Wavelengths assignment on a ring all-optical metropolitan area network*

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Abstract

We consider an all-optical network which topology is a ring interconnecting n nodes. Each node can be connected as sender and receiver on ℓ of the λ wavelengths of the ring. Each pair of nodes has to share at least β common wavelengths. The load of a wavelength is equal to the number of nodes connected to it.

In this paper, we give some bounds and some (near)optimal polynomial strategies for assigning wavelengths to nodes for given constraints n, ℓ, λ and β.

Keywords : *all-optical ring, wavelengths assignment, bounds, polynomial algorithms, combinatorial problems.*

1 Introduction

In the past few years, there has been growing interest in wide area "All-Optical Networks" because of the large bandwidth of optical fiber and the use of wavelength division multiplexing (WDM) [1, 4]. This is mainly due to the evolution of the WDM and Time-WDM transmission technologies proposed by many factories (Alcatel, Ciena, Fujitsu, Hitachi,..) [4, 8]. From their point of view, we will quickly be able to use optical channels with 128 wavelengths to obtain a bandwidth of many hundred Gbit/s, with a good ratio "signal over noise".

Thus, all-optical packet networks are an attractive possibility to conceive a telecommunication network made of a WAN connecting metropolitan area networks (MAN) (see european research projects SONATA, MEPHISTO, DAVID [5]). In many of these networks, the topology of the MAN is a ring, because of fault tolerance and communication properties [7].

Thus, in the model we consider in this paper, the MAN interconnects several rings (and connects the rings to the outside world through the WAN) on a same global node. Each ring serves several nodes, allowing them to send and receive packets on one or several wavelengths. We focus only on

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the point-to-point communication on the same ring. It is likely that a given node will not make use of the full capacity available, which implies that the Transmitters/Receivers of the node need not be fully equipped - which may be an appealing cost saving, especially during the early network deployment.

Now, for load balancing reasons (both in a ring, and among rings), it would be advisable to have a regular repartition of all available wavelengths among all nodes, such that each pair of nodes share at least one wavelength to exchange data. Thus, the problem we deal with here can be described as follows.

Wavelengths allocation problem description

Consider a network which topology is a directed ring with n nodes $\{d:0,\dots,d:n\}$. The number of available wavelengths on this ring is λ, denoted $\mathcal{L}=\{l:0,\dots,l:\lambda\}$. The number of transmitters and receivers in each node is equal to ℓ. Each pair transmitter-receiver is connected to one same wavelength. A wavelength assignment configuration C is given by the discrete variables $t_{i,j}$ and $r_{i,j}$, with $1\le i\le n$ and $1\le j\le\lambda$ defined as follows : $t_{i,j}=1$ (resp. $r_{i,j}=1$) if the node i is able to send (resp. receive) packets on wavelength j and 0 otherwise. Thus, by definition $\sum\limits_j t_{i,j}=\sum\limits_j r_{i,j}=\ell$.

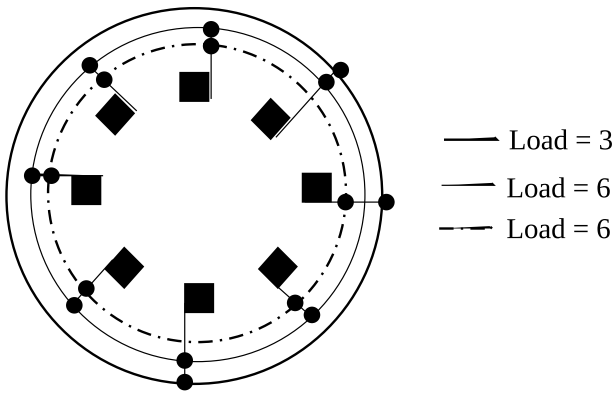

Figure 1: A (non optimal) configuration for $n=8$, $\lambda=3$, $\ell=2$ and $\beta=1$

The *load* of a wavelength k is defined by the number of transmitters connected on it, i.e., $load(k)=\sum\limits_{1\le i\le n} t_{i,k}$. The load of the assignment C is defined by $load(C)=\max\limits_{1\le k\le\lambda} load(k)$.

To ensure the communication efficiency, we introduce a parameter β called *minimal point-to-point bandwidth (MPB)*, with $1\le\beta\le\ell$. We want the configuration to guarantee that for each pair i,i' of nodes, there are at least β common wavelengths between the transmitters of i and the receivers of i' (see figure 1). More precisely, we say that an assignment configuration C is *β-balanced* iff for each ordered pair i,i', with $1\le i,i'\le n$,

$$|\{l_k\in\mathcal{L}:\ t_{i,k}=r_{i',k}=1\}|$$

Problem: WAVELENGTHS-ASSIGNMENT (n,λ,ℓ,β,c)

Given : `a ring with` n `nodes,` λ `wavelengths,` ℓ `transmitters/receivers per node, a MPB` β `and an integer` $c \geq 1$
Question : `Does there exist a` β`-balanced configuration` C `such that` $load(C) \leq c$?

It is still an open problem to know if this problem is NP-complete or not (see [6] for definition).

Our results : we deal with the optimisation problem related to `Wavelengths-Assignment`, i.e., finding the minimal load of a configuration for a given instance $(n, \lambda, \ell, \beta, c)$. We first give a general lower bound for this minimal load. We then present a polynomial construction of a configuration for a particular case of instances (i.e., with $\ell \geq \lambda \left\lfloor \frac{\lambda}{\beta} \right\rfloor$). This construction gives us a 2-approximation algorithm for this case. At the end, we show that the lower bound of Theorem 1 is tight by giving a construction of optimal load configurations for a non trivial restricted set of instances. Local Variables:

2 Bounds and polynomial strategies

This section is devoted to compute a lower bound for a β-balanced configuration and an upper bound for particular cases.

2.1 Lower Bound

Theorem 1 *Let A be a directed ring with n nodes. Let λ and ℓ be respectively the number of wavelengths and the number of transmitters. For any β-balanced configuration C, we have* $load(C) \geq max(\lceil \frac{\ell * n}{\lambda} \rceil, \lceil \frac{\beta*(n-1)}{\ell} + 1 \rceil)$.

Proof. 1 Each node has ℓ transmitters and thus there are $\ell * n$ transmitters over λ wavelengths. We can deduce that for any β-balanced configuration C, we have $load(C) \geq \frac{\ell * n}{\lambda}$.

Let u be a node in the directed ring. By definition, node u has ℓ transmitters and it can send (or receive) only on ℓ wavelengths. Moreover, it has at least β common wavelengths with each node of the ring. Thus, there are $\beta * (n-1)$ transmitters belonging to all other nodes of the ring plus ℓ transmitters of u over ℓ wavelengths. So we can deduce that for any β-balanced configuration C, we have $load(C) \geq \frac{\beta*(n-1)}{\ell} + 1$. □

2.2 An upper bound for particular case.

Proposition 1 *Let A be a directed ring with n nodes. Let λ and ℓ be respectively the number of wavelengths and the number of transmitters, with $\lambda \geq \lceil \frac{x}{2} \rceil (\ell - \beta) + \ell$ where $x = \left\lfloor \frac{\ell}{\beta} \right\rfloor$. A β-balanced configuration C can be polynomially computed for A, with*

$$2\lceil n/(x+1) \rceil \geq load(C) \geq \lceil n/(x+1) \rceil.$$

The lower bound of Proposition 1 follows easily from Theorem 1. Indeed, we can notice that, in this case, we have, by definition, $\beta(x+1) \geq \ell \geq \beta x$ and thus, $\frac{1}{\beta x} \geq \frac{1}{\ell} \geq \frac{1}{\beta(x+1)}$. From Theorem 1, we can deduce that any β-balanced configuration C, we has this load less than $\frac{\beta*(n-1)}{\ell} + 1$. As $\frac{\beta*(n-1)}{\ell} + 1 \geq \frac{n-1}{x+1} + 1$, the lower bound of Proposition 1 is proven.

Before proving the proposition, we first build a β-balanced configuration C for the case where $\lceil \ell/2 \rceil = \beta$ and $\lambda \geq 3\beta$. we generalise the construction for the case where $(x+1)\beta \geq \ell \geq x\beta$, and $\lambda \geq \frac{\ell(\beta+1)}{2}$.

Case where $\left\lfloor \frac{\ell}{\beta} \right\rfloor = 2$ and $\lambda \geq 2\ell - \beta$

We build a β-balanced configuration C as follow.

Let r be an integer such that $n \ mod \ 3 = r$. The set of the nodes is split into three subsets called V_1, V_2, V_3. r of them contain $\lceil n/3 \rceil$ vertices and the others contain $\lfloor n/3 \rfloor$ vertices.

Nodes in V_1 and V_2 have one transmitter on β wavelengths $\{l : 0, \ldots, l : \beta - 1\}$. Nodes in V_1 and V_3 have one transmitter on β wavelengths $\{l : \beta, \ldots, l : 2\beta - 1\}$. Nodes in V_2 and V_3 have one transmitter on β wavelengths $\{l : 2\beta, \ldots, l : 3\beta - 1\}$.

Let q be an integer such that $\ell \ mod \ \beta = q$. Nodes in V_1 and V_2 have one transmitter on β wavelengths $\{l : 3\beta, \ldots, l : 3\beta + q - 1\}$.Nodes in V_3 have one transmitter on β wavelengths $\{l : 3\beta + q, \ldots, l : 3\beta + 2q - 1\}$.

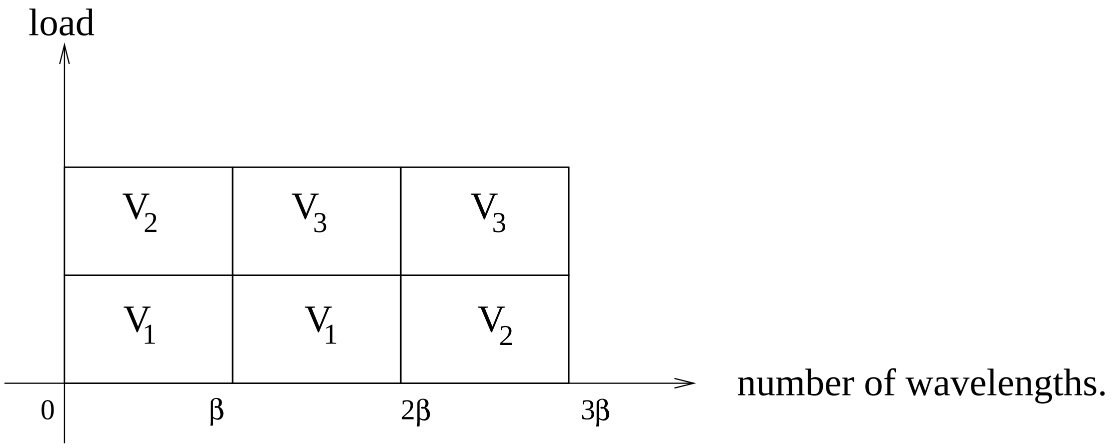

Figure 2: Configuration C where $\frac{\ell}{\beta} = 2$ and $\lambda \geq 2l - \beta$

This configuration C (see figure 2) is β-balanced and its load is equals to $2\lceil n/3 \rceil$.

Lemma 1 *Wet C^* be the β-balanced configuration which minimises this load,*

$$2\lceil n/3 \rceil \geq load(C^*) \geq \lceil n/3 \rceil$$

Proof of Proposition 1 (i.e., case $x = \left\lfloor \frac{\ell}{\beta} \right\rfloor$, and $\lambda \geq \left\lceil \frac{x}{2} \right\rceil (\ell - \beta) + \ell$)

We can now generalise the method given for the previous case. We build a β-balanced configuration C as follow. The set of the nodes is split into $(x+1)$ subsets called $V_1, V_2, \ldots V_{x+1}$. Let r be an integer such that $n \ mod \ (x+1) = r$. r of them contain $\lceil n/(x+1) \rceil$ vertices and the others contain $\lfloor n/(x+1) \rfloor$ vertices.

The main idea, is that β wavelengths is associated to each couple of $\{V_1, V_2, \ldots V_{x+1}\}$. Thus, we define some groups of wavelengths as follows:

1. $\forall 1 \leq i < j \leq x+1$, $W(i,j)$ is a set containing β different wavelengths.

2. $\forall 1 \leq i \leq \lceil \frac{x+1}{2} \rceil$, $W'(i)$ is a set containing q different wavelengths where $\ell = x\beta + q$.

3. all these groups have no common wavelength.

Let us describe the construction of this configuration. for any integer i, j such that $i < j \leq x+1$, nodes in V_i and V_j have one transmitter on β wavelengths in $W(i,j)$ After this operation, every node has $x\beta$ transmitters on β wavelengths among first $\frac{x(x+1)}{2}\beta$ wavelengths. As every node should have ℓ transmitters, this construction is not still completed. Let q be an integer such that $\ell = x\beta + q$.

- for any integer $i = 1, \cdots, \lfloor \frac{x+1}{2} \rfloor$, nodes in V_{2i-1} and V_{2i} have one transmitter on q wavelengths of $W'(i)$.
- if $(x+1)$ is odd, nodes in $V_{(x+1)}$ have one transmitter on q wavelengths of $W'(\lceil \frac{x+1}{2} \rceil)$.

The load of this β-balanced configuration is $2 \times \lceil n/(x+1) \rceil$. Proposition 1 is proved. □

3 An optimal 1-configuration scheme for a restricted parameter space

Proposition 2 *Consider any prime number $p \geq 3$. Let A be a directed ring with n nodes, $p^2 \leq n \leq p^3$. Let $\lambda = p(p+1)$ and $\ell = p+1$ be respectively the number of wavelengths and the number of transmitters. An optimal 1-configuration for A, with load $\left\lceil \frac{n}{p} \right\rceil$, can be polynomially build.*

First, we give a lower bound for the case. And finally, for this case, we give a construction of 1-configuration which its load is equal to the lower bound.

The lower bound is given by applying Theorem 1: the load of 1-configuration for this case is greater or equal to $\left\lceil \frac{n}{p} \right\rceil = (max(\lceil \frac{(p+1)*n}{p(p+1)} \rceil, \lceil \frac{(n-1)}{p+1} + 1 \rceil))$.

Now, we give a construction for two particular cases.

The basic scheme: case where $n = p^2$

Suppose that the number of nodes in a ring that wish to participate in the allocation of the wavelength is a square of a prime number $n = p^2$. In this case a perfect allocation of $p(p+1)$ wavelengths can be performed. Each node is assigned $p+1$ wavelengths and each wavelength is shared by p nodes in a way that there is a single common wavelength for each pair of nodes.

The way this scheme works is the following: the p^2 nodes are arranged in a $p \times p$ square matrix. Using this matrix the wavelengths are assigned to the nodes. In total $p+1$ groups of wavelengths, each group containing p different wavelengths are used. The first group of wavelengths, denoted by W_r, is assigned to the p rows of the matrix, a different wavelength per row (see Table 2(a)). The second group, denoted by W_c, is assigned to the p columns of the matrix, a different wavelength per column (see Table 2(b)). The subsequent groups of wavelengths are assigned to the diagonals of the matrix. The $p-1$ groups of diagonals with shifts from 1 to $p-1$. See Table 2(c) for the assignment of wavelengths to the 1-shift diagonals, Table 2(d) for the 2-shift diagonals, Table 2(e) for the 3-shift diagonals, and finally Table 2(f) for the 4-shift diagonals. In each case the entries in the principal i-shift diagonal are given in boldface. These $p-1$ last groups of wavelengths are denoted by W_d^i for i from 1 to $p-1$, where the index i corresponds to the shift of the diagonal.

d:0	d:1	d:2	d:3	d:4
d:5	d:6	d:7	d:8	d:9
d:10	d:11	d:12	d:13	d:14
d:15	d:16	d:17	d:18	d:19
d:20	d:21	d:22	d:23	d:24

Table 1: $n = 25$ nodes arranged in a square matrix.

Note that, this technic works if p is a prime number but does not work if p can be composite number. Tables 1 to 3 show a perfect allocation scheme for $n = p^2 = 5^2 = 25$ nodes.

In Table 3 we can see the wavelengths allocated to each node. We can notice that there is a single common wavelength between any pair of nodes. It is enough to compare the wavelengths in two different entries of the table.

Although this schemes seems to be extremely restricted and applicable to such a narrow parameter space that seems to be of little use, it would be interesting to notice 30 wavelengths can be perfectly assigned to for $n = 25$ nodes, so that each node is connected to 6 wavelengths (meaning that there is need for 6 transceivers per node), each wavelength is assigned to exactly 5 nodes, and as a result there is a single common wavelengths per pair of nodes. These parameter values resemble to real for a MAN network. The only two parameters that may need to be relaxed are the number of nodes per MAN ring and the number of common wavelengths per pair of nodes.

Proof of Proposition 2 (i.e., case where $(p^2 < n \leq p^3$ **).**

Now, let k be an integer such that $(k-1)p^2 < n \leq kp^2$. In this case, the lower bound is equal to $max(\frac{\ell * n}{\lambda}, \frac{\beta*(n-1)}{\ell} + 1) = \lceil n/p \rceil$.

First, we consider a direct ring A' of p^2 nodes. The perfect allocation C' for the ring A' is computed using the previous technique. Thus, the load of the perfect allocation C' equals to p.

Let r be an integer such that $n = (k-1)p^2 + r$.

Now, we will consider a direct ring A of n nodes and we will build an perfect allocation for the ring A from allocation C. r nodes of ring A' represents a group of k nodes of ring A and $p^2 - r$ nodes of ring A' is associated a group of $k-1$ nodes of ring A. Now, we compute the load of configuration C. By construction, the load of allocation C is greater than $load(C') * k (= pk)$. As $kp \leq load(C)$, the load of allocation C is pk. □

4 Conclusion

The lower bound given in Theorem 1 is tight (see Proposition 2), but one open question is to know whether it can be changed to improve the result of Proposition 1. In spite of many efforts, the lower bound given in Theorem 1 seems to be difficult to improve.

Proposition 2 gives an optimal results for restrictive parameter space. One interesting question is : how can we generalise to increase the number of nodes, without changing the number of wavelengths, and by altering as little as possible the perfect allocation described in the previous paragraph? First answers can be the following:

l:0	l:0	l:0	l:0	l:0
l:1	l:1	l:1	l:1	l:1
l:2	l:2	l:2	l:2	l:2
l:3	l:3	l:3	l:3	l:3
l:4	l:4	l:4	l:4	l:4

(a)

l:5	l:6	l:7	l:8	l:9
l:5	l:6	l:7	l:8	l:9
l:5	l:6	l:7	l:8	l:9
l:5	l:6	l:7	l:8	l:9
l:5	l:6	l:7	l:8	l:9

(b)

l:10	l:11	l:12	l:13	l:14
l:14	**l:10**	l:11	l:12	l:13
l:13	l:14	**l:10**	l:11	l:12
l:12	l:13	l:14	**l:10**	l:11
l:11	l:12	l:13	l:14	**l:10**

(c)

l:15	l:16	l:17	l:18	l:19
l:18	l:19	**l:15**	l:16	l:17
l:16	l:17	l:18	l:19	**l:15**
l:19	**l:15**	l:16	l:17	l:18
l:17	l:18	l:19	**l:15**	l:16

(d)

l:20	l:21	l:22	l:23	l:24
l:22	l:23	l:24	**l:20**	l:21
l:24	**l:20**	l:21	l:22	l:23
l:21	l:22	l:23	l:24	**l:20**
l:23	l:24	**l:20**	l:21	l:22

(e)

l:25	l:26	l:27	l:28	l:29
l:26	l:27	l:28	l:29	**l:25**
l:27	l:28	l:29	**l:25**	l:26
l:28	l:29	**l:25**	l:26	l:27
l:29	**l:25**	l:26	l:27	l:28

(f)

Table 2: The allocation of wavelengths to (a) rows, (b) columns, (c) 1-shift diagonals, (d) 2-shift diagonals, (e) 3-shift diagonals, and (f) 4-shift diagonals of the square matrix.

1. Give l wavelengths to each row, column, diagonal. This multiplies the number of wavelengths by l, but make the number of shared wavelengths per pair of nodes equal to l.
2. Give more than a single wavelength per row, column, diagonal. Choose wavelengths from the same group i.e., W_r, W_c, W_d^i.

This generalisation slightly alters the perfect allocation obtained in Proposition 2.

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l:0,5,10,15,20,25	l:0,6,11,16,21,26	l:0,7,12,17,22,27	l:0,8,13,18,23,28	l:0,9,14,19,24,29
l:1,5,14,18,22,26	l:1,6,10,19,23,27	l:1,7,11,15,24,28	l:1,8,12,16,20,29	l:1,9,13,17,21,25
l:2,5,13,16,24,27	l:2,6,14,17,20,28	l:2,7,10,18,21,29	l:2,8,11,19,22,25	l:2,9,12,15,23,26
l:3,5,12,19,21,28	l:3,6,13,15,22,29	l:3,7,14,16,23,25	l:3,8,10,17,24,26	l:3,9,11,18,20,27
l:4,5,11,17,23,29	l:4,6,12,18,24,25	l:4,7,13,19,20,26	l:4,8,14,15,21,27	l:4,9,10,16,22,28

Table 3: Final wavelength assignment to nodes.

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