Learning Shape Metrics based on Deformations and Transport

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Introduction

Motivation : shape metrics : needed for shape evolutions, shape matchings, shape priors...; how to choose **the right metric ?**

Aim : estimate a suitable metric automatically from a training set of shapes

Difficulties :

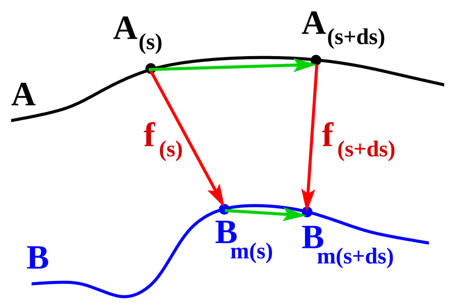
- sets of shapes : **high-dimensional** and **sparse** (human silhouettes $\geqslant$ 30 dim.)
- much **variability** : no meaningful *mean* shape
- probable deformations differ depending on the shape of interest
- no reliable matching between very different shapes; topological changes
- kernel methods : no explicit deformation priors + unaffordable density (high dim.)

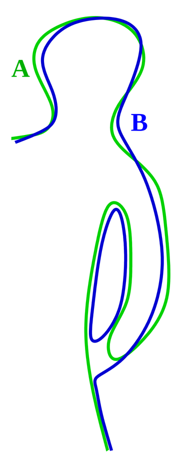
Method : search for the **optimal metrics**, based on:

- **matchings between close shapes**, possibly **topologically different**
- **transport**, from matchings, with **reliability weights**
- **increased density** with transported deformations
- **inner products fit empirical distributions** of deformations (local PCAs)
- regularizer : *shape* $\mapsto$ *metric* is **smooth** for Kullback-Leibler div.
- result is **global optimum of a criterion on metrics**

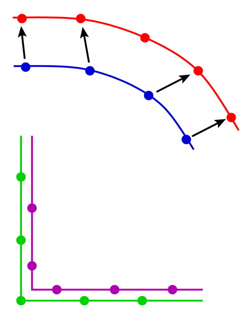
Matching close shapes

$$E_{match}(m) = \|B \circ m - A\|^2_{H^1_\alpha} + \gamma \|\angle(\mathbf{n}_A, \mathbf{n}_{B \circ m})\|^2_{L^2}$$





- Optimization : dynamic time warping
- Possible matching to $\varnothing$ (vanishing points)
- Oversampling of targets
- Convergence proof in the simple case when sampling rates get finer



Why over-sample targets.

Link between Kullback-Leibler and PCA

In the tangent space of one shape :

Empirical distribution of deformations : $\mathcal{D}_{emp} \;=\; \sum_j \; w_j \; \delta_{\mathbf{f}_j}$

possibly smoothed by a kernel : $\mathcal{D}^{\mathcal{K}}_{emp}(\mathbf{f}) \;=\; \sum_j \; w_j \, \mathcal{K}(\mathbf{f}_j - \mathbf{f})$

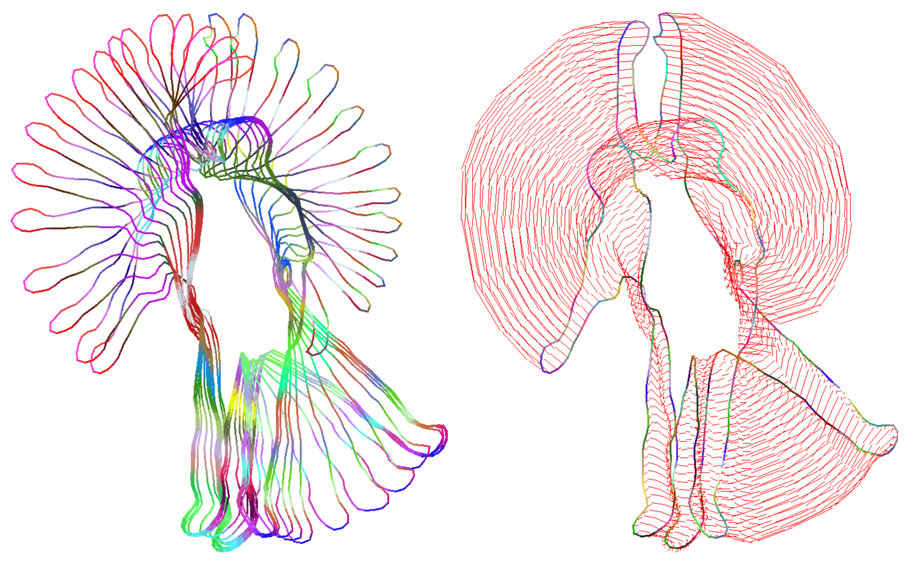
Given a inner product of reference P_0 (here H^1_α),

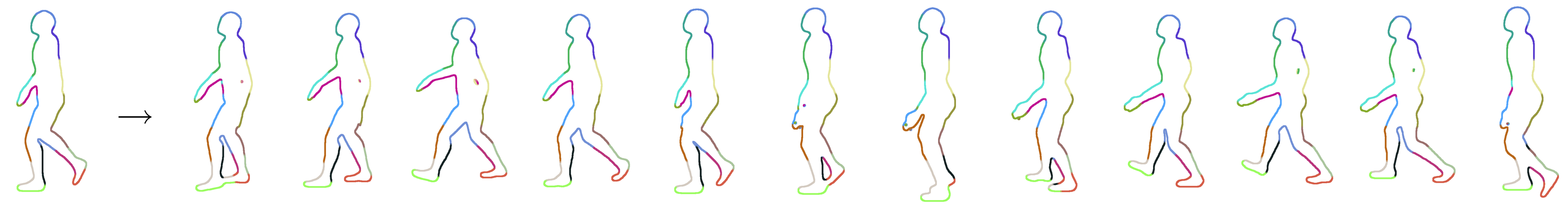
Inner product P ($\mathcal{C}^0$ wrt. P_0) = Gaussian distribution : $\mathcal{D}_P(\mathbf{f}) \;\propto\; e^{-\|\mathbf{f}\|^2_P}$

$\inf_P \; KL(\mathcal{D}_P | \mathcal{D}^{(\mathcal{K})}_{emp}) \quad \Longrightarrow \quad P_0$, weighted PCA$(\mathbf{f}_j, w_j)$

In case of $\mathcal{K}$, add second moment matrix of $\mathcal{K}$ to correlation matrix

Transport : information propagation





Set of shapes (S_i) $\Longrightarrow$ weighted graph

Pairwise matching costs : $C^m_{ij} = E_{match}(m_{i\to j})$

Local transport : $\forall\; h : S_j \to \mathcal{X}$,

$$T^L_{S_j \to S_i}(h) : S_i \to \mathcal{X} \;:\; s \mapsto h\left(m_{i\to j}(s)\right)$$

Reliability : $w^L_{ij} = e^{-\alpha_T C^m_{ij}}$

Optimal path from S_{i_0} to S_{j_0} : $(i_0, i_1, \dots, i_k = j_0)$

$$C^G_{i_0 j_0} = \textstyle\sum_n C^m_{i_n i_{n+1}}$$

Possibility of individualized paths $C^m_{ij}(h)$

Optimal transport : $T^G_{i_0 \to j_0} = T^L_{i_{k-1}\to j_0} \circ \dots \circ T^L_{i_1 \to i_2} \circ T^L_{i_0 \to i_1}$

Reliability : $w^G_{ij} = e^{-\alpha_T C^G_{ij}}$ (series of small reliable steps)

Inner products : deformation priors

Samples from the training set (video, 9s, 24Hz)

Local deformations : $\mathbf{f}_{i\to j} \;=\; S_j \circ m_{i\to j} - S_i$

Shape of interest : S_k

Transport : $\mathbf{f}^k_{i\to j} \;=\; T_{i\to k}(\mathbf{f}_{i\to j}), \quad w^k_{i\to j} \;=\; w^L_{ij}\; w^G_{ik}$

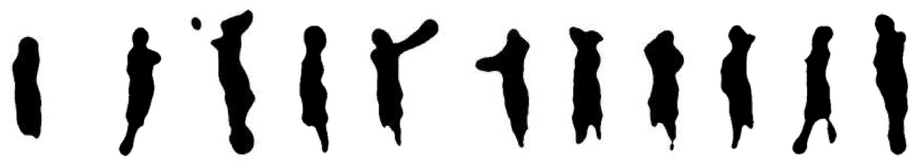
Modes from H^1_α, weighted PCA :

$$\inf_{\langle \mathbf{e}_n | \mathbf{e}_{n'} \rangle_{H^1_\alpha} = \delta_{n=n'}} \; \sum_{i,j} w^k_{i\to j} \left\| \mathbf{f}^k_{i\to j} - \sum_n \left\langle \mathbf{f}^k_{i\to j} \,|\mathbf{e}_n \right\rangle_{H^1_\alpha} \mathbf{e}_n \right\|^2_{H^1_\alpha}$$

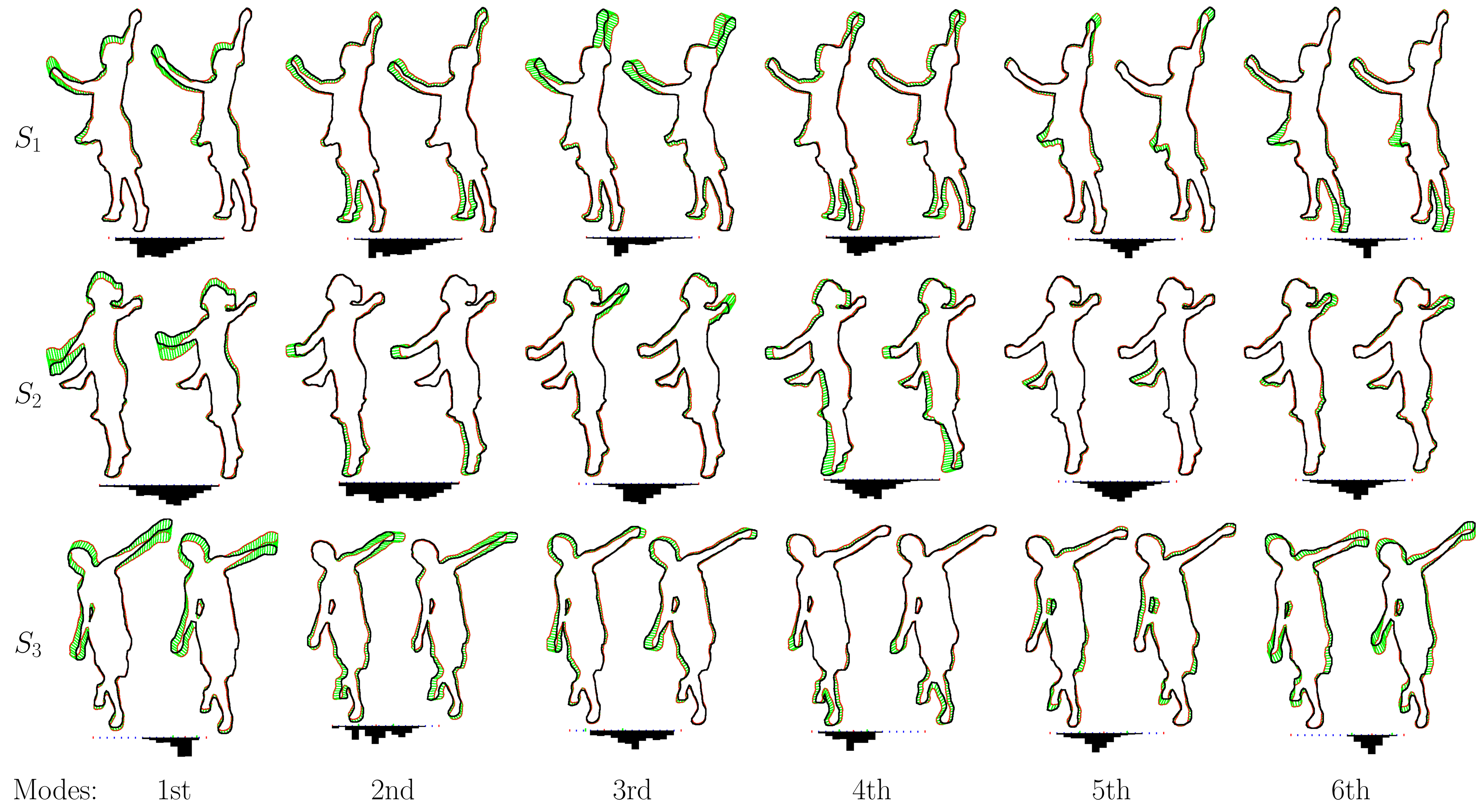
Modes $(\mathbf{e}_n, \lambda_n) \;\Longrightarrow\;$ inner product $\langle \cdot\,|\cdot \rangle_{P_k}$

Use of inner products as shape matching energy :

$$\inf_{m,\,\overrightarrow{a}} \;\; C(\overrightarrow{a}) + \frac{1}{\lambda^2_{\text{noise}}} \left\| B \circ m - A - \sum_n a_n \mathbf{e}_n \right\|^2_{H^1_\alpha}$$



Comparison: PCA on level-sets: mean + first modes



Optimum of a criterion on metrics

In general, **no best smooth direction field** (hairy ball theorem)

Criterion for a smooth metric: $\sum_{i,k} \; w^G_{ik} \;\; KL\left(\mathcal{D}_{P_k} \;\middle|\; T_{i\to k}(\mathcal{D}_{emp_i})\right)$

with $\mathcal{D}_{emp_i} = \sum_j \; w^L_{ij}\; \delta_{\mathbf{f}_{i\to j}}$ and $T_{i\to k}(\delta_{\mathbf{f}}) = \delta_{T_{i\to k}(\mathbf{f})}$

$= \sum_k KL(\mathcal{D}_{P_k} | \mathcal{D}^T_{emp_k})$, sum of independent terms, where $\mathcal{D}^T_{emp_k} = \sum_{i,j} w^k_{i\to j}\; \delta_{\mathbf{f}^k_{i\to j}}$

$\Longrightarrow$ our inner products

Criterion for smooth probability distributions:

$$\inf_g \;\; \sum_i \|g_i - g^0_i\|^2_{L^2(T_i)} \;+\; \sum_{ij} w_{ij} \,\|T_{i\to j}(g_i) - g_j\|^2_{L^2(T_j)}$$

with $g_i = d\mathcal{D}_{P_i}/d\mu$, $g^0_i = d\mathcal{D}_{emp_i}/d\mu$

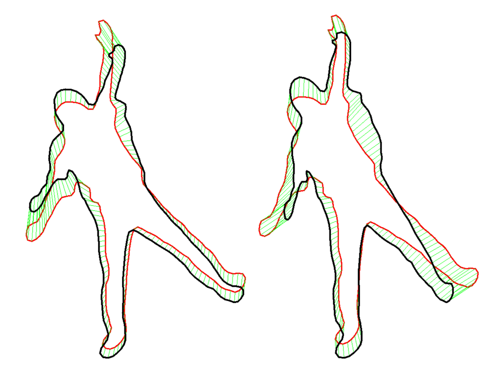
Inner products that best fit the optimal g $\;\Longrightarrow\;$ ours

Discussion

	require a mean	handle high variability	global coherency	handle sparse sets	explicit defor-mation prior	criterion on metrics
mean + modes (PCA)	yes	no	-	-	yes	yes
kNN + local PCAs	no	yes	no	no	yes	no
kernels on distances	no	yes	yes	no	no	yes
transport + KL reg.	no	yes	yes	yes	yes	yes

Conclusions :

- Transport = density enhancer, more representative neighborhoods
- Particular suitability to video datasets
- Link Kullback-Leibler $\Longleftrightarrow$ PCA



5NN and 10NN means