Overview

1. Network model and motivation
2. Non uniformly random scheduler
3. Data collection in Population Protocols
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1. Network model and motivation
2. Non uniformly random scheduler
3. Data collection in Population Protocols
   - Lower bounds on expected convergence time
   - Analytical results on time complexities of data collection protocols
   - Energy-efficient protocol
   - Numerical results
Population Protocols

1. Anonymous agents
2. Interaction in pairs: $p \rightarrow q$
3. Asymmetric: initiator, responder
4. Scheduler: order of interaction
Population Protocols

1. Anonymous agents
2. Interaction in pairs:
3. Asymmetric: initiator, responder
4. Scheduler: order of interaction

Examples of passively Mobile Sensor Network
- ZebraNet (wildlife tracking)
- EMMA (pollution monitoring)
Enhanced Population Protocols: Non uniformly random scheduler $S(P)$, $P \in R^{n \times n}$

- Uniform random scheduler: $P_{i,j} = 1/n(n-1)$
- Non-uniform random scheduler: general probability distribution $P_{i,j}$
- Motivation: differing mobility patterns, differing speeds
- Every agent starts with an initial value.
- Data collection is complete when the base station has all values.
- Values can be transferred from agent to agent.
Lower bounds on the expected convergence time

Theorem

The expected convergence time of any protocol solving data collection with non-uniformly random scheduler is \( \Omega(n \log n) \).

Theorem

The expected convergence time of any protocol solving data collection is \( \Omega(\max_i \frac{1}{\sum_{j=1}^n (P_{i,j} + P_{j,i})}) \).
TTF Protocol: Transfer to the Faster [Beauquier et al, PODC’10]

Transfer all values from $j$ to $i$ if $i$ is faster than $j$
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- define $x_i(t) =$ number of data held by agent $i$ at step $t$
TTF Protocol: Transfer to the Faster [Beauquier et al, PODC’10]

Transfer all values from \( j \) to \( i \) if \( i \) is faster than \( j \)

\begin{align*}
\text{faster} &= \text{smaller cover time} \ (= \text{time to meet all agents}) \\
\text{define } x_i(t) &= \text{number of data held by agent } i \text{ at step } t \\
\text{then } x(t) &= W(t) \cdot x(t-1) \text{ where }
\end{align*}

\[
W(t) = \begin{pmatrix}
1 & 0 & \ldots & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & 1 & \ddots & 0 \\
0 & 0 & \ldots & 0 & \ldots & 1
\end{pmatrix}
\]
TTF Protocol

Transfer all values from \( j \) to \( i \) if \( i \) is faster than \( j \)

\[
\text{then } x(t) = W(t) \cdots W(1) \cdot x(0)
\]

• convergence speed of matrix product \( W(t) \cdots W(1) \) depends on the second eigenvalues of the \( W(\tau) \)

Theorem

The expected convergence time of the TTF protocol is

\[
O \left( \frac{n \log n}{\log \lambda_2(\tilde{W})^{-1}} \right)
\]

where \( \tilde{W} \) is the expected value w.r.t. \( P_{i,j} \) of a matrix associated to the matrices \( W(\tau) \).
TTF Protocol

- this upper bound on the data collection time of TTF is quite loose
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During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.
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**Decide whether to execute TTF**

with probability $p_i$
During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.

Transfer if $i$ is faster than $j$
During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.
Lazy TTF ($p$)

During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.

When $p$ is a vector of all ones, lazy TTF($p$) = TTF

When $p$ is a vector of all zeros, infinite time but zero energy consumption

Energy/Time trade off
Lazy TTF (p)

During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.

Theorem

The expected convergence time of the TTF protocol is

$$O \left( \frac{n \log n}{\log \lambda_2(\tilde{W}_p)^{-1}} \right).$$
Lazy TTF (p)

During an interaction as initiator, do nothing with probability $p_i$, otherwise execute TTF.

**Theorem**

The expected convergence time of the TTF protocol is

$$O \left( \frac{n \log n}{\log \lambda_2(\tilde{W}_p)^{-1}} \right).$$

Choose $p$: optimize the upper bound on the gathering time.
\[ OP_1 : \min_{p \in \mathbb{R}^n} \lambda_2(\tilde{W}) \]

subject to \( s \).

\( Eq. \ (1) \)

\[ 0 \leq p_i \leq 1 \quad \forall i \in \{1, \ldots, n\} \]

equivalent to

\[ OP_2(\text{convex}) : \min_{p \in \mathbb{R}^n, s} \quad s \]

subject to

\[ sl - \tilde{W} \succeq 0 \]

\( Eq. \ (1) \)

\[ 0 \leq p_i \leq 1 \quad \forall i \in \{1, \ldots, n\} \]

Solving \( OP_2 \Rightarrow \hat{p} \).
For small systems, the expected convergence time $T_E(TTF)$ and $T_E(lazy\ TTF(\hat{\rho}))$ can be calculated directly via the Markov chain.

\[ \mathcal{E}: \text{Total energy consumption of a protocol} \]

\[
\mathcal{E}(TTF) = 2T_E(TTF) \cdot \mathcal{E}_{wkp} \\
\mathcal{E}(\text{lazyTTF}(\hat{\rho})) = 2T_E(\text{lazyTTF}(\hat{\rho})) \times \sum_i \sum_j (P_{i,j}\hat{p}_i + P_{j,i}\hat{p}_j) \cdot \mathcal{E}_{wkp}.
\]
For small systems, the expected convergence time $T_E(TTF)$ and $T_E(lazy\ TTF(\hat{p}))$ can be calculated directly via the Markov chain.

**$E$: Total energy consumption of a protocol**

\[
E(TTF) = 2 \cdot T_E(TTF) \cdot E_{wkp}
\]
\[
E(lazy\ TTF(\hat{p})) = 2 \cdot T_E(lazy\ TTF(\hat{p})) \times \sum_i \sum_j (P_{i,j} \hat{p}_i + P_{j,i} \hat{p}_j) \cdot E_{wkp}.
\]

Each system of size $n$, $S(n)$: 10000 schedulers randomly generated

\[
Gap(T_E, n) = \left( \sum_{s \in S(n)} \frac{T_E^s(lazy\ TTF(\hat{p}^s)) - T_E^s(TTF)}{T_E^s(TTF)} \right) / 10000
\]

and

\[
Gap(E, n) = \left( \sum_{s \in S(n)} \frac{E^s(lazy\ TTF(\hat{p}^s)) - E^s(TTF)}{E^s(TTF)} \right) / 10000.
\]
<table>
<thead>
<tr>
<th>Size n</th>
<th>$\text{Gap}(T_E, n)$</th>
<th>$\text{Gap}(E, n)$</th>
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<tbody>
<tr>
<td>4</td>
<td>11.60%</td>
<td>-15.32%</td>
</tr>
<tr>
<td>5</td>
<td>17.10%</td>
<td>-23.60%</td>
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<tr>
<td>6</td>
<td>22.04%</td>
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<tr>
<td>7</td>
<td>26.31%</td>
<td>-36.99%</td>
</tr>
<tr>
<td>8</td>
<td>27.41%</td>
<td>-39.07%</td>
</tr>
</tbody>
</table>

**Table:** Gaps on time and energy.
Conclusions:

- Initiate the study of non uniformly random scheduler in the context of population protocols
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- Give explicit lower bounds on expected convergence time of any data collection protocol
- Give analytical results for two distributed data collection protocols (a known TTF and a new parametrized energy efficient protocol)
Conclusions:

- Initiate the study of non uniformly random scheduler in the context of population protocols
- Give explicit lower bounds on expected convergence time of any data collection protocol
- Give analytical results for two distributed data collection protocols (a known TTF and a new parametrized energy efficient protocol)
- Present numerical results to show the efficiency of the new protocol
Thanks for your attention!