

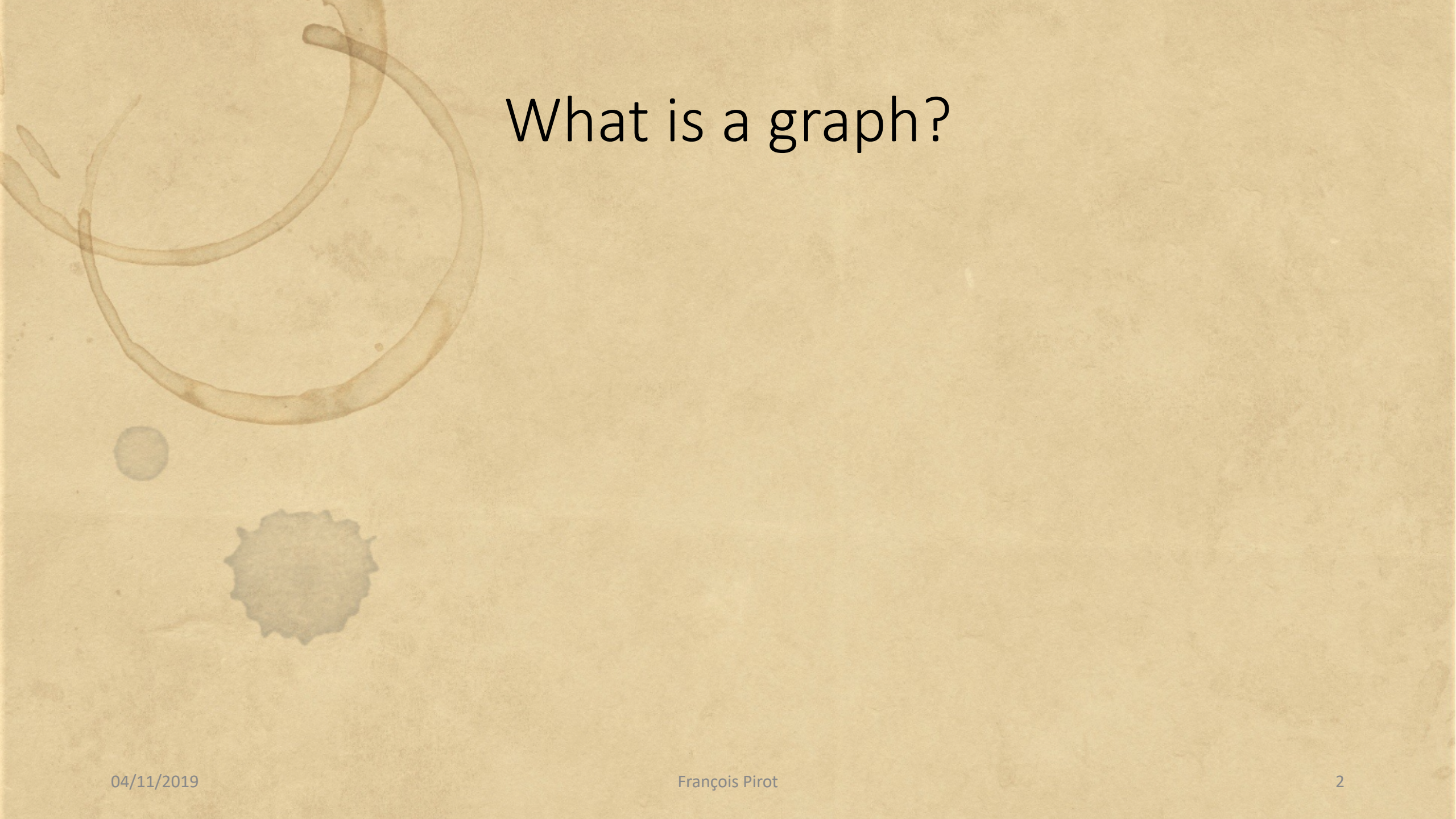
Colouring Sparse Graphs

François Pirot

04/11/2019

Under the supervision of

Miguel Couceiro, Eric Cator, Ross Kang, and Jean-Sébastien Sereni.

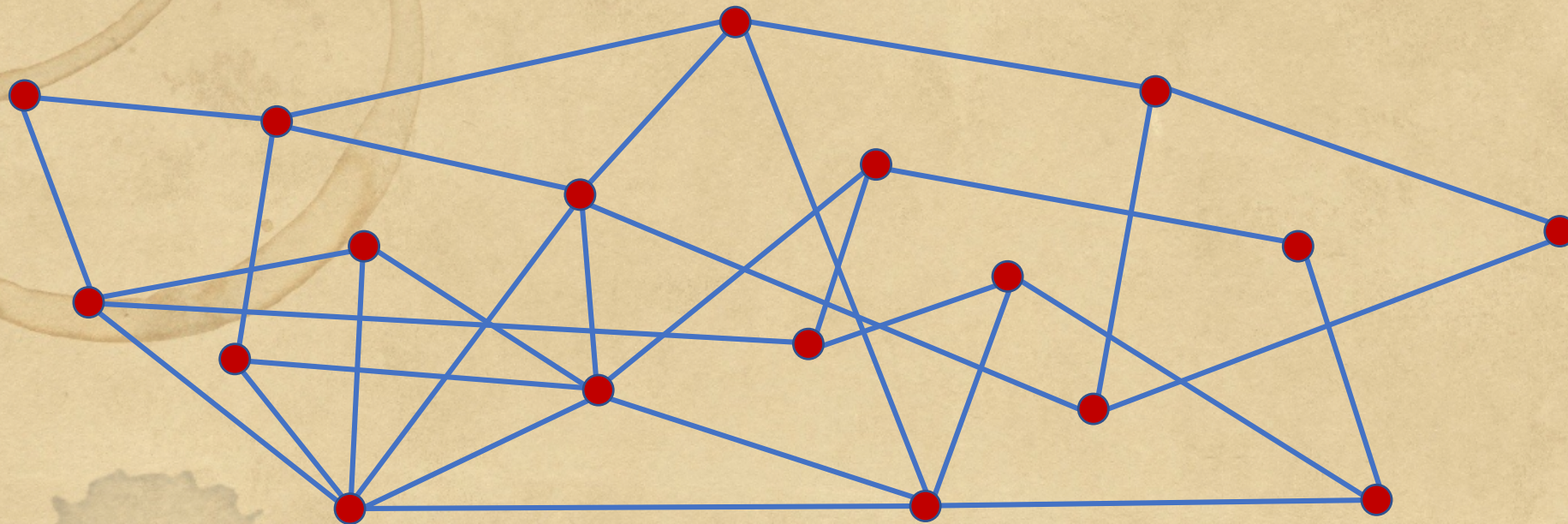


What is a graph?

What is a graph?

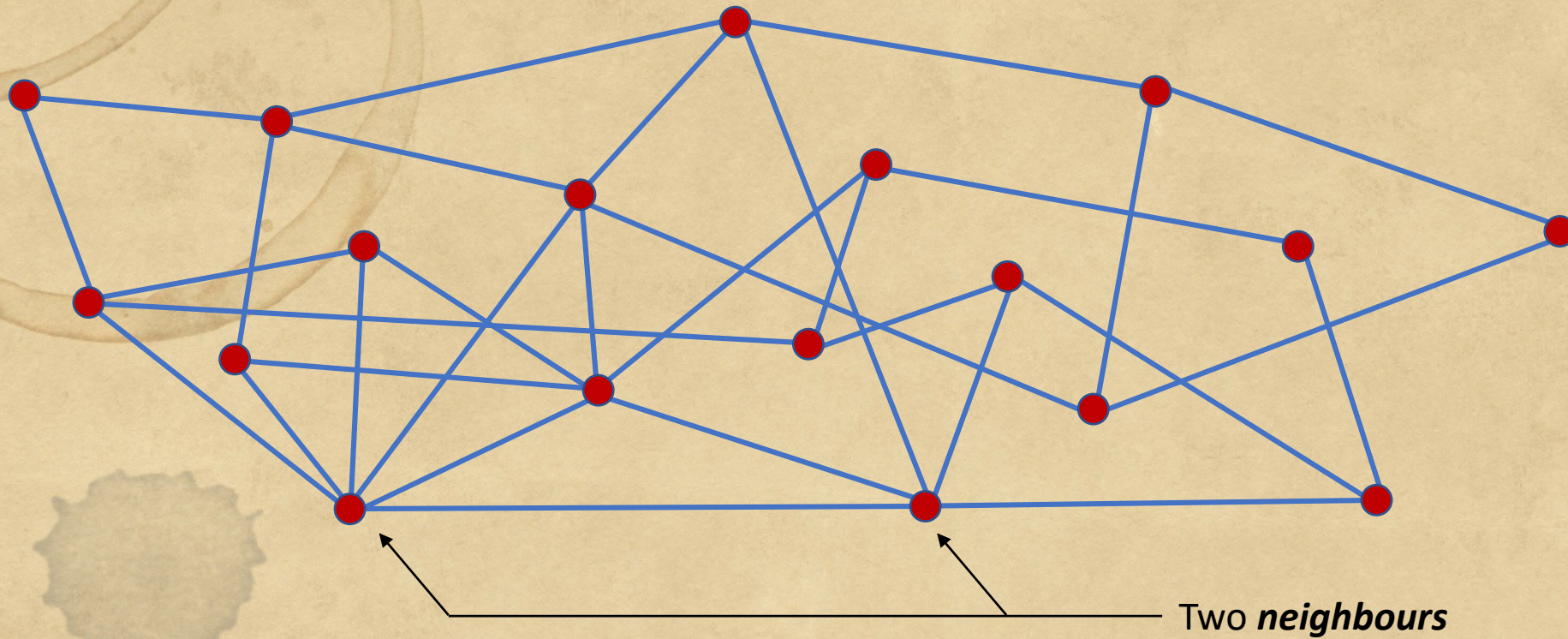
- Nodes / Vertices

What is a graph?



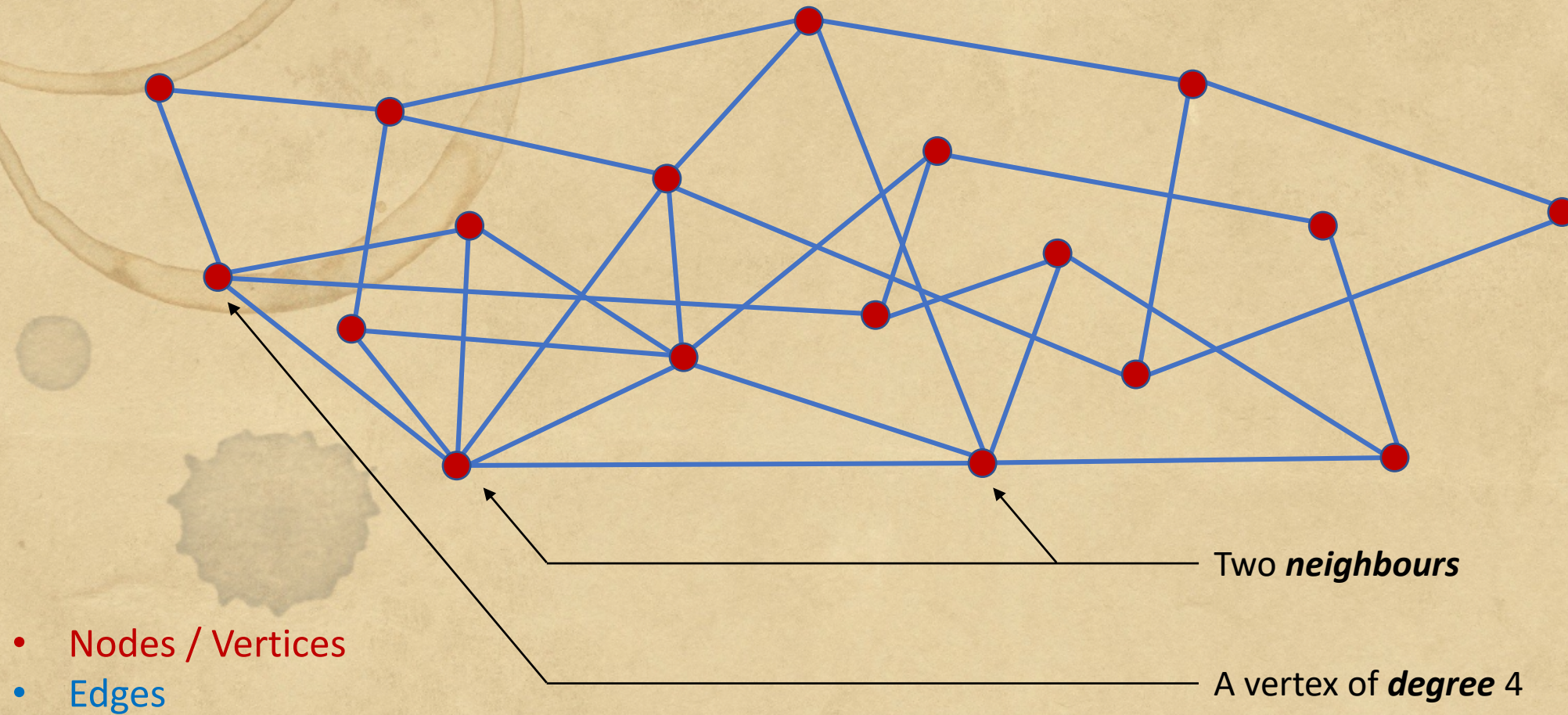
- Nodes / Vertices
- Edges

What is a graph?

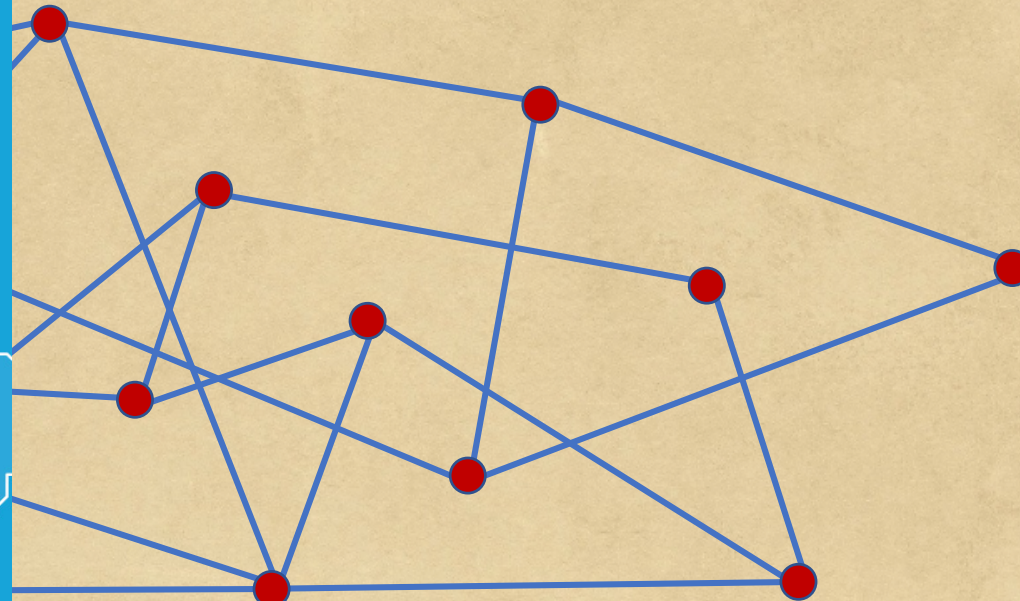


- Nodes / Vertices
- Edges

What is a graph?



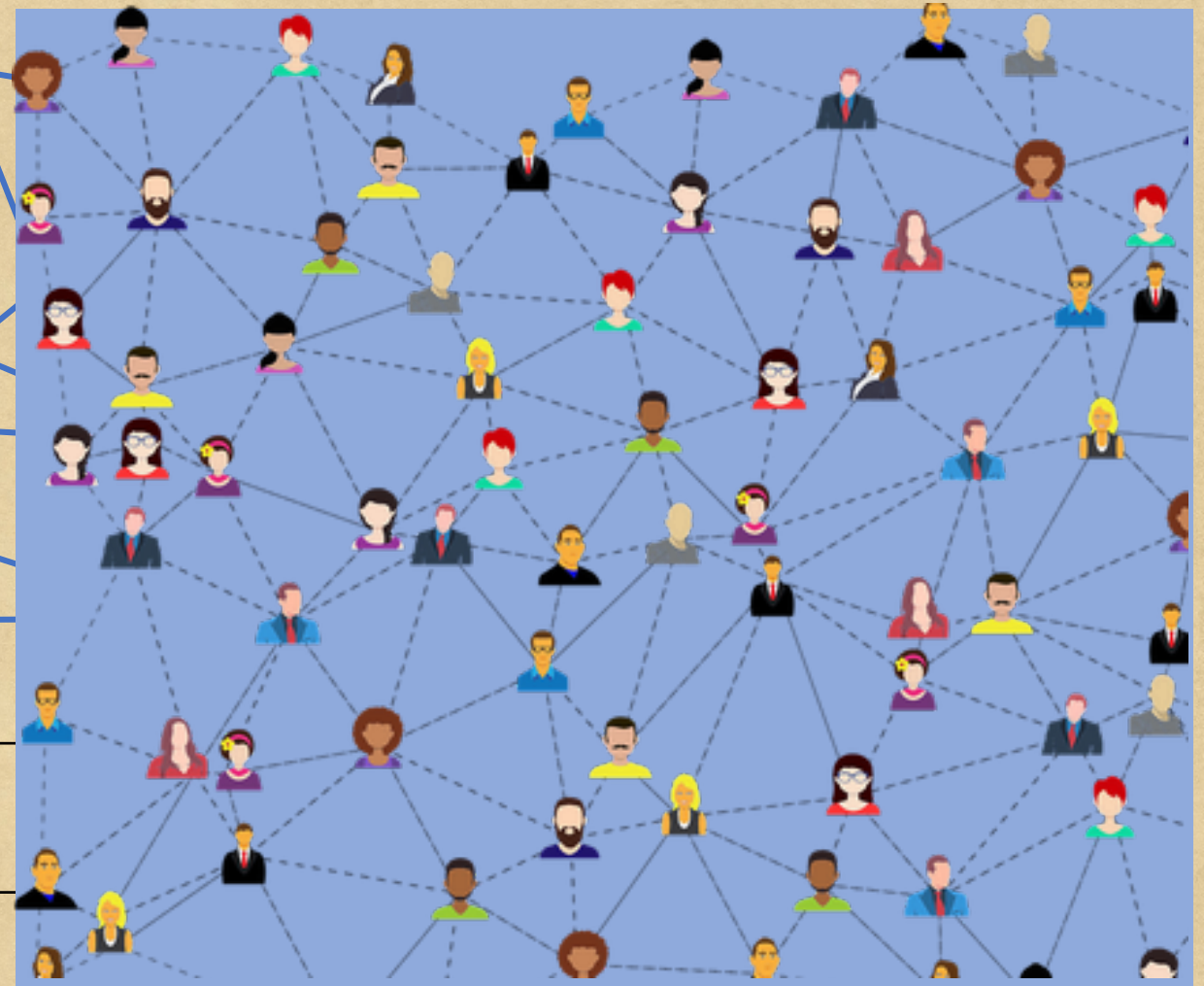
What is a graph?



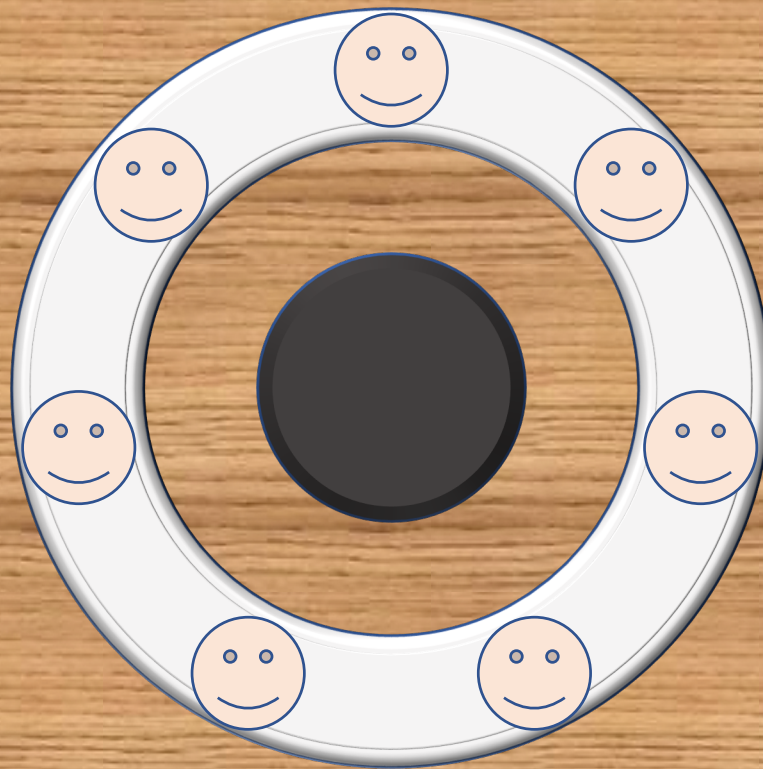
Two *neighbours*

A vertex of *degree* 4

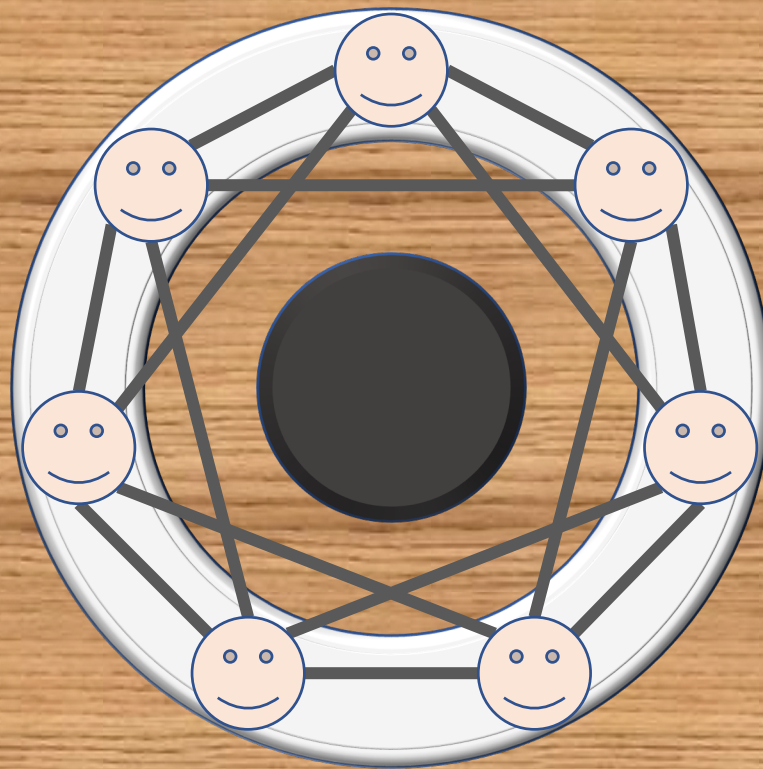
What is a graph?



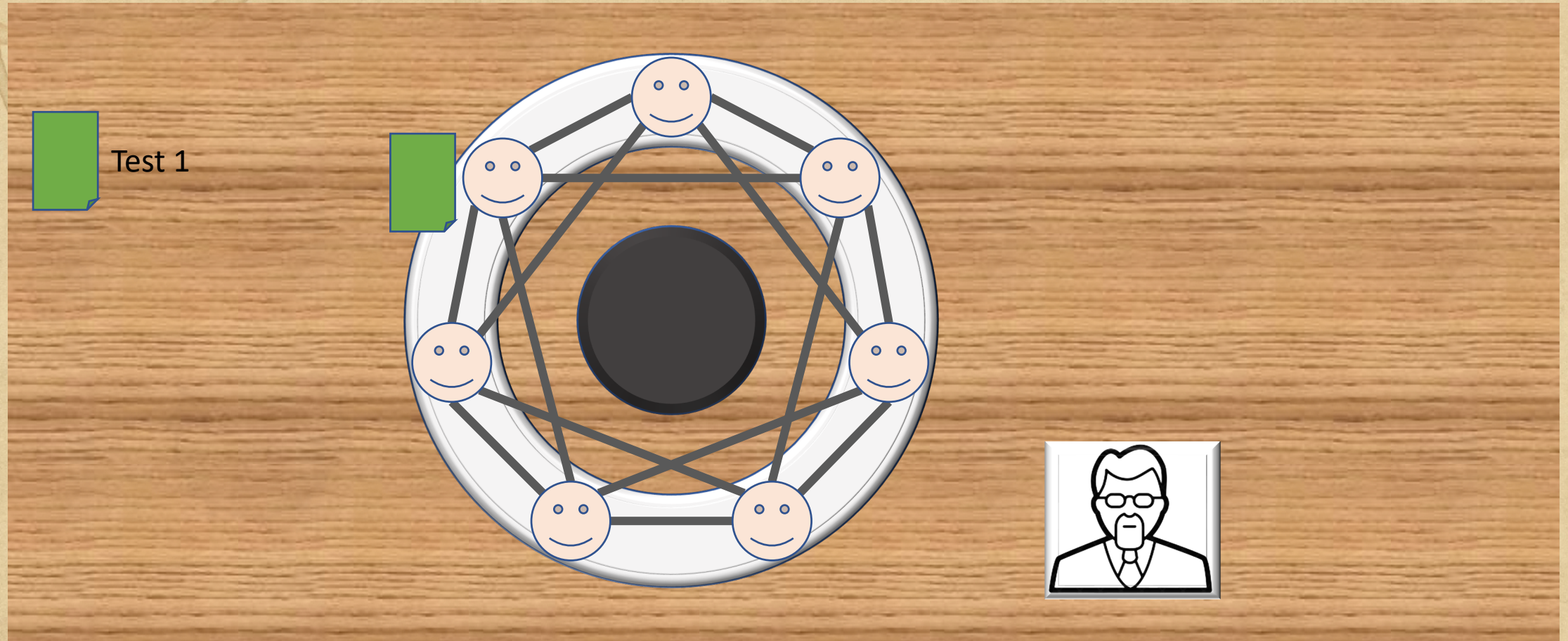
A cheating-proof test



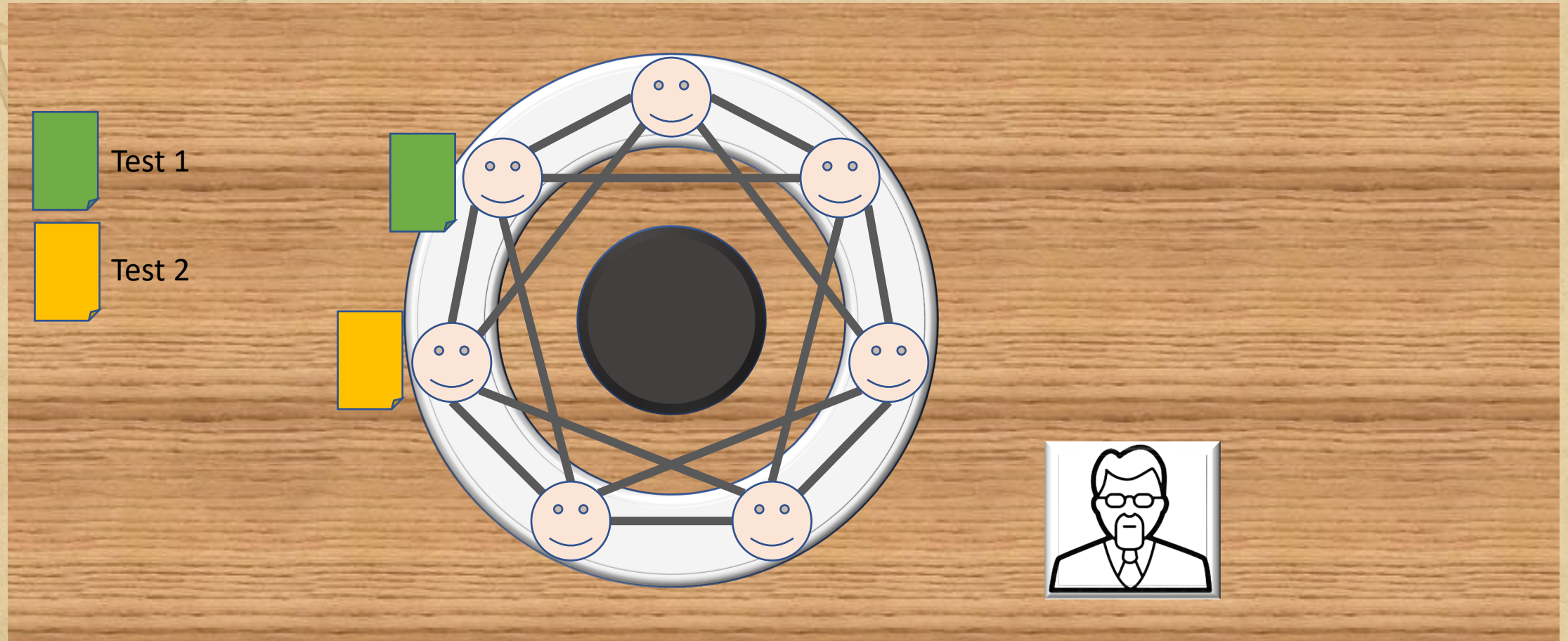
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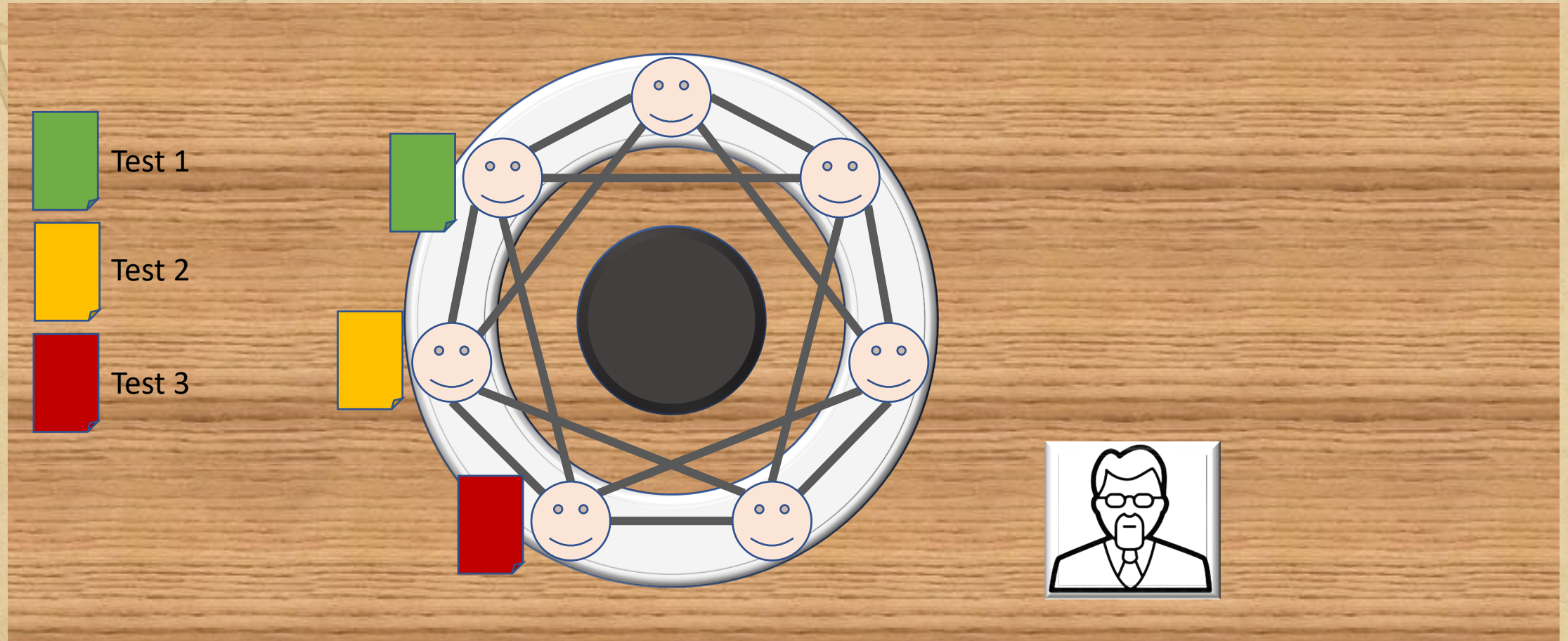
A cheating-proof test



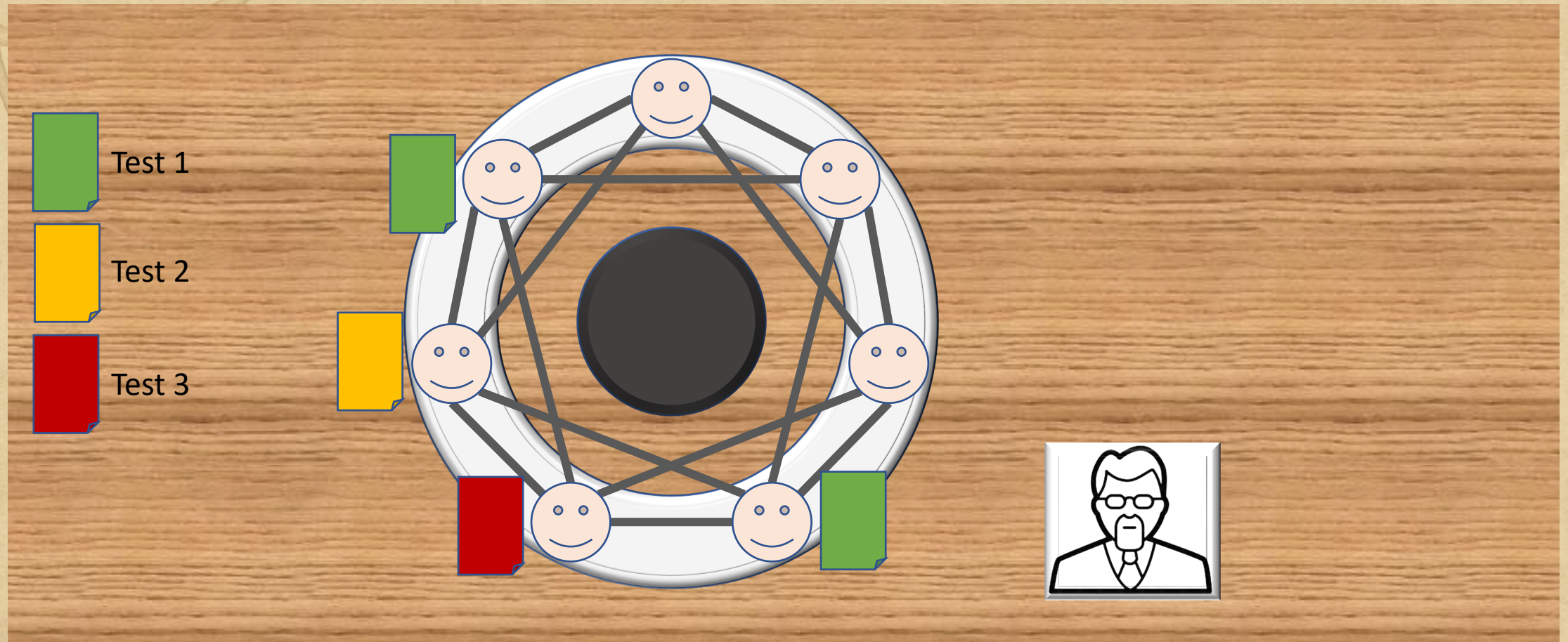
A cheating-proof test



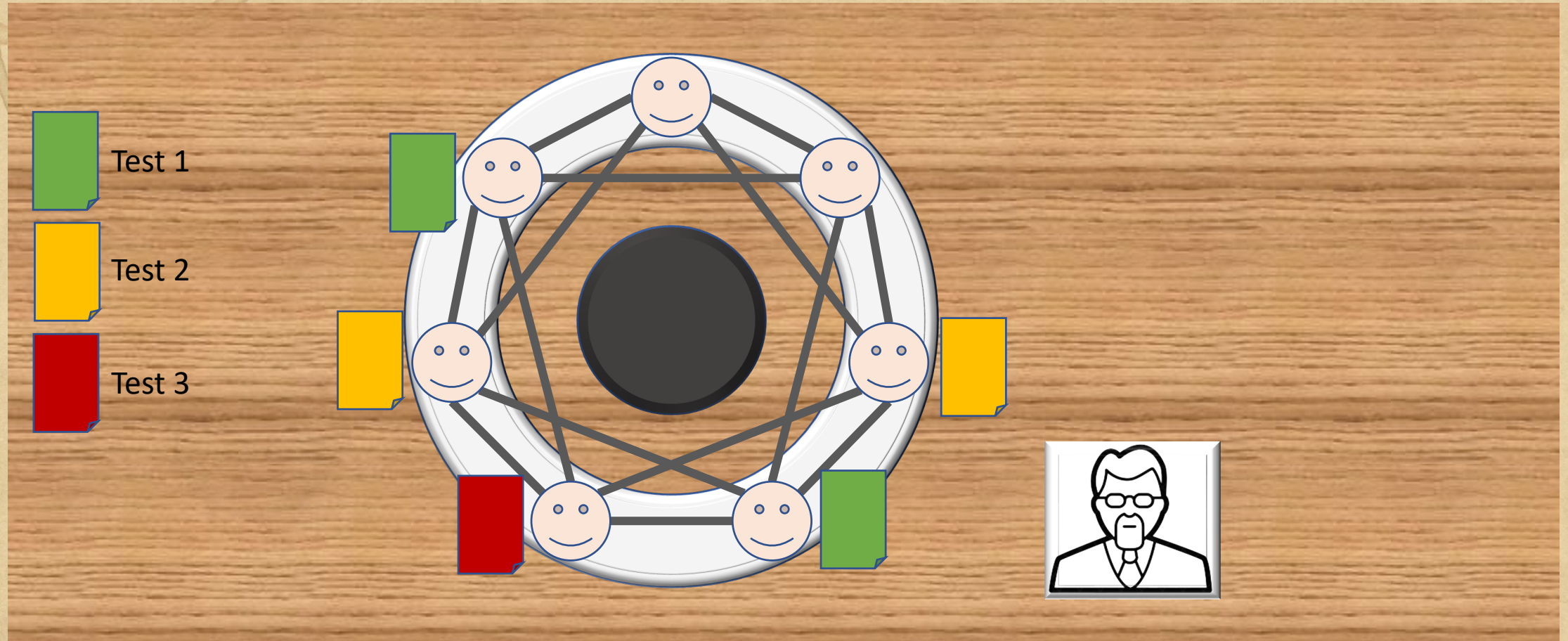
A cheating-proof test



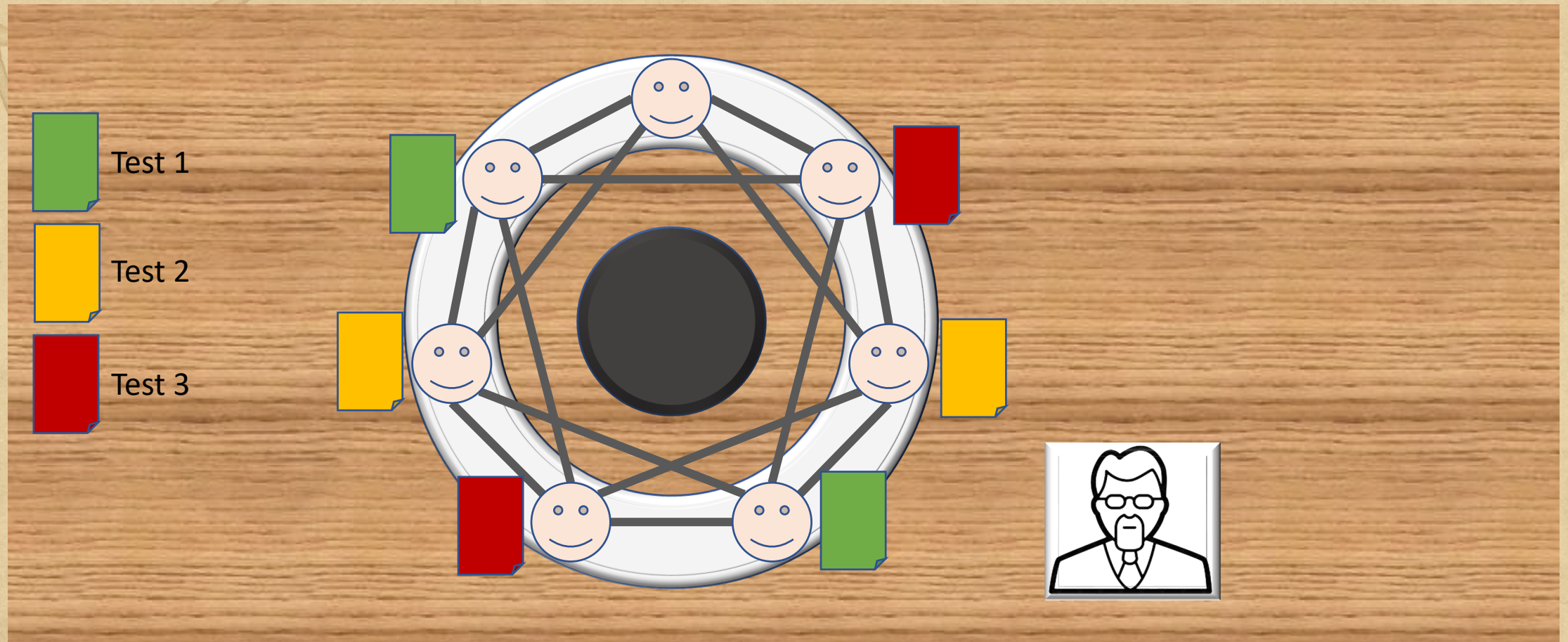
A cheating-proof test



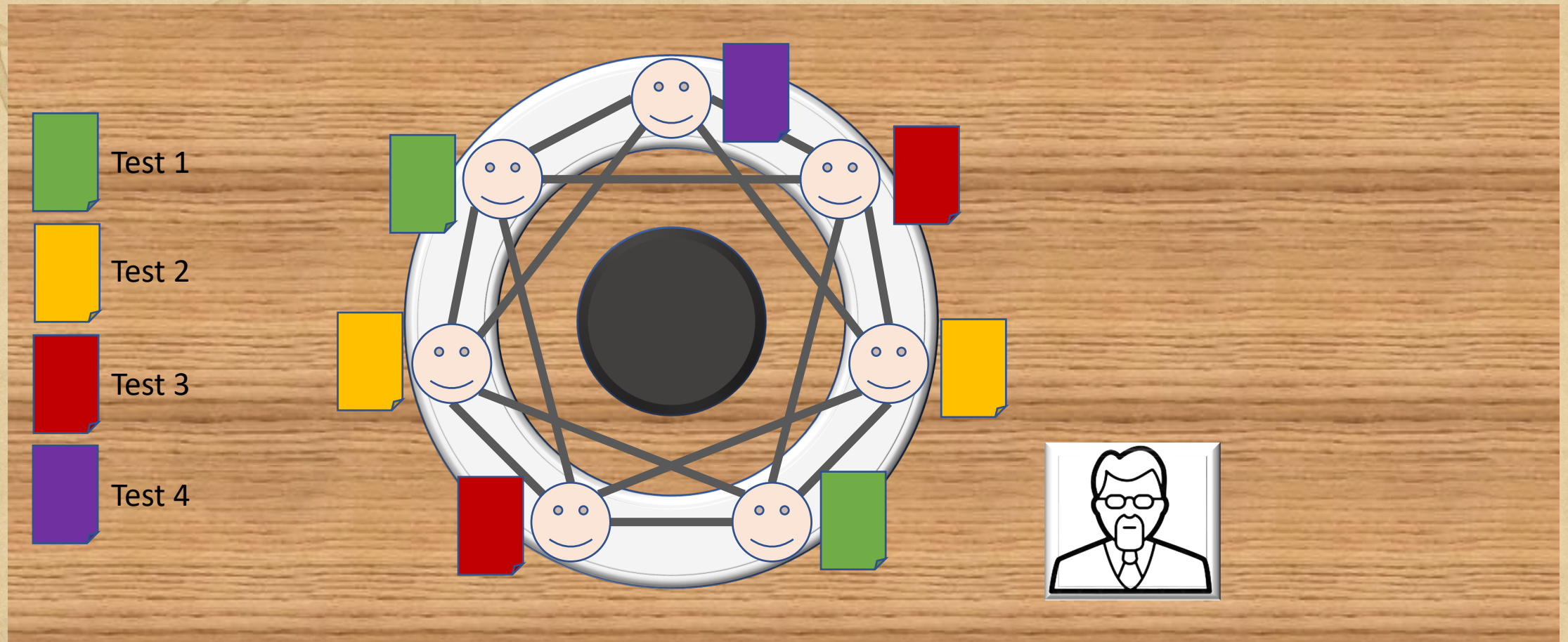
A cheating-proof test



A cheating-proof test



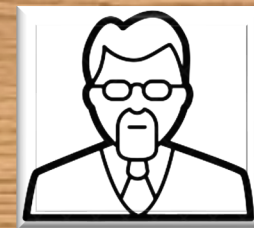
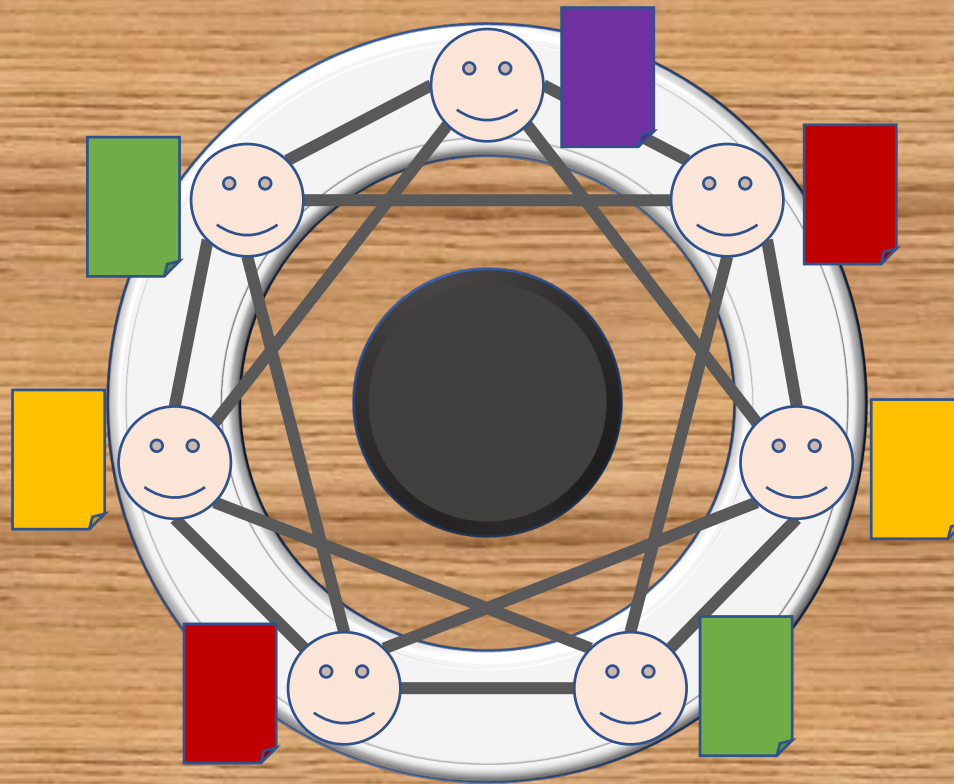
A cheating-proof test



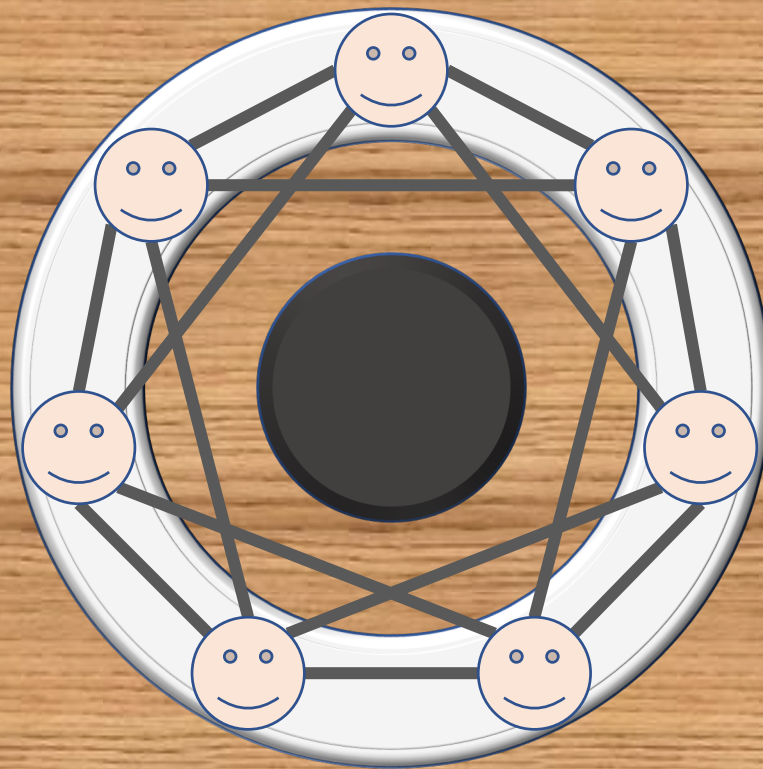
A cheating-proof test

Chromatic number: $\chi = 4$

- Test 1
- Test 2
- Test 3
- Test 4



A cheating-proof test

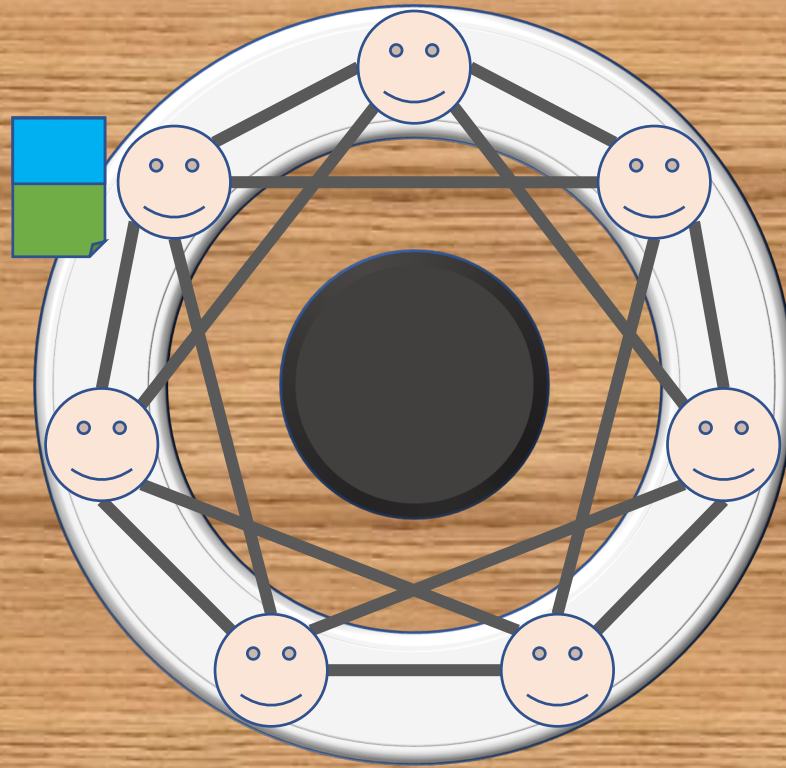


Chromatic number: $\chi = 4$

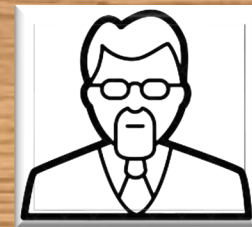


A cheating-proof test

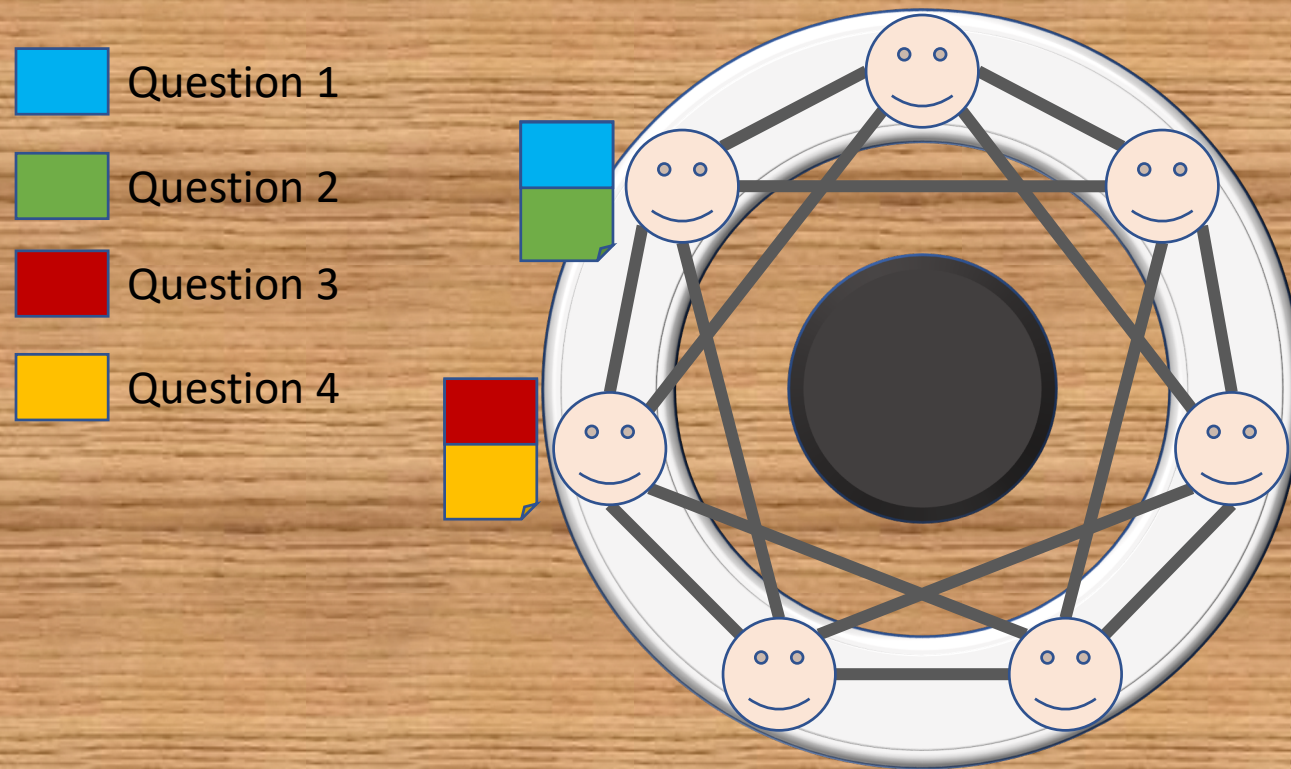
 Question 1
 Question 2



Chromatic number: $\chi = 4$



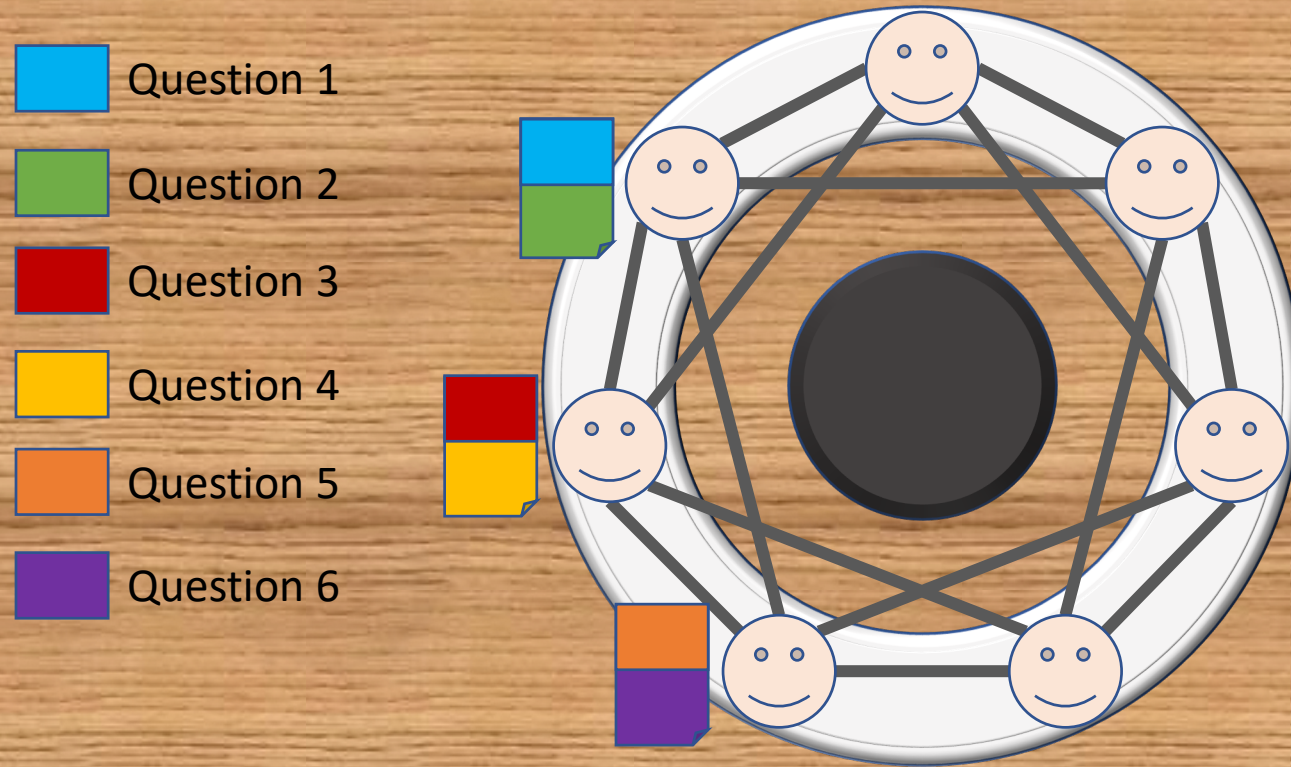
A cheating-proof test



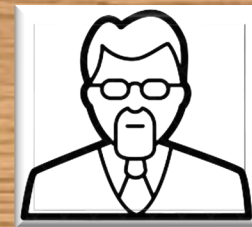
Chromatic number: $\chi = 4$



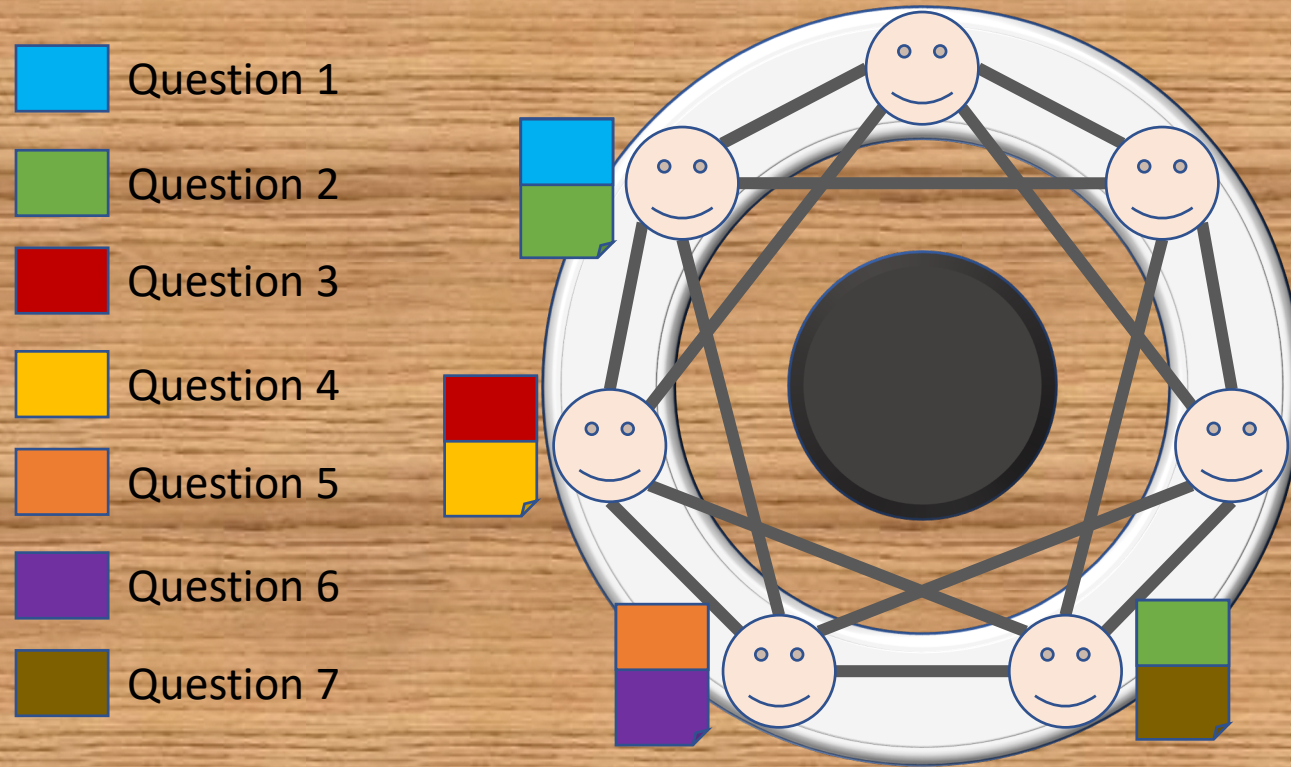
A cheating-proof test



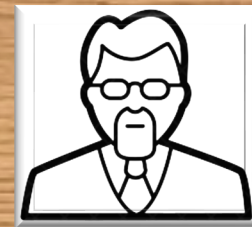
Chromatic number: $\chi = 4$



A cheating-proof test

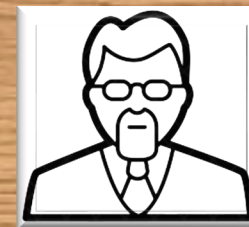
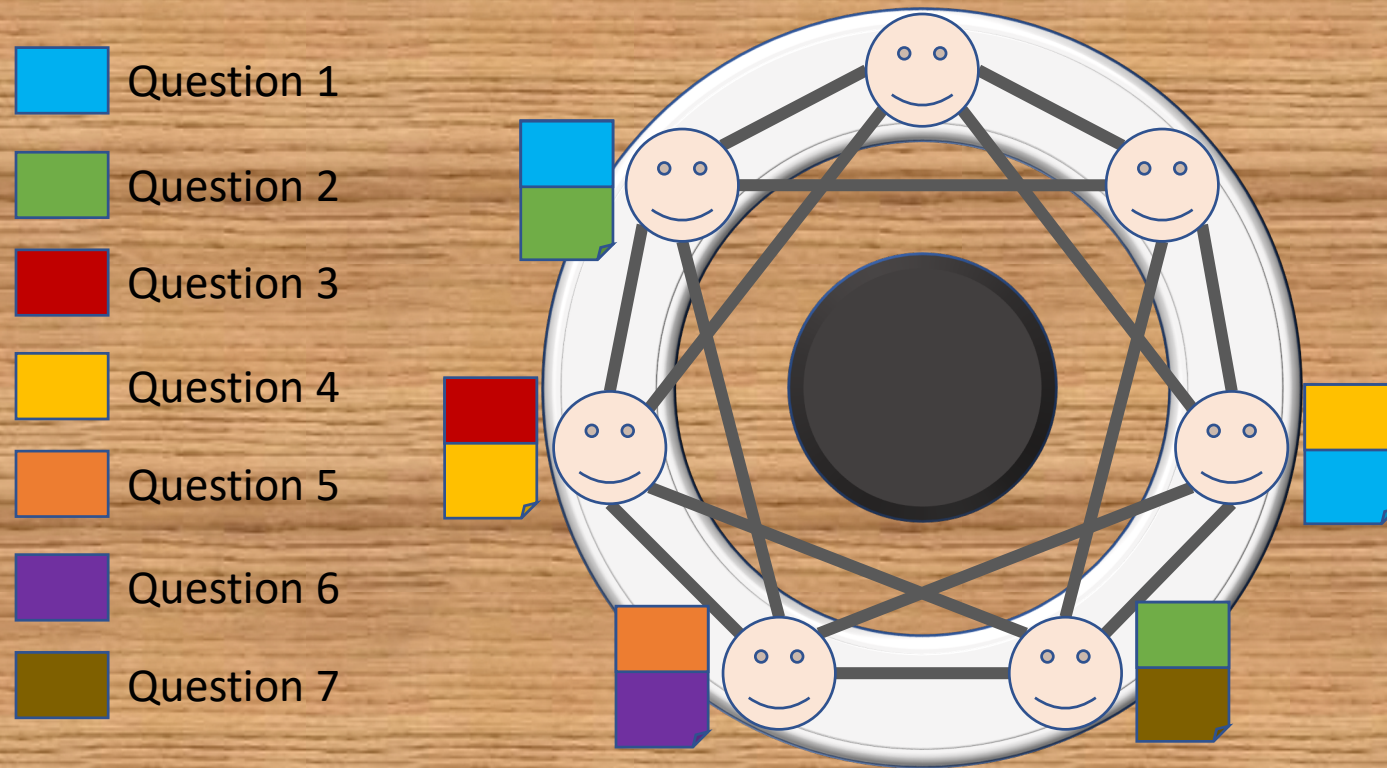


Chromatic number: $\chi = 4$



A cheating-proof test

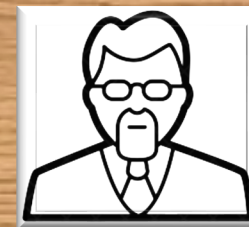
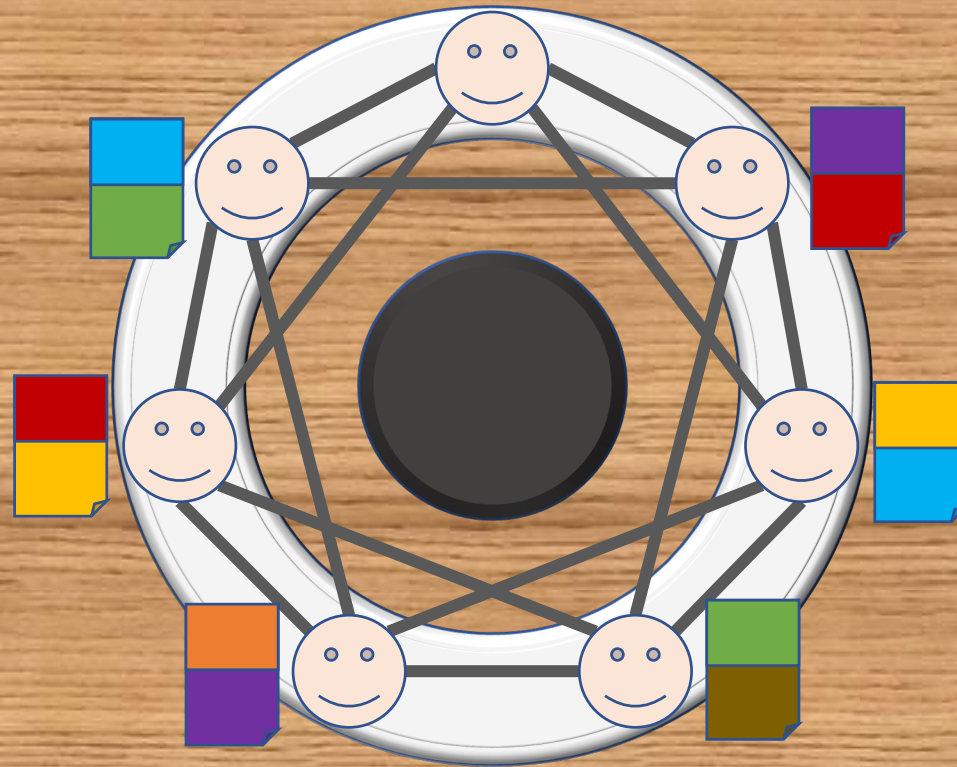
Chromatic number: $\chi = 4$



A cheating-proof test

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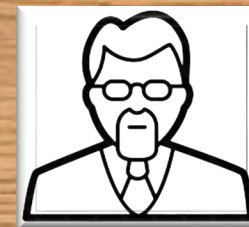
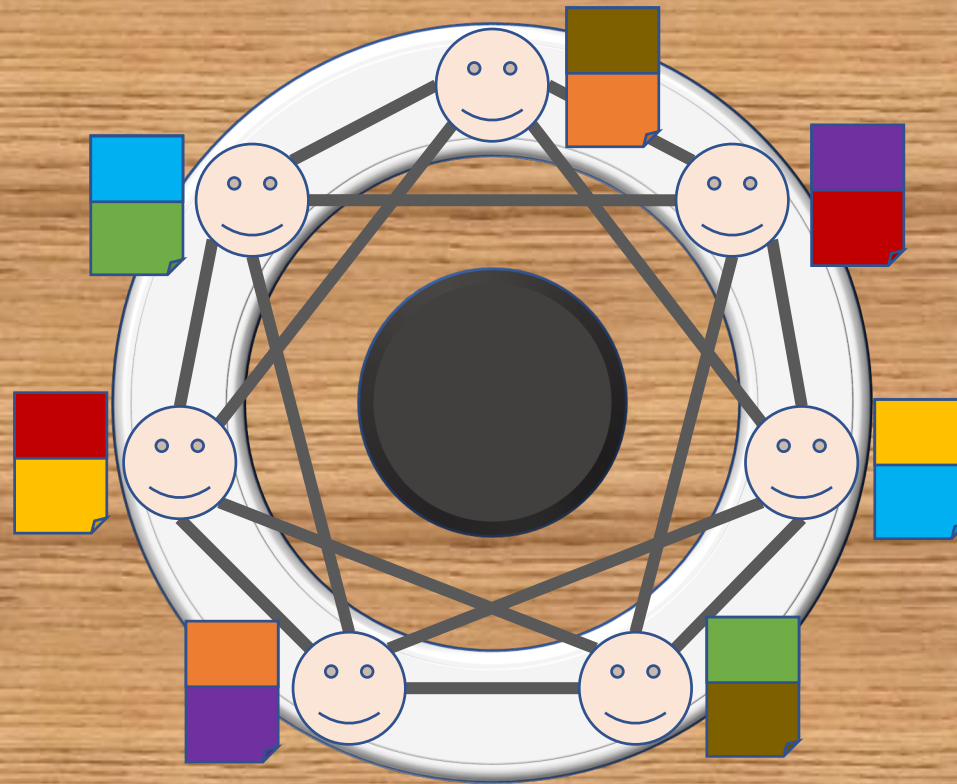
-  Question 1
-  Question 2
-  Question 3
-  Question 4
-  Question 5
-  Question 6
-  Question 7



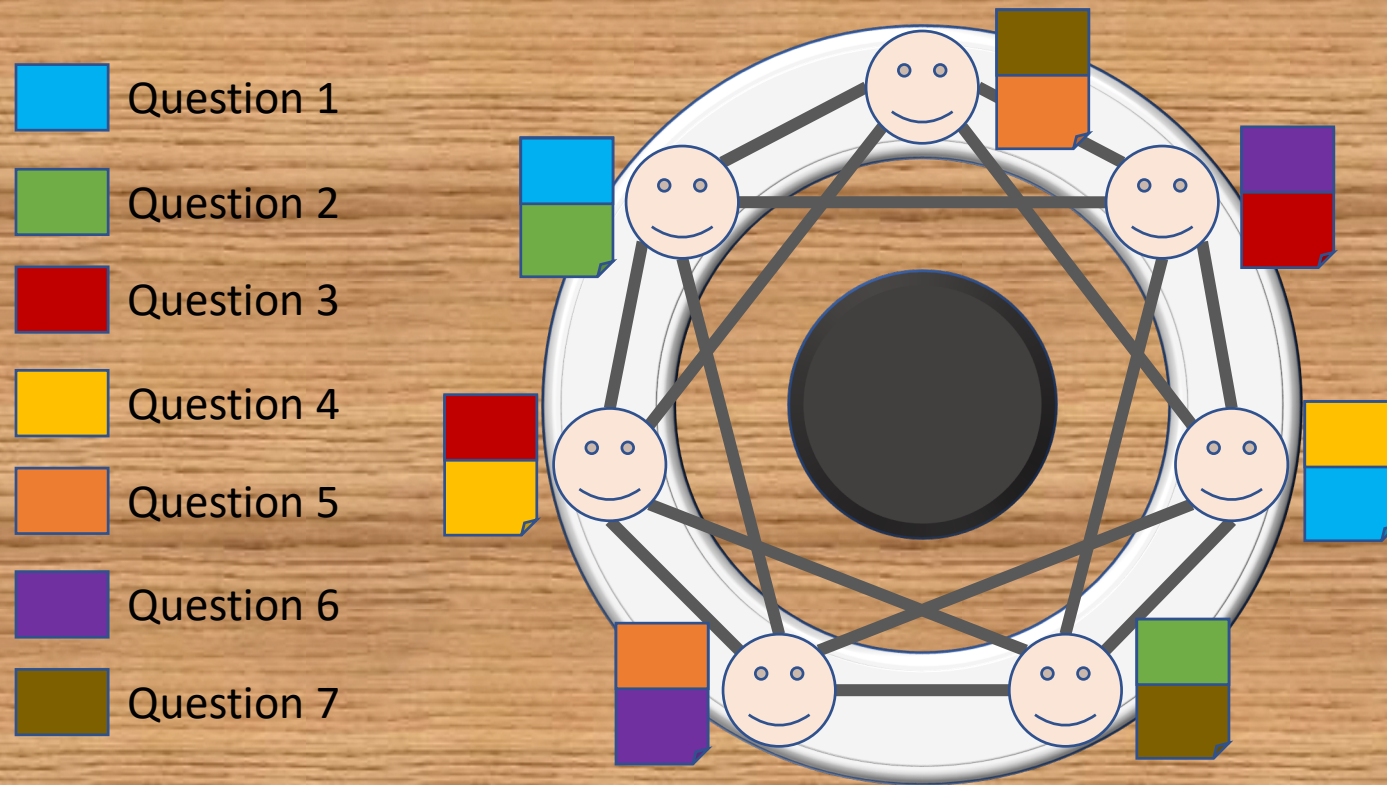
A cheating-proof test

Chromatic number: $\chi = 4$

-  Question 1
-  Question 2
-  Question 3
-  Question 4
-  Question 5
-  Question 6
-  Question 7



A cheating-proof test



Chromatic number: $\chi = 4$

Fractional chromatic number: $\chi_f = 7/2$



The (fractional) chromatic number

- The perfect tool in resource management

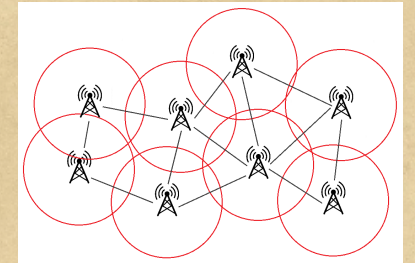
The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing



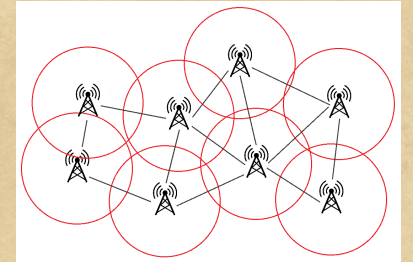
The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation



The (fractional) chromatic number

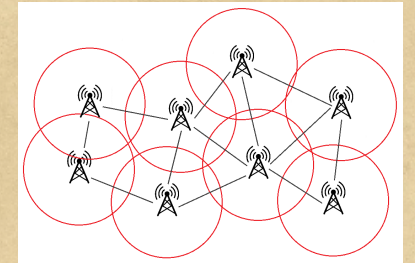
- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation
 - Time tables



The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation
 - Time tables
 - Many more...

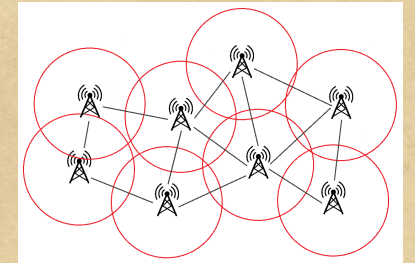
5	3			7			
6			1	9	5		
	9	8				6	
8				6			3
4			8		3		1
7				2			6
	6					2	8
			4	1	9		5
				8			7
						7	9



The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation
 - Time tables
 - Many more...
- Really hard to compute

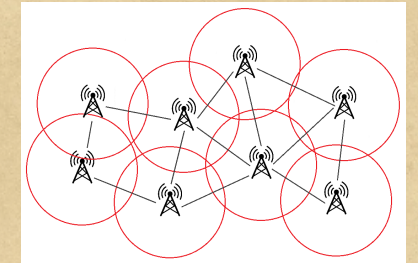
5	3			7			
6			1	9	5		
	9	8				6	
8				6			3
4			8		3		1
7				2			6
	6					2	8
			4	1	9		5
				8		7	9



The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation
 - Time tables
 - Many more...
- Really hard to compute
 - NP-hard

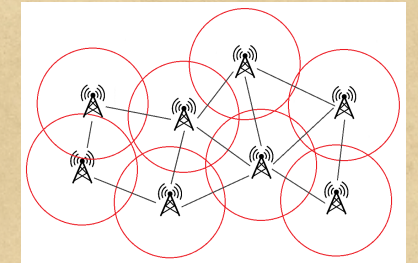
5	3			7			
6			1	9	5		
	9	8				6	
8				6			3
4			8		3		1
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			4	1	9		5
				8		7	9



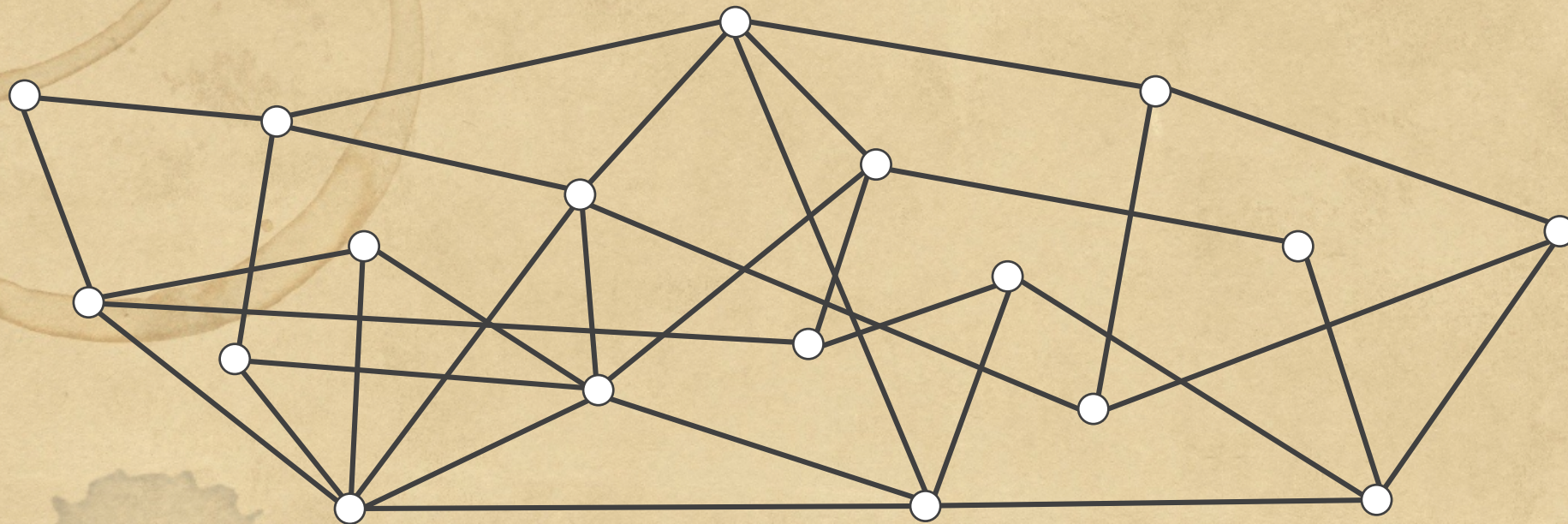
The (fractional) chromatic number

- The perfect tool in resource management
 - Parallel computing
 - Frequency allocation
 - Time tables
 - Many more...
- Really hard to compute
 - NP-hard
 - Hard to approximate

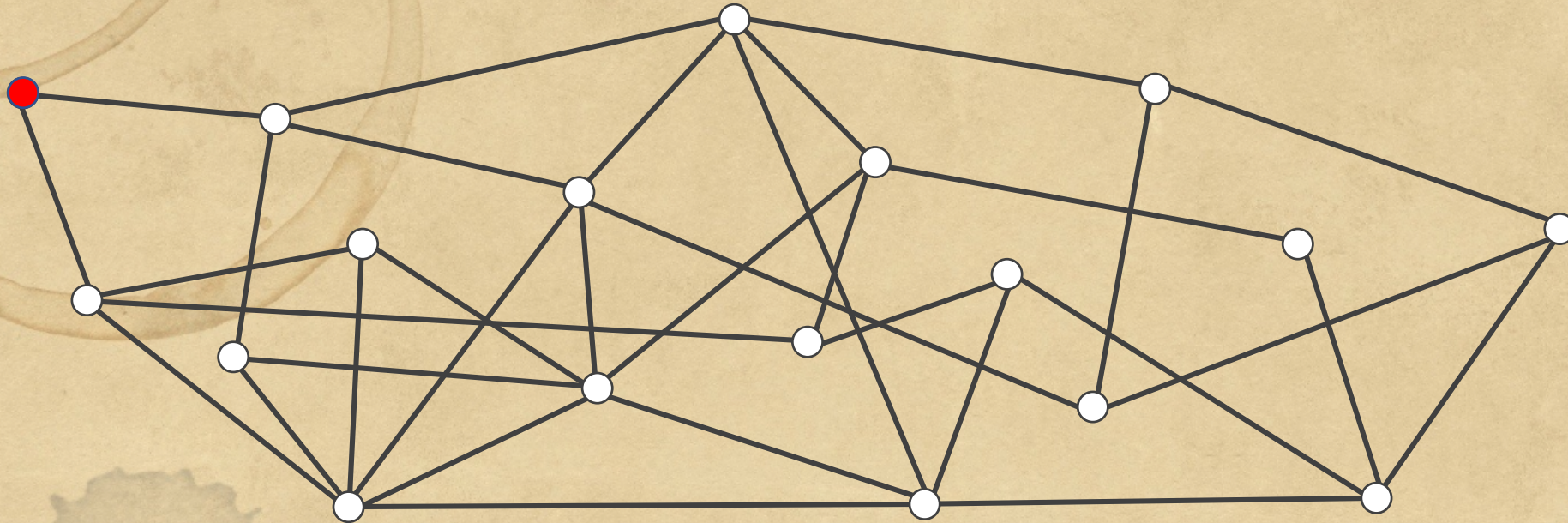
5	3			7			
6			1	9	5		
	9	8				6	
8				6			3
4			8		3		1
7				2			6
	6					2	8
			4	1	9		5
				8		7	9



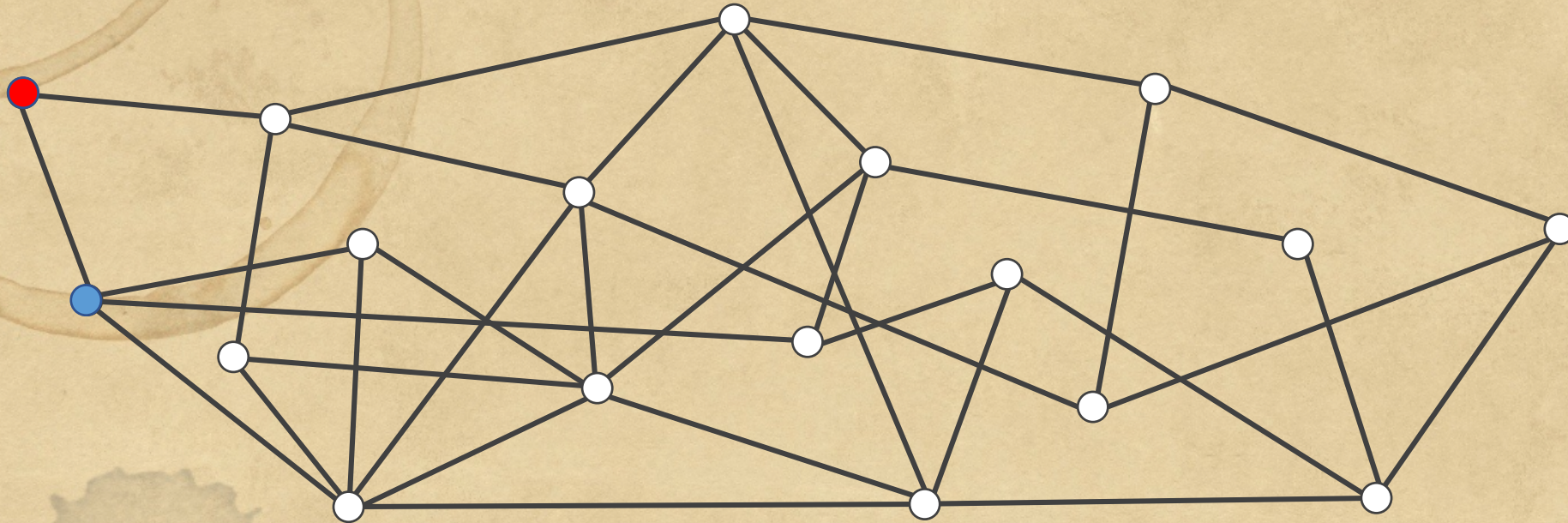
Greedy colouring



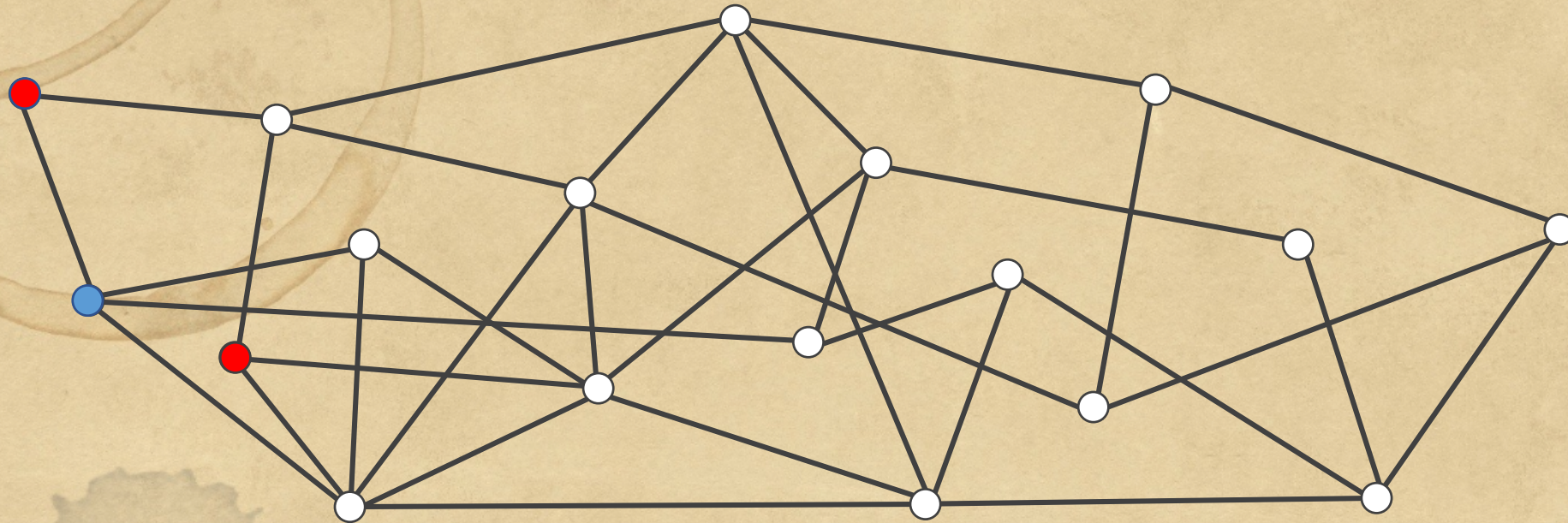
Greedy colouring



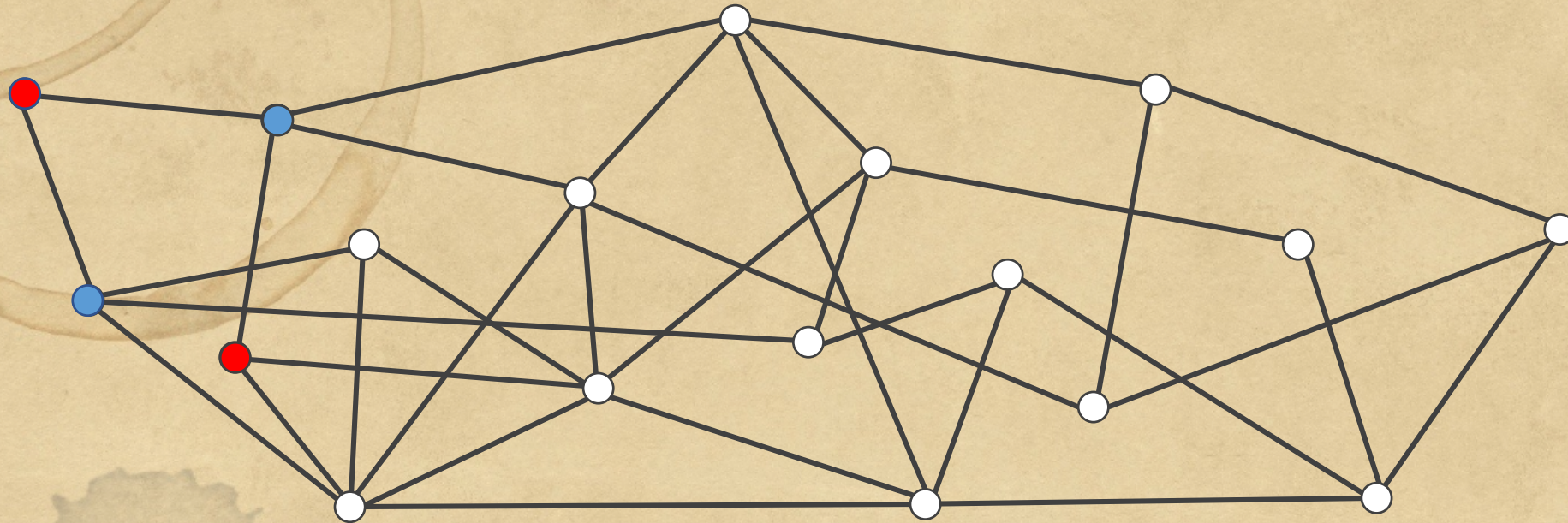
Greedy colouring



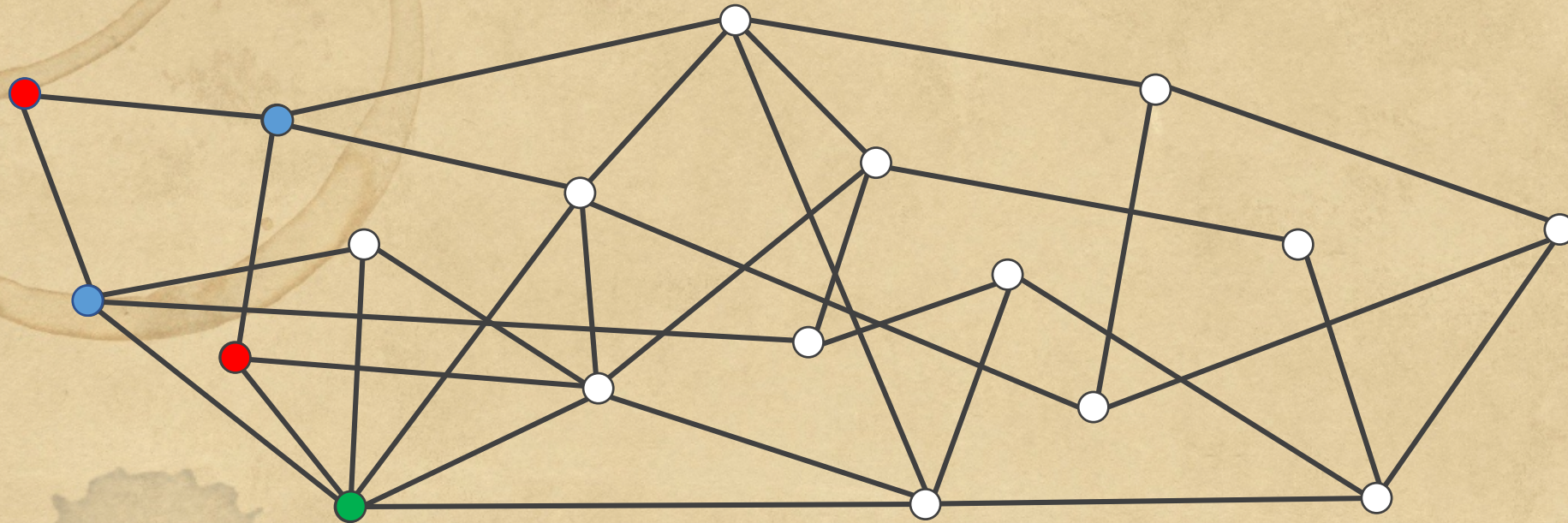
Greedy colouring



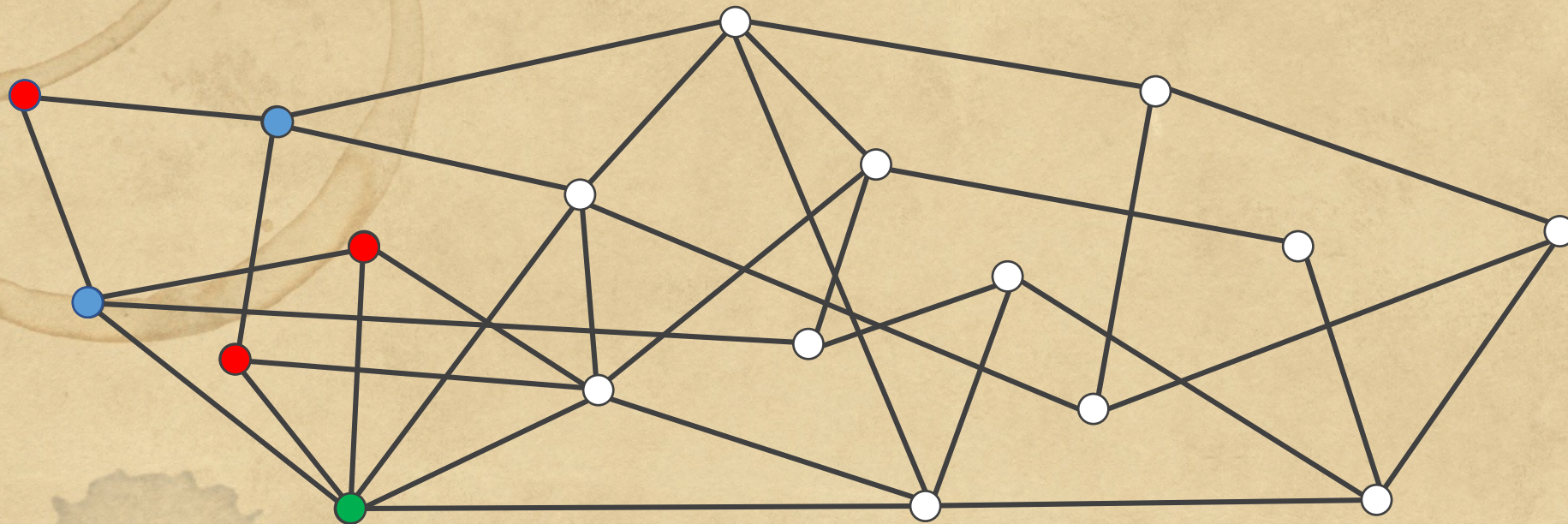
Greedy colouring



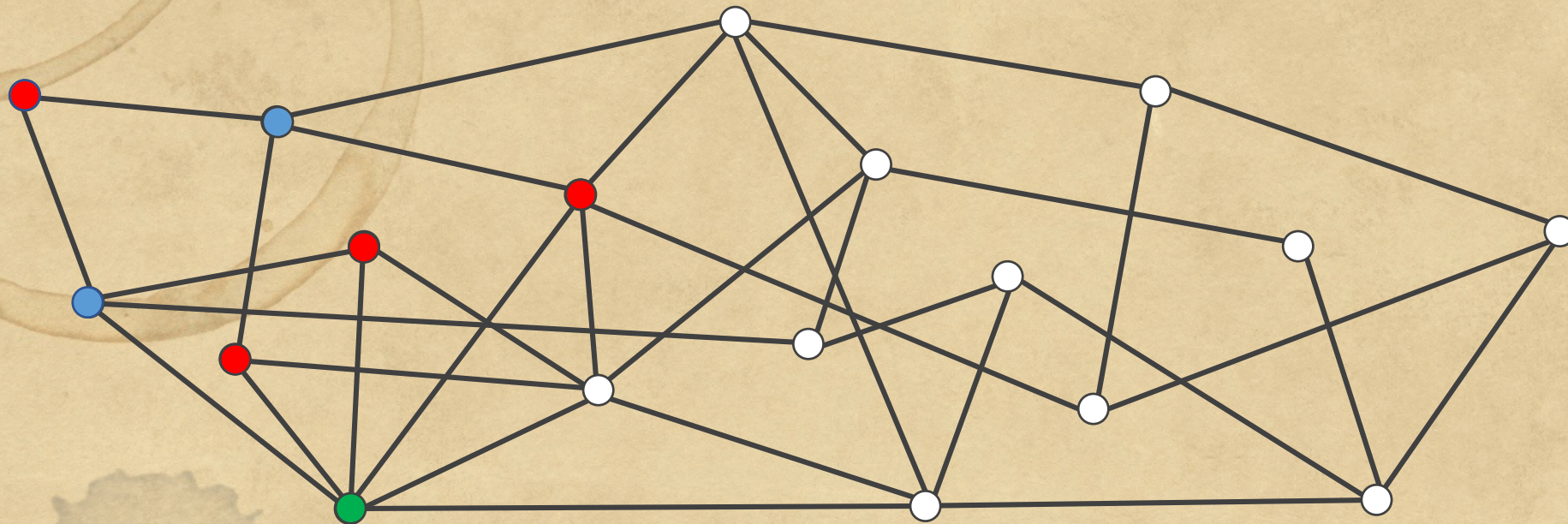
Greedy colouring



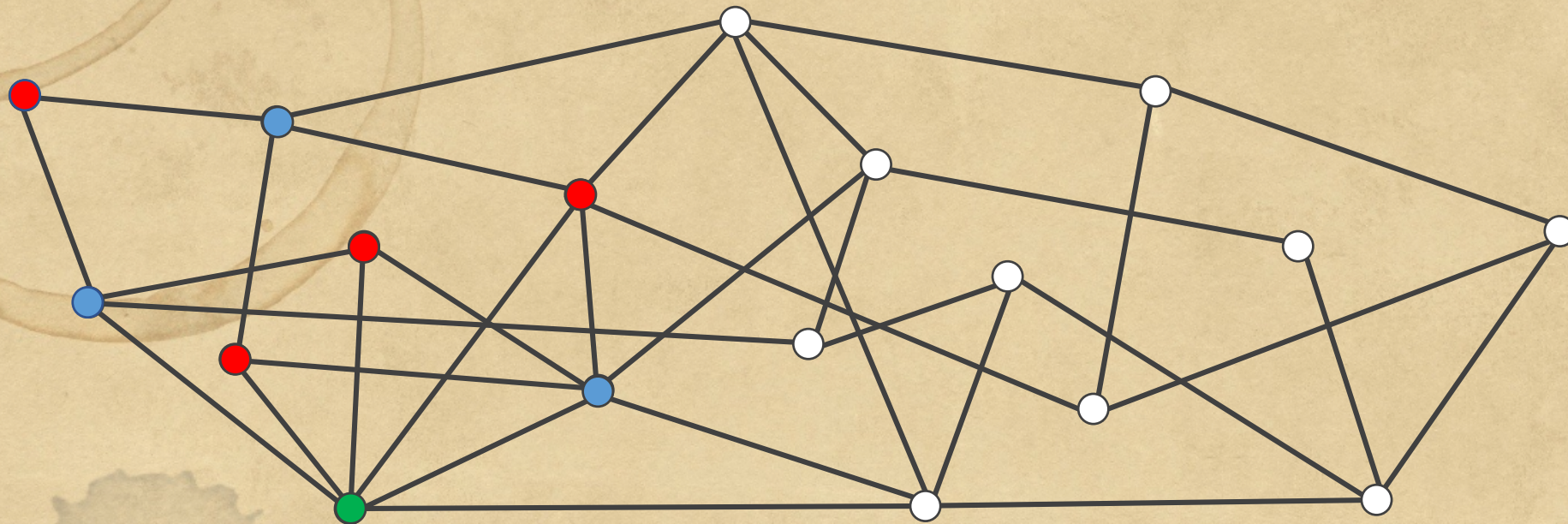
Greedy colouring



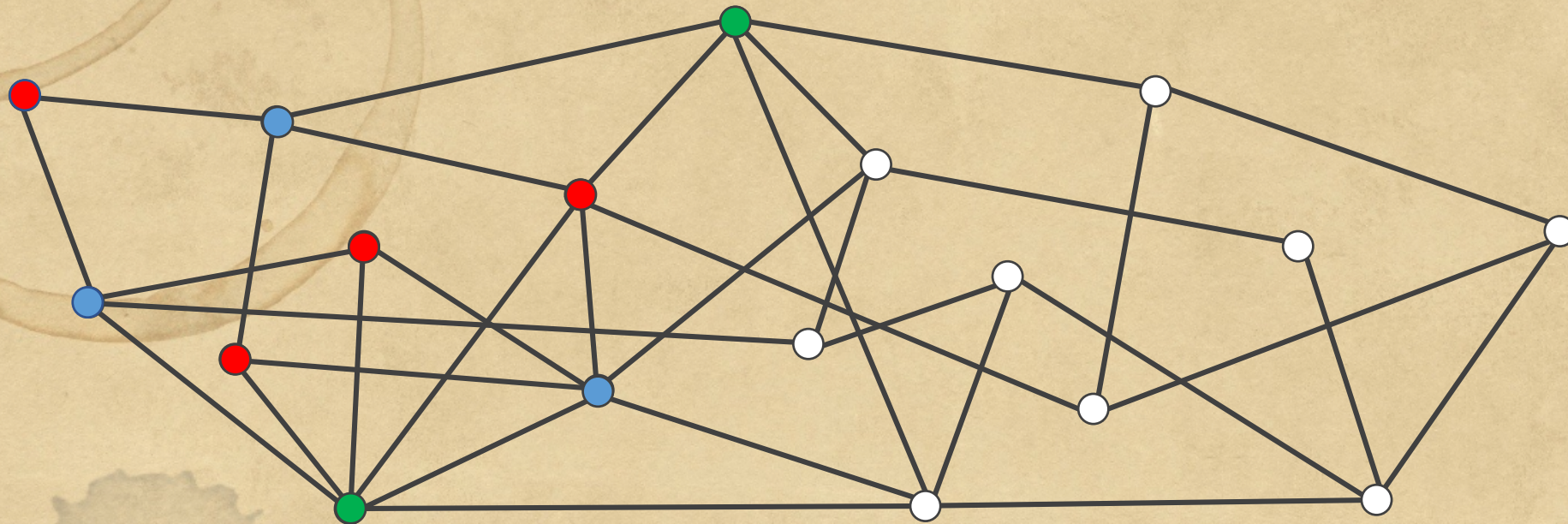
Greedy colouring



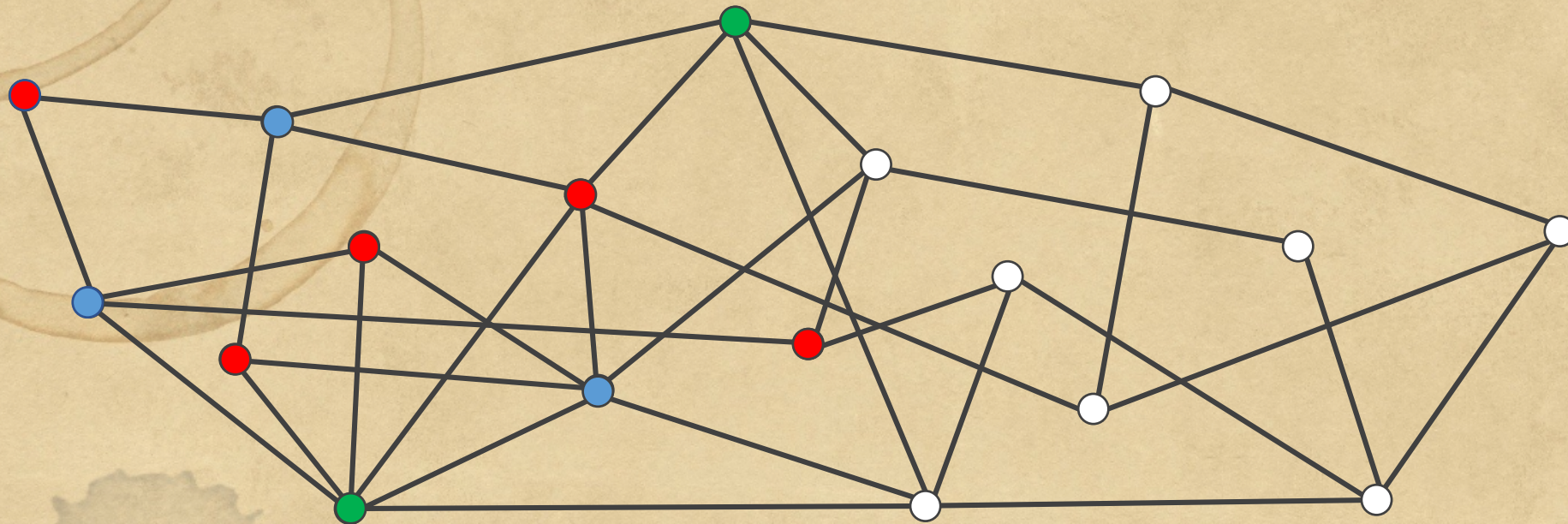
Greedy colouring



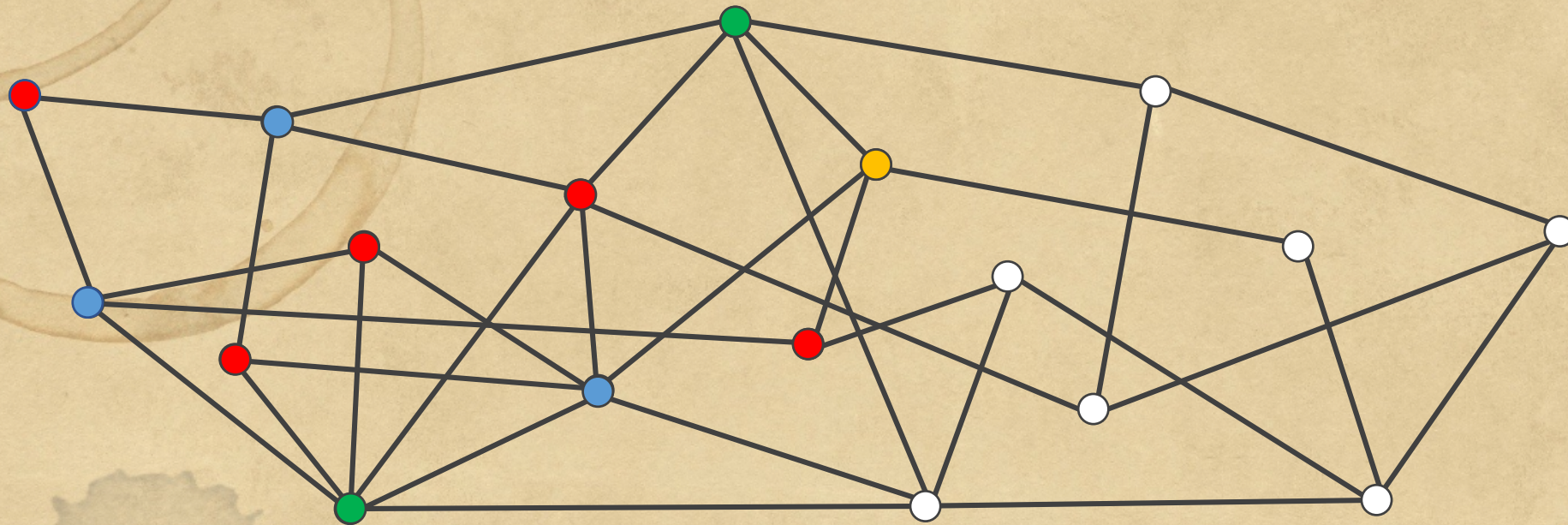
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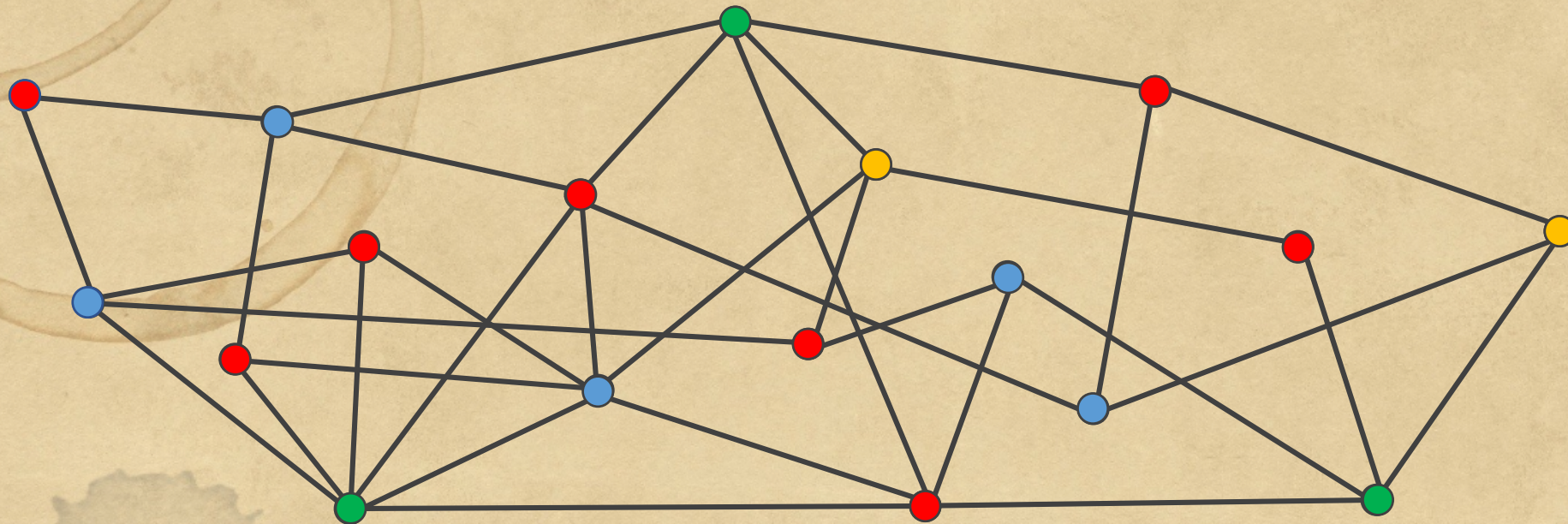
Greedy colouring



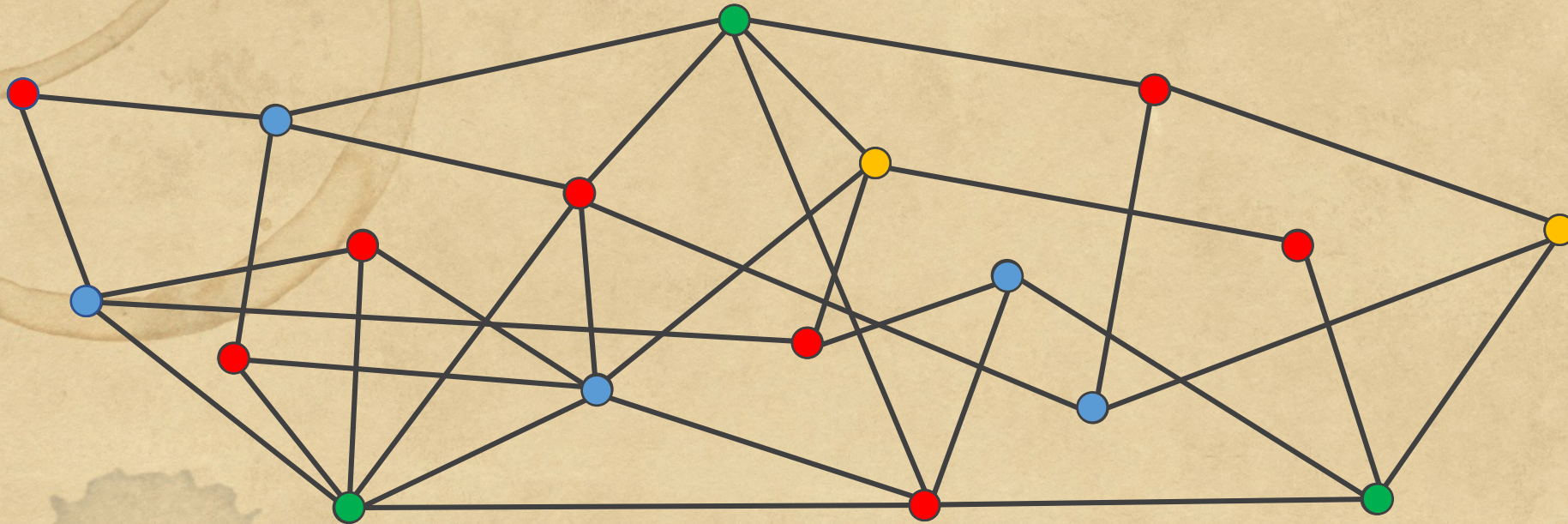
Greedy colouring



Greedy colouring

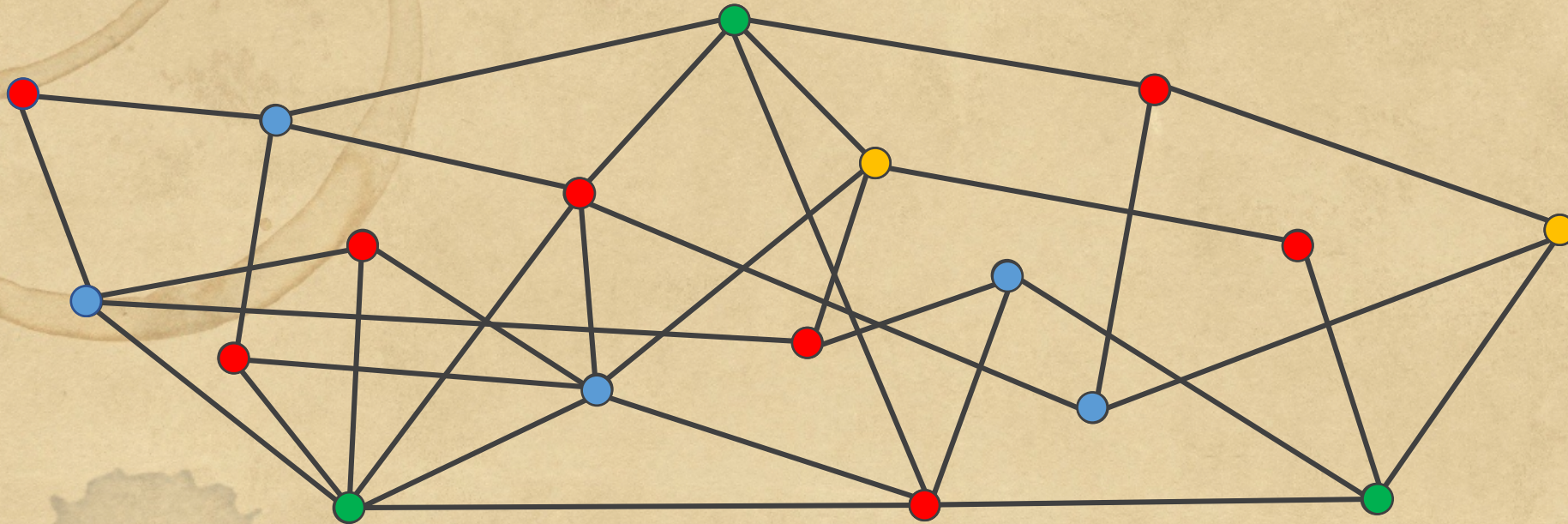


Greedy colouring

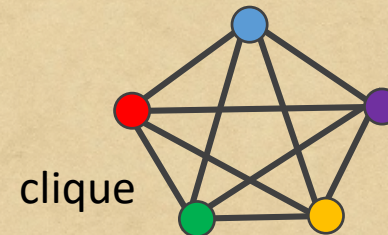


- Worst scenario: $\Delta + 1$ colours, where Δ is the maximum degree (here $\Delta = 6$).
- Sometimes this is best possible.

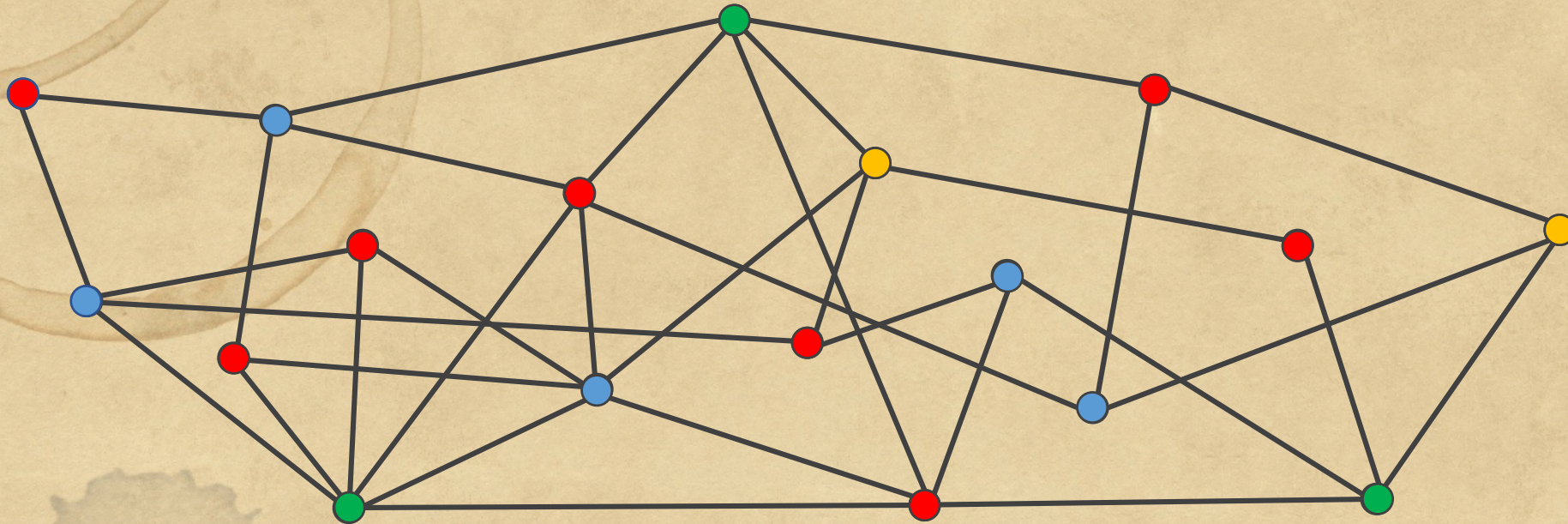
Greedy colouring



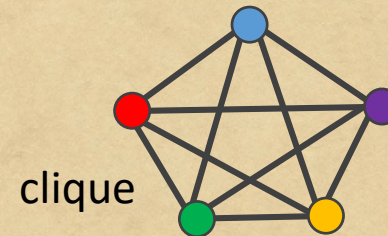
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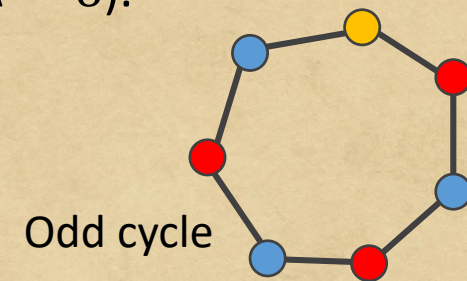
Greedy colouring



- Worst scenario: $\Delta + 1$ colours, where Δ is the maximum degree (here $\Delta = 6$).
- Sometimes this is best possible.



clique



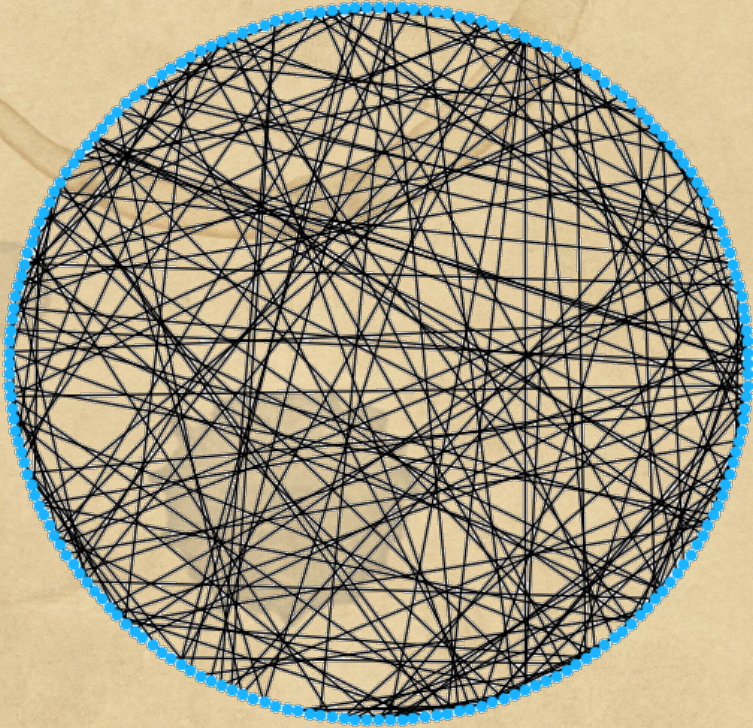
Odd cycle



Can we do (much) better?

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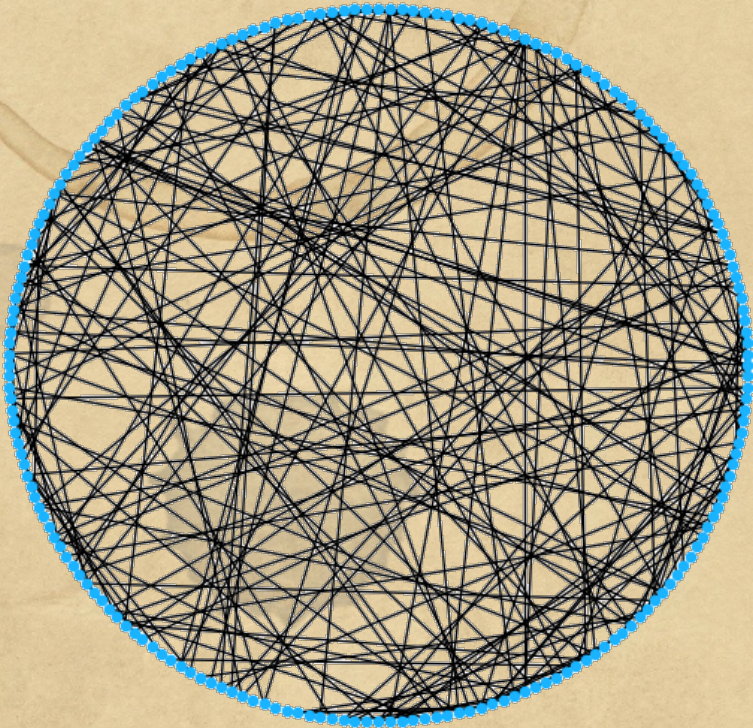
- Random graphs



Can we do (much) better?

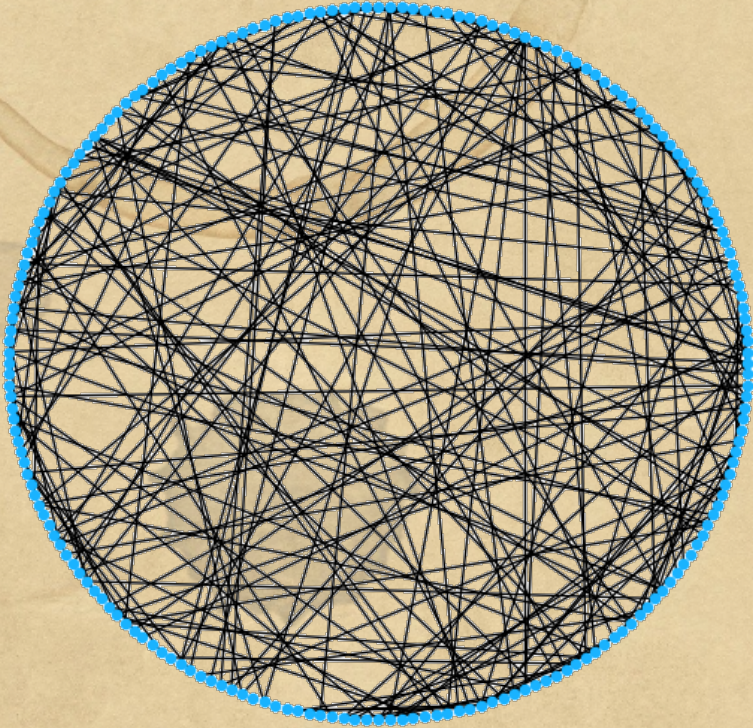
- Random graphs

- From local to global



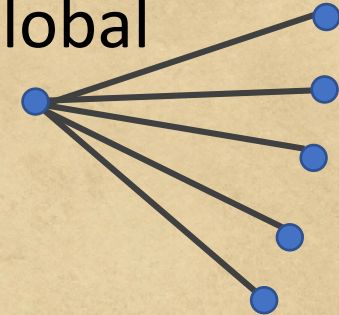
Can we do (much) better?

- Random graphs



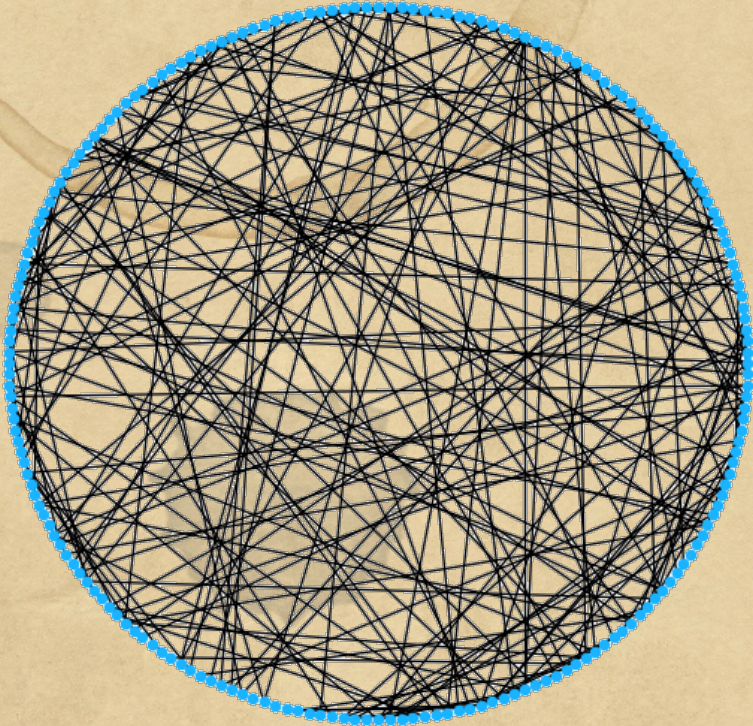
- From local to global

- No triangle.



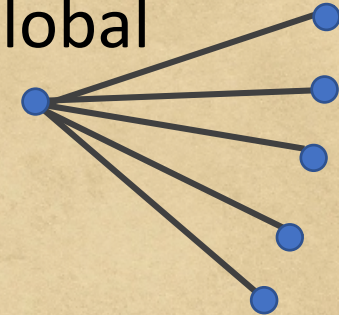
Can we do (much) better?

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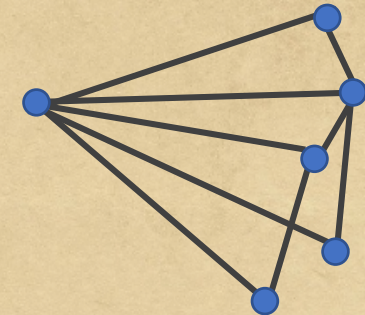


- From local to global

- No triangle.

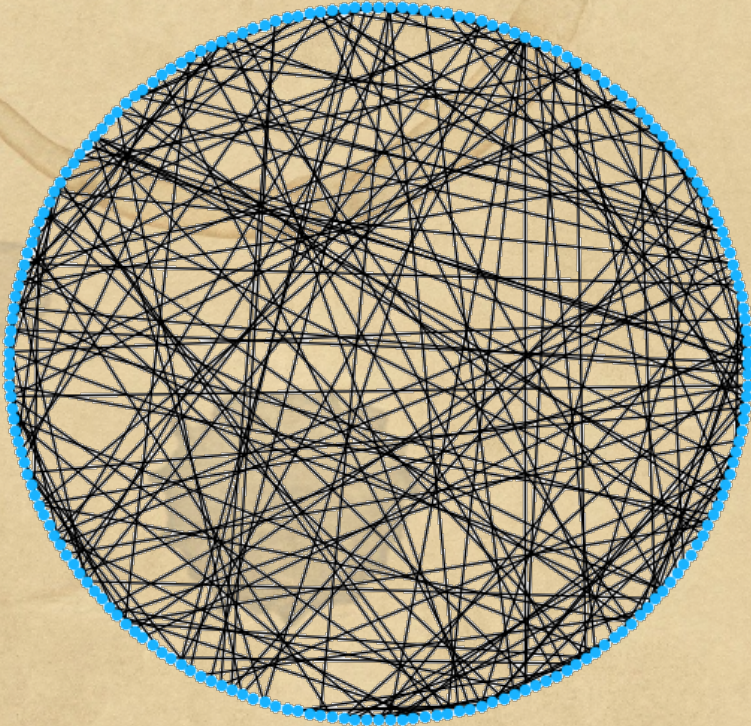


- Few triangles.



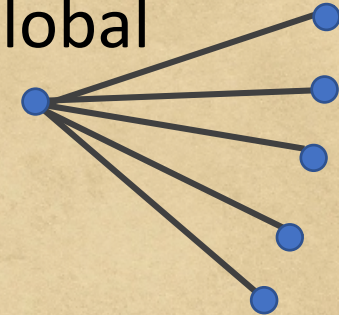
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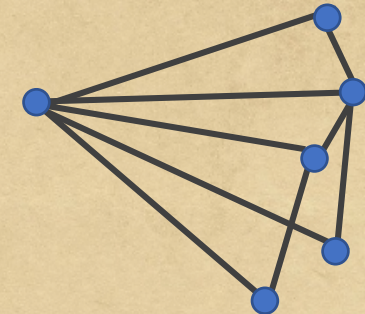


- From local to global

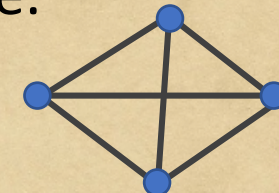
- No triangle.



- Few triangles.



- No large clique.



Thank you!

