The Why/Krakatoa/Caduceus Platform for Deductive Program Verification

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Introduction

Provers based on HOL are suitable tools to verify **purely functional** programs (see other lectures)

But how to verify an **imperative program** with your favorite prover?

for instance this one

```
t(a,b,c){int d=0,e=a&~b&~c,f=1;if(a)for(f=0;d=(e-=d)&-e;f+=t(a-d,(b+d)*2,(
c+d)/2));return f;}main(q){scanf("%d",&q);printf("%d\n",t(~(~0<<q),0,0));}
```

Usual methods

- Floyd-Hoare logic
- Dijkstra's weakest preconditions

- could be formalized in the prover (deep embedding)
- could be applied by a tactic (shallow embedding)

$\Rightarrow$ would be **specific to this prover**

Which programming language?

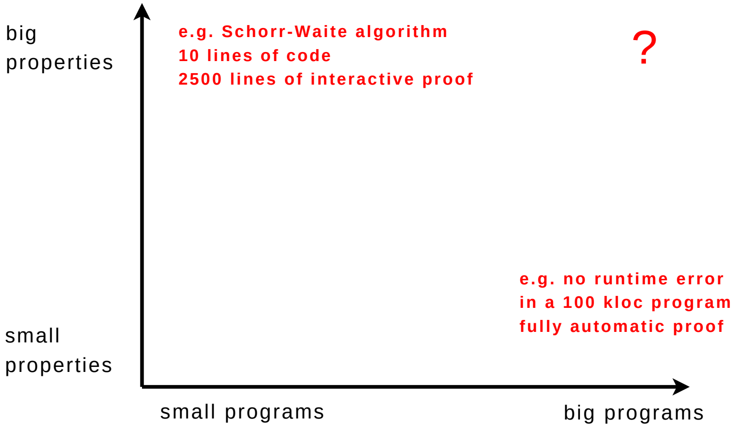
a realistic existing programming language such as C or Java?

- many constructs $\Rightarrow$ many rules
- would be **specific to this language**

The ProVal project — `http://proval.lri.fr/`

- general **goal**: prove behavioral properties of **pointer programs**
- pointer program = program manipulating data structures with **in-place mutable fields**
- we currently focus on **C** and **Java** programs

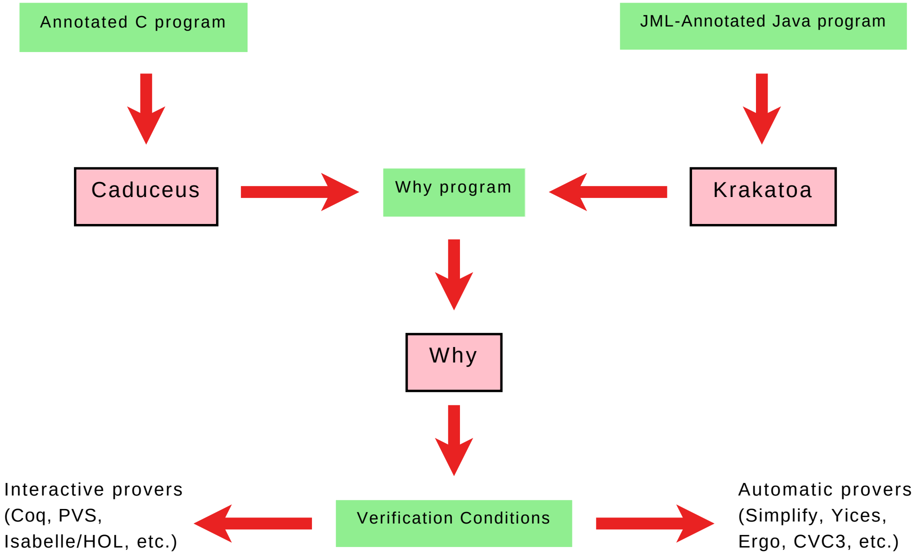
What kind of properties



Principles

- specification as **annotations** at the source code level
  - JML (Java Modeling Language) for Java
  - our own language for C (mostly JML-inspired)
- generation of **verification conditions** (VCs)
  - using Hoare logic / weakest preconditions
  - other similar approaches: static verification (ESC/Java, SPEC#), B method, etc.
- **multi-prover** approach
  - off-the-shelf provers, as many as possible
  - automatic provers (Simplify, Yices, Ergo, etc.)
  - proof assistants (Coq, PVS, Isabelle/HOL, etc.)

Platform Overview



Outline

1. An intermediate language for program verification
   1. syntax, typing, semantics, proof rules
   2. the Why tool
   3. multi-prover approach
2. Verifying C and Java programs
   1. specification languages
   2. models of program execution
3. A challenging case study

part I

An Intermediate Language for Program Verification

Basic Idea

makes program verification

- prover-independent but prover-aware
- language-independent

so that we can use it to verify C, Java, etc. programs with HOL provers but also with FO decision procedures

The essence of Hoare logic: assignment rule

$$\{ \ P[x \leftarrow E] \ \} \ x \ := \ E \ \{ \ P \ \}$$

1. absence of aliasing
2. side-effects free E **shared** between program and logic

Data types

Any purely applicative data type from the logic can be used in programs

Example: a data type `int` for integers with constants 0, 1, etc. and operations +, ∗, etc.
The pure expression 1+2 belongs to both programs and logic

A single data structure: the **reference** (mutable variable) containing **only pure values**, with no possible alias between two different references

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ML syntax

No distinction between expressions and statements
⇒ less constructs
⇒ less rules

dereference	`!`x
assignment	x `:=` e
local variable	`let` x `=` e_1 `in` e_2
local reference	`let` x `= ref` e_1 `in` e_2
conditional	`if` e_1 `then` e_2 `else` e_3
loop	`while` e_1 `do` e_2 `done`

sequence e_1`;` e_2 ≡ `let _ =` e_1 `in` e_2

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Annotations

- $\texttt{assert}\ \{p\};\ e$
- $e\ \{p\}$

Examples:

- $\texttt{assert}\ \{x > 0\};\ 1/x$
- $x\ \texttt{:=}\ 0\ \{!x = 0\}$
- $\texttt{if}\ !x > !y\ \texttt{then}\ !x\ \texttt{else}\ !y\ \{\mathit{result} \geq !x \land \mathit{result} \geq !y\}$
- $x\ \texttt{:=}\ !x + 1\ \{!x > \texttt{old}(!x)\}$

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Loop invariant and variant

- while e_1 do {invariant p variant t} e_2 done

Example:

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while !x < N do
  { invariant !x ≤ N variant N − !x }
  x := !x + 1
done
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Auxiliary variables

Used to denote the **intermediate** values of variables

Example: $\ldots\ \{!x = X\}\ \ldots\ \{!x > X\}\ \ldots$

We will use **labels** instead

- new construct L:e
- new annotation `at`(t, L)

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$$\begin{array}{l} \quad \vdots \\ L : \texttt{while}\ \ldots\ \texttt{do}\ \{\ \texttt{invariant}\ !x \geq \texttt{at}(!x, L)\ \ldots\ \} \\ \qquad \ldots \\ \quad \texttt{done} \end{array}$$

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Functions

A function declaration introduces a **precondition**

- $\texttt{fun}\ (x : \tau) \rightarrow \{p\}\ e$
- $\texttt{rec}\ f\ (x_1 : \tau_1) \ldots (x_n : \tau_n) : \beta\ \{\texttt{variant}\ t\} = \{p\}\ e$

Example:

$$\texttt{fun}\ (x : \texttt{int}\ \mathrm{ref}) \rightarrow \{!x > 0\}\ x\ \texttt{:=}\ !x - 1\ \{!x \geq 0\}$$

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Modularity

A function declaration extends the ML function type with a **precondition**, an **effect** and a **postcondition**

$$f : \; x : \tau_1 \rightarrow \{p\} \, \tau_2 \; \texttt{reads} \, x_1, \ldots, x_n \; \texttt{writes} \, y_1, \ldots, y_m \, \{q\}$$

Example:

$$swap : \; x : \texttt{int} \; \mathsf{ref} \rightarrow y : \texttt{int} \; \mathsf{ref} \rightarrow$$
$$\{\} \, \texttt{unit} \; \texttt{writes} \, x, y \, \{!x = \texttt{old}(!y) \wedge \; !y = \texttt{old}(!x)\}$$

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Exceptions

Finally, we introduce **exceptions** in our language

- a more realistic ML fragment
- to interpret abrupt statements like `return`, `break` or `continue`

new constructs

- `raise` $(E\ e) : \tau$
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The notion of postcondition is extended

$$\texttt{if } x < 0 \texttt{ then raise } \text{Negative} \texttt{ else } \mathit{sqrt}\ x$$
$$\{\ \mathit{result} \geq 0 \mid \text{Negative} \Rightarrow x < 0\ \}$$

So is the notion of effect

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Loops and exceptions

We can replace the `while` loop by an **infinite loop**

- `loop` e {`invariant` p `variant` t}

and simulate the `while` loop using an exception

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while e1 do {invariant p variant t} e2 done  ≡
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simpler constructs ⇒ simpler typing and proof rules

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Summary

Types

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\begin{array}{lcl}
\tau & ::= & \beta \mid \beta \text{ ref} \mid (x : \tau) \rightarrow \kappa \\
\kappa & ::= & \{p\}\, \tau\ \epsilon\, \{q\} \\
q & ::= & p; E \Rightarrow p; \ldots; E \Rightarrow p \\
\epsilon & ::= & \texttt{reads}\ x, \ldots, x\ \texttt{writes}\ x, \ldots, x\ \texttt{raises}\ E, \ldots, E
\end{array}
$$

Annotations

$$
\begin{array}{lcl}
t & ::= & c \mid x \mid !x \mid \phi(t, \ldots, t) \mid \texttt{old}(t) \mid \texttt{at}(t, L) \\
p & ::= & \text{True} \mid \text{False} \mid P(t, \ldots, t) \\
 & \mid & p \Rightarrow p \mid p \wedge p \mid p \vee p \mid \neg\, p \mid \forall x : \beta.p \mid \exists x : \beta.p
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Programs

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u & ::= & c \mid x \mid \texttt{!}x \mid \phi(u,\dots,u) \\
e & ::= & u \\
 & \mid & x \texttt{ := } e \\
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 & \mid & L\texttt{:}e \\
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Typing

A typing judgment

$$\Gamma \vdash e : (\tau, \epsilon)$$

Rules given in the notes (page 24)

The main purpose is to **exclude aliases**

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Semantics

Call-by-value semantics, with left to right evalutation

Big-step operational semantics given in the notes (page 26)

Proof Rules: Weakest Preconditions

We define the predicate $wp(e, q)$, called the weakest precondition for program e and postcondition q

Property: If $wp(e, q)$ holds, then e terminates and q holds at the end of execution (and all inner annotations are verified)

The converse holds for the fragment without loops and functions

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Definition of $wp(e, q)$

We actually define $wp(e, q; r)$ where

- q is the "normal" postcondition
- $r \equiv E_1 \Rightarrow q_1; \ldots; E_n \Rightarrow q_n$ is the set of "exceptional" post.

Basic constructs

$$wp(u, q; r) = q[result \leftarrow u]$$

$$wp(x \texttt{ := } e, q; r) = wp(e, q[result \leftarrow \texttt{void}; !x \leftarrow result]; r)$$

$$wp(\texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2, q; r) = wp(e_1, wp(e_2, q; r)[x \leftarrow result]; r)$$

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Basic constructs

$$wp(u, q; r) = q[result \leftarrow u]$$

$$wp(x \texttt{ := } e, q; r) = wp(e, q[result \leftarrow \texttt{void}; \texttt{!}x \leftarrow result]; r)$$

$$wp(\texttt{let } x \texttt{ = } e_1 \texttt{ in } e_2, q; r) = wp(e_1, wp(e_2, q; r)[x \leftarrow result]; r)$$

$$wp(\texttt{let } x \texttt{ = ref } e_1 \texttt{ in } e_2, q; r) = wp(e_1, wp(e_2, q; r)[\texttt{!}x \leftarrow result]; r)$$

$$\begin{array}{l} wp(\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3, q; r) = \\ \quad wp(e_1, \texttt{if } result \texttt{ then } wp(e_2, q; r) \texttt{ else } wp(e_3, q; r); r) \end{array}$$

$$wp(L\!: e, q; r) = wp(e, q; r)[\mathrm{at}(t, L) \leftarrow t]$$

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$$wp(e_1, \texttt{if } result \texttt{ then } wp(e_2, q; r) \texttt{ else } wp(e_3, q; r); r)$$

$$wp(L\texttt{:}e, q; r) = wp(e, q; r)[\texttt{at}(t, L) \leftarrow t]$$

Traditional rules

Assignment of a side-effects free expression

$$wp(x \texttt{ := } u, q) = q[\texttt{!}x \leftarrow u]$$

Exception-free sequence

$$wp(e_1\texttt{ ; }e_2, q) = wp(e_1, wp(e_2, q))$$

Traditional rules

Assignment of a side-effects free expression

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Exception-free sequence

$$wp(e_1 \texttt{;}\ e_2, q) = wp(e_1, wp(e_2, q))$$

Exceptions

$$wp(\texttt{raise}\ (E\ e) : \tau, q; r) = wp(e, r_E; r)$$

$$wp(\texttt{try}\ e_1\ \texttt{with}\ E\ x \rightarrow e_2\ \texttt{end}, q; r) = \\ wp(e_1, q; E \Rightarrow wp(e_2, q; r)[x \leftarrow result]; r)$$

Exceptions

$$wp(\texttt{raise}\ (E\ e) : \tau, q; r) = wp(e, r_E; r)$$

$$\begin{array}{l} wp(\texttt{try}\ e_1\ \texttt{with}\ E\ x \rightarrow e_2\ \texttt{end}, q; r) = \\ \quad wp(e_1, q; E \Rightarrow wp(e_2, q; r)[x \leftarrow \mathit{result}]; r) \end{array}$$

Annotations

$$wp(\texttt{assert } \{p\};\ e, q; r) = p \wedge wp(e, q; r)$$

$$wp(e\ \{q'; r'\}, q; r) = wp(e, q' \wedge q; r' \wedge r)$$

Annotations

$$
\begin{aligned}
wp(\texttt{assert}\ \{p\};\ e, q; r) &= p \wedge wp(e, q; r) \\
wp(e\ \{q'; r'\}, q; r) &= wp(e, q' \wedge q; r' \wedge r)
\end{aligned}
$$

Loops

$$wp(\texttt{loop}\ e\ \{\texttt{invariant}\ p\ \texttt{variant}\ t\}, q; r) = \\ p\ \wedge\ \forall\omega.\ p \Rightarrow wp(L\!:e, p \wedge t < \texttt{at}(t, L); r)$$

where ω = the variables (possibly) modified by e

Usual while loop

$wp(\texttt{while}\ e_1\ \texttt{do}\ \{\texttt{invariant}\ p\ \texttt{variant}\ t\}\ e_2\ \texttt{done}, q; r)$
$= p\ \wedge\ \forall\omega.\ p \Rightarrow$
$wp(L\!:\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ \texttt{raise}\ E, p \wedge t < \texttt{at}(t, L), E \Rightarrow q; r)$
$= p\ \wedge\ \forall\omega.\ p \Rightarrow$
$wp(e_1, \texttt{if}\ \mathit{result}\ \texttt{then}\ wp(e_2, p \wedge t < \texttt{at}(t, L))\ \texttt{else}\ q, r)[\texttt{at}(x, L) \leftarrow x]$

Loops

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$$wp(\texttt{loop}\ e\ \{\texttt{invariant}\ p\ \texttt{variant}\ t\}, q; r) = \\ p\ \wedge\ \forall\omega.\ p \Rightarrow wp(L\!:e, p \wedge t < \texttt{at}(t, L); r)$$

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Loops

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Usual while loop

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$$wp(e_1, \texttt{if}\ \mathit{result}\ \texttt{then}\ wp(e_2, p \wedge t < \texttt{at}(t, L))\ \texttt{else}\ q, r)[\texttt{at}(x, L) \leftarrow x]$$

Functions

$$wp(\texttt{fun}\ (x:\tau) \rightarrow \{p\}\ e, q; r) \quad = q\ \wedge\ \forall x.\forall \rho. p \Rightarrow wp(e, \mathsf{True})$$

$$wp(\texttt{rec}\ f\ (x_1:\tau_1)\dots(x_n:\tau_n):\tau\ \{\texttt{variant}\ t\} = \{p\}\ e, q; r)$$
$$= q\ \wedge\ \forall x_1.\dots.\forall x_n.\forall \rho. p \Rightarrow wp(L{:}e, \mathsf{True})$$

when computing $wp(L{:}e, \mathsf{True})$, f is assumed to have type

$$(x_1:\tau_1) \rightarrow \cdots \rightarrow (x_n:\tau_n) \rightarrow \{p \wedge t < \mathtt{at}(t, L)\}\ \tau\ \epsilon\ \{q\}$$

Functions

$$wp(\texttt{fun}\ (x:\tau) \rightarrow \{p\}\ e, q; r) \quad = q\ \wedge\ \forall x.\forall \rho.p \Rightarrow wp(e, \mathrm{True})$$

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$$(x_1:\tau_1) \rightarrow \cdots \rightarrow (x_n:\tau_n) \rightarrow \{p \wedge t < \texttt{at}(t, L)\}\, \tau\ \epsilon\, \{q\}$$

Function call

Simplified using

$$e_1\ e_2 \equiv \texttt{let}\ x_1\ \texttt{=}\ e_1\ \texttt{in}\ \texttt{let}\ x_2\ \texttt{=}\ e_2\ \texttt{in}\ x_1\ x_2$$

Assuming

$$x_1 \ :\ (x : \tau) \rightarrow \{p'\}\, \tau'\ \epsilon\, \{q'\}$$

we define

$$wp(x_1\ x_2, q) = p'[x \leftarrow x_2]\ \wedge\ \forall \omega.\forall result.(q'[x \leftarrow x_2] \Rightarrow q)[\texttt{old}(t) \leftarrow t]$$

Function call

Simplified using

$$e_1\ e_2 \equiv \texttt{let}\ x_1\ \texttt{=}\ e_1\ \texttt{in}\ \texttt{let}\ x_2\ \texttt{=}\ e_2\ \texttt{in}\ x_1\ x_2$$

Assuming

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we define

$$wp(x_1\ x_2, q) = p'[x \leftarrow x_2]\ \wedge\ \forall \omega. \forall result.(q'[x \leftarrow x_2] \Rightarrow q)[\texttt{old}(t) \leftarrow t]$$

Outline

1. An intermediate language for program verification
   1. syntax, typing, semantics, proof rules
   2. **the Why tool**
   3. multi-prover approach
2. Verifying C and Java programs
   1. specification languages
   2. models of program execution
3. A challenging case study

The Why Tool

This intermediate language is implemented in the Why tool

input = polymorphic first-order logic declarations + programs

output = logical declarations + goals, in the syntax of the selected prover

Logical Declarations

```
type t

logic zero : t

logic succ : t -> t

logic le : t, t -> prop

axiom a : forall x:t. le(zero,x)

goal g : le(zero, succ(zero))
```

Programs

```
parameter x : int ref

parameter g :
  b:t -> { x>=0 } t writes x { result=succ(b) and x=x@+1 }

let h (a:int) (b:t) =
  { x>=0 }
    if !x = a then x := 0;
    g (succ b)
  { result=succ(succ(b)) }

exception E
exception F of int
```

Usage

it is a compiler:

- `why --coq f.why` to produce a re-editable Coq file `f_why.v`
- `why --simplify f.why` to produce a Simplify script `f_why.sx`
- etc.

the following provers/formats are supported:

- Coq, PVS, Isabelle/HOL, HOL-light, HOL4, Mizar
- Simplify, Ergo, SMT (Yices, CVC3, etc.), CVC-Lite, haRVey, Zenon

there is a graphical user interface, `gwhy`

Example: Dijkstra's Dutch national flag

Goal: to sort an array where elements only have three different values (blue, white and red)

Algorithm

0	b	i	r	n
BLUE	WHITE	...to do...	RED	

$flag(t,\ n)\ \equiv$

$b \leftarrow 0$

$i \leftarrow 0$

$r \leftarrow n$

while $i < r$

case $t[i]$

BLUE : *swap* $t[b]$ *and* $t[i]$; $b \leftarrow b + 1$; $i \leftarrow i + 1$

WHITE : $i \leftarrow i + 1$

RED : $r \leftarrow r - 1$; *swap* $t[r]$ *and* $t[i]$

Correctness proof

we want to prove

- **termination**
- **absence of runtime error** = no array access out of bounds
- **behavioral correctness** = the final array is sorted **and** contains the same elements as the initial array

Modelization

We model

- colors using an **abstract datatype**
- arrays using references containing **functional arrays**

An abstract type for colors

```
type color

logic blue : color
logic white : color
logic red : color

predicate is_color(c:color) = c=blue or c=white or c=red

parameter eq_color  :
  c1:color -> c2:color ->
    {} bool { if result then c1=c2 else c1<>c2 }
```

Functional arrays

```
type color_array

logic acc : color_array, int -> color
logic upd : color_array, int, color -> color_array

axiom acc_upd_eq :
  forall a:color_array. forall i:int. forall c:color.
    acc(upd(a,i,c),i) = c

axiom acc_upd_neq :
  forall a:color_array. forall i,j:int. forall c:color.
    i <> j -> acc(upd(a,j,c),i) = acc(a,i)
```

Array bounds

```
logic length : color_array -> int

axiom length_update :
  forall a:color_array. forall i:int. forall c:color.
    length(upd(a,i,c)) = length(a)

parameter get :
  t:color_array ref -> i:int ->
    { 0<=i<length(t) } color reads t { result=acc(t,i) }

parameter set :
  t:color_array ref -> i:int -> c:color ->
    { 0<=i<length(t) } unit writes t { t=upd(t@,i,c) }
```

Array bounds

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logic length : color_array -> int

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  t:color_array ref -> i:int -> c:color ->
    { 0<=i<length(t) } unit writes t { t=upd(t@,i,c) }
```

The swap function

```
let swap (t:color_array ref) (i:int) (j:int) =
  { 0 <= i < length(t) and 0 <= j < length(t) }
  let u = get t i in
  set t i (get t j);
  set t j u
  { t = upd(upd(t@,i,acc(t@,j)), j, acc(t@,i)) }
```

5 proofs obligations

- 3 automatically discharged by Why
- 2 left to the user (and automatically discharged by Simplify)

The swap function

```
let swap (t:color_array ref) (i:int) (j:int) =
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```

5 proofs obligations

- 3 automatically discharged by Why
- 2 left to the user (and automatically discharged by Simplify)

Function code

```
let dutch_flag (t:color_array ref) (n:int) =
  let b = ref 0 in
  let i = ref 0 in
  let r = ref n in
  while !i < !r do
     if eq_color (get t !i) blue then begin
       swap t !b !i;
       b := !b + 1;
       i := !i + 1
     end else if eq_color (get t !i) white then
       i := !i + 1
     else begin
       r := !r - 1;
       swap t !r !i
     end
  done
```

Function specification

```
let dutch_flag (t:color_array ref) (n:int) =
  { 0 <= n and length(t) = n and
    forall k:int. 0 <= k < n -> is_color(acc(t,k)) }
  ⋮
  { (exists b:int. exists r:int.
      monochrome(t,0,b,blue) and
      monochrome(t,b,r,white) and
      monochrome(t,r,n,red))
    and permutation(t,t@,0,n-1) }
```

The monochrome property

```
predicate monochrome(t:color_array,i:int,j:int,c:color) =
  forall k:int. i<=k<j -> acc(t,k)=c
```

The permutation property

```
logic permutation : color_array, color_array, int, int -> prop

axiom permut_refl : forall t:color_array. forall l,r:int.
  permutation(t,t,l,r)

axiom permut_sym : forall t1,t2:color_array. forall l,r:int.
  permutation(t1,t2,l,r) -> permutation(t2,t1,l,r)

axiom permut_trans : forall t1,t2,t3:color_array. forall l,r:i
  permutation(t1,t2,l,r) -> permutation(t2,t3,l,r) ->
  permutation(t1,t3,l,r)

axiom permut_swap : forall t:color_array. forall l,r,i,j:int.
  l <= i <= r -> l <= j <= r ->
  permutation(t, upd(upd(t,i,acc(t,j)), j, acc(t,i)), l, r)
```

The permutation property

```
logic permutation : color_array, color_array, int, int -> prop

axiom permut_refl : forall t:color_array. forall l,r:int.
  permutation(t,t,l,r)

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axiom permut_trans : forall t1,t2,t3:color_array. forall l,r:i
  permutation(t1,t2,l,r) -> permutation(t2,t3,l,r) ->
  permutation(t1,t3,l,r)

axiom permut_swap : forall t:color_array. forall l,r,i,j:int.
  l <= i <= r -> l <= j <= r ->
  permutation(t, upd(upd(t,i,acc(t,j)), j, acc(t,i)), l, r)
```

Loop invariant

```
⋮
init:
while !i < !r do
   { invariant
       0 <= b <= i and i <= r <= n and
       monochrome(t,0,b,blue) and
       monochrome(t,b,i,white) and
       monochrome(t,r,n,red) and
       length(t) = n and
       (forall k:int. 0 <= k < n -> is_color(acc(t,k))) and
       permutation(t,t@init,0,n-1)
     variant
       r - i }
   ⋮
done
```

Proof obligations

11 proof obligations

- loop invariant holds initially
- loop invariant is preserved and variant decreases (3 cases)
- `swap` precondition (twice)
- array access within bounds (twice)
- postcondition holds at the end of function execution

All automatically discharged by Simplify!

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   1. specification languages
   2. models of program execution
3. A challenging case study

Discharging the Verification Conditions

we want to use **off-the-shelf provers**, as many as possible

requirements

- first-order logic
- equality and arithmetic
- quantifiers (memory model, user algebraic models)

Provers Currently Supported

automatic decision procedures

- provers *a la* Nelson-Oppen
  - Simplify, Yices, Ergo
  - CVC Lite, CVC3
- resolution-based provers
  - haRVey, rv-sat
- tableaux-based provers
  - Zenon

interactive proof assistants

- Coq, PVS, Isabelle/HOL, HOL4, HOL-light, Mizar

Typing Issues

verification conditions are expressed in polymorphic first-order logic

need to be **translated** to logics with various type systems:

- unsorted logic (Simplify, Zenon)
- simply sorted logic (SMT provers)
- parametric polymorphism (CVC Lite, PVS)
- polymorphic logic (Ergo, Coq, Isabelle/HOL)

Typing Issues

erasing types is unsound

```
type color

logic white,black : color

axiom color: forall c:color. c=white or c=black
```

$$\forall c,\ c = \texttt{white} \lor c = \texttt{black} \ \vdash\ \bot$$

Type Encoding

several type encodings are used

- monomorphization
  - each polymorphic symbol is replace by several monomorphic types
  - may loop
- usual encoding "types-as-predicates"
  - $\forall x, \mathtt{nat}(x) \Rightarrow P(x)$
  - does not combine nicely with most provers
- new encoding with **type-decorated terms**
  Handling Polymorphism in Automated Deduction (CADE 21)

Trust in Prover Results

- some provers apply the de Bruijn principle and thus are **safe**
  - Coq, HOL family
- most provers **have to be trusted**
  - Simplify, Yices
  - PVS, Mizar
- some provers output **proof traces**
  - Ergo, CVC family, Zenon

Provers Collaboration

most of the time, we run the various provers **in parallel**, expecting at least one of them to discharge the VC

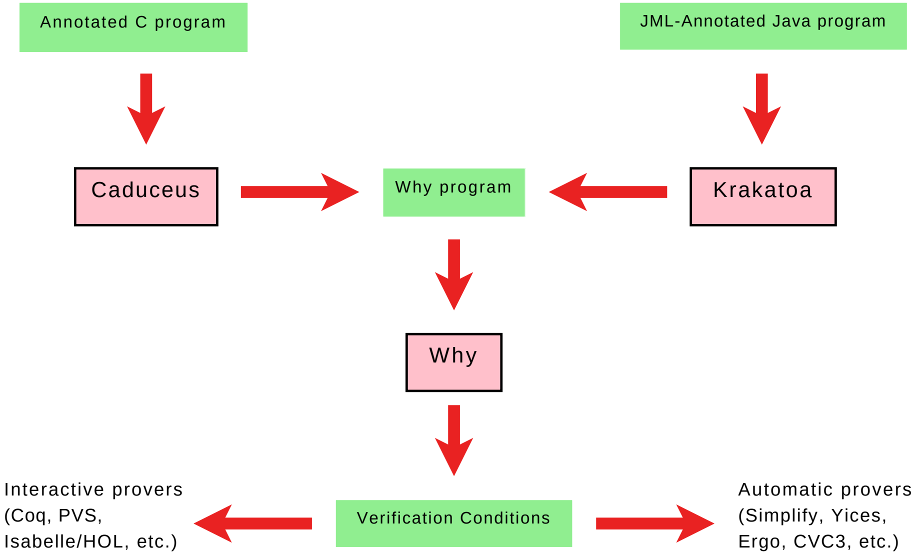
if not, we turn to interactive theorem provers

- no real collaboration between automatic provers
- from Coq or Isabelle, one can call automatic theorem provers
  - proofs are checked when available
  - results are trusted otherwise

part II

Verifying C and Java Programs

Platform Overview



Outline

1. An intermediate language for program verification
   1. syntax, typing, semantics, proof rules
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   3. multi-prover approach
2. Verifying C and Java programs
   1. **specification languages**
      - how to formally specify behaviors
   2. models of program execution
3. A challenging case study

Which language to specify behaviors?

Java already has a specification language: **JML** (Java Modeling Language) used in runtime assertion checking tools, ESC/Java, JACK, LOOP, CHASE

JML allows to specify

- precondition, postcondition and side-effects for methods
- invariant and variant for loops
- class invariants
- model fields ($\sim$ ghost code)

Which language to specify behaviors?

we designed a similar language for **C programs**, largely inspired by JML

additional features:

- pointer arithmetic
- algebraic models
  - any axiomatized theory can be used in specifications
  - no runtime assertion checking
- floating-point arithmetic
  - round errors can be specified

A First Example: Binary Search

binary search: search a sorted array of integers for a given value

famous example; see J. Bentley's *Programming Pearls*:
most programmers are wrong on their first attempt to write binary search

Binary Search (code)

```
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1, p = -1;
  while (l <= u ) {
    int m = (l + u) / 2;
    if (t[m] < v)
      l = m + 1;
    else if (t[m] > v)
      u = m - 1;
    else {
      p = m; break;
    }
  }
  return p;
}
```

Binary Search (spec)

we want to prove:

1. absence of runtime error
2. termination
3. behavioral correctness

Binary Search (spec)

```
/*@ requires
  @   n >= 0 &&
  @   \valid_range(t,0,n-1) &&
  @   \forall int k1, int k2;
  @     0 <= k1 <= k2 <= n-1 => t[k1] <= t[k2]
  @*/
int binary_search(int* t, int n, int v) {
  ...
}
```

Binary Search (spec)

```
/*@ requires
  @   n >= 0 &&
  @   \valid_range(t,0,n-1) &&
  @   \forall int k1, int k2;
  @     0 <= k1 <= k2 <= n-1 => t[k1] <= t[k2]
  @ ensures
  @   (\result >= 0 && t[\result] == v) ||
  @   (\result == -1 && \forall int k;
  @                          0 <= k < n => t[k] != v)
  @*/
int binary_search(int* t, int n, int v) {
  ...
}
```

Binary Search (spec)

```
/*@ requires ...
  @ ensures  ...
  @*/
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1, p = -1;
  /*@ variant u-l
    @*/
  while (l <= u ) {
    ...
  }
}
```

Binary Search (spec)

```
/*@ requires ...
  @ ensures  ...
  @*/
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1, p = -1;
  /*@ invariant
    @   0 <= l && u <= n-1 && p == -1 &&
    @   \forall int k;
    @     0 <= k < n => t[k] == v => l <= k <= u
    @ variant u-l
    @*/
  while (l <= u ) {
    ...
  }
}
```

Binary Search (proof)

DEMO

Algebraic Models

in JML, annotations are written using **pure Java code**
this is mandatory to perform **runtime assertion checking**

but it is often convenient to introduce **axiomatized theories** in order to annotate programs, that is

- abstract types
- function symbols, w or w/o definitions
- predicates, w or w/o definitions
- axioms

Example: Priority Queues

static data structure for a **priority queue** containing integers

```
void clear();       // empties the queue
void push(int x);   // inserts a new element
int  max();         // returns the maximal element
int  pop();         // removes and returns the maximal element
```

Bags

```
//@ type bag

//@ logic bag empty_bag()

//@ logic bag singleton_bag(int x)

//@ logic bag union_bag(bag b1, bag b2)

/*@ logic bag add_bag(int x, bag b)
  @   { union_bag(b, singleton_bag(x)) } */

//@ logic int occ_bag(int x, bag b)

/*@ predicate is_max_bag(bag b, int m) {
  @   occ_bag(m, b) >= 1 &&
  @   \forall int x; occ_bag(x,b) >= 1 => x <= m
  @ } */
```

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  @ } */
```

Bags

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  @   occ_bag(m, b) >= 1 &&
  @   \forall int x; occ_bag(x,b) >= 1 => x <= m
  @ } */
```

Priority Queues (spec)

```
//@ logic bag model() { ... }

//@ ensures model() == empty_bag()
void clear();

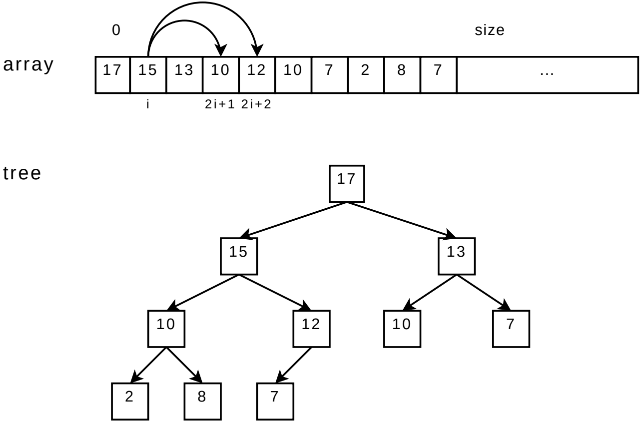
//@ ensures model() == add_bag(x, \old(model()))
void push(int x);

//@ ensures is_max_bag(model(), \result)
int max();

/*@ ensures is_max_bag(\old(model()), \result) &&
  @         \old(model()) == add_bag(\result, model()) */
int pop();
```

Implementing Priority Queues

implementation: heap encoded in an array



bag { 2, 7, 7, 8, 10, 10, 12, 13, 15, 17 }

Trees

```
//@ type tree

//@ logic tree Empty()

//@ logic tree Node(tree l, int x, tree r)
```

Heaps

```
//@ predicate is_heap(tree t)

//@ axiom is_heap_def_1: is_heap(Empty())

/*@ axiom is_heap_def_2:
  @   \forall int x; is_heap(Node(Empty(), x, Empty()))
  @*/

/*@ axiom is_heap_def_3:
  @   \forall tree ll; \forall int lx;
  @   \forall tree lr; \forall int x;
  @     x >= lx => is_heap(Node(ll, lx, lr)) =>
  @     is_heap(Node(Node(ll, lx, lr), x, Empty()))
  @*/
```

...

Trees and Bags

```
//@ logic bag bag_of_tree(tree t)

/*@ axiom bag_of_tree_def_1:
  @   bag_of_tree(Empty()) == empty_bag()
  @*/

/*@ axiom bag_of_tree_def_2:
  @   \forall tree l; \forall int x; \forall tree r;
  @     bag_of_tree(Node(l, x, r)) ==
  @     add_bag(x, union_bag(bag_of_tree(l), bag_of_tree(r)))
  @*/
```

Trees and Arrays

```
//@ logic tree tree_of_array(int *t, int root, int bound)

/*@ axiom tree_of_array_def_2:
  @   \forall int *t; \forall int root; \forall int bound;
  @     0 <= root < bound =>
  @     tree_of_array(t, root, bound) ==
  @     Node(tree_of_array(t, 2*root+1, bound),
  @          t[root],
  @          tree_of_array(t, 2*root+2, bound))
  @*/
```

...

Priority Queues (spec)

```
#define MAXSIZE 100

int heap[MAXSIZE];

int size = 0;

//@ invariant size_inv : 0 <= size < MAXSIZE

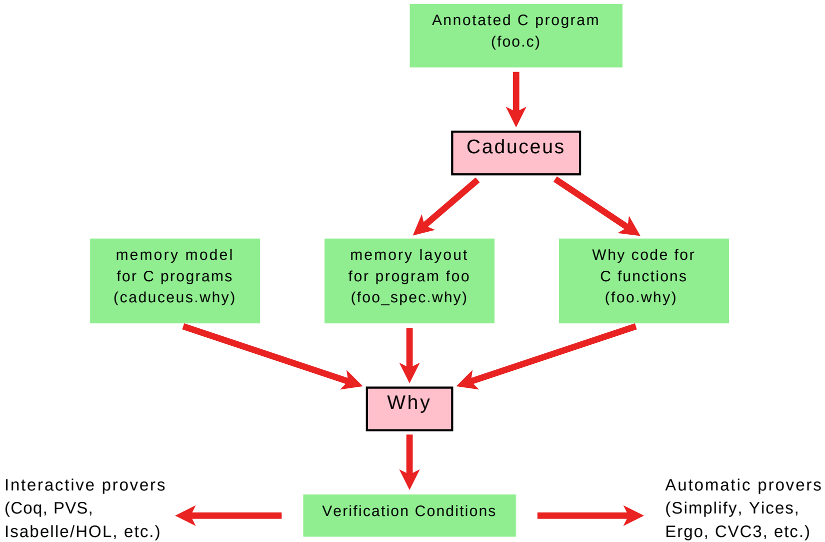
//@ invariant is_heap: is_heap(tree_of_array(heap, 0, size))

/*@ logic bag model()
  @   { bag_of_tree(tree_of_array(heap, 0, size)) } */
```

Outline

1. An intermediate language for program verification
   1. syntax, typing, semantics, proof rules
   2. the Why tool
   3. multi-prover approach
2. Verifying C and Java programs
   1. specification languages
   2. **models of program execution**
      - translation of pointer programs to alias-free Why programs
3. A challenging case study

Generating the Verification Conditions



To Pointer Programs to Alias-Free Programs

naive idea: model the **memory as a big array**

using the theory of arrays

$$\texttt{acc} : \texttt{mem}, \texttt{int} \rightarrow \texttt{int}$$
$$\texttt{upd} : \texttt{mem}, \texttt{int}, \texttt{int} \rightarrow \texttt{mem}$$

$$\forall m\, p\, v,\ \texttt{acc}(\texttt{upd}(m, p, v), p) = v$$
$$\forall m\, p_1\, p_2\, v,\ p_1 \neq p_2 \Rightarrow \texttt{acc}(\texttt{upd}(m, p_1, v), p_2) = \texttt{acc}(m, p_2)$$

Naive Memory Model

then the C program

```
int x;
int y;
x = 0;
y = 1;
//@ assert x == 0
```

becomes

$$m := \mathtt{upd}(m, x, 0);$$
$$m := \mathtt{upd}(m, y, 1);$$
$$\mathtt{assert}\ \mathtt{acc}(m, x) = 0$$

the verification condition is

$$\mathtt{acc}(\mathtt{upd}(\mathtt{upd}(m, x, 0), y, 0), x) = 0$$

Memory Model for Pointer Programs

we use the **component-as-array** model (Burstall-Bornat)

each structure/object field is mapped to a different array

relies on the property **“two different fields cannot be aliased”**

strong consequence: prevents pointer casts and unions (a priori)

Benefits of the Component-As-Array Model

```
struct S { int x; int y; } p;
...
p.x = 0;
p.y = 1;
//@ assert p.x == 0
```

becomes

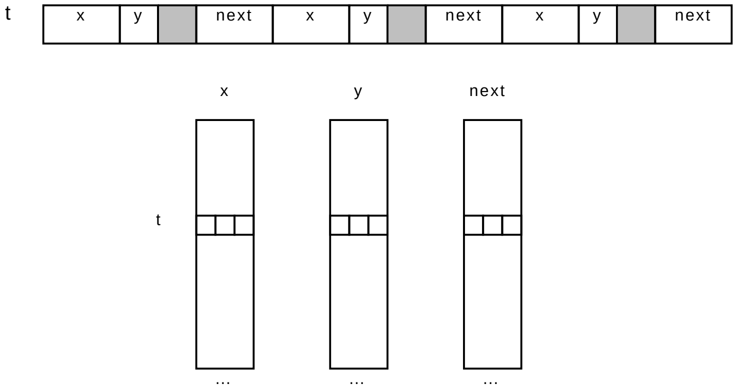
$x := \mathtt{upd}(x, p, 0);$
$y := \mathtt{upd}(y, p, 1);$
$\mathtt{assert}\ \mathtt{acc}(x, p) = 0$

the verification condition is

$$\mathtt{acc}(\mathtt{upd}(x, p, 0), p) = 0$$

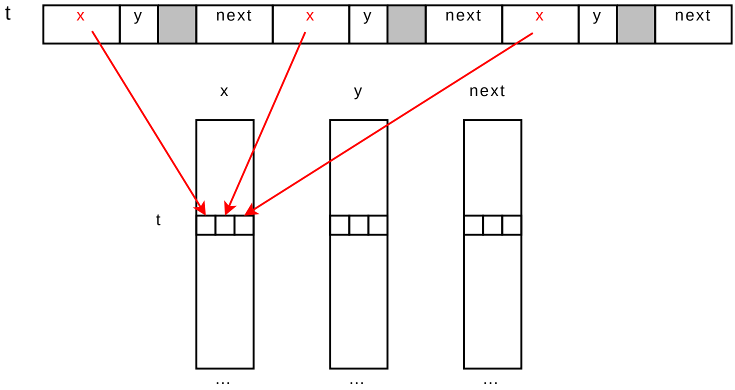
Component-As-Array Model and Pointer Arithmetic

```
struct S { int x; short y; struct S *next; } t[3];
```



Component-As-Array Model and Pointer Arithmetic

```
struct S { int x; short y; struct S *next; } t[3];
```



Separation Analysis

on top of Burstall-Bornat model, we add some **separation analysis**

- each pointer is assigned a **zone**
- zones are **unified** when pointers are assigned / compared
- functions are **polymorphic** wrt zones

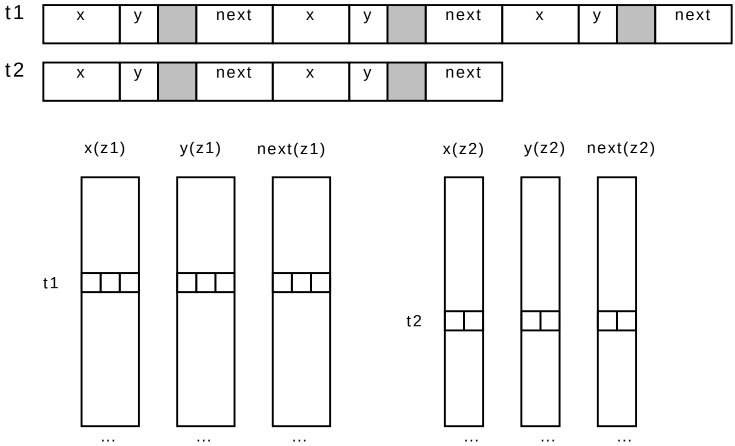
similar to ML-type inference

then the component-as-array model is refined according to zones

Separation Analysis for Deductive Verification (HAV'07)

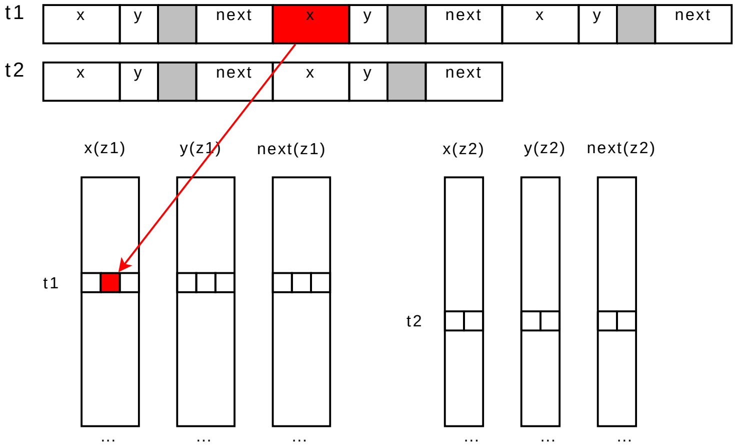
Separation Analysis

```
struct S { int x; short y; struct S *next; } t1[3], t2[2];
```



Separation Analysis

```
struct S { int x; short y; struct S *next; } t1[3], t2[2];
```



Example

little challenge for program verification proposed by P. Müller:

> *count the number n of non-zero values in an integer array t, then copy these values in a freshly allocated array of size n*

`t`	2	1	0	4	0	5	3	0

`count=5`

`u`	2	1	4	5	3

P. Müller's Example (code)

```
void m(int t[], int length) {
  int count=0, i, *u;

  for (i=0 ; i < length; i++)
    if (t[i] > 0) count++;

  u = (int *)calloc(count,sizeof(int));
  count = 0;

  for (i=0 ; i < length; i++)
    if (t[i] > 0) u[count++] = t[i];
}
```

P. Müller's Example (spec)

```
void m(int t[], int length) {
  int count=0, i, *u;
  //@ invariant count == num_of_pos(0,i-1,t) ...
  for (i=0 ; i < length; i++)
    if (t[i] > 0) count++;
  //@ assert count == num_of_pos(0,length-1,t)
  u = (int *)calloc(count,sizeof(int));
  count = 0;
  //@ invariant count == num_of_pos(0,i-1,t) ...
  for (i=0 ; i < length; i++)
    if (t[i] > 0) u[count++] = t[i];
}
```

P. Müller's Example (proof)

12 verification conditions

- without separation analysis: 10/12 automatically proved
- with separation analysis: 12/12 automatically proved

DEMO

Integer Arithmetic

up to now, we did not consider integer arithmetic

there are basically three ways to model arithmetic

- **exact**: all computations are interpreted using mathematical integers; thus it **assumes** that there is no overflow
- **bounded**: the user have to **prove** that there is no integer overflow
- **modulo**: overflows are possible and modulo arithmetic is used; it is **faithful** to machine arithmetic

Overflows in Binary Search

we proved binary search using exact arithmetic

let us prove that there is no overflow

DEMO

Modelling Integer Arithmetic

difficulty: we do not want to lose the ability of provers to handle arithmetic

thus we cannot simply axiomatize machine arithmetic using new abstract data types

Bounded Arithmetic

consider signed 32-bit integers

```
type int32

logic of_int32: int32 -> int

axiom int32_domain:
  forall x:int32.
    -2147483648 <= of_int32(x) <= 2147483647

parameter int32_of_int:
  x:int ->
    { -2147483648 <= x <= 2147483647 }
    int32
    { of_int32(result) = x }
```

Bounded Arithmetic

consider the C fragment

```
(x + 1) * y
```

it is translated into

```
int32_of_int
  ((of_int32
     (int32_of_int
       ((of_int32 x) + (of_int32 (int32_of_int 1)))))
   *
   (of_int32 y))
```

Bounded Arithmetic

in practice, most proof obligations are easy to solve

```
int f(int n) {
  int i = 0;
  while (i < n) {
    ...
    i++;
  }
}
```

we do not even need to insert annotations

Modulo Arithmetic

```
type int32

logic of_int32: int32 -> int

axiom int32_domain:
  forall x:int32. -2147483648 <= of_int32(x) <= 2147483647

logic mod_int32: int -> int

parameter int32_of_int:
  x:int -> { } int32 { of_int32(result) = mod_int32(x) }

axiom mod_int32_id:
  forall x:int.
    -2147483648 <= x <= 2147483647 -> mod_int32(x) = x
...
```

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3. **A challenging case study**

A challenging case study

challenge for **the verified program of the month**:

```
t(a,b,c){int d=0,e=a&~b&~c,f=1;if(a)for(f=0;d=(e-=d)&-e;f+=t(a-d,(b+d)*2,(
c+d)/2));return f;}main(q){scanf("%d",&q);printf("%d\n",t(~(~0<<q),0,0));}
```

appears on a web page collecting C signature programs

due to Marcel van Kervinck (author of MSCP, a chess program)

Unobfuscating...

```
int t(int a, int b, int c) {
  int d, e=a&~b&~c, f=1;
  if (a)
    for (f=0; d=e&-e; e-=d)
      f += t(a-d, (b+d)*2, (c+d)/2);
  return f;
}

int main(int q) {
  scanf("%d", &q);
  printf("%d\n", t(~(~0<<q), 0, 0));
}
```

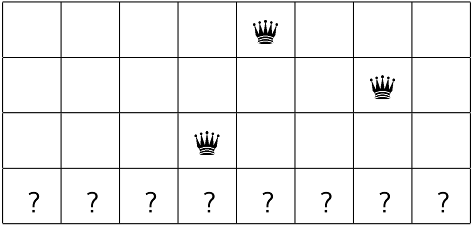
this program reads an integer *n*
and prints the number of solutions to the *n*-queens problem

How does it work?

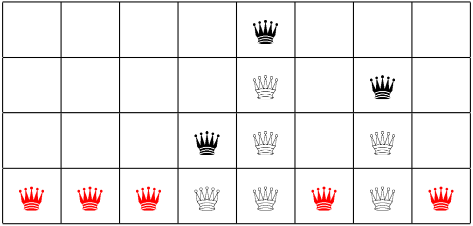
- backtracking algorithm (no better way to solve the n-queens)
- integers used as **sets** (bit vectors)

integers	sets
`0`	$\emptyset$
`a&b`	$a \cap b$
`a+b`	$a \cup b$, when $a \cap b = \emptyset$
`a-b`	$a \setminus b$, when $b \subseteq a$
`~a`	$\complement a$
`a&-a`	$min\_elt(a)$, when $a \neq \emptyset$
`~(~0<<n)`	$\{0, 1, \ldots, n-1\}$
`a*2`	$\{i+1 \mid i \in a\}$, written $S(a)$
`a/2`	$\{i-1 \mid i \in a \wedge i \neq 0\}$, written $P(a)$

What a, b and c mean

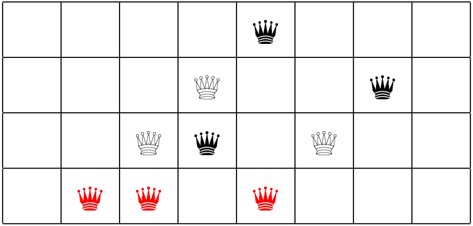


What a, b and c mean



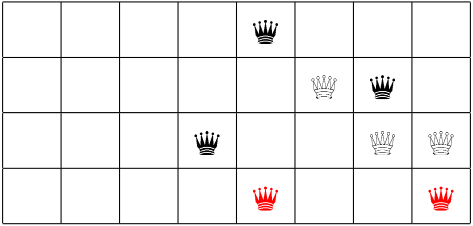
a = columns to be filled = 11100101_2

What a, b and c mean



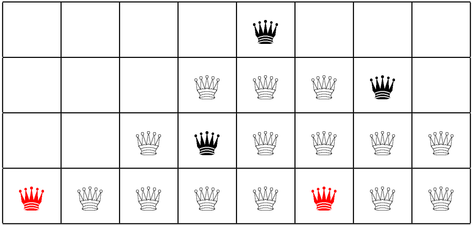
b = positions to avoid because of left diagonals = 01101000_2

What a, b and c mean



c = positions to avoid because of right diagonals = 00001001_2

What a, b and c mean



$a\&\tilde{}b\&\tilde{}c$ = positions to try = 10000100_2

Now it is clear

```
int t(int a, int b, int c) {
  int d, e=a&~b&~c, f=1;
  if (a)
    for (f=0; d=e&-e; e-=d)
      f += t(a-d, (b+d)*2, (c+d)/2);
  return f;
}

int queens(int n) {
  return t(~(~0<<n), 0, 0);
}
```

Abstract finite sets

```
//@ type iset

//@ predicate in_(int x, iset s)

/*@ predicate included(iset a, iset b)
  @ { \forall int i; in_(i,a) => in_(i,b) } */

//@ logic iset empty()

//@ axiom empty_def : \forall int i; !in_(i,empty())
```

...

total: **66 lines** of functions, predicates and axioms

C ints as abstract sets

```
//@ logic iset iset(int x)

/*@ axiom iset_c_zero : \forall int x;
  @   iset(x)==empty() <=> x==0 */

/*@ axiom iset_c_min_elt :
  @   \forall int x; x != 0 =>
  @     iset(x&-x) == singleton(min_elt(iset(x))) */

/*@ axiom iset_c_diff : \forall int a, int b;
  @   iset(a&~b) == diff(iset(a), iset(b)) */
```

...

total: **27 lines**

Termination

```
int t(int a, int b, int c){
  int d, e=a&~b&~c, f=1;
  if (a)
    //@ variant card(iset(e-d))
    for (f=0; d=e&-e; e-=d) {
      f += t(a-d,(b+d)*2,(c+d)/2);
    }
  return f;
}
```

3 verification conditions, all proved automatically

similarly for the termination of the recursive function:
7 verification conditions, all proved automatically

Soundness

how to express that we compute the right number,
since the program is not storing anything,
not even the current solution?

answer: by introducing **ghost code** to perform the missing operations

Ghost code

ghost code can be regarded as regular code, as soon as

- ghost code does not modify program data
- program code does not access ghost data

ghost data is purely logical $\Rightarrow$ ne need to check the validity of pointers

Program instrumented with ghost code

```
//@ int** sol;
//@ int s;
//@ int* col;
//@ int k;

int t(int a, int b, int c) {
  int d, e=a&~b&~c, f=1;
  if (a)
    for (f=0; d=e&-e; e-=d) {
      //@ col[k] = min_elt(d);
      //@ k++;
      f += t3(a-d, (b+d)*2, (c+d)/2);
      //@ k--;
    }
  //@ else
  //@   store_solution();
  return f;
}
```

Program instrumented with ghost code (cont'd)

```
/*@ requires solution(col)
  @ assigns  s, sol[s][0..N()-1]
  @ ensures  s==\old(s)+1 && eq_sol(sol[\old(s)], col)
  @*/
void store_solution();

/*@ requires
  @   n == N() && s == 0 && k == 0
  @ ensures
  @   \result == s &&
  @   sorted(sol, 0, s) &&
  @   \forall int* t; solution(t) <=>
  @       (\exists int i; 0<=i<\result && eq_sol(t,sol[i]))
  @*/
int queens(int n) { return t(~(~0<<n), 0, 0); }
```

Finally, we get...

256 lines of code and specification

regarding VCs:

- main function `queens`: **15** verification conditions
  - **all** proved automatically (Simplify, Ergo or Yices)
- recursive function `t`: **51** verification conditions
  - **42** proved automatically: 41 by Simplify, 37 by Ergo and 35 by Yices
  - **9** proved manually using Coq (and Simplify)

Conclusion

Summary

the Why/Krakatoa/Caduceus platform features

- behavioral specification languages for C and Java programs, at source code level
- deductive program verification using original memory models
- multi-provers backend (interactive and automatic)

free software under GPL license; see http://why.lri.fr/

successfully applied on both

- academic case studies (Schorr-Waite, N-queens, list reversal, etc.)
- industrial case studies (Gemalto, Dassault Aviation, France Telecom)

Other Features

other features not covered in this lecture

- **floating point arithmetic**
  - allows to specify rounding and method errors
  - *Formal Verification of Floating-Point Programs* (ARITH 18)
- **pruning strategies** to help decision procedures on large VCs
  - *A Graph-based Strategy for the Selection of Hypotheses* (FTP 2007)

Ongoing Work & Future Work

ongoing work

- ownership: when class/type invariants must hold?
- C unions & pointer casts

future work

- verification of ML programs