Mesurer la hauteur d'un arbre

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H

comme

hauteur d'un arbre

```
type tree = E | N of tree * tree
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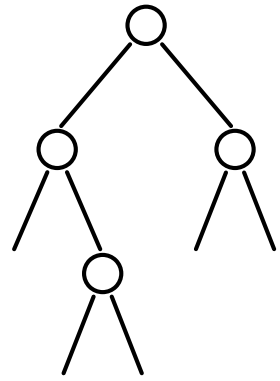

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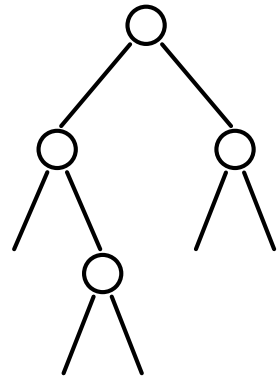
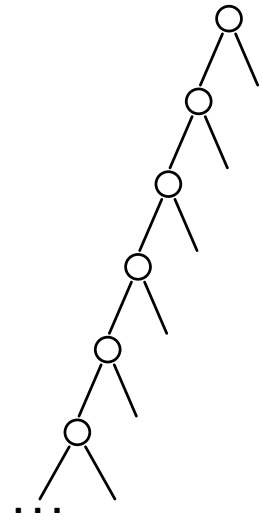


```
let rec height = function
  | E          -> 0
  | N (l, r) -> 1 + max (height l) (height r)
```



```
Stack overflow during evaluation (looping recursion?).
```

« il suffit d'utiliser une pile »

des solutions ad hoc

```
let rec bfs m next = function
  | [] when next = [] -> m
  | []                -> bfs (m+1) []           next
  | E         :: curr -> bfs m     next         curr
  | N (l, r) :: curr  -> bfs m     (l::r::next) curr
```

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```

```
let height t = bfs 0 [] [t]
```

parcours en profondeur

```
let rec dfs m = function
  | []                    -> m
  | (n, E) :: s           -> dfs (max n m) s
  | (n, N (l,r)) :: s -> dfs m ((n+1,l)::(n+1,r)::s)
```

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```

```
let height t = dfs 0 [0, t]
```

des solutions génériques

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let rec hcps t k = match t with
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```
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une « pile » sur le tas

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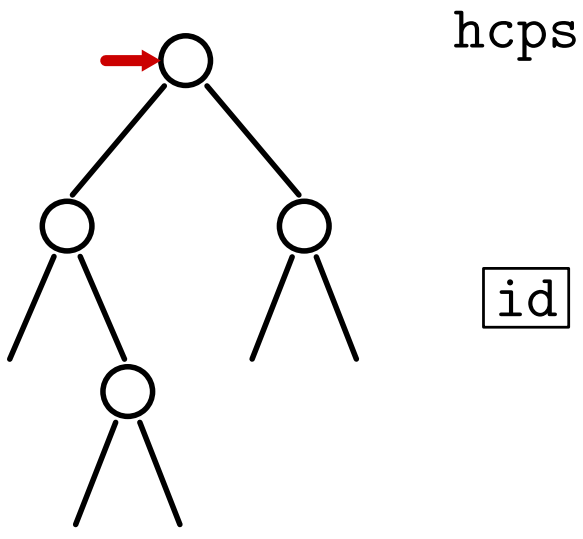

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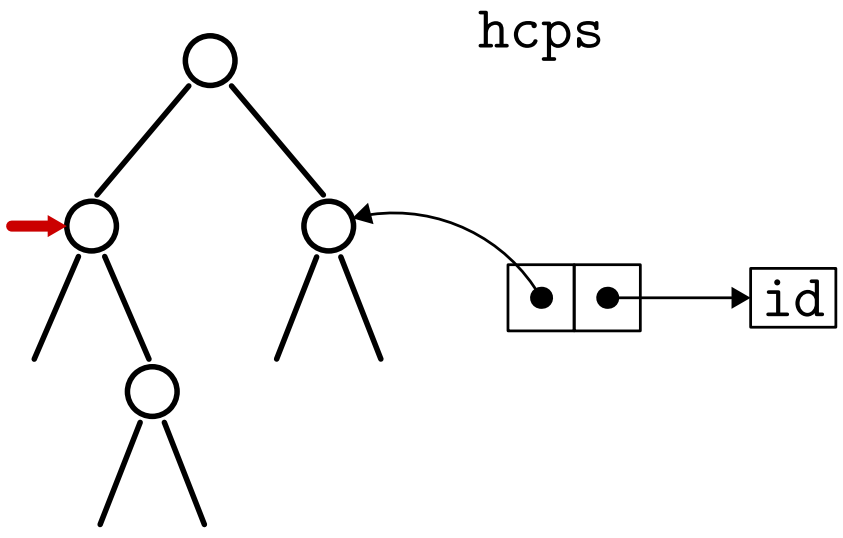

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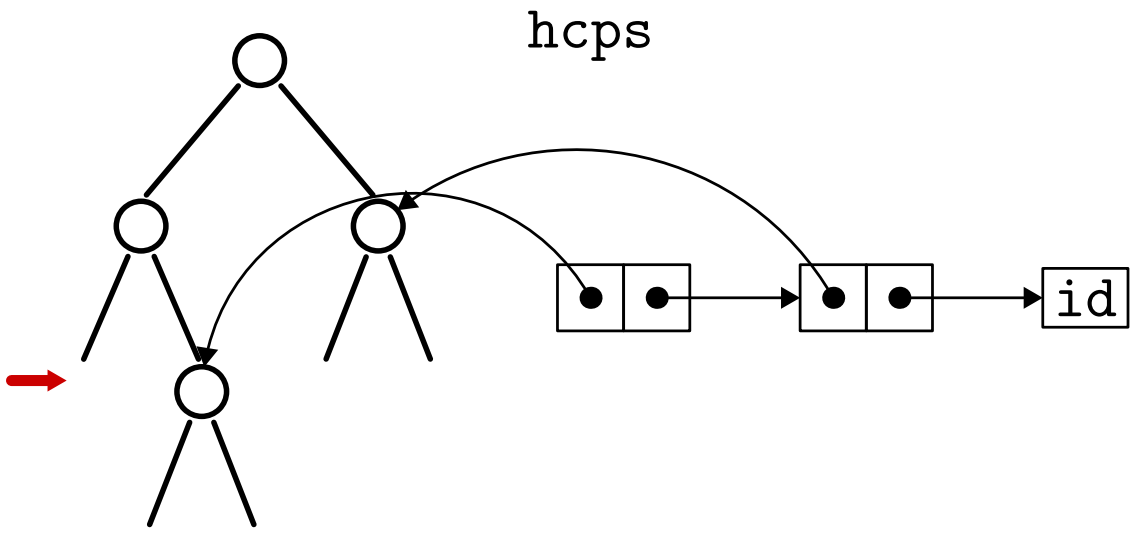

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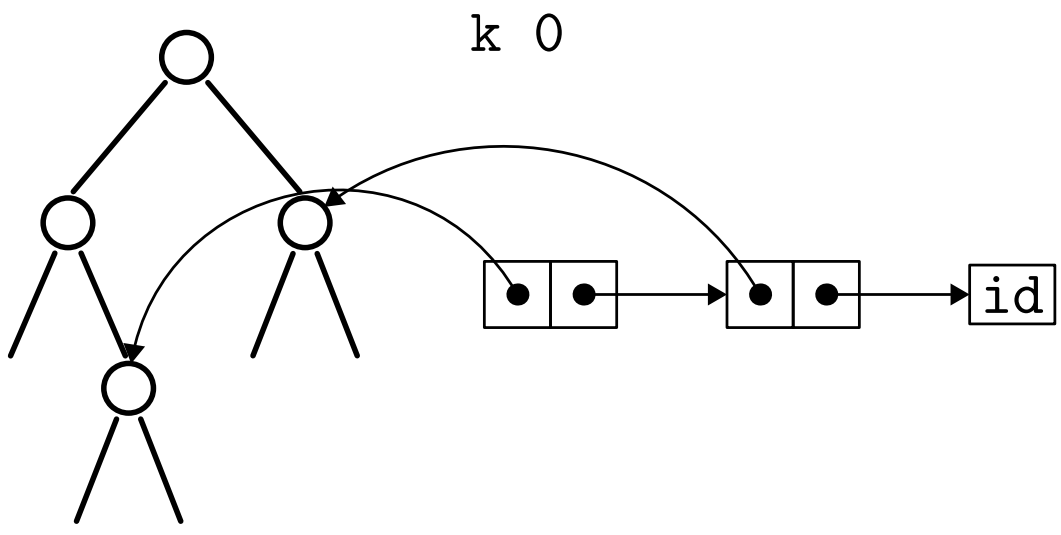

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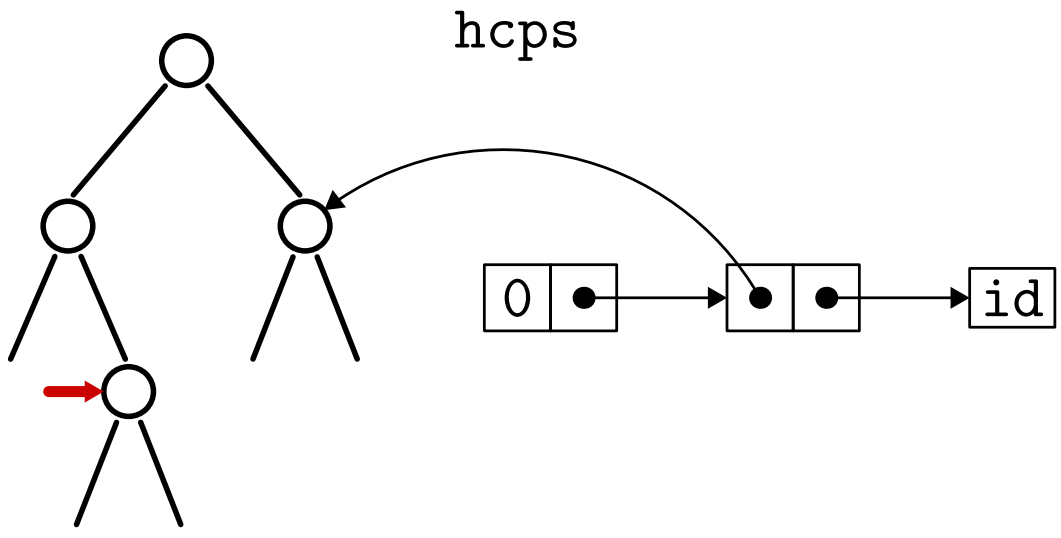

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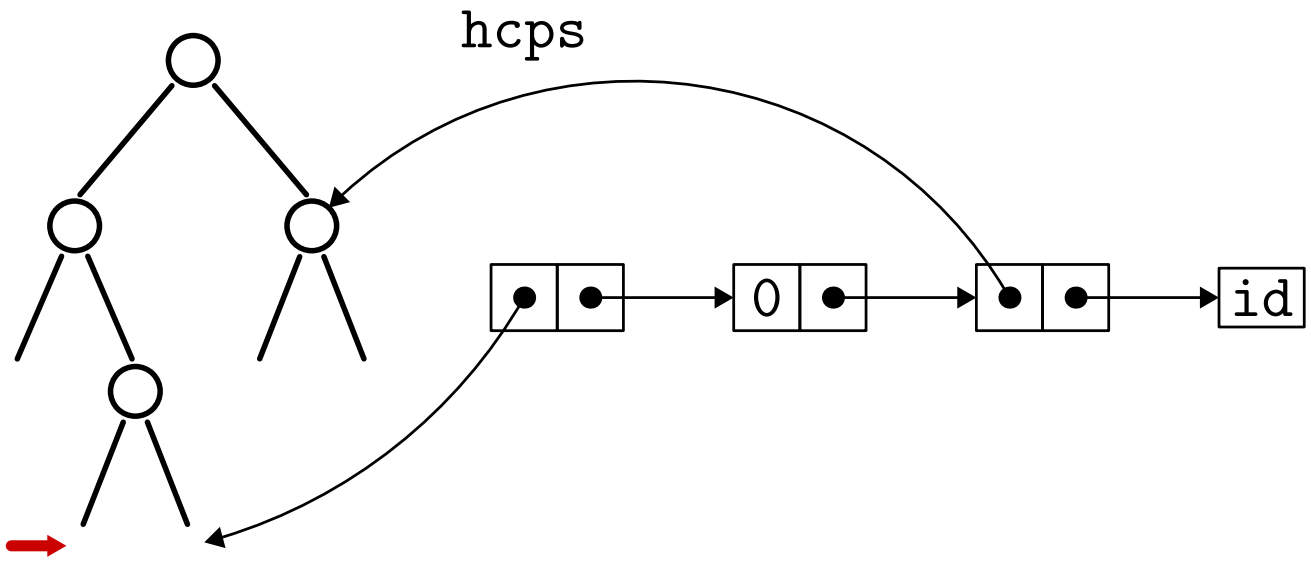

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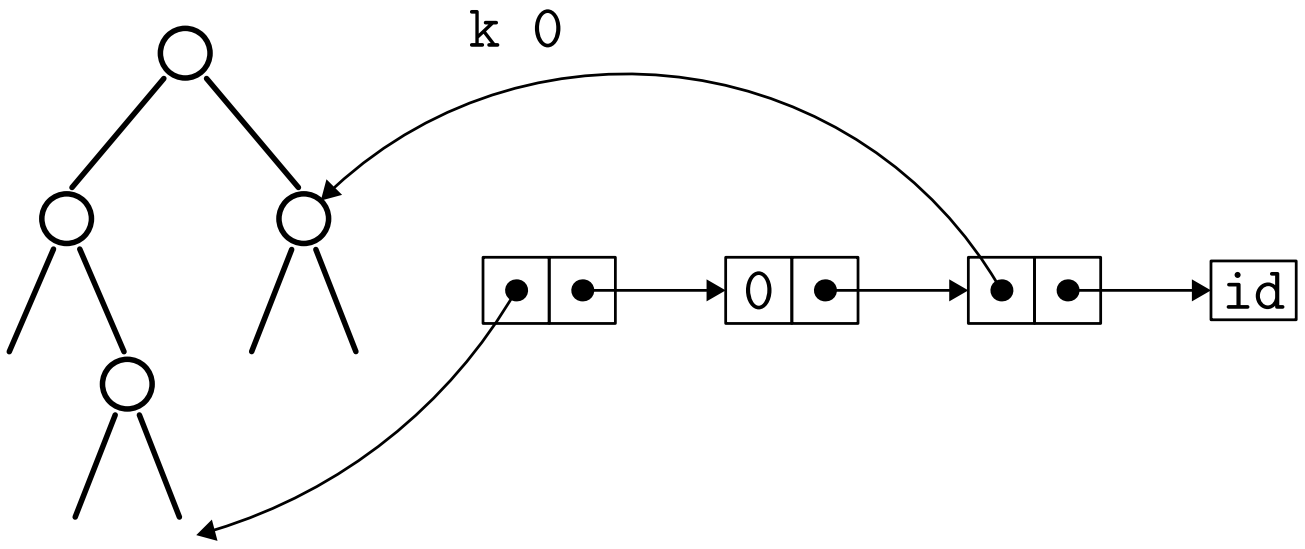

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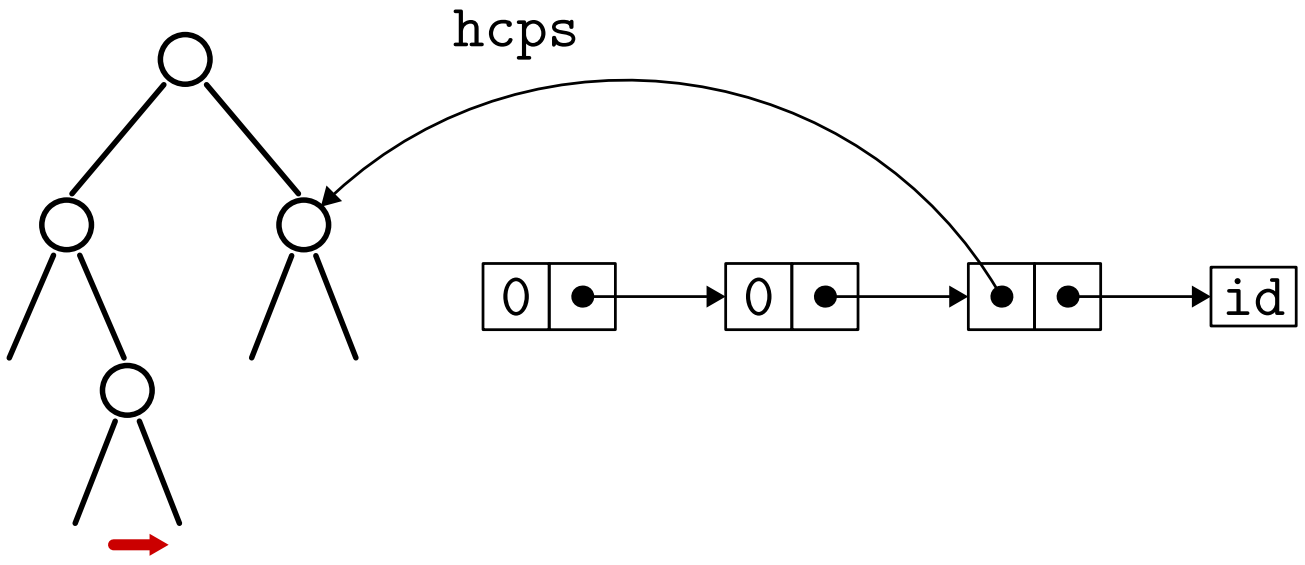

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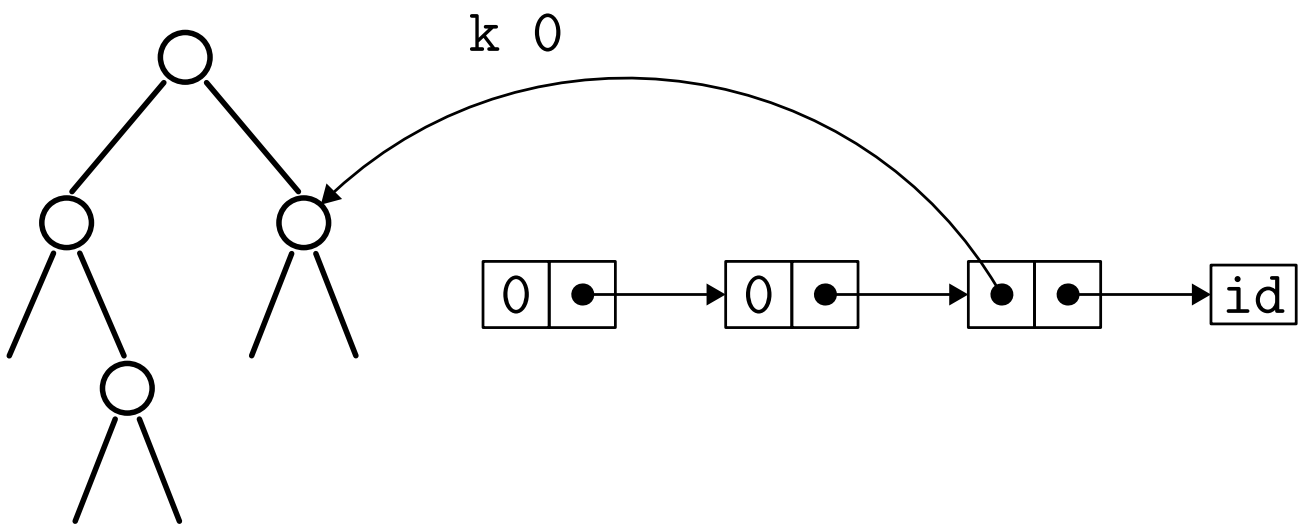

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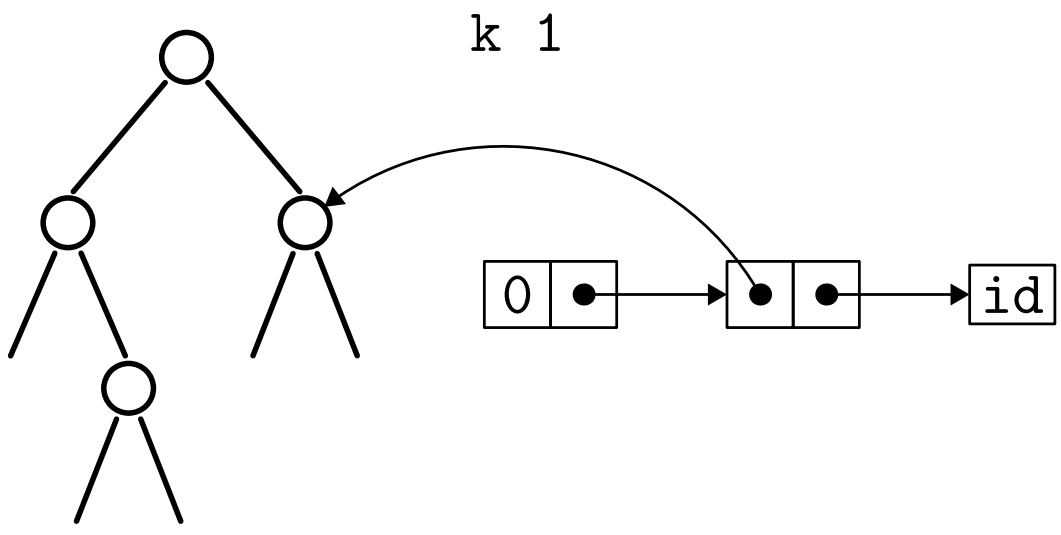

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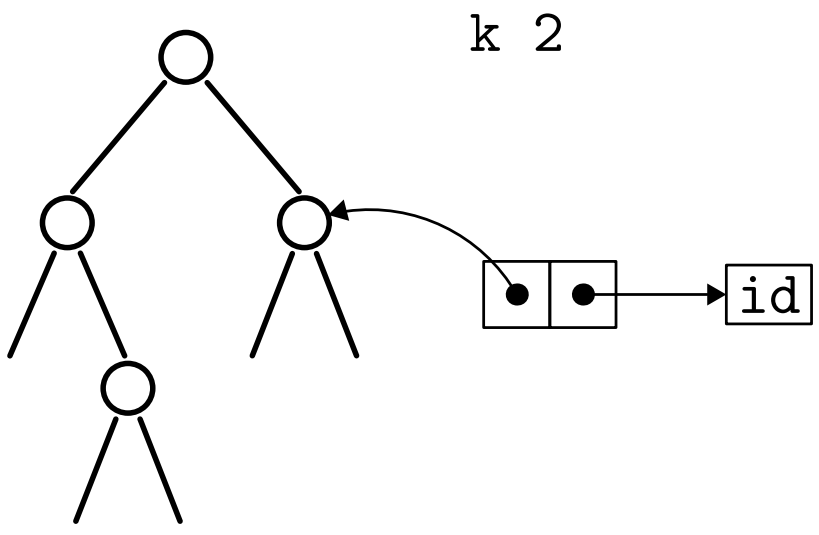

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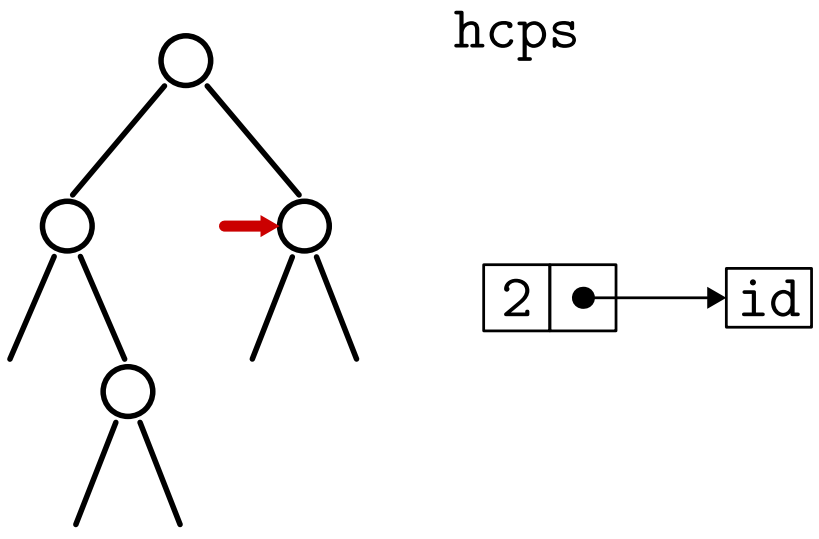

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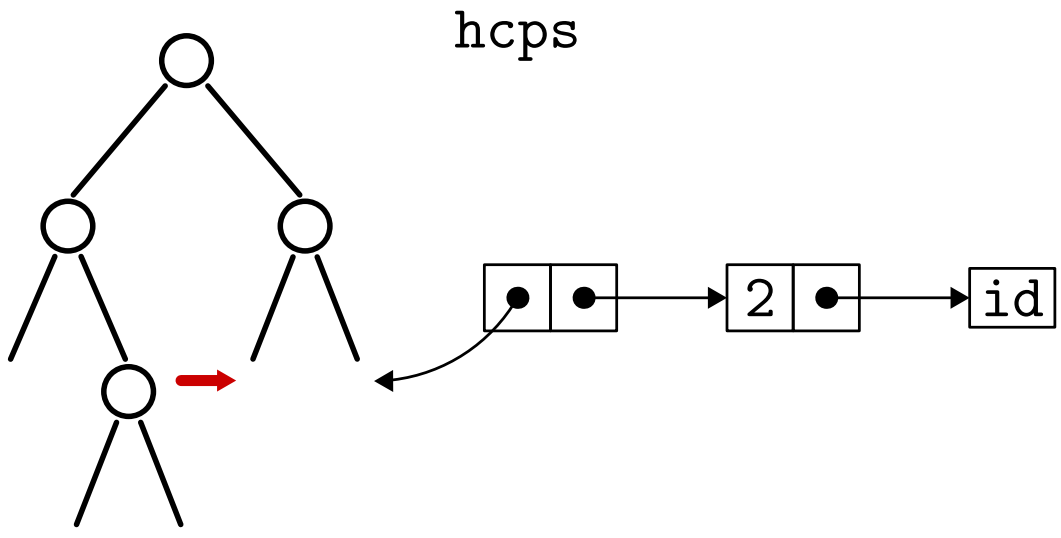

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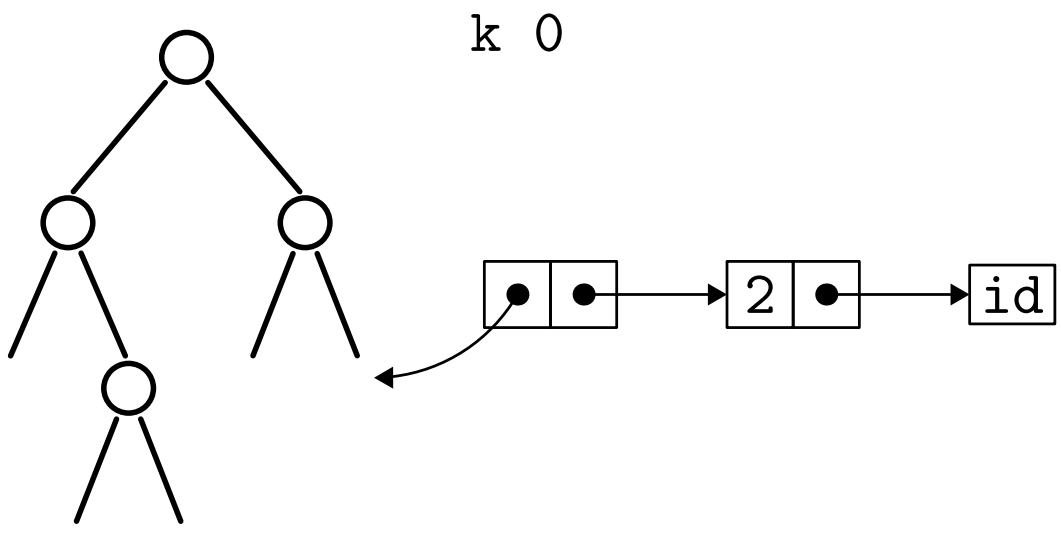

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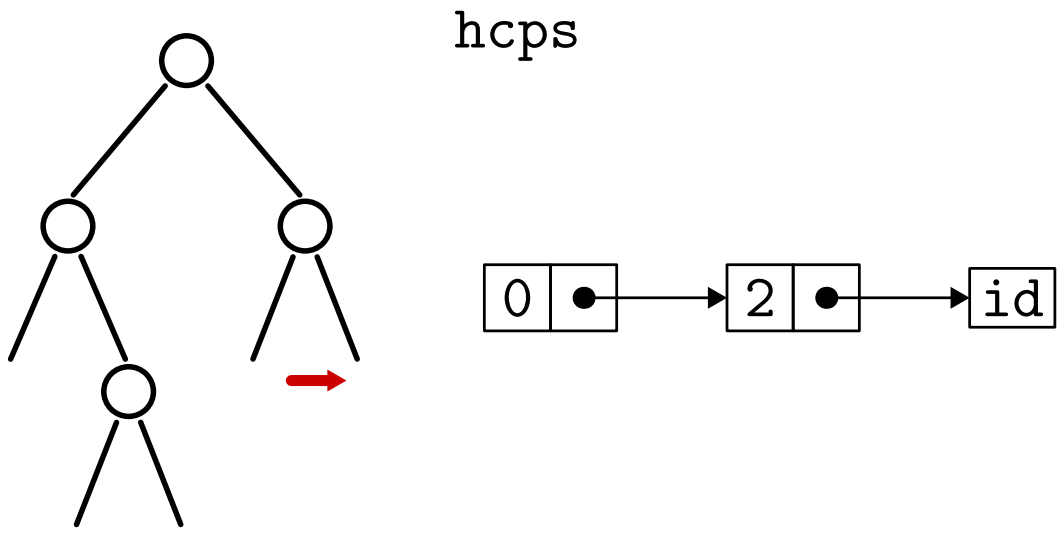

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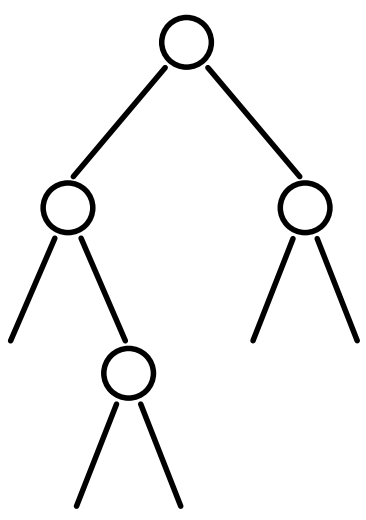
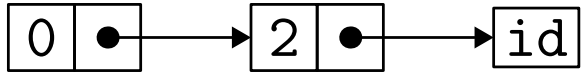


k 0



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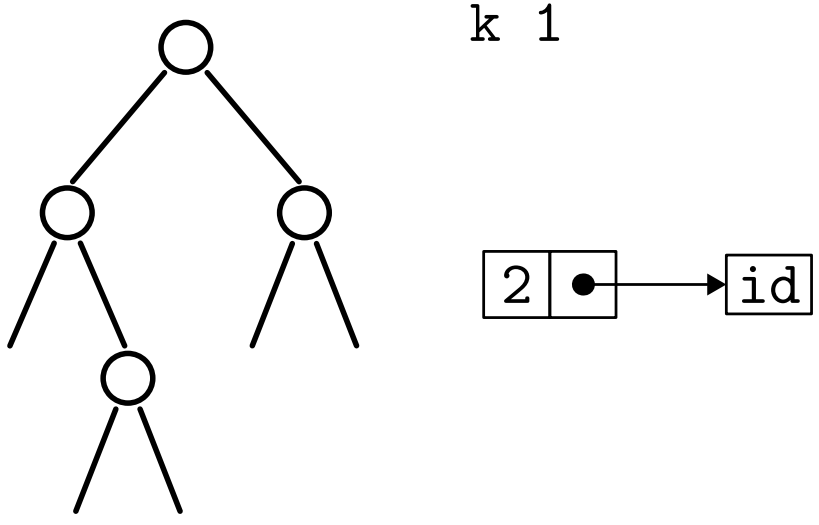

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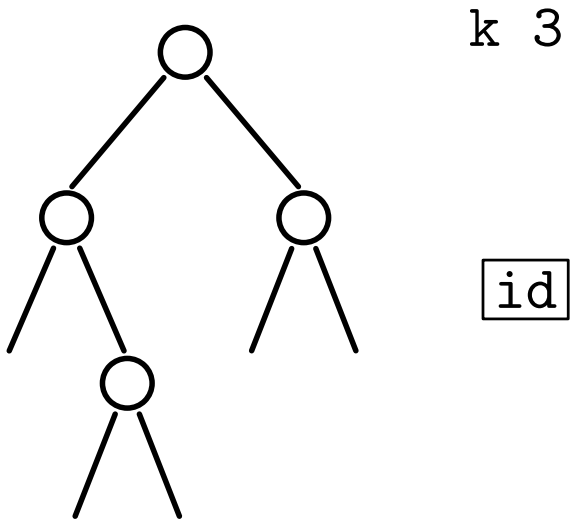

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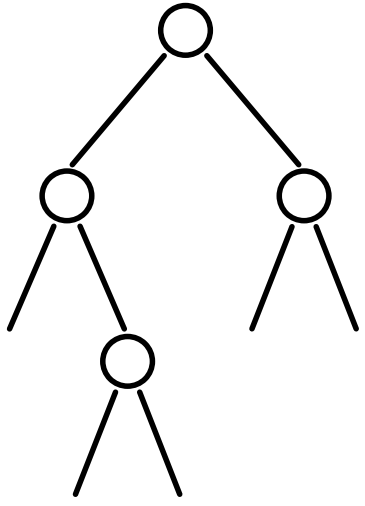

3

et dans un autre langage ?

après tout, on a des fonctions anonymes en Java, C++, etc.

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```
int hcps(tree t, function<int(int)> const&k) {
  if (t == NULL) return k(0);
  return hcps(t->left,  [t, &k](int hl) {
  return hcps(t->right, [hl,&k](int hr) {
  return k(1 + (hl > hr ? hl : hr)); }); });
}
```

malheureusement, les appels terminaux ne sont pas optimisés

défonctionnalisation (Reynolds, 1972)

```
type cont =
  | Kid
  | Kleft  of tree * cont
  | Kright of int  * cont
```

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```
type cont =
  | Kid
  | Kleft  of tree * cont
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```
let rec hcps t k = match t with
  | E         -> apply k 0
  | N (l, r) -> hcps l (Kleft (r, k))
```

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```
and apply k v = match k with
  | Kid              -> v
  | Kleft  (r,  k) -> hcps r (Kright (v, k))
  | Kright (hl, k) -> apply k (1 + max hl v)
```

```
let hdefun t = hcps t Kid
```

s'adapte plus facilement à d'autres langages

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```
struct Kont {
  enum   { Kid, Kleft, Kright } kind;
  union  { struct Node *r; int hl; };
  struct Kont *kont;
};

int hcps(struct Node *t, struct Kont *k) { ... }
int apply(struct Kont *k, int v) { ... }
```

gcc/clang optimisent ici tous les appels terminaux

encore une autre solution

parcourir l'arbre avec un zipper (Huet, 1997)

version	
`height`	1,28

version	
`bfs`	0,94
opt. `bfs`	0,58
`dfs`	2,32
opt. `dfs`	0,52

version	
`cps`	2,55
`defun`	1,80
`zipper`	3,75

(ici avec OCaml uniquement)

si l'arbre est mutable, on peut faire un parcours infixe en place :

> *Joseph M. Morris.*
> *Traversing binary trees simply and cheaply.*
> *Information Processing Letters, 9(5) :197–200, 1979.*

(modifie et restaure la structure de l'arbre)

s'adapte facilement pour calculer la hauteur

éviter le débordement de pile est rapidement compliqué

avec OCaml (ou Haskell), on est plutôt bien loti !

```
   type tree = E |
  N of tree * tree
let rec aux t k =match
t with | E -> k 0 |N(l,
 r) -> aux l(fun hl ->
aux r (fun hr ->k (1 +
   max hl hr)))let
        height
        (t)=
         aux
         (t)(
         fun
         (h)
        ->h )
```