Shape Statistics for Image Segmentation with Prior

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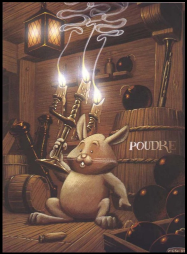
Introduction

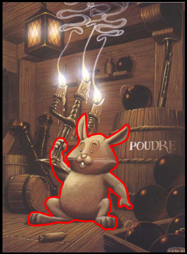
Image Segmentation

- Find a contour in a given image
- The best curve for a given segmentation criterion
- Criterion based on color homogeneity, texture, edge detectors, etc.



Image

$\longrightarrow$



Segmentation

Introduction

Image Segmentation

- Find the best contour for a given criterion

Variational Method

- Energy E to minimize with respect to a curve C
- Compute the derivative of the energy
- Gradient descent: $\partial_t C = -\nabla E(C)$
- Initialization $\rightarrow$ local minimum
- Other methods: graph cuts (suitable for few energies)

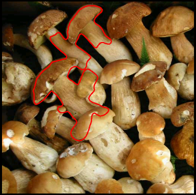
Introduction

Image Segmentation

- Find the best contour for a given criterion

Variational Method

- Minimize criterion by gradient descent with respect to the contour
- Most criteria: no shape information



Introduction

Image Segmentation

- Find the best contour for a given criterion

Variational Method

- Minimize criterion by gradient descent with respect to the contour

Shape Statistics

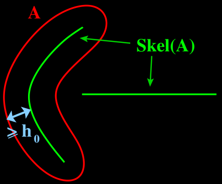
- Sample set of contours from already segmented images
- Shape variability ?
- Shape prior ?

Introduction
Shapes and Shape Metrics
Set of Shapes
Topological equivalence
Variational Shape Warping
Gradient Descent
Generalized Gradients
Approximation of the Hausdorff distance
Statistics
Mean and Modes of Variation
Examples
Images
Segmentation with prior
Shape prior (shape probability)
Starfish example
Boletus example

I - Shapes and Shape Metrics

Set of Shapes

- A shape: a smooth set of points in $\mathbb{R}^n$
- $\mathcal{C}^2$: seen as a function from its parameterization into $\mathbb{R}^n$
- $\mathcal{F}(h_0)$: distance to its skeleton $\geqslant h_0$ [D&Z]
  - curvature $\leqslant \kappa_0 = 1/h_0$
  - no double point: distance between two different parts $\geqslant h_0$



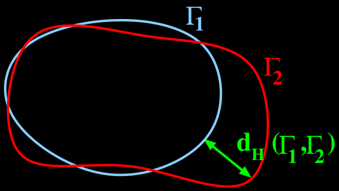
[D&Z]: M.C. Delfour & J.-P. Zolésio, *Shapes and Geometries*, 2000

Shape Metrics

- **Explicit**

$$d_H(\Gamma_1, \Gamma_2) \;=\; \max\left\{ \sup_{\mathbf{x}\in\Gamma_1} d_{\Gamma_2}(\mathbf{x}),\; \sup_{\mathbf{x}\in\Gamma_2} d_{\Gamma_1}(\mathbf{x}) \right\}$$

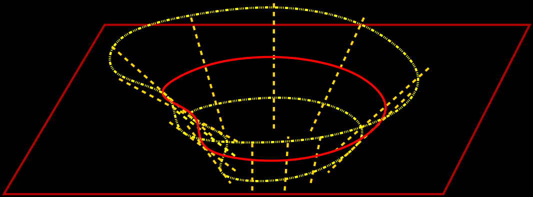
with
$d_{\Gamma_1}(\mathbf{x}) = \inf_{\mathbf{y}\in\Gamma_1} d(\mathbf{x},\mathbf{y})$



Shape Metrics

- Explicit - **Implicit**

$$d_{W^{1,2}}(\Gamma_1, \Gamma_2)^2 = \left\| \tilde{d}_{\Gamma_1} - \tilde{d}_{\Gamma_2} \right\|^2_{L^2(\Omega,\mathbb{R})} + \left\| \nabla \tilde{d}_{\Gamma_1} - \nabla \tilde{d}_{\Gamma_2} \right\|^2_{L^2(\Omega,\mathbb{R}^n)}$$



Shape Metrics

- Explicit - Implicit - **Path-based** [T&Y]

$$\underset{v,\ \substack{v(0,\cdot)=\Gamma_1 \\ v(1,\cdot)=\Gamma_2}}{\arg\min} \int_t \left\| \frac{\partial}{\partial t} v(t,\cdot) \right\|^2_{H^1(\Omega,\mathbb{R}^n)} dt$$

[T&Y]: All work by A. Trouvé & L. Younes

Topological equivalence

On the previous set of smooth shapes:

- Hausdorff distance
- L^2 or $W^{1,2}$ norm between the signed distance functions
- area of the symmetric difference

These metrics are topologically equivalent !

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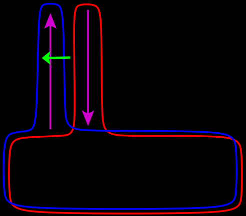
These metrics are topologically equivalent !

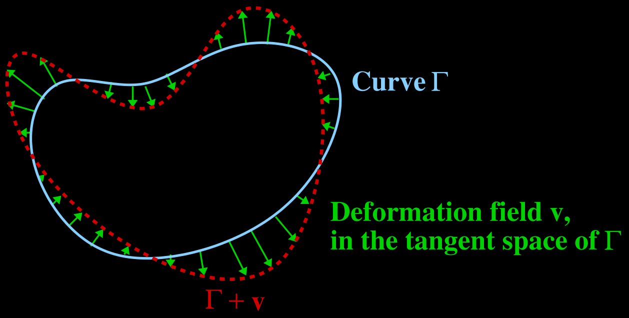
- Same notion of convergence
- Qualitatively different behaviour at greater scales
- Hausdorff distance: more geometrical sense



II - Variational Shape Warping

Shape Gradient

Directional derivative: $\mathcal{G}_\Gamma\left(E(\Gamma), \mathbf{v}\right) = \lim_{\varepsilon \to 0} \frac{E(\Gamma + \varepsilon\, \mathbf{v}) - E(\Gamma)}{\varepsilon}$



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Gradient: field ∇E, $\quad \forall\ \mathbf{v} \in F,\ \ \mathcal{G}_\Gamma\left(E(\Gamma), \mathbf{v}\right) = \langle \nabla E |\ \mathbf{v} \rangle_F$

Usual tangent space: $F = L^2$:

$$\langle f | g \rangle_{L^2} = \int_\Gamma f(\mathbf{x}) \cdot g(\mathbf{x})\ d\Gamma(\mathbf{x})$$

Gradient Descent Scheme

- Build minimizing path:

$$\Gamma(0) = \Gamma_1$$

$$\frac{\partial \Gamma}{\partial t} = -\nabla_\Gamma^F E(\Gamma)$$

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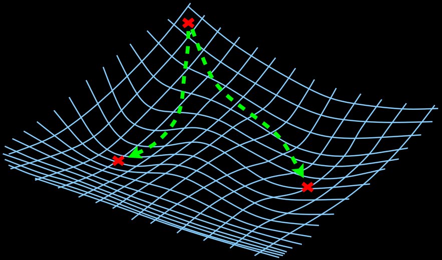

- Change the inner product $F \implies$ change the minimizing path

[C&P]: G. Charpiat, J.-P. Pons, R. Keriven & O. Faugeras, ICCV 2005
[SYM]: G. Sundaramoorthi, A.J. Yezzi & A. Mennucci, VLSM 2005
[T98]: A. Trouvé, IJCV 1998!

Gradient Descent Scheme

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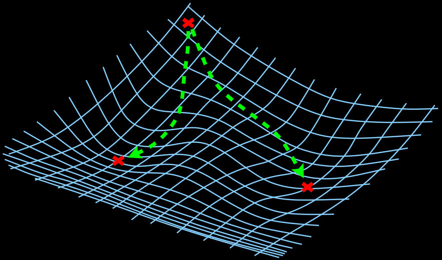


- Change the inner product $F \implies$ change the minimizing path
- $-\nabla^F_\Gamma E(\Gamma) = \arg\min_{\mathbf{v}} \left\{ \mathcal{G}_\Gamma\left(E(\Gamma), \mathbf{v}\right) + \frac{1}{2} \|\mathbf{v}\|^2_F \right\}$
- F as a prior on the minimizing flow

Generalized Gradients: Spatially Coherent Flows

- L^2 inner product

$$\langle f \,|\, g \rangle_{L^2} = \int_\Gamma f(x) \cdot g(x) \, d\Gamma(x)$$

Generalized Gradients: Spatially Coherent Flows

- L^2 inner product
- H^1 inner product

$$\langle f \,|g\rangle_{L^2} = \int_\Gamma f(x)\cdot g(x)\, d\Gamma(x)$$

$$\langle f \,|g\rangle_{H^1} = \langle f \,|g\rangle_{L^2} + \langle \partial_x f \,|\partial_x g\rangle_{L^2}$$

$$\nabla^{H^1} E = \arg\inf_u \left\| u - \nabla^{L^2} E \right\|^2_{L^2} + \|\partial_x u\|^2_{L^2}$$

Generalized Gradients: Spatially Coherent Flows

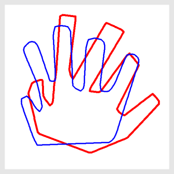
- L^2 inner product
- H^1 inner product
- Set S of prefered transformations (e.g. rigid motion)
  Projection on S: P
  Projection orthogonal to S: $Q \quad (P + Q = Id)$

$$\langle f \,|\, g \rangle_S \;=\; \langle P(f) \,|\, P(g) \rangle_{L^2} + \alpha \, \langle Q(f) \,|\, Q(g) \rangle_{L^2}$$

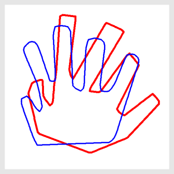
Generalized Gradients: Spatially Coherent Flows

- L^2 inner product
- H^1 inner product
- Set of prefered transformations (e.g. rigid motion)
- Example: two different warpings for the Hausdorff distance

$$\frac{\partial \Gamma}{\partial t} = -\nabla_\Gamma d_H(\Gamma, \Gamma_2)$$



usual



rigidified

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Generalized Gradients: Spatially Coherent Flows

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- Gaussian smoothing of the L^2 gradient: symmetric positive definite

Extension to non-linear criteria

- $-\nabla_{\Gamma}^{F} E(\Gamma) = \arg\min_{\mathbf{v}} \left\{ \mathcal{G}_{\Gamma}\big(E(\Gamma), \mathbf{v}\big) + \frac{1}{2} \|\mathbf{v}\|_{F}^{2} \right\}$

Extension to non-linear criteria

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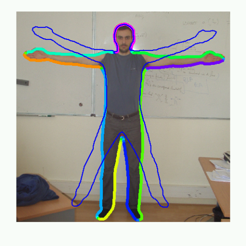
- Example: semi-local rigidification



$$w_x \colon y \in \Omega \quad \mapsto \quad A(x)(y - C(x))^\perp + T(x)$$

$$v(x) = w_x(x)$$

$$R(T, A, C) = \|v\|^2_{L^2} + \left\| \|D_x w_x(\cdot)\|_{L^2(\Omega)} \right\|^2_{L^2(\Gamma)}$$

Differentiable approximation of the Hausdorff distance

- Hausdorff distance:

$$d_H(\Gamma_1, \Gamma_2) = \max \left\{ \sup_{\mathbf{x}\in\Gamma_1} d_{\Gamma_2}(\mathbf{x}), \sup_{\mathbf{x}\in\Gamma_2} d_{\Gamma_1}(\mathbf{x}) \right\}$$

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- Replace $\sup_{\mathbf{x}\in\Gamma} f(\mathbf{x})$ by $\Psi^{-1}\left(\frac{1}{|\Gamma|} \int_\Gamma \Psi\Big(f(\mathbf{x})\Big)\, d\mathbf{x} \right)$
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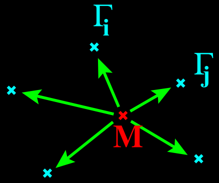
- In practice: $\Psi(a) = a^\alpha$. Similar trick for inf and max.
- The approximation tends to the Hausdorff distance.
- The approximation error can be expressed as an analytic function of the parameters.

III - Mean and Modes of Variation

- Previous framework: to warp a shape onto another one
- Given a set $(\Gamma_i)_{1 \leqslant i \leqslant N}$ of shapes: their mean M ?
- center of mass: M minimizes $\displaystyle\sum_{i=1,\cdots,N} d_H(M, \Gamma_i)^2$
- N fields $\beta_i = \nabla_M \left(d_H(M, \Gamma_i)^2 \right)$



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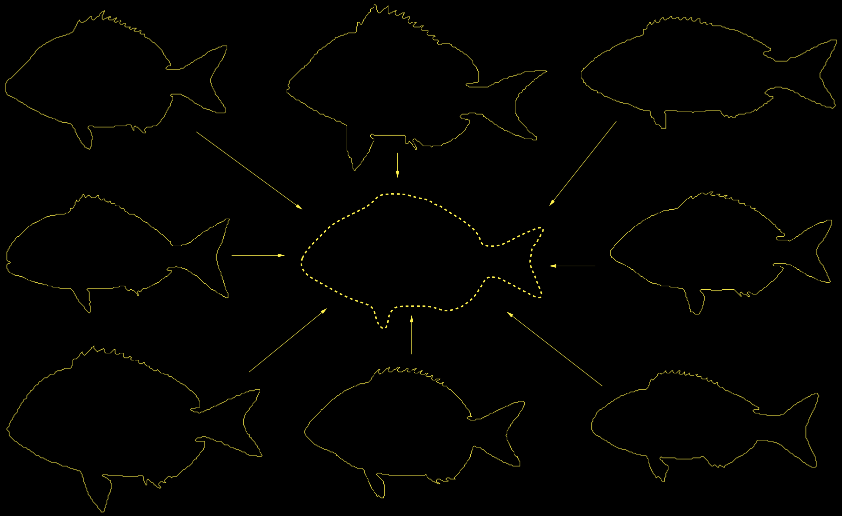
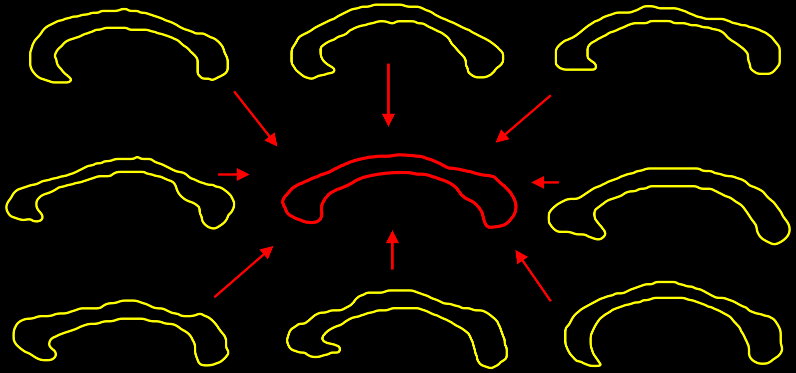
- N fields $\beta_i \; = \; \nabla_M \left(d_H(M, \Gamma_i)^2 \right)$
- Covariance matrix $\Lambda_{i,j} = \langle \beta_i | \beta_j \rangle_M$
- PCA on instantaneous deformation fields β_i:
  diagonalize $\Lambda \quad \Longrightarrow \quad$ characteristical modes m_k

Examples

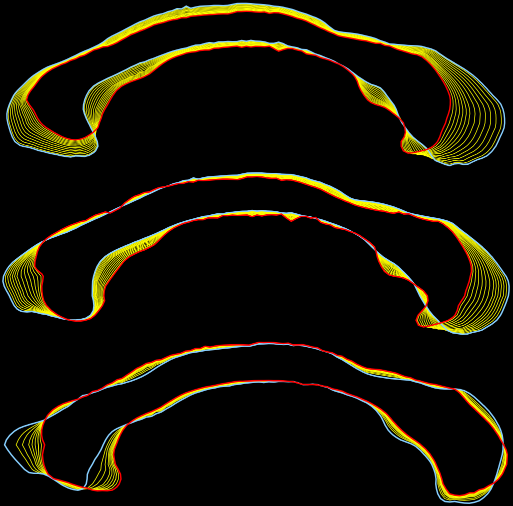


Mean of eight fish.

Example: set of 2D corpi callosi contours



First modes of deformation:



Images

- Same approach for a sample of images (instead of contours)
- Compute the mean and then statistics on deformation
- To each image I_i, associate a diffeomorphism h_i
- Warped images: $I_i \circ h_i$

I_i

$\xrightarrow{h_i}$

$I_i \circ h_i$

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I_i  $\xrightarrow{h_i}$  $I_i \circ h_i$

I_j  $\xrightarrow{h_j}$  $I_j \circ h_j$

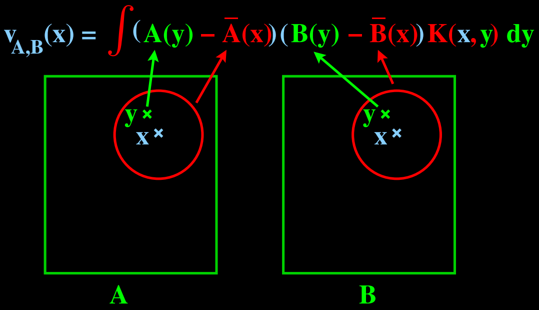
$$\left.\begin{matrix} I_i \circ h_i \\ I_j \circ h_j \end{matrix}\right\} LCC(I_i \circ h_i, I_j \circ h_j)$$

- Similarity between two images: $LCC(I_i \circ h_i, I_j \circ h_j)$ where:

$$LCC(A,B) = \int_\Omega \frac{v_{A,B}(\mathbf{x})^2}{v_A(\mathbf{x})\ v_B(\mathbf{x})}\, d\mathbf{x}$$

with

$v_A(\mathbf{x})$: local variance of A in a gaussian neighborhood of $\mathbf{x}$.



[HER]: G. Hermosillo, PhD Thesis, 2002

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with
$v_A(\mathbf{x})$: local variance of A in a gaussian neighborhood of $\mathbf{x}$.
- Find (multi-scale !) best diffeomorphisms which minimize
$$\frac{1}{n-1} \sum_{i \neq j} LCC(I_i \circ h_i, I_j \circ h_j) + \sum_k R(h_k)$$

Images

The first 5 images I_i.

The first 5 warped images $I_i \circ h_i$.

Images

The last 5 images.

The last 5 warped images.

Images

Raw mean (pixel by pixel) of the previous ten faces

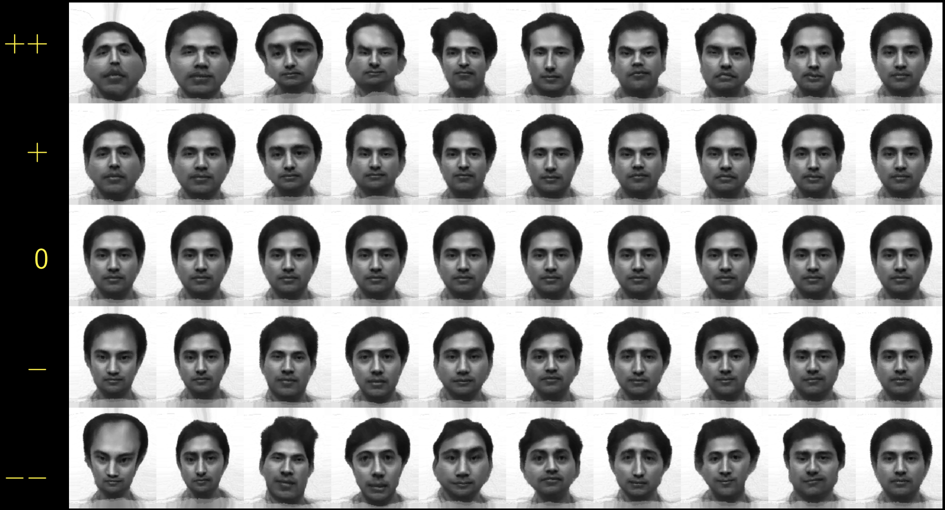
Mean of the previous warped ten faces

One of the ten faces

Characteristic modes of deformation:

- spatial modes (statistics on h_i)
- intensity modes (statistics on $I_i \circ h_i$)
- combined modes (both)

Images



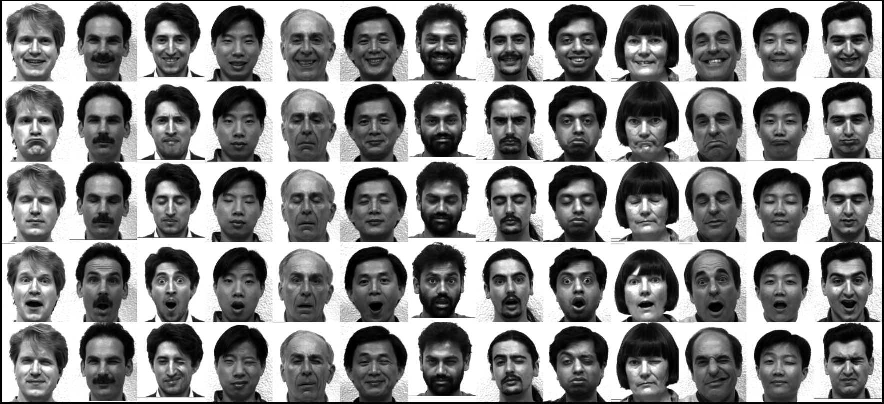
Characteristic modes of deformation (a column = a mode)

Images

Each column represents a mode, applied to their mean image with amplitude $\alpha = \{\sigma_k, -\sigma_k\}$.

Animations for the first two modes:

Expression recognition task



Expression recognition task (with J.-Y. Audibert):

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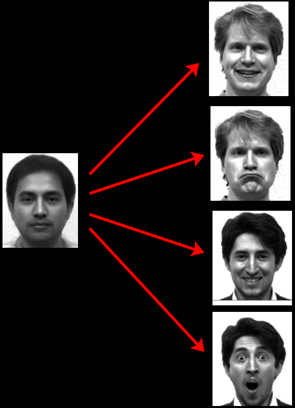

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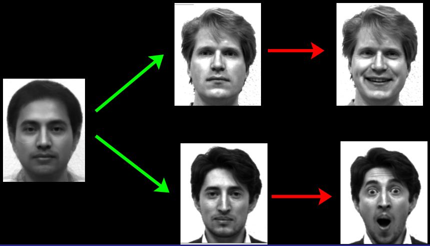
- SVM on diffeomorphisms from a new normal face to the same new face with expression (after alignment on the mean)



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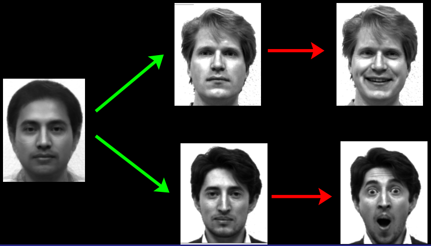

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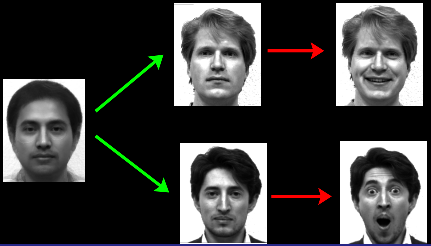

- cross-validation error: 12 on 65
- comparison: SVM on intensity variations between normal and expressive faces (without alignment): 17 errors

Images

Expression recognition mistakes are labeled.

IV - Image segmentation

Shape priors

- Rigid registration of the mean: no shape variability.

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Shape priors

- Rigid registration of the mean: no shape variability.
- PCA on signed distance function
  [LGF]: M. Leventon, E. Grimson & O. Faugeras, ICCV 2000
  [R&P]: M. Rousson & N. Paragios, ECCV 2002

IV - Image segmentation

Shape priors

- Rigid registration of the mean: no shape variability.
- Parzen method: $P(C) = \sum_i exp\left(-d(C, C_i)^2 / 2\sigma^2\right)$

  [CRE]: D. Cremers, T. Kohlberger & C. Schnörr, PR 2003

IV - Image segmentation

Shape priors

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- Parzen method on the fields $\alpha_i = -\nabla_M E^2(M, C_i)$
  $P(C) = P(\alpha) = \sum_i exp\left(-\|\alpha - \alpha_i\|^2_{L^2}/2\sigma^2\right)$

IV - Image segmentation

Shape priors

- Rigid registration of the mean: no shape variability.
- PCA on fields α_i: gaussian eigenmodes β_k
  $P(C) = P(\alpha) =$
  $$\prod_k exp\left(-\langle \alpha \,|\beta_k \rangle^2 /2\sigma_k^2\right) \quad \times exp\left(-\|N(\alpha)\|^2/2\sigma_n^2\right)$$

Shape prior (shape probability)

- Invariance to rigid motion:
  Maximization with respect to shape C and rigid motion R
  $$P\Big(R(C)\Big)$$

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  Maximization with respect to shape C and rigid motion R
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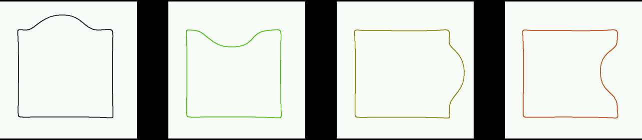
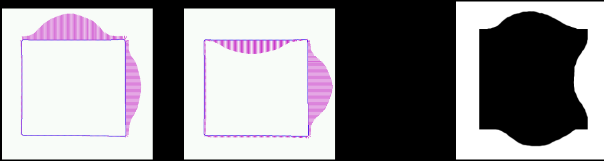
- Field priors require the computation of the second cross-derivative of the distance:
  $$\nabla_C \nabla_M E^2(C, M)$$

Shape prior (shape probability)

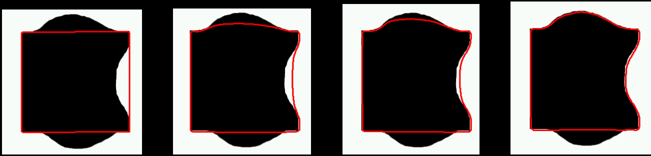
Toy example: learning set



Significant modes and new image



Segmentation with the Gaussian eigenmode prior

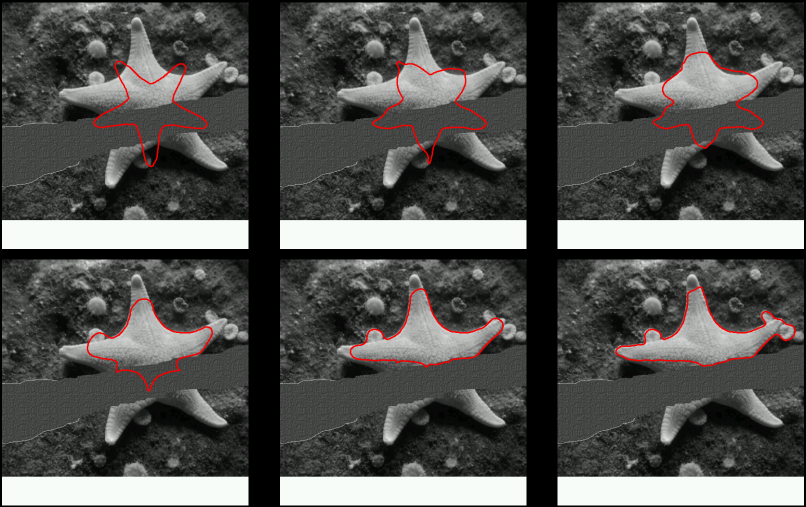


Learning set of 12 starfish

Starfish example

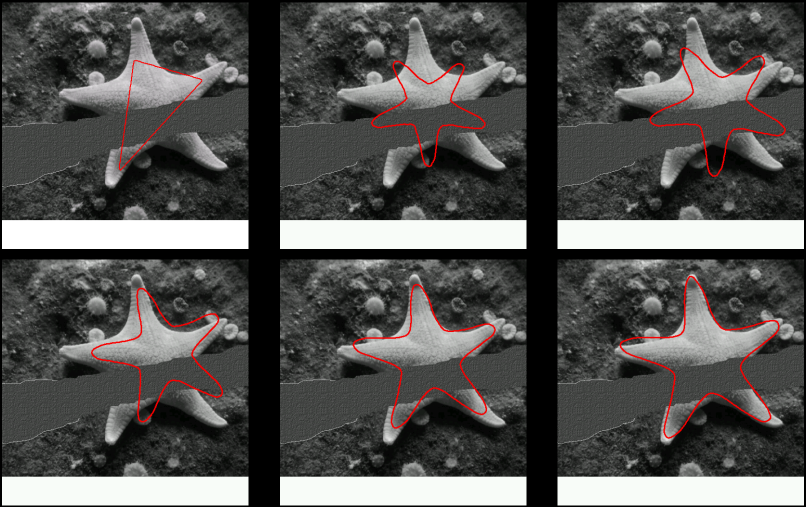
The mean of the set of starfish with its first six eigenmodes.

Starfish example



Segmentation without prior (intensity region histogram criterion).

Starfish example

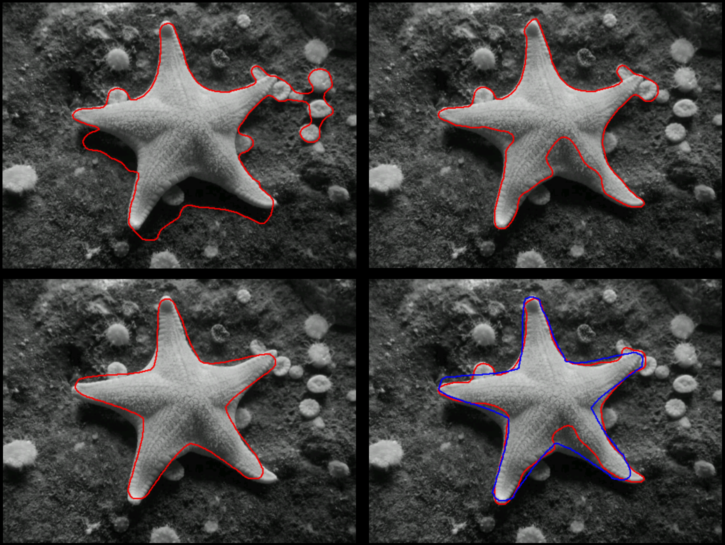


Rigid registration of the mean (same criterion).

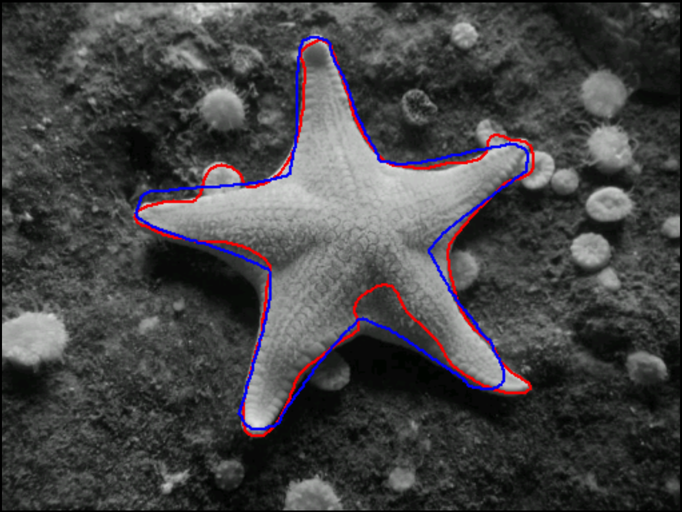
Starfish example

without shape prior (for two different initializations)

with the mean (without and with noise)

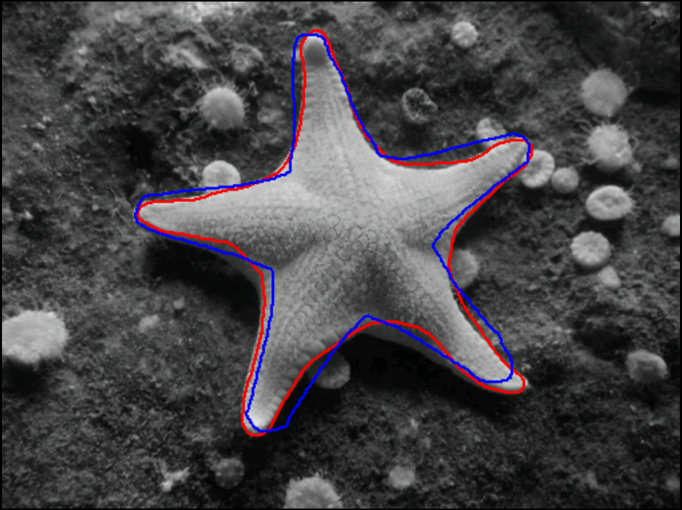


Starfish example



Mean (+ noise)

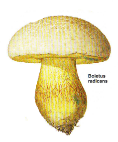
Starfish example



Mean + eigenmodes.

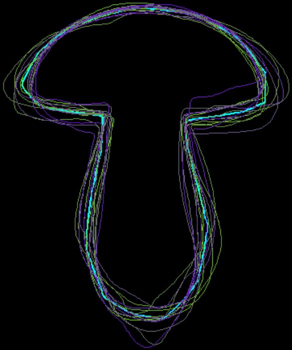
Boletus example

Some of the 14 mushrooms

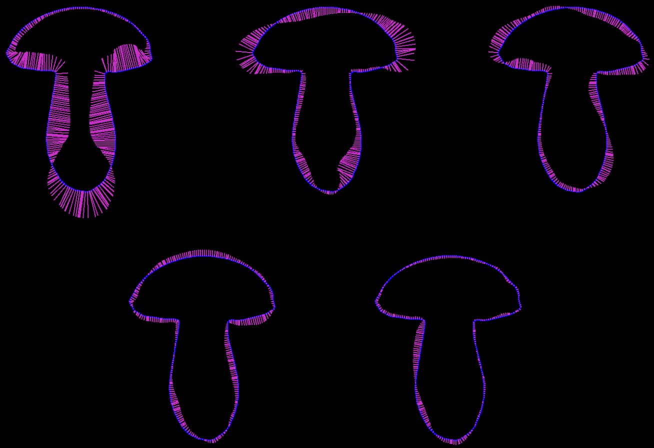


Automatic alignment while computing the mean

$\longrightarrow$



First modes

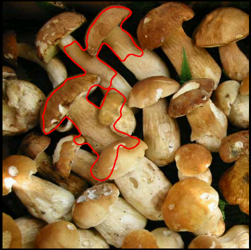


Boletus example

Segmentation task
(color region histogram criterion)

Initialization

Result:



without

with shape prior

Summary

- Set of shapes and shape metrics
  - Topological equivalence of usual metrics

References:

- *Approximations of shape metrics and application to shape warping and empirical shape statistics*, in *Foundations of Computational Mathematics*, Feb. 2005.

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- Warping via a gradient descent
  - Importance of the inner product (priors on minimizing flows)
  - Extension to non-linear priors

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- Gradient of the approximation of the Hausdorff distance vs. "gradient" of the distance itself

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- Hausdorff distance vs. kernel distances
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- Path-based distances vs. gradient of a distance with special inner products
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- Shape prior for segmentation vs. object detection
- Image classification vs. shape classification and image segmentation

Thank you for your attention !

References:

- G. Charpiat, O. Faugeras & R. Keriven, *Approximations of shape metrics and application to shape warping and empirical shape statistics*, in *Foundations of Computational Mathematics*, Feb. 2005.
- G. Charpiat, P. Maurel, J.-P. Pons, R. Keriven & O. Faugeras, *Generalized Gradients: Priors on Minimization Flows*, in *IJCV* (already online).
- G. Charpiat, O. Faugeras & R. Keriven (& J.-Y. Audibert), *Image Statistics based on Diffeomorphic Matching*, in *ICCV 2005*.