Deterministic Regular Expressions in Linear Time

reasoning about deterministic regular expressions without building the automata

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NICTA
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Context

DTD and XML Schema use regular expressions to specify which sequence of children may appear below a node.

```
<!ELEMENT book (author,chapter*,index?)>
```

Constraint: regular expression must be deterministic.

We provide new algorithms to:

- Check if a regular expression is deterministic.
- Decide the membership problem for deterministic regular expressions.

Outline

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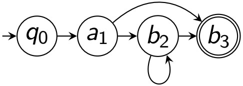
Structure of regular expressions

ab^*b

abb^*

Structure of regular expressions

$a_1 b_2^* b_3$



b_3 *follows* a_1, b_2 *follows* $a_1 \ldots$

$a_1 b_2 b_3^*$

Structure of regular expressions

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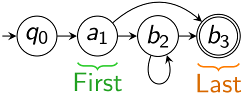

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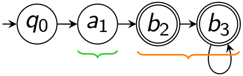
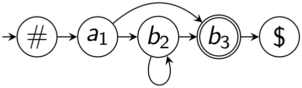


Structure of regular expressions

$\# a_1 b_2^* b_3 \$$



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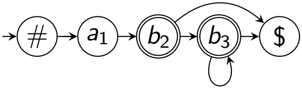
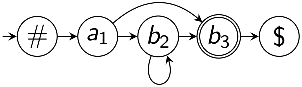


Deterministic regular expressions (a.k.a. one-unambiguous)

Expression is *non deterministic* if:

$$b_i \longrightarrow a_j \qquad a_k \qquad (j \neq k)$$

$\#a_1 b_2^* b_3 \$$
$\Rightarrow$ *non deterministic*



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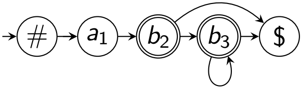

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Expression is *non deterministic* if:

$$\begin{tikzcd} b_i \arrow[r, bend left] \arrow[rr, bend left] & a_j & a_k & (j \neq k) \end{tikzcd}$$

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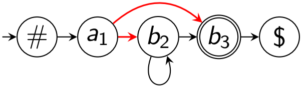

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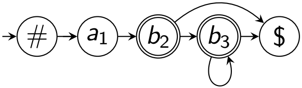

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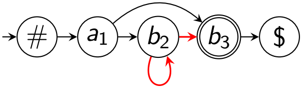

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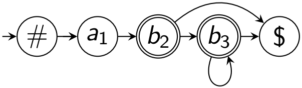

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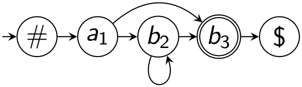
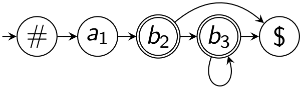


Ambiguity parsing $w = ab$

$\#a_1 b_2 b_3^*\$$
$\Rightarrow$*deterministic*



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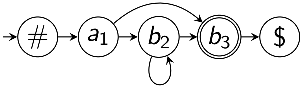

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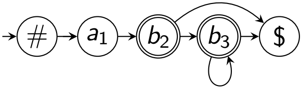

$e = (a+b)b?(ab)^* \quad ?$
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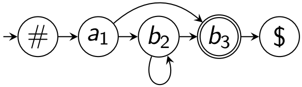

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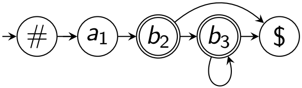

$e = (a + b)b?(ab)^*$ $\quad\Rightarrow$ *deterministic*
$e = (ab+ba?)^*$ $\quad\Rightarrow$ *non deterministic:* $w = ba$

Outline

Problems of interest

Testing determinism:

Input: expression e,
Question: is e deterministic?

Membership:

Input: word w, deterministic expression e,
Question: $w \in L(e)$?

Scenario: big expression, big alphabet.

Remark:

size of e $=$ number of nodes in the parse tree
$\simeq$ number of positions.

Problems of interest

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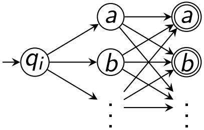
Straightforward solution through Glushkov automaton.
Build Glushkov in $O(|\Sigma| \times |e|)$ [Brüggeman-Klein TCS'93].
$\Longrightarrow$ (quadratic in $|e|$)

Number of transitions of Glushkov can be quadratic:

$e = (a + b + c \ldots)(a + b + c \ldots),$
$e' = (a + b + c \ldots)^*,$
$e'' = (a?b?c? \ldots)$



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Can we do better?

Summary

	Glushkov	Our results
testing determinism	$O(\|\Sigma\| \times \|e\|)$	$O(\|e\|)$
membership	$O(\|\Sigma\| \times \|e\| + \|w\|)$	$O(\|e\| + \|w\| \log\log(e))$
⋆ k-occurrence		$O(k \times \|w\|)$
⋆ restrictions on $+$		$O(\|e\| + \|w\|)$
⋆ star free		$O(\|e\| + \|w\|)$

This talk

Roadmap

Preprocessing

We do not construct automaton. Instead we work on parse tree which we preprocess in linear time to build some pointers+datastructures.

- **Testing determinism.**
  We search a witness for non-determinism in e: pair of two positions with same label that follow a common position.

$$\begin{tikzcd} b_i \arrow[r] \arrow[rr, bend left] & a_j & a_k \end{tikzcd} \qquad (j \neq k)$$

We limit the number of pairs examined to $O(|e|)$ using skeleta from [Bojańczyk and Parys JACM'11].

- **Testing membership.**
  We simulate transitions on-the-fly using the pointers from preprocessing.

Roadmap: transition simulation

We want a procedure for transition simulation:

Input:

a position a_i in the expression,

a letter b (the next letter of the word)

Output:

the unique b-labeled position that follows a_i

Outline

Some tools

LCA preprocessing [Harel&Tarjan, SICOMP'84]

One can preprocess a tree in linear time, to answer lowest common ancestor (LCA) queries and ancestor queries in constant time.

(Input for preprocessing: t)

LCA query:

Input: two nodes n_1, n_2 in t

Output: the lowest common ancestor (LCA) of n_1 and n_2

Ancestor query:

Input: two nodes n_1, n_2 in t

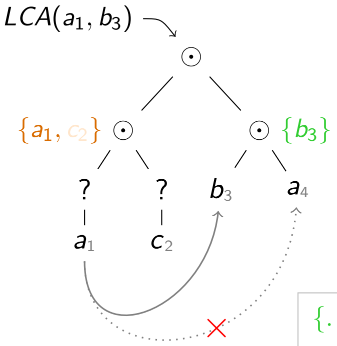
Question: is n_1 an ancestor of n_2?

Work on parse tree

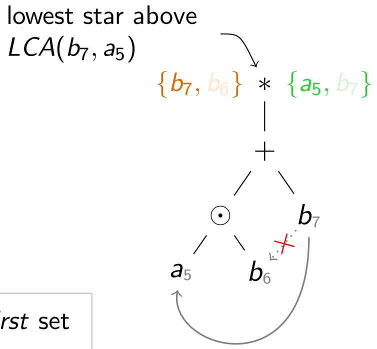
Goal: Given two positions a_i and b_j,
test if b_j follows a_i in constant time.

Work on parse tree: Follow

When does b_j follow a_i?



Case ❶ : $\odot$



Case ❷ : $*$

$\{\dots\}$	:	*First* set
$\{\dots\}$	:	*Last* set

Work on parse tree: Follow (2)

Assuming that after some preprocessing we can test *First* and *Last* in constant time,

Theorem

We can test if b_j follows a_i in constant time.

Work on parse tree: Follow (2)

Assuming that after some preprocessing we can test *First* and *Last* in constant time,

Theorem

We can test if b_j follows a_i in constant time.

Preprocessing:

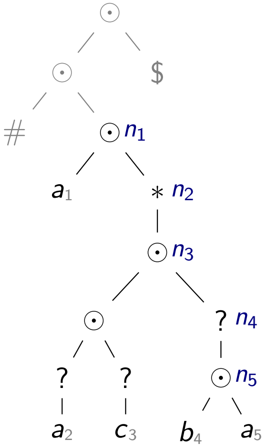
pointer to lowest $*$ ancestor of each node.
build LCA, *First* and *Last* structures.

Algo. to test if b_j follows a_i:

compute $LCA(b_j, a_i)$
follow the $*$ pointer (for case 2: $*$)
test that a_i, b_j in *First* and *Last* of appropriate nodes

New objective: test if $a_i \in First(n)$ in constant time

Work on parse tree: First and Last



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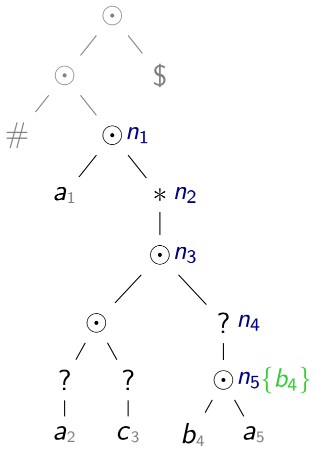
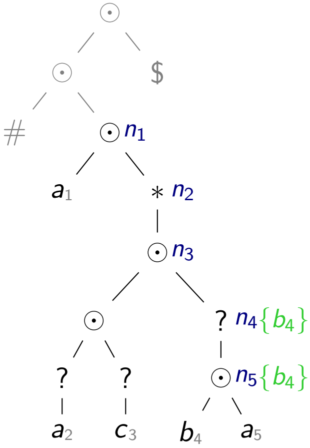


$\{a_2, \ldots\}$: *First* set

Work on parse tree: First and Last



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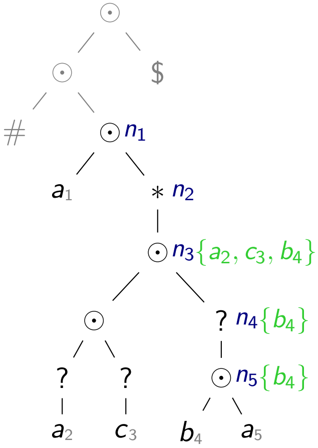

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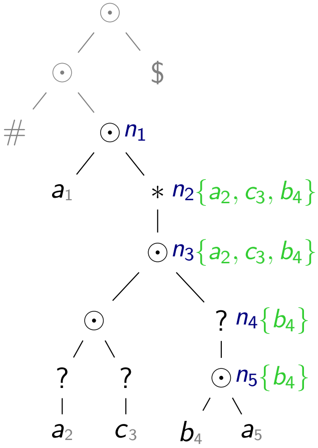

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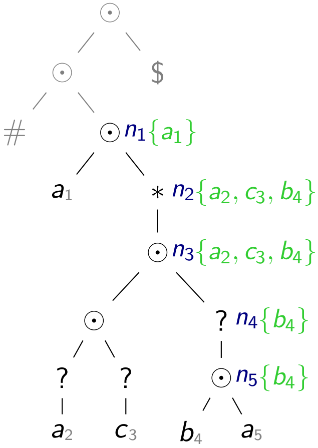

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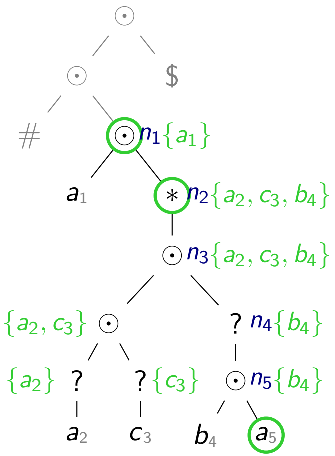

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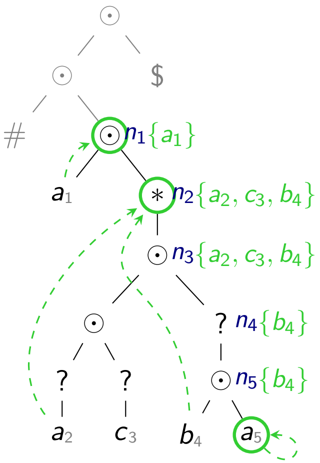
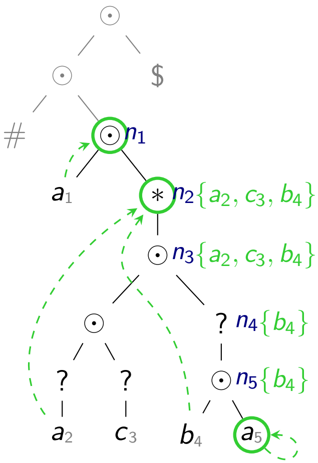


$\{a_2, \ldots\}$: *First* set

: a *SupFirst* pointer

Work on parse tree: First and Last

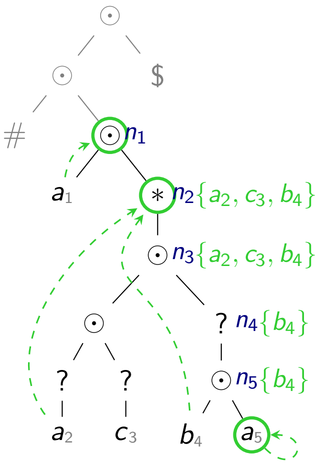


> We compute *SupFirst* pointers in simple traversal.

$b_4 \in First(n_3)$ because
 n_2 ancestor of n_3
 and n_3 ancestor of b_4.

$a_5 \notin First(n_3)$

Work on parse tree: First and Last



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✓ We can test if $a_i \in \mathit{First}(n)$ in constant time.

Symmetrically, we can test if $a_i \in \mathit{Last}(n)$ in constant time.

Reminder

Theorem

We can test if b_j follows a_i in constant time.

Outline

Simple case: k-occurrence expression

Theorem

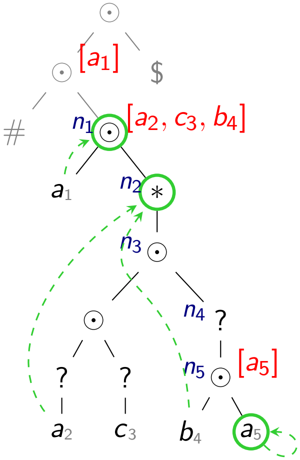
For deterministic k-occurrence expression, the membership problem can be solved in $O(k|w|)$ after $O(|e|)$ preprocessing.

each transition simulated in $O(k)$:

$$e = (ba)^* \boxed{c}\, acabcc?b$$

General case

We put color a in $parent(SupFirst(a_i))$ and store a_i as the witness for color a in that node.



OBSERVATION: positions followed by a_5 are below n_5, those followed by a_2 or b_4 are below n_1...

: a *SupFirst* pointer

$[a_5]$: map $a \mapsto a_5$ (color a, witness a_5)

Finding the right ancestor

We can use "nearest colored ancestor queries".

Nearest colored ancestor [Muthukrishnan et al. 96]

We can preprocess a tree t in expected linear time $O(|t|)$ to answer nearest colored ancestor queries in $O(\log \log |t|)$.

Expected time because of hashmaps, but becomes worst-case linear using lazy arrays.

Evaluation algorithm

Repeatedly jump to the nearest ancestor with color a, and test if its witness follows.

Why is it linear?

Use amortization argument

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$\star$ restrictions on $+$		$O(\|e\| + \|w\|)$
$\star$ star free		$O(\|e\| + \|w\|)$

This talk

Future work

- Close the $\log\log$ gap for membership.
- Searching regular pattern instead of matching (KMP...)

Thanks for your attention!

For a few dollars more...

Questions are most welcome!

...but there is no guarantee for the answer