}, i \in \mathcal{M})$ by proving that $\mathbf{T}$ is a contraction. A contraction is defined [11] as follows: let (X, d) be a metric space, $f: X \to X$ is a contraction if there exists a constant $k \in [0,1)$ such that $\forall x, y \in X, d(f(x), f(y)) \leq k d(x, y)$, where $d(x, y) = \|x - y\| = \max_i |x_i - y_i|$.

We first show that $\forall n \in \mathcal{N}, \mathbf{T'^n} \triangleq (\mathbf{T_i'^n}, i \in \mathcal{M})$ defined as follows is a contraction:

$$\gamma_i(t+1) \triangleq \mathbf{T_i'^n}(\gamma(\mathbf{t})) = \frac{1-\sum_{j\in\mathcal{M}, a_j=a_i, j\neq i}\gamma_j(t)}{\pi_n\theta_i n_0 B_n}. \tag{11}$$

Noticing that

$$d(f(x),f(y)) = ||f(x)-f(y)|| \leq \left|\left|\frac{\partial f}{\partial x}\right|\right| \cdot ||x-y|| = \left|\left|\frac{\partial f}{\partial x}\right|\right| d(x,y),$$

it suffices to show that the Jacobian $\left|\left|\frac{\partial f}{\partial x}\right|\right| \leq k$. In our case, it suffices to show that $||J||_\infty \leq k$, where $J = \{J_{ij}\}$ is the Jacobian of $\mathbf{T}'^{\,\mathbf{n}}$ defined by $J_{ij} = \frac{\partial \gamma_i(t+1)}{\partial \gamma_j(t)}$.

It follows from (11) that

$$J_{ij} = \begin{cases} -\frac{1}{\pi_n\theta_i n_0 B_n} & a_j = n, j \neq i \\ 0 & \text{otherwise} \end{cases}.$$

Hence if $\pi_n > M/(\theta_i n_0 B_n), \forall n \in \mathcal{N}$, it holds that

$$||J||_\infty = \max_{i\in\mathcal{M}} \sum_{j\in\mathcal{M}} |J_{ij}| \leq \frac{M}{\pi_n\theta_i n_0 B_n} < 1,$$

which shows that $\mathbf{T}'^n$ is a contraction.

We now prove that $\mathbf{T}$ with $\mathbf{T_i} = \max_{n\in\mathcal{N}} \mathbf{T_i'^n}$ is a contraction. To this end, given any $\gamma^1 \triangleq (\gamma_i^1, i \in \mathcal{M})$ and $\gamma^2 \triangleq (\gamma_i^2, i \in \mathcal{M})$, let $\|\gamma^1 - \gamma^2\| = |\gamma^1{}_{i\_1} - \gamma^2{}_{i\_1}|$ and $||\mathbf{T}(\gamma^\mathbf{1}) - \mathbf{T}(\gamma^\mathbf{2})|| = |\mathbf{T}_{\mathbf{i_2}}'^{n_1}(\gamma^\mathbf{1}) - \mathbf{T}_{\mathbf{i_2}}'^{n_2}(\gamma^\mathbf{2})|$. Without loss of generality, assume that $\mathbf{T}_{\mathbf{i_2}}'^{n_1}(\gamma^\mathbf{1}) \geq \mathbf{T}_{\mathbf{i_2}}'^{n_2}(\gamma^\mathbf{2})$. It follows from $\mathbf{T_i} = \max_{n\in\mathcal{N}} \mathbf{T_i'^n}$ that $\mathbf{T}_{\mathbf{i_2}}'^{n_2}(\gamma^\mathbf{2}) \geq \mathbf{T}_{\mathbf{i_2}}'^{n_1}(\gamma^\mathbf{2})$. Recall that $\mathbf{T}'^{n_1}$ is a contraction, it then holds that

$$||\mathbf{T}(\gamma^\mathbf{1}) - \mathbf{T}(\gamma^\mathbf{2})|| = |\mathbf{T}_{\mathbf{i_2}}'^{n_1}(\gamma^\mathbf{1}) - \mathbf{T}_{\mathbf{i_2}}'^{n_2}(\gamma^\mathbf{2})| \leq |\mathbf{T}_{i2}'^{n_1}(\gamma^1) - \mathbf{T}_{\mathbf{i_2}}'^{n_1}(\gamma^\mathbf{2})| \leq ||\mathbf{T}'^{n_1(\gamma^1)} - \mathbf{T}'^{n_1}(\gamma^\mathbf{2})|| \leq \frac{M}{\pi_n\theta_i n_0 B_n} ||\gamma^1 - \gamma^2||.$$

Hence under the condition in the theorem, the best response update $\mathbf{T}$ is a contraction. Both the uniqueness and the convergence is guaranteed. □

Theorem 3 establishes the sufficient condition for the uniqueness and convergence of the NE in the power auction game for the considered scenario. Under the sufficient condition, the unique NE is also stable in that any deviated point from the NE will be dragged back to the NE under best response update. However, the condition is only sufficient, not necessary and may be too stringent in some cases. Hence, it is possible that even the above condition is not met, the best response update still converges to the NE.

In the general cases, due to the complexity of the power auction game in which each SU has to solve a two-dimensional, non-continuous and non-decomposable optimization problem, we do not have a formal proof of the uniqueness of the NE and the convergence under the best response update. However, our experiment results show that the convergence is achieved in the vast majority of cases (cf. Sect. 6.3).

3.3 The two-level game model

To get more insight on the structure of the auction game, we introduce and analyze in this subsection the following two-level game model: the lower level bidding game under fixed PU setting (Definition 1) and the higher level PU selection game (Definition 2).

Definition 1 Given a fixed PU setting $\mathbf{a} = \{a_i, i \in \mathcal{M}\}$, the bidding game, denoted as $G^B{}_{NSA}(\mathbf{a})$ and $G^B{}_{NPA}(\mathbf{a})$ for the SINR and power auction respectively, is a tuple $(\mathcal{M}, \mathcal{A}, \{S_i, i \in \mathcal{M}\})$, where the SU set $\mathcal{M}$ is the player set, $\mathcal{A} = [0, +\infty)^M$ is the strategy set, $\{S_i\}$ is the utility function set with S_i being the surplus function. Each player

(SU) i selects its strategy (bid) $b_i \geq 0$ to maximize its utility S_i.

The above defined bidding game can be analyzed in the same way as the single-PU bidding game presented in [11] with the following result on the NE.

Lemma 4 *For the SINR auction (for the power auction under the approximation (5)) with $\beta_n > 0, \forall n \in \mathcal{N}$, there exists a threshold price vector $\pi^{sb}{}_{th}(\mathbf{a})$ ($\pi^{pb}{}_{th}(\mathbf{a})$) such that there exists a NE to which the best response update converges if the price vector $\pi > \pi^{sb}{}_{th}(\mathbf{a})$ ($\pi > \pi^{pb}{}_{th}(\mathbf{a})$), there is no NE otherwise.*

Proof The proof for the SINR auction follows immediately from Theorem 1 and Proposition 6 in [11]. For the power auction, we show that under the condition in the lemma, the best response function has the same structure as that in the SINR auction in [11] whose convergence to NE is proven (Theorem 1 in [11]). To this end, recall that under (5), the utility function can be written as

$$S_i = U_i(\gamma_i) - \frac{\pi g_{i0} I_i}{h_{ii}} \gamma_i,$$

where I_i is independent of b_i. For each SU i, its the best response $b_i = B(b_{-i})$ satisfies the following equations:

$$\begin{cases} b_i = +\infty & \text{if } \pi \leq \frac{h_{ii}}{I_i g_{i0}} U_i'\left(\frac{P_{a_i} h_{ii}}{n_0 B_{a_i} g_{ia_i}}\right) \\ \pi = \frac{h_{ii} U_i'(\gamma_i)}{I_i g_{i0}} & \text{if } \frac{h_{ii}}{I_i g_{i0}} U_i'\left(\frac{P_{a_i} h_{ii}}{n_0 B_{a_i} g_{ia_i}}\right) < \pi < \frac{h_{ii}}{I_i g_{i0}} U_i'(0) \\ b_i = 0 & \text{if} \pi \geq \frac{h_{ii}}{I_i g_{i0}} U_i'(0) \end{cases} \tag{12}$$

Noticing the structural similarity between (12) and (22) in [11], we can establish the existence of NE and the convergence to the NE under the best response update (12). □

Definition 2 The PU selection game, denoted as $G^{PU}{}_{NSA}$ and $G^{PU}{}_{NPA}$ for the SINR and power auction respectively, is a tuple $(\mathcal{M}, \mathcal{A} = \{A_i\}, \{\widehat{S}_i, i \in \mathcal{M}\})$, where $\mathcal{M}$ is the player set, $A_i = \mathcal{N}$ is the strategy set of SU i, the utility function of SU i is defined as $\widehat{S}_i(a_i, a_{-i}) \triangleq S_i(\mathbf{a}, \mathbf{b}^*)$ where $\mathbf{b}^*(\mathbf{a}) = \{b_i^*(\mathbf{a}), i \in \mathcal{M}\}$ denotes the NE of the bidding game under the PU setting $\mathbf{a}$. Each player (SU) i selects its strategy (PU) $a_i \in \mathcal{N}$ to maximize $\widehat{S}_i$.

To analyze the PU selection game, we write the optimization problem of each SU i as

$$\max_{a_i} \widehat{S}_i(a_i, a_{-i}) = \max_{a_i} S_i(\mathbf{a}, b^*(a)).$$

Noticing that in the bidding game under PU setting $\mathbf{a}$, it holds that $S_i(\mathbf{a}, \mathbf{b}^*(\mathbf{a})) = \max_{b\_i} S_i(a_i, b_i)$, we have

$$\max_{a_i} \widehat{S}_i(a_i, a_{-i}) = \max_{a_i} \max_{b_i} S_i(a_i, b_i),$$

which, according to Lemma 1, is the same optimization problem as for the global auction game analyzed previously. Hence, we can map the NE of the PU selection game and the corresponding bidding game to the NE of the global auction game, as stated in the following theorem.

Theorem 4 *Any (pure) NE of the auction game can be mapped to a (pure) NE of the PU selection game $\mathbf{a}^*$ and the corresponding NE of the bid game $\mathbf{b}^*(\mathbf{a}^*)$ under the PU setting $\mathbf{a}^*$, i.e., any pure NE of the power auction game can be expressed as $\mathbf{s}^* = (a_i^*, b_i^*(\mathbf{a}^*), i \in \mathcal{M})$.*

By decomposing the global auction game into the PU selection game and the bidding game, we introduce a two-level architecture into the spectrum auction problem, in which the higher level PU selection game is a finite strategy game. This hierarchicalization can help us analyze the spectrum auction in more complex scenarios, as explored in the next section.

4 Revenue allocation among PUs

Our analysis so far investigates the competition among resource demanders, i.e., SUs. A natural yet crucial question is how PUs (resource providers) interact with each other to maximize their revenue. In this section, we provide a systematic study on this issue.

In order to make our analysis tractable, we consider logarithmic utilities, i.e., $U_i(\gamma_i) = \theta_i ln(\gamma_i)$. In the SINR auction, recall Lemma 2 and Corollary 1, with sufficiently high prices at PUs, the NE SINR at the receiver of SU i γ_i^* is given by the following equation:

$$\gamma_i^* = \frac{\theta_i}{\pi_{a_i^*} g_{ia_i^*}}, \quad \forall i \in \mathcal{M},$$

where $a_i^* = \operatorname{argmin}_{n \in \mathcal{N}} \pi_n g_{in}$.

In the power auction, under the approximation (5), the NE transmission power of SU i p_i^* is given by the following equation:

$$p_i^* = \frac{\theta_i}{\pi_{a_i^*} g_{ia_i^*}}, \quad \forall i \in \mathcal{M},$$

where $a_i^* = \operatorname{argmin}_{n \in \mathcal{N}} \pi_n g_{in} I_i / h_{ii}$.

Recall the payment of SU i to PU a_i^*, in the SINR auction, we have

$$C_i = \pi_{a_i^*} g_{ia_i^*} \gamma_i^* = \theta_i.$$

In the power auction, we have

$$C_i = \pi_{a_i^*} g_{ia_i^*} p_i^* = \theta_i.$$

In both cases, the global revenue of PU n, denoted by R_n, can be calculated as

$$R_n = \sum_{a_i^*=n} C_i = \sum_{a_i^*=n} \theta_i.$$

The total revenue of all PUs is thus

$$R_{tot} = \sum_{n\in\mathcal{N}} R_n = \sum_{i\in\mathcal{M}} \theta_i,$$

The engineering implications behind the above results are: (1) the revenue that a PU gets from any SU i connecting to it is θ_i; (2) the revenue of PU is independent of the total resource it processes (i.e., the temperature interference constraint P_n); (3) the total revenue R_{tot} is constant regardless of the price set by PUs; however, by choosing a lower price, a PU can increase its revenue by attracting more SUs. Consequently, each PU faces the following tradeoff: choosing a lower price increases its revenue, but setting a price too low will not ensure the convergence of the system. In other words, a price war among PUs is clearly catastrophic as it will lead to system instability. Therefore, the research question posed in the beginning of this section can be translated into the question of how to allocate the total revenue R_{tot} among PUs in a fair and efficient way.

We address this problem by proposing the following guideline.

The prices of PUs are chosen in such a way that the revenue of each PU is proportional to its resource (interference temperature threshold): i.e., $\forall n_1, n_2 \in \mathcal{N}, R_{n_1}/R_{n_2} = P_{n_1}/P_{n_2}$.

The design rationale of the proposed guideline are trifold: (1) the revenue allocation following the guideline is proportional fair with weight $\{P_n\}$ (intuitively and naturally, a PU putting more resource to allocate to SUs deserves more revenue); (2) from an economic point of view, a PU providing more resource tends to set a lower price; in our context, this leads to more revenue; (3) the guideline can encourage the PUs to provide more resource to SUs, given that without a stimulating mechanism, a PU has no incentive to do so as its revenue depends only on the number of SUs choosing it and their utility parameters $\{\theta_i\}$.

In subsequent study, we study how to set the prices under the proposed guideline. Instead of performing a formal mathematic study, we proceed our analysis by focusing on an illustrative example for the SINR auction, which we argue is enough to provide an in-depth insight on the revenue allocation among PUs and the resulting interaction on price setting. More extensive investigation on the general case and the power auction case consists of a significant extension of the current work and is subject of our ongoing research.

In the considered scenario (Fig. 1), SUs with random θ_i are uniformly distributed in an area of 1×1 with two PUs located at (0.25, 0.5) and (0.75, 0.5). The channel gain $h_{ij} = d_{ij}^{-\alpha}$, where d_{ij} is the distance between i and j, α is the path loss exponent. Recall Lemma 2 and Corollary 1, the expected revenue of PU 1 R_1 and that of PU 2 R_2 has the following relation:

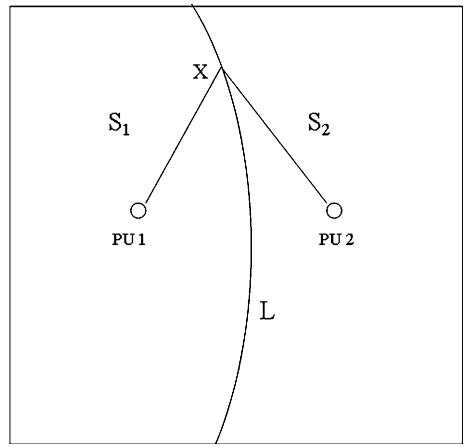

Fig. 1 An illustrative scenario

$$\frac{R_1}{R_2} = \frac{S_2}{S_1},$$

where S_1 and S_2 denote the area of the two regions separated by the curve L (cf Fig. 1). Let $d_1(X)$ and $d_2(X)$ denote the distance between any point X on L and PU 1 and PU 2, respectively, L is determined by

$$\pi_1(d_1(X))^{-\alpha} = \pi_2(d_2(X))^{-\alpha}.$$

That is, all SUs with its sender located at the left of L are better off choosing PU 2 and all SUs with its sender located at the right of L are better off choosing PU 1. It then follows from the proposed guideline that π_1 and π_2 can be calculated by

$$\frac{S_2}{S_1} = \frac{P_1}{P_2}.$$

Note that the above equation has unlimited number of solutions. In fact, if (π_1^0, π_2^0) is a solution, then $(k\pi_1^0, k\pi_2^0)$ with $k > 0$ also satisfies the equation. To guarantee the system convergence, k should be set to a sufficiently high value. More specifically, we develop an algorithm (Algorithm 2) based on Monte-Carlo method to compute the prices under the proposed guideline.

As a desirable property, Algorithm 2 can be run once at the deployment of the cognitive network. Furthermore, the execution of the algorithm can be done locally without explicit message exchange overhead among PUs. After that, to accommodate the dynamics of SUs, π_1 and π_2 can be adjusted while fixing the ratio between them to ensure the convergence of the system.

Algorithm 2 Revenue allocation among PUs: price calculation

Initialization: Set π_1 to some high value, set N to some high integer, set ϵ to some small value. Set $(\pi_2^{min}, \pi_2^{max}) = (0, +\infty)$. Set the iteration index $t = 0$

repeat
  Randomly choose $\pi_2(t) = (\pi_2^{min} + \pi_2^{max})/2$.
  Calculate S_2/S_1 under the price setting $(\pi_1, \pi_2(t))$ using Monte-Carlo method, as follows:
  Set $N_1 = 0$.
  for $i = 0, i < N$ **do**
    Randomly select a point P in the network
    Calculate the distance between P and the two PUs, denoted as d_1, d_2.
    if $\pi_1(d_1)^{-\alpha} < \pi_2(d_2)^{-\alpha}$ **then**
      $N_1 = N_1 + 1$
    end if
  end for
  if $N_1/(N - N_1) < P_1/P_2$ **then**
    $\pi_2^{max} = \pi_2(t)$
  else
    $\pi_2^{min} = \pi_2(t)$
  end if
  $t = t + 1$.
until $|N_1/(N - N_1) - P_1/P_2| < \epsilon$

5 Spectrum auction with free spectrum bands

Until now, we have analyzed the spectrum auction game in which the unlicensed SUs purchase spectrum resource from licensed PUs. In this section, we extend our auction framework to the more challenging scenario with free spectrum bands. In such context, the SUs have the choice between accessing the licensed spectrum bands owned by PUs which is charged as a function of the enjoyed SINR or received power at PUs, and switching to the unlicensed spectrum bands which are free of charge but become more crowded when more SUs operate in these spectrum bands. Consequently, the SUs should strike a balance between accessing the free spectrum bands with probably more interference and paying for communication gains by staying with the licensed bands. In the subsequent study, we assume that there is one free spectrum band available for all SUs. However, the sequel analysis can be extended to the scenario of multiple free spectrum bands.

We start with the SINR auction. In the new scenario with a free band, we define the spectrum band set $\mathcal{N} = \{1, \ldots, N, N+1\}$ where band 1 to N are the licensed bands processed by PU 1 to N, band $N + 1$ denotes the free band with bandwidth B_{N+1}. Compared with the previous analysis without free spectrum band, each SU i has an additional choice of switching to band $N + 1$ and the corresponding utility is

$$S_i(N+1) = U_i(\gamma_i), \tag{13}$$

where γ_i is the SINR of SU i. It is obvious to see that all SUs operating at B_{N+1} transmits at its maximum power, denoted as p_j^{max}, $j \in \mathcal{M}$, to maximize their utility. Hence

$$\gamma_i = \frac{p_i^{max} h_{ii}}{n_0 B_{N+1} + \sum_{j \neq i, a_j = N+1} p_j^{max} h_{ji}}.$$

From Corollary 1, each SU i faces the choice of accessing the licensed band with minimum effective price and the free band $N + 1$. As in Definition 1 and 2, we can define the corresponding PU selection game and bidding game in the new context[5]. The PU selection game is a finite strategy game and hence has at least one pure or mixed NE. By performing the same analysis as that in Sect. 3.3, we can establish a mapping between a NE of the auction game and a NE of the PU selection game in the new context.

We then address the problem of how to reach a NE of the PU selection game, which is also a NE of the global auction game. We first notice that the myopic best response update in the PU selection game is not guaranteed to converge to a NE. In fact, during the course of PU selection, the SUs may notice that the utility of accessing a licensed spectrum is higher than staying in the free spectrum, and thus switch to the licensed spectrum accordingly. Since the SUs do this simultaneously, the free spectrum becomes under-loaded and the SUs will switch back to the free spectrum in the next iteration. This phenomenon, in which a player keeps switching between two strategies, is known as ping-pong effect.

To eliminate the ping-pong effect, we develop an algorithm based on the no-regret learning to converge to a correlated equilibrium (CE) of the PU selection game, which is shown to be a CE of the global auction game, too. Before presenting the algorithm, we first provide a brief introduction on CE and no-regret learning.

5.1 Overview of correlated equilibrium

The concept of CE was proposed by Nobel Prize winner, Aumann [6], in 1974. It is more general than NE. The idea is that a strategy profile is chosen randomly according to a certain distribution. Given the recommended strategy, it is to the players' best interests to conform with this strategy. The distribution is called CE, formally defined as follows.

[5] For the free band, there is no bidding game, or alternatively, we can define a dumb bidding game for the free band, at the NE of which each SU choosing the free band submits 0 as bid and the utility is given by (13).

Definition 3 Let $G = (\mathcal{N}, (\Sigma_i, i \in \mathcal{N}), (S_i, i \in \mathcal{N}))$ be a finite strategy game, where $\mathcal{N}$ is the player set, Σ_i is the strategy set of player i and S_i is the utility function of i, a probability distribution p is a correlated equilibrium of G if and only if $\forall i \in \mathcal{N}, r_i \in \Sigma_i$, it holds that

$$\sum_{r_{-i}\in\Sigma_{-i}} p(r_i, r_{-i})[S_i(r_i', r_{-i}) - S_i(r_i, r_{-i})] \le 0, \quad \forall r_i' \in \Sigma_i,$$

or equivalently,

$$\sum_{r_{-i}\in\Sigma_{-i}} p(r_{-i}|r_i)[S_i(r_i', r_{-i}) - S_i(r_i, r_{-i})] \le 0, \quad \forall r_i' \in \Sigma_i,$$

where Σ_{-i} and r_{-i} denote the strategy space and the strategy of the players except i.

The second formula means that when the recommendation to player i is to choose strategy r_i, then choosing strategy $r_i' \neq r_i$ cannot lead to a higher expected payoff to i.

The CE set is nonempty, closed and convex in every finite strategy game. Moreover, every NE is a CE and corresponds to the special case where $p(r_i, r_{-i})$ is a product of each individual player's probability for different strategies, i.e., the play of the different players is independent.

5.2 Overview of no-regret learning

The no-regret learning algorithm [5] is also termed regret-matching algorithm. The stationary solution of the no-regret learning algorithm exhibits no regret and the probability of choosing a strategy is proportional to the "regret" for not having chosen that strategy. For any two strategies $r_i \neq r_i'$ at any time T, the regret of player i for not playing r_i' is

$$R_i^T(r_i, r_i') \triangleq \max(D_i^T(r_i, r_i'), 0), \tag{14}$$

where

$$D_i^T(r_i, r_i') \triangleq \frac{1}{T}\sum_{t\le T}(S_i^t(r_i', r_{-i}) - S_i^t(r_i, r_{-i})). \tag{15}$$

$D_i^T(r_i, r_i')$ has the interpretation of average payoff that player i would have obtainned, if it had played r_i' every time in the past instead of r_i. $R^T{}_i(r_i, r_i')$ is thus a measure of the average regret. The probability that player i chooses r_i is a linear function of the regret. For every period T, define the relative frequency of players' strategy $\mathbf{r}$ played till T periods of time as follow:

$$z_T(r) \triangleq \frac{1}{T} N(T, \mathbf{r}),$$

where $N(T, \mathbf{r})$ denotes the number of periods before T that the players' strategy is $\mathbf{r}$. *As an important property, z_T is guaranteed to converge almost surely (with probability one) to a set of CE in no-regret learning algorithm.*

Algorithm 3 No-regret learning algorithm: SINR auction

Initialization: For each SU i, let p denote a random number between 0 and 1 and $a_i^* = \min_{n\in\mathcal{N}} \pi_n g_{in}$ (if a_i^* is not a singleton, randomly choose one), set $p^0_{a_i=a_i^*} = p$ and $p^0_{a_i=N+1} = 1 - p_{a_i^*}$. Let T_0 be a sufficient iteration duration. Let μ be a constant larger than the payoff upper bound of the SUs.

for $t = kT_0, k = 1, 2, 3, \ldots$ **do**

Select spectrum a_i with probability $p^t{}_i(a_i)$ and use best-response update to converge to the NE of the bidding game.

When the NE is achieved after sufficient time, update the average regret R_i^t.

Let a_i^t denote the spectrum which SU i selects for iteration t, calculate p_i^{t+1} as:

$$\begin{cases} p_i^{t+1}(a_i) = \frac{1}{\mu} R_i^t, & \forall a_i \in \mathcal{N},\ a_i \neq a_i^t \\ p_i^{t+1}(a_i) = 1 - \sum_{n\in\mathcal{N}, n\neq a_i^t} p_i^{t+1}(n), & a_i = a_i^t \end{cases}$$

end for

5.3 Proposed algorithm based on no-regret learning

In this subsection, we develop an algorithm (Algorithm 3) based on no-regret learning and prove its convergence to a CE of the SINR auction game.

Theorem 5 *There exists a threshold price vector* πth *such that if the price vector* $\pi > \pi$th, *the proposed algorithm converges surely to a CE of the SINR auction game.*

Proof It follows from the structure of the bidding game that a threshold price vector πth exists such that if the price vector $\pi > \pi$th, the convergence to the NE of the bidding game is guaranteed under the given spectrum setting. It then follows from the convergence property of the no-regret learning that the proposed algorithm converges surely to a CE of the PU selection game, denoted as $\mathbf{p}$, i.e.,

$$\sum_{a_j\in\mathcal{N}, j\in\mathcal{M}, j\neq i} p(a_{-i}|a_i)[S_i((a_i', b_i'^*), (a_{-i}, b_{-i}'^*)) \\ - S_i((a_i, b_i^*), (a_{-i}, b_{-i}^*))] \le 0, \\ \forall a_i' \in \mathcal{N},$$

where b_i^* and $b_i'^*$ is the strategy of SU i at the NE of the bidding game under the spectrum setting (a_i, a_{-i}) and (a_i', a_{-i}), respectively. It follows from the NE definition of the bidding game that

$$S_i((a_i', b_i'^*), (a_{-i}, b_{-i}'^*)) = \max_{\gamma_i} U_i(\gamma_i) - \pi_{a_i'} g_{ia_i'} \gamma_i.$$

On the other hand, we have

$$S_i((a_i', b_i'), (a_{-i}, b_{-i}^*)) \le \max_{\gamma_i} U_i(\gamma_i) - \pi_{a_i'} g_{ia_i'} \gamma_i, \forall b_i' \ge 0.$$

Hence, it holds that

$$\sum_{a_j \in \mathcal{N}, j \in \mathcal{M}, j \neq i} p(a_{-i}|a_i)[S_i((a_i', b_i'), (a_{-i}, b_{-i}^*)) - S_i((a_i, b_i^*), (a_{-i}, b_{-i}^*))] \leq 0, \quad \forall a_i' \in \mathcal{N}, \forall b_i \geq 0,$$

indicating that $\mathbf{p}$ is also a CE of the SINR auction game. □

As a desirable property, Algorithm 3 can be implemented distributedly such that each SU i only needs to know the price vector π, its own channel gain h_{ii} and that between S_i and each PU n g_{in}. The best response update of the bidding game can be implemented distributedly at each SU i based on the knowledge of h_{ii} and g_{in}, the measurement of n_0 and the SINR γ_i, as detailed in [11]. We then show that the average regret can be calculated at each SU without any other information. Noticing (15) and recall the utility function of the PU selection game in Definition 2, it suffices to show that at each iteration $t, \Gamma_i^t(a_i^t, a_{-i}^t) \triangleq \sum_{k \leq t} \widehat{S}_i(a_i^t, a_{-i}^k), \forall a_i^t \in \mathcal{N}$ can be calculated distributedly.

In fact, at each iteration k, $\widehat{S}_i$ can be calculated as

$$S_i^k = \begin{cases} U_i\left(\frac{h_{ii}p_i^{max}}{I_i^{N+1}}\right) & a_i^t = N+1 \\ U_i(\gamma_{ia_i^t}^*) - \pi_{a_i^t} g_{ia_i^t} \gamma_{ia_i^t}^* & a_i^t \neq N+1 \end{cases},$$

where $\gamma_{ia_i^t}^* = U_i'^{-1}(\pi_{a_i^t} g_{ia_i^t}), I_i^{N+1}$ is the interference experienced by SU i when choosing the free band, which can be measured locally. Γ_i^t can then be calculated by induction as

$$\Gamma_i^t = \begin{cases} U_i^t(a_i^t, a_{-i}^t) & t = 1 \\ \Gamma_i^{t-1}(a_i^t, a_{-i}^{t-1}) + U_i^t(a_i^t, a_{-i}^t) & t > 1 \end{cases}$$

Consequently, the average regret can then be calculated based on only local measurement, which leads to the entirely distributed implementation of the proposed algorithm.

For the power auction, a similar distributed algorithm based on no-regret learning can be derived with convergence to a CE.

6 Simulation analysis

In this section, we conduct simulations to evaluate the performance of the proposed auction framework and demonstrate some intrinsic properties of the proposed auction framework, especially the fairness and efficiency, which are not explicitly addressed in the analytical part of the paper. After presenting the simulation setting, we introduce a reference power allocation scheme, called NAIVE, to which our proposed auction mechanisms are compared. In the first set of simulations, we consider an illustrative scenario to compare the SINR, power auctions with the NAIVE scheme. In the second set of simulations, we focus on the power auction in realistic network configurations with and without free spectrum band.

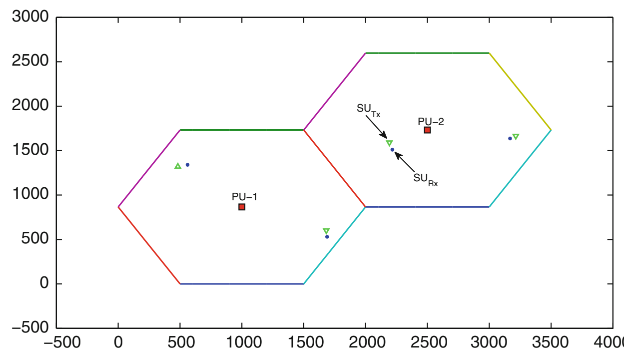

Fig. 2 Simulation setting

6.1 Simulation parameters and reference scheme

In our simulations conducted using Matlab, we consider a network of two PUs and multiple SUs (transmitter-receiver pairs). PUs can be seen as two access points or base stations covering two hexagonal cells, as shown in Fig. 2. They can accept a certain amount of interference while allowing SUs to communicate during uplink PU transmissions. The SUs are randomly distributed in the network.

In all simulations, we set $B_n = 5$ MHz and $P_n = 2n_0B_n$ $\forall$ n. We adopt a typical urban path-loss model (C2 NLOS WINNER model [18] for WiMAX) with carrier frequency $f_c = 3.5$ GHz and path-loss exponent $\alpha = 3.5$. Shadowing effect is neglected.

In order to show the performance gain brought by our solutions, we introduce a reference power allocation scheme termed NAIVE. In NAIVE, SUs choose the furthest PU based on the knowledge of channel gains g_{in}[6]. Each PU n then allocates power $p_i = P_n/(M_n g_{in})$ to SU i choosing it, where M_n is the number of SU choosing PU n. In the scenario with a free band, the SUs in the NAIVE scheme switch to the free band with certain probability p_{free} (we analyze the cases $p_{free} = 1/2$ and $p_{free} = 1/3$). This simple scheme serves as the reference scheme for performance comparison.

6.2 Illustrative example: SINR and power auctions

We start with an illustrative example to compare the SINR, power auctions and the NAIVE scheme. We consider the fixed network configuration illustrated in Fig. 2 with two PUs and four SUs with $\theta_i \in [1, 20], \forall i$. There is no free band in this example. The prices $\pi_1 = \pi_2$ are optimized by dichotomy.

[6] The rationale of the choice is that choosing the furthest PU causes the least interference at the PU.

We study the dynamics of the spectrum auction game under the best-response update. In the SINR auction, each SU chooses the PU with the minimum effective price (cf. Corollary 1) and then iteratively adjust its bid. Figure 3 (left) shows the convergence of allocated power to SUs. After about 40 iterations, convergence is reached. Compared with the SINR auction where the choice of PU is done at the very first iteration and is not modified afterwards, in the power auction, the effective price is given by Lemma 3. As one part of the effective price, I_i changes from one iteration to another depending on the strategy of other SUs s_{-i}, thus the choice of PU may also vary from one iteration to another. However, as shown in Fig. 3 (right), the final allocated power of each SU converges after about 10 iterations and the choice of PU is stablized. Compared with the SINR auction, the power auction converges in a faster but less smooth way.

In Fig. 4, we focus on the efficiency and fairness of the considered schemes by studying the average utility per SU and the Jain's fairness index [19]. The Jain's index is computed based on the normalized utility U_i/θ_i. From the results, we observe that the SINR auction slightly outperforms the NAIVE scheme in terms of average utility, with a much more significant difference in terms of fairness. The power auction, on the other hand, has a very good performance in terms of both efficiency and fairness.

6.3 Realistic experiment: power auction

We now turn to more realistic scenarios. We focus on the power auction as it achieves the best performance in the above illustrative example. The power auction is also more natural and realistic in that SUs pay for the interference they create to PUs instead of the SINR they get as in the

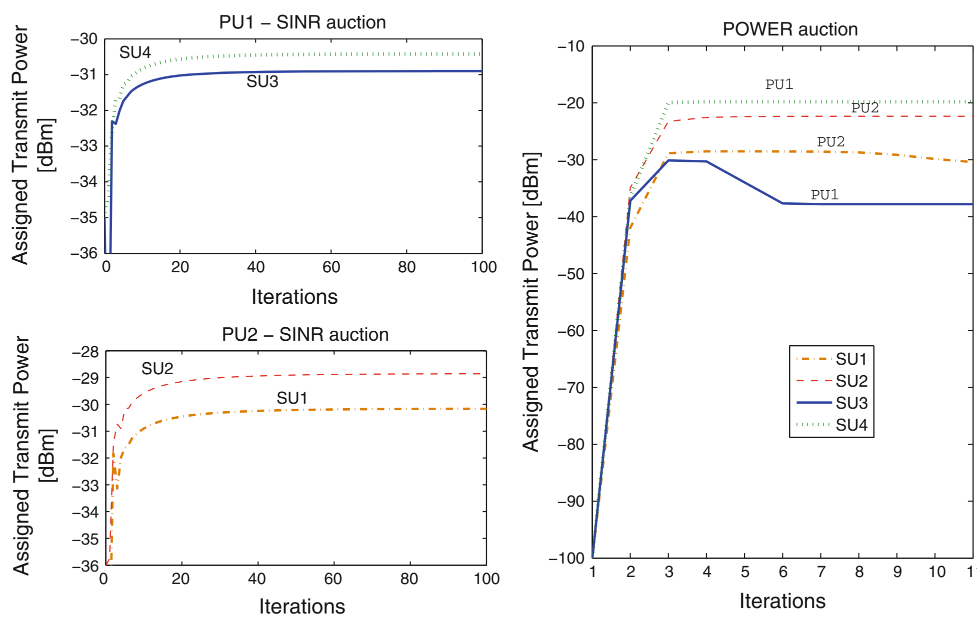

Fig. 3 Convergence of allocated power in the SINR auction (*left*) and power auction (*right*); final PU choice is shown for each SU with power auction

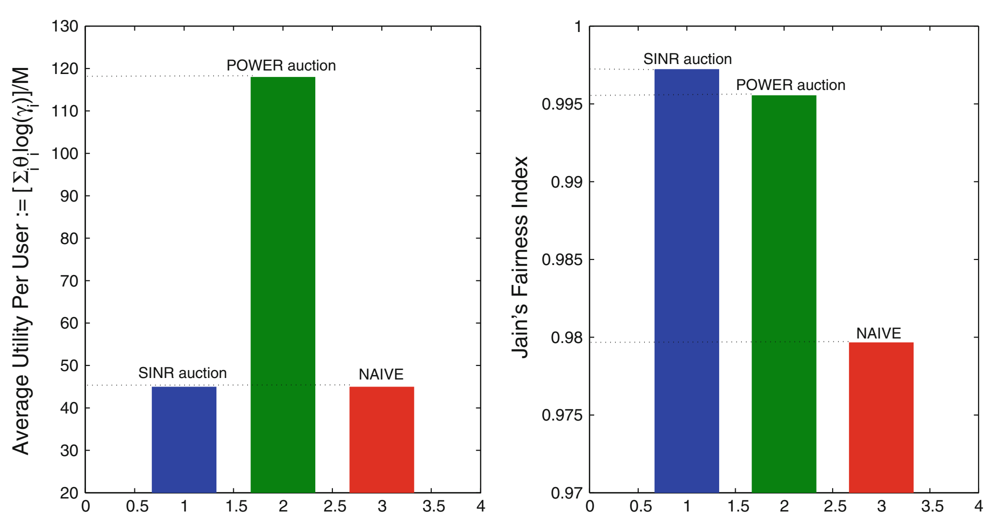

Fig. 4 Average utility per SU (*left*) and Jain's fairness index calculated based on U_i/θ_i (*right*) for SINR, power auctions and NAIVE

SINR auction. In our simulation, the transmitters of SUs are randomly located in each of the two cells. the receivers are randomly drawn in a disk with radius 100 m whose center is the corresponding transmitter. We run Monte Carlo simulations with 1,000 snapshots. At each snapshot, SU locations are randomly drawn with θ_i randomly drawn in [1, 20].

6.3.1 Convergence

As explained in Sect. 3.2, the best-response update is not guaranteed to converge. We thus study the convergence probability. We consider that the convergence is achieved if the best-response update in the power auction converges within 100 iterations, otherwise we consider that the auction does not converge. Figure 5 shows the probability of convergence as a function of the number of SU under this criterion: in the vast majority cases (more precisely, in more than 95% cases), convergence is achieved. In the subsequent simulations, in case of non-convergence, the results are based on the allocated power values after 100 iterations.

6.3.2 Load balancing

Figure 6 shows a scenario in which PU 1 fixes its price $\pi_1 = 10^{30}$ and PU 2 varies its price π_2 in the range [10^{25}, 10^{35}]. The total number of SUs M is set to 40. As shown in the figure, the number of SUs choosing PU 1 increases with π_2. The results demonstrate the benefit of the proposed power auction framework in load balancing by adjusting the prices of PUs. This feature is obviously not possible in NAIVE.

6.3.3 Efficiency and fairness

We now focus on two key performance metrics: efficiency and fairness. To this end, we compare the power auction

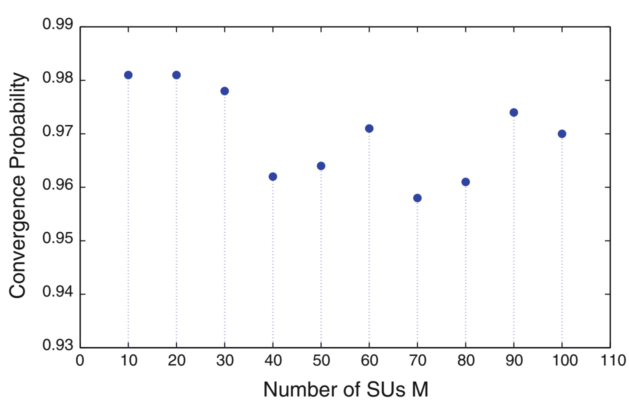

Fig. 5 Convergence probability of power auction

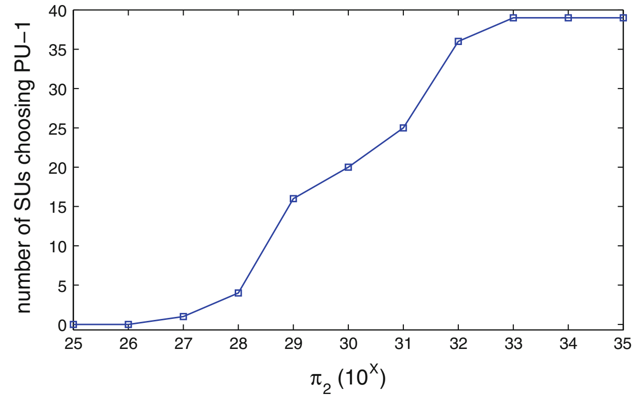

Fig. 6 Number of SUs choosing PU 1 as a function of π_2

and the NAIVE scheme in terms of average utility per SU and the Jain fairness index in two configurations. In the first configuration *M/2 system*, half of SUs are geographically located in cell 1 and the other half in cell 2. In the second configuration *M-2 system*, the number of SUs in cell 2 is constant ($M_2 = 2$), while the number of SU in cell 1 is variable in cell 1 ($M_1 = M - 2$). The two configurations represent two typical network scenarios, the balanced one with a homogeneous distribution of SUs and the unbalanced one with a heterogeneous distribution of SUs. As for the illustrative example, we set $\pi_1 = \pi_2$ and choose the price by dichotomy for the given number of SUs.

Figure 7 (left) shows that the average utility per SU is almost the same in the two configurations in the power auction (see the M/2=M-2 MultiPU Power curve in the figure) and is always higher than that in the NAIVE scheme. Figure 8 shows that the Jain fairness index (calculated in the same way as in the illustrative example) of power auction is always above that of NAIVE. In particular, in the unbalanced scenario, the power auction outperforms significantly the NAIVE scheme.

6.3.4 Power auction with a free band

We now study the power auction and the proposed no-regret learning algorithm (Sect. 5.3) by introducing a free band of 5 MHz. $p_i^{max} = 20$ dBm, $\forall i \in \mathcal{M}$. In the simulation, SUs in the NAIVE scheme choose the free band with probability $p_{free} = 1/2$ or $p_{free} = 1/3$ and emit at the maximum power p_i^{max}. The power allocation of SUs staying in licensed bands follows the same way as in the scenario without free band.

Figure 7 (right) shows the average utility of the power auction and NAIVE. As can be observed, compared with the scenario without free band, the average utility in NAIVE is slightly degraded even a new band is introduced. In contrast, the no-regret learning algorithm results a higher utility when the free band is added. Consequently, the

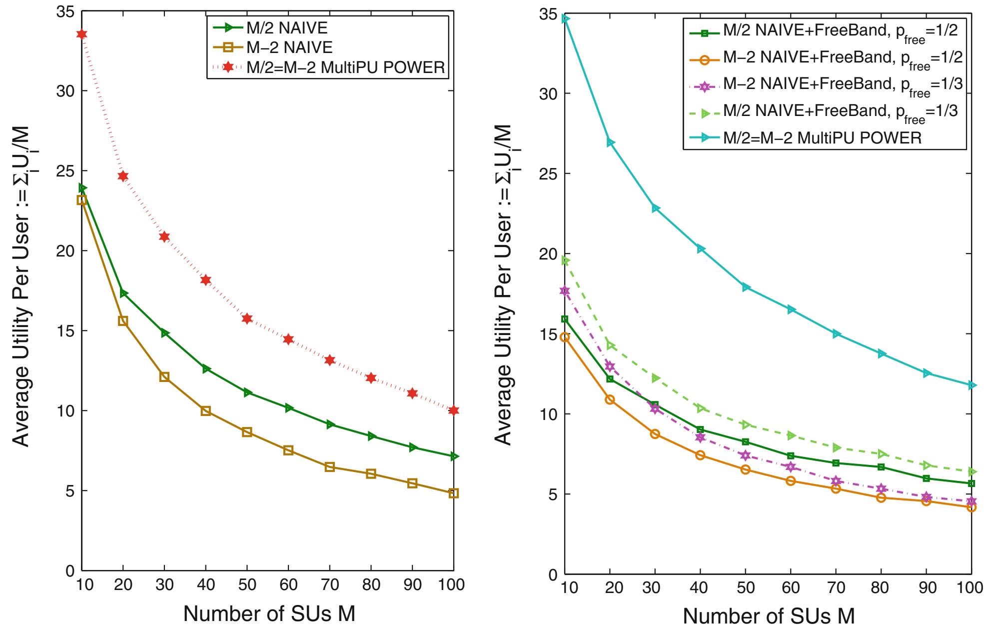

Fig. 7 Utility comparison in balanced (*M/2*) and unbalanced (*M-2*) scenarios between power auction and NAIVE with (*left*) and without (*right*) free band

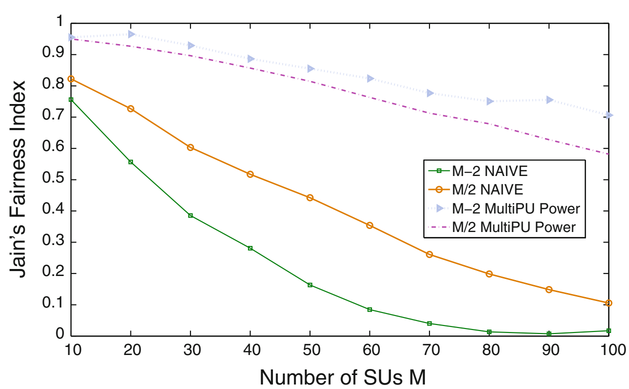

Fig. 8 Fairness comparison in balanced (*M/2*) and unbalanced (*M-2*) scenario between power auction and NAIVE

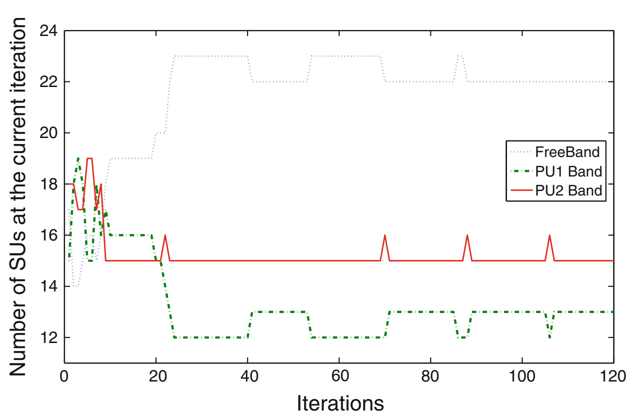

Fig. 9 Evolution of number of SUs choosing PU1, PU2 and the free band

utility gap between the power auction and NAIVE is more significant in the scenario with free band. Furthermore, we observe the convergence of the no-regret learning algorithm. Figure 9 shows the evolution of the number of SUs choosing PU1, PU2 and the free band for $M = 50$. The results demonstrate the benefit of the proposed no-regret learning algorithm to converge to an equilibrium with reasonable network efficiency in a distributed fashion.

7 Conclusion

In this paper, we proposed an auction framework for cognitive radio networks to allow unlicensed SUs to share the available spectrum of licensed PUs, subject to the interference temperature constraint at each PU. We provided an in-depth analysis on the resulting multiple-PU multiple-SU non-cooperative auction game. We then extended the proposed auction framework to the more challenging scenario with free spectrum bands by developing an algorithm based on no-regret learning to reach a CE of the auction game. The proposed algorithm, which can be implemented distributedly based on local observation, is especially suited in decentralized adaptive learning environments as cognitive radio networks. The simulation results demonstrate the effectiveness of the proposed auction framework in achieving high efficiency and fairness in spectrum allocation.

Acknowledgments This work is supported by the project TEROPP (technologies for TERminal in OPPortunistic radio applications) funded by the French National Research Agency (ANR). Part of the

work was presented at the 29th IEEE Conference on Computer Communications (IEEE Infocom 2010) [1].

References

1. Chen, L., Iellamo, S., Coupechoux, M., & Godlewski, P. (March 2010). An auction framework for spectrum allocation with interference constraint in cognitive radio networks. In *Proceedings of IEEE infocom*.
2. Haykin, S. (2005). Cognitive radio: Brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications, 23*(2), 201–220.
3. Buddhikot, M. (April 2007). Understanding dynamic spectrum access: Models, taxonomy and challenges. In *Proceedings of. IEEE DySPAN*.
4. Spectrum policy task force report. (2002). *Federal communications commission*.
5. Hart, S., & Mas-Colell, A. (2000). A simple adaptive procedure leading to correlated equilibrium. *Econometrica, 68*(5), 1127–1150
6. Aumann, R. J. (1972). Subjectivity and correlation in randomized strategy. *Journal of Mathematical Economics, 1*(1), 67–96.
7. Krishna, V. (2002). *Auction theory*. Dordrecht: Academic Press.
8. Zhou, X., & Zheng, H. (2009) Trust: A general framework for truthful double spectrum auctions. In *Proceedings of IEEE infocom*.
9. Zhu, J., & Liu, K. J. R. (2008). Multi-stage pricing game for collusion-resistant dynamic spectrum allocation. *IEEE Journal on Selected Areas in Communications, 26*(1), 182–191.
10. Kasbekar, G. S., & Sarkar, S. (2009). Spectrum auction framework for access allocation in cognitive radio networks. In *Proceedings of ACM Mobihoc*.
11. Huang, J., Berry, R., & Honig M. L. (2006). Auction-based spectrum sharing. *Mobile Networks and Applications (MONET), 11*, 405–418.
12. Gandhi, S., Buragohain, C., Cao, L., Zheng, H., & Suri. S. (April 2007). A general framework for wireless spectrum auctions. In *Proceedings of IEEE DySPAN*.
13. Wu, Y., Wang, B., Liu, K. J. R., & Clancy, T. C. (2009). A scalable collusion-resistant multi-winner cognitive spectrum auction game. *IEEE Transactions on Communications, 57*(12), 3805–3816.
14. Zhou, X., Gandhi, S., Suri, S., & Zheng, H. (2008). ebay in the sky: Strategy-proof wireless spectrum auctions. In *Proceedings of ACM MobiCom*.
15. Zhou, X., & Zheng, H. (2010) Breaking bidder collusion in large-scale spectrum auctions. In *Proceedibgs of ACM MobiHoc*.
16. Myerson, R. B. (1991). *Game theory: Analysis of conflict*. Cambridge, MA: Harvard University Press.
17. Rosen, J. B. (1965). Existence and uniqueness of equilibrium points for concave n-person games. *Econometrica, 33*(3), 520–534.
18. Srinivasan, R. (2008). *IEEE 802.16m evaluation methodology document (EMD)*. IEEE.
19. Jain, R., Chiu, D., & Hawe W. (1984). A quantitative measure of fairness and discrimination for resource allocation in shared computer systems. DEC Research Report TR-301.

Author Biographies

Lin Chen currently works as assistant professor in the department of computer science of the University of Paris-Sud XI. He received his B.E. degree in Radio Engineering from Southeast University, China in 2002 and the Engineer Diploma from Telecom ParisTech, Paris in 2005. He also holds a M.S. degree of Networking from Paris University. His main research interests include security and cooperation enforcement in wireless networks, modeling and control for wireless networks and game theory.

Stefano Iellamo works as research and development Engineer at TELECOM ParisTech. He completed his Master's degree in Telecommunications Engineering at Politecnico di Milano in 2009. In the Computer and Network Science department of ENST his work mainly focuses on Dynamic Spectrum Access for Cognitive Radio Networks.

Marceau Coupechoux is Associate Professor at Ecole Nationale Supérieure des Télécommunications in Paris (ENST, TELECOM ParisTech) since 2005. He obtained his master from ENST in 1999 and from University of Stuttgart, Germany in 2000, and his Ph.D. from Institut Eurecom, Sophia-Antipolis, France, in 2004. From 2000 to 2005, he was with Alcatel-Lucent (Research & Innovation and then in the Network Design department). In the Computer and Network Science department of ENST, he is working on cellular networks, wireless networks and ad hoc networks, focusing mainly on layer 2 protocols, scheduling and resource management.

Philippe Godlewski was born in Paris, France in November 29, 1949. He received the Engineer's degree from Télécom Paris (ENST) in 1973 and the PhD in 1976 from University Paris 6. He is professor at ENST (or presently Telecom ParisTech). His fields of interest include Cellular Networks, Air Interface Protocols, Multiple Access Techniques, Error Correcting Codes, Communication and Information Theory.