Graph Algorithms

TD4 : Matchings

Throughout this TD, given a graph G, n is its number of vertices, and m its number of edges.

1 Consequences of Hall's Theorem

Let G be a d-regular bipartite graph (all degrees in G equal d).

1. Show that G contains a perfect matching.
2. Show that $\chi'(G) = d$.

2 Vertex Cover

Let G be a graph. We denote $\nu(G)$ the size of a maximum matching in G, and $\tau(G)$ the size of a minimum vertex cover of G.

1. Show that $\nu(G) \leq \tau(G) \leq 2\nu(G)$.
2. Write a polynomial algorithm that returns a 2-approximation of a minimal vertex cover of G.

3 More on Kőnig's Theorem

1. Prove that the following is an equivalent statement of Kőnig's Theorem. For every bipartite graph H on n vertices, $\alpha(H) = n - \nu(H)$.
2. Write an algorithm that returns a maximum independent set of any given bipartite graph. We suppose that we have access to an algorithm `maxMatching` that returns a maximum matching of any (bipartite) input graph on n vertices in time $O(n^{2.5})$.