Speed-Accuracy Tradeoff: A Formal Information-Theoretic Transmission Scheme (FITTS)

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TOCHI Paper, ACM CHI 2019, Glasgow, UK

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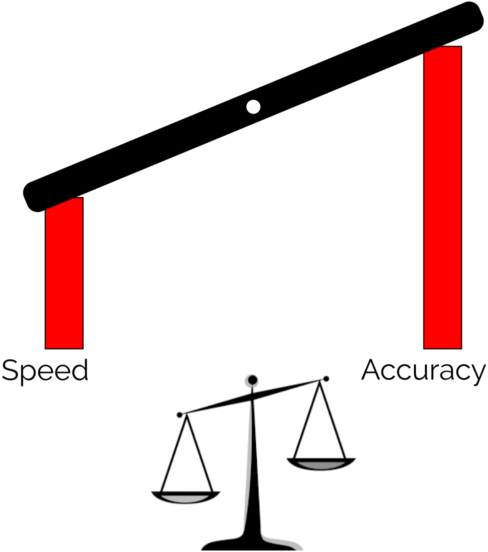

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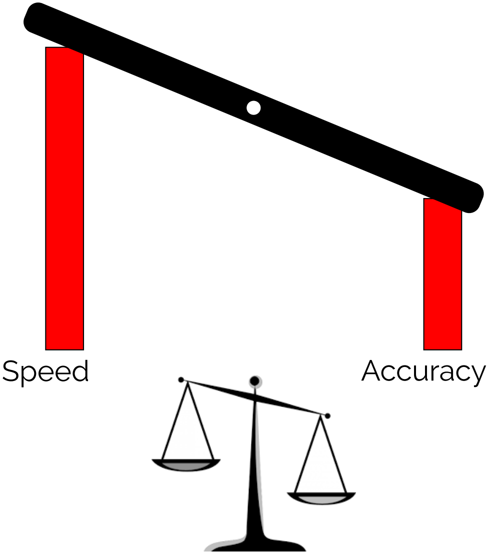
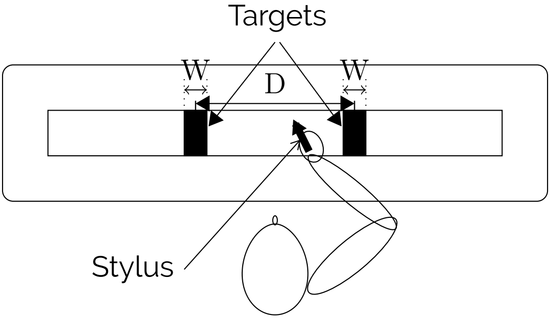


Quantifying the Speed-Accuracy Tradeoff

Fitts (1954)



Index of Difficulty: $\mathrm{ID} = \log(1 + \mathrm{D}/\mathrm{W})$ (bit)

Fitts' law for Movement Time:

$\mathrm{MT} = a + b\,\mathrm{ID}$

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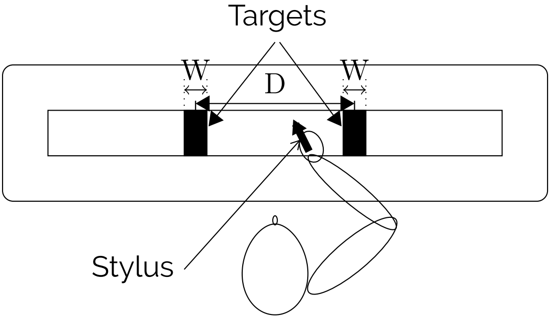

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Fitts' law for Movement Time:

$\mathrm{MT} = a + b\ \mathrm{ID}$

Vary according to participant, device

Fitts' Law in HCI

$$\mathrm{MT} = a + b\,\mathrm{ID}$$

- Evaluate (by estimating values *a* and *b*):
  1. Device performance
  2. User performance

Fitts' Law in HCI

$$\mathrm{MT} = a + b\,\mathrm{ID}$$

- Evaluate (by estimating values *a* and *b*):
  1. Device performance
  2. User performance

- Predict (by assuming values for *a* and *b*)
  1. Movement time for a pointing task
  2. Complex interaction times (optimize interfaces, keyboards)

Even though Fitts' law is widely used in HCI, it suffers from many issues.

I have selected 3 problems for this presentation

Problem 1: Many different formulations

How many can you think of ?

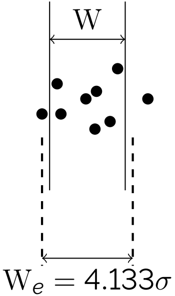
Problem 1: Many different formulations

$$\mathrm{MT} = a + b \log_2(2\mathrm{D}/\mathrm{W})$$
$$\mathrm{MT} = a + b \log_2(\mathrm{D}/\mathrm{W})$$
$$\mathrm{MT} = a + b \log_2(1/2 + \mathrm{D}/\mathrm{W})$$
$$\mathrm{MT} = a + b\,\mathrm{D} + c(1/\mathrm{W} - 1)$$
$$\mathrm{MT} = a + b\sqrt{\mathrm{D}}$$
$$\mathrm{MT} = a + (\mathrm{D}/\mathrm{W})^b$$
$$\mathrm{MT} = a + b \log_2 \mathrm{D} + c \log_2 \mathrm{W}$$
$$\mathrm{MT} = \ldots$$

More than 15 different formulations !

Problem 2: Sometimes Participants Miss the Target

Solution : Correct W a-posteriori to account for misses (usually $W_e = 4.133\sigma$)
(Crossman 1957, MacKenzie 1992)



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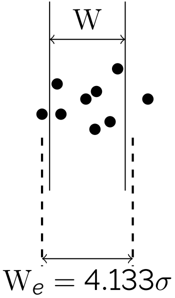
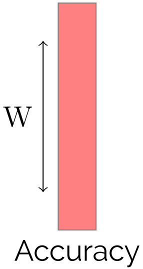


Refer to paper for a critique

Problem 2: Noisy manipulation of Accuracy



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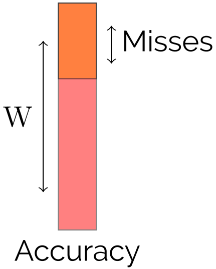
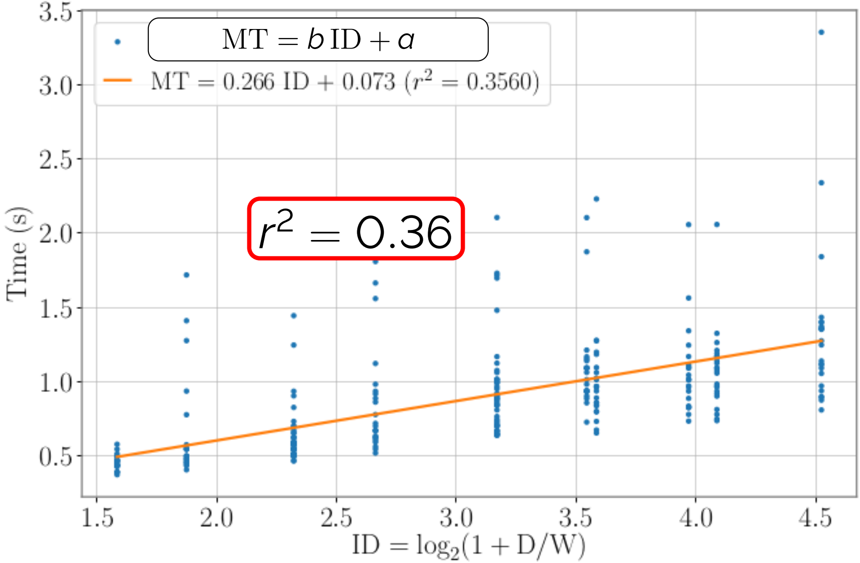


Accuracy's definition in Fitts' task is not accurate...

Problem 3: Regression



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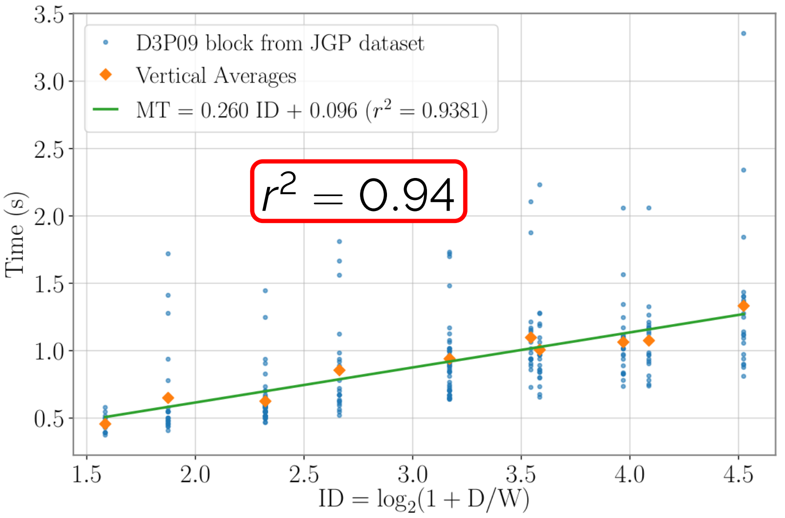
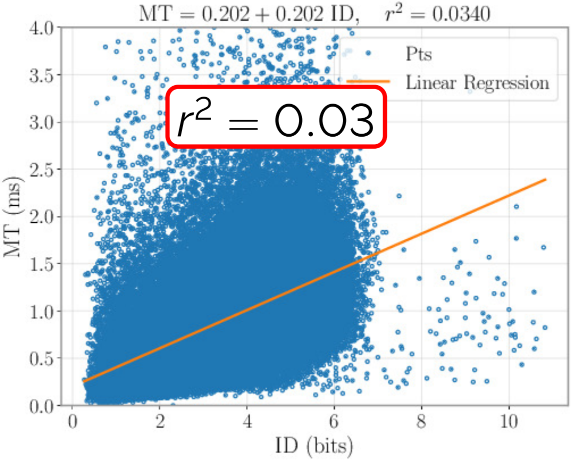


Fitts' law is a poor model for MT and can only model **average** MT

Problem 3: Regression



How can high variability data be handled (e.g. data from a field experiment) ?

Goal of this work:

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A theoretical framework for the speed-accuracy tradeoff:

- simple
- rigorous
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- address problems 1, 2 and 3

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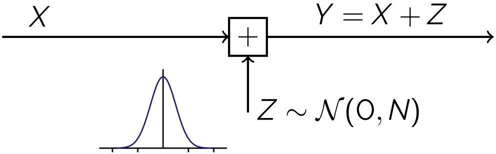
Tool used: Shannon's Information Theory

- Historically, Fitts' law is derived from an analogy with Shannon's capacity formula
- The channel capacity theorem can be interpreted as a speed-accuracy tradeoff

Capacity of the Gaussian Channel:

Capacity:

$$C = \max_X I(X;Y)$$



- **Shannon's Theorem 17 (1948)**:

$$C = B_W \log\left(1 + \frac{P}{N}\right) \text{ bit/sec.}$$

- Tradeoff between time (B_W) and bits $\log\left(1 + \frac{P}{N}\right)$

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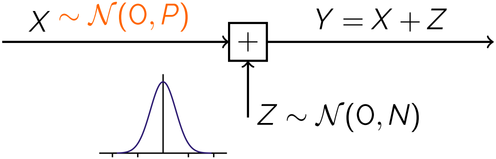

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A closer look at Fitts' 1954 analogy

Fitts' 1954 **analogy**: Humans have finite rate communication channels

- " The average amplitude of a human movement is equivalent to signal " $\longrightarrow P \equiv \mathrm{D}$
- " Movement variability is equivalent to noise " $\longrightarrow N \equiv \mathrm{W}$
- B_W is expressed in Hertz $\equiv s^{-1} \simeq \mathrm{MT}^{-1}$
- $\mathrm{MT} \propto \frac{1}{C} \log(1 + \mathrm{D}/\mathrm{W})$ (1954) $\simeq a + b \log(1 + \mathrm{D}/\mathrm{W})$ (1964)

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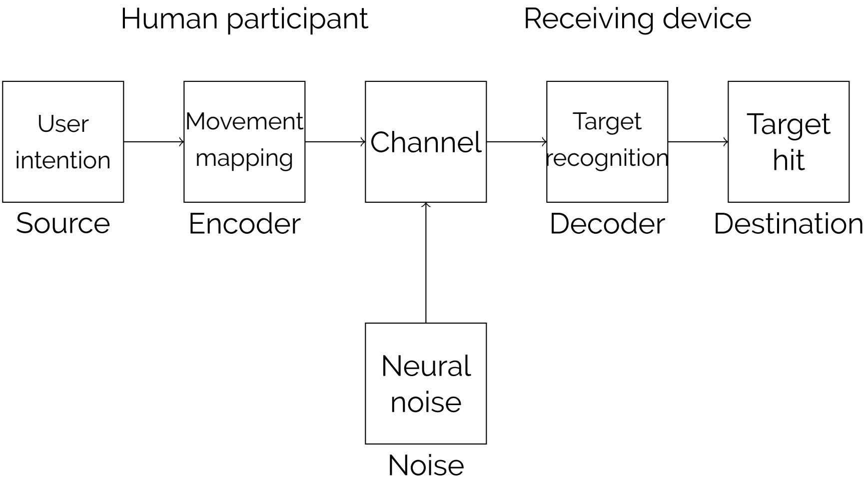
- Why equate a **variance** ($P \propto X^2$) to **amplitude** D ?
- What is the channel model of the aiming task?
- Which input? Which output?

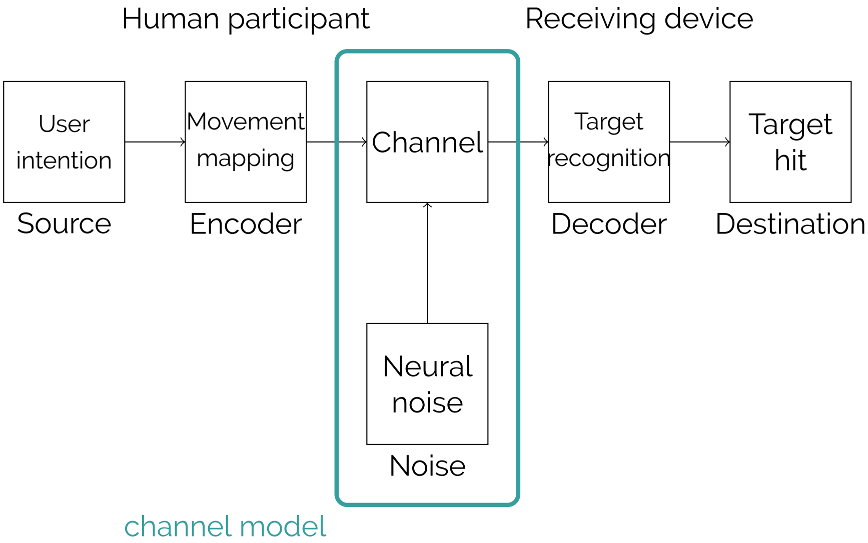
Our model (FITTS) answers these questions.

FITTS: a Formal Information-Theoretic Transmission Scheme

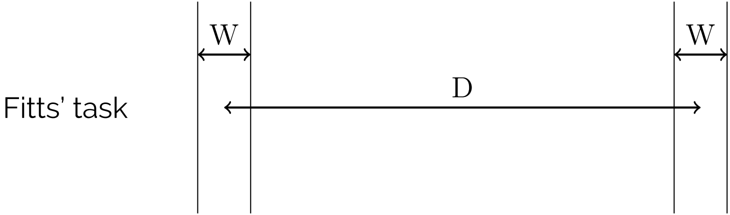
Communication Model



Communication Model



Constraints on X, Y and Z



- Hit the target: aim at the center and allow variability less than $\frac{W}{2}$ (*no miss*):

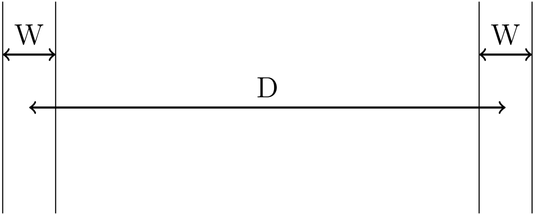
$$-\frac{W}{2} \leq Z \leq \frac{W}{2}$$

- The amplitude range of the input signal is D:

$$-\frac{D}{2} \leq X \leq \frac{D}{2}$$

Constraints on X, Y and Z

Fitts' task



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- Additive noise model: $Y = X + Z$

$$-\frac{D+W}{2} \leq Y \leq \frac{D+W}{2} \quad \text{(triangular inequality)}$$

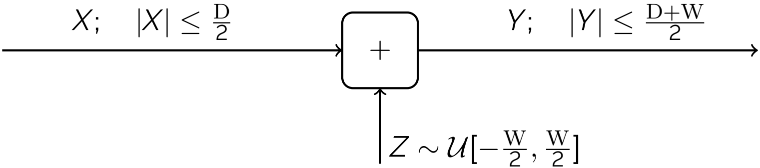
Which noise Z ?

- We assumed $|Z|$ is bounded by $\frac{W}{2}$, but choosing the noise distribution is not evident
- **Principle of Maximum Entropy** (Jaynes 1957): The probability distribution that best represents the current state of knowledge is the one with *largest* entropy (i.e., *the worst noise*!)

The maximum entropy distribution for $-\frac{W}{2} \leq Z \leq \frac{W}{2}$ is the uniform distribution $\mathcal{U}[-\frac{W}{2}, \frac{W}{2}]$:

$$p_Z(z) = \frac{1}{W}, \qquad z \in [-\frac{W}{2}, \frac{W}{2}]$$

Uniform Channel Model

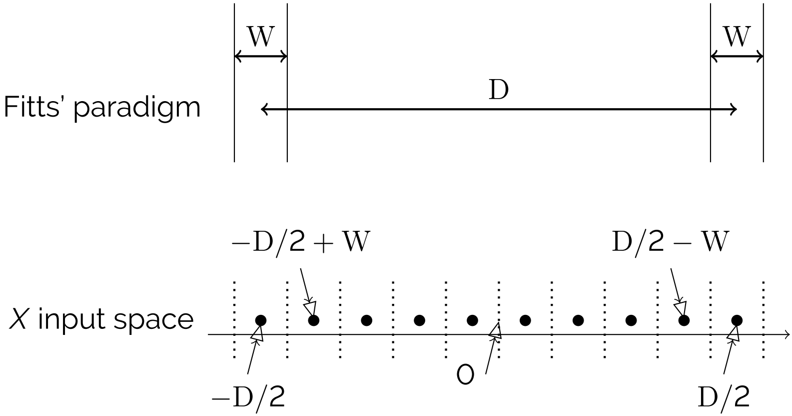


- Capacity of the uniform channel:

$$C = \max_X I(X;Y) = \log(1+\mathrm{D}/\mathrm{W}) = \mathrm{ID} \text{ (MacKenzie 1989)}$$

- Discrete uniform input X reaches capacity
- **Addresses Problem 1 (Formulations)**

Capacity Achieving Distribution



Aiming is choosing

C as a Lower Bound

- Uniform noise = maximum entropy noise = worst noise.

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  What if the noise is not uniform, but e.g. Gaussian ?

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- Uniform noise = maximum entropy noise = worst noise.

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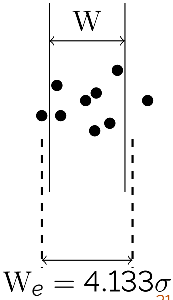
What if the noise is not uniform, but e.g. Gaussian ?

- With a Gaussian noise, limited to $\mathrm{W} = 4.133\sigma$, the resulting capacity C' differs by at most 0.2 bit:

$$\boxed{C \leq C' \leq C + 0.2 \text{ bit}}$$

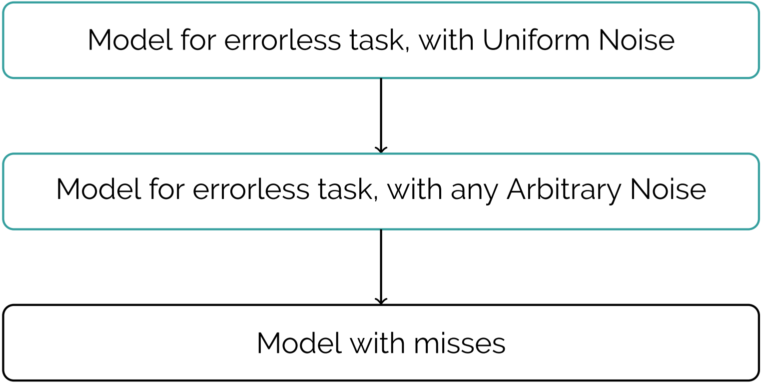
(Crossman-MacKenzie correction)



Model for errorless task, with Uniform Noise

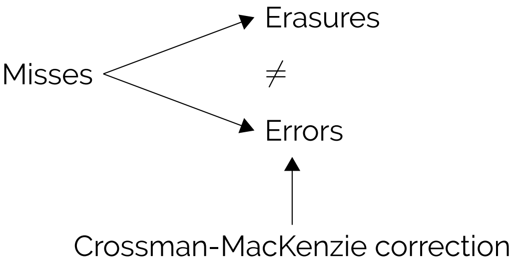
↓

Model for errorless task, with any Arbitrary Noise


Model for errorless task, with Uniform Noise
Model for errorless task, with any Arbitrary Noise
Model with misses

Distinguishing Errors from Erasures

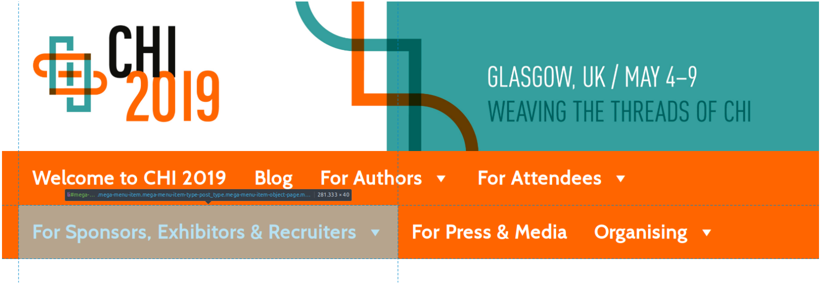
Participants sometimes miss the target (rate ε).



$$W_e = 4.133\sigma$$

- Using σ = considering the distance from each endpoint to the target's center
- 100% of movements involve metrical errors (of varying amplitude)

Erasures



In GUIs, what matters is

- **Whether or not** the click falls in the intended area
- Not **where** the click takes place
- Not go **as close** to the center of the target as possible

Dichotomous hit or miss corresponds to erasures in information-theory.

Capacity for the Channel with Erasures

We show that

$$C = (1 - \varepsilon)\log_2(1 + \mathrm{D}/\mathrm{W}) = (1 - \varepsilon)\mathrm{ID} = \mathrm{ID}(\varepsilon),$$

with ε the miss rate.

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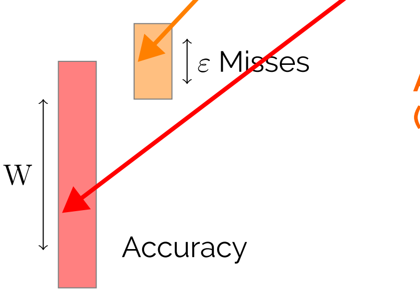

Addresses Problem 2 (Misses)

Comparison with Crossman-MacKenzie correction

Crossman-Mackenzie	**Erasure Model**
$\mathrm{ID}_e = \log(1 + \frac{\mathrm{D}}{4.133\sigma})$	$\mathrm{ID}(\varepsilon) = (1 - \varepsilon)\mathrm{ID}$
Mistakes = Metrical Errors	Mistakes = Erasures
Continuous	Discrete Binary
Gaussian hypothesis	No hypothesis
$\lim_{\varepsilon \to 0} \mathrm{ID}_e = \infty$	$\mathrm{ID}(0) = (1 - 0)\mathrm{ID} = \mathrm{ID}$

Which Movement Time ?

A complete model for movement time (MT) in Fitts' task …

- Channel capacity gives **minimum** MT ($C = \max I(X; Y)$)

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Average-minimum time

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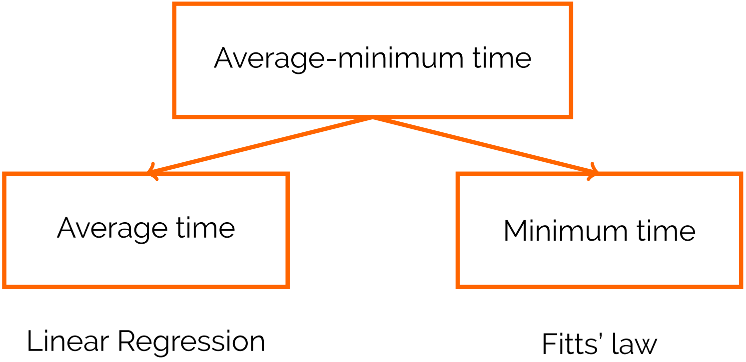
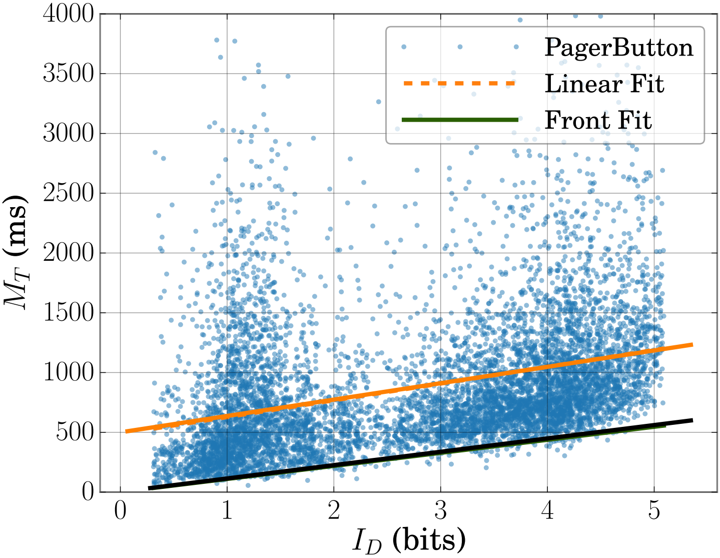


Example — Field Study



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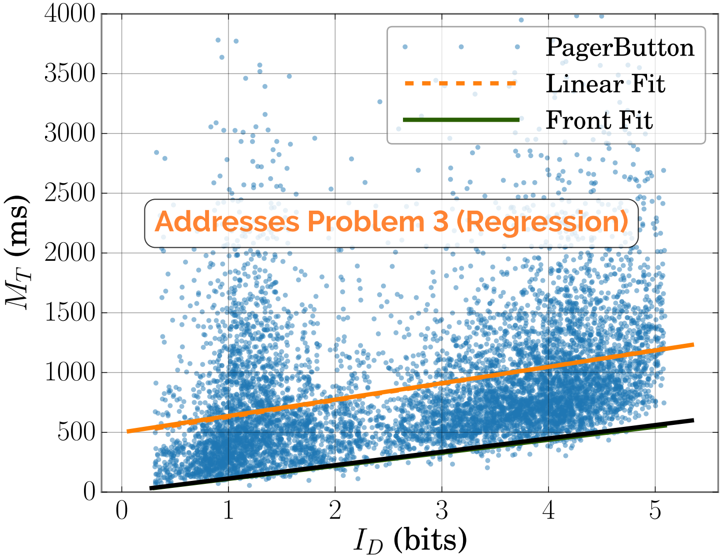

Conclusion

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FITTS is a “black-box” model for endpoints:

- only concerned with MT (not about the full trajectory)
- Not based on empirical findings (neuroscience, psychology)
- Based on (abstract) information theory
- No feedback taken into account

Additional problem: Compute the *minimum* time regression

Speed-Accuracy Tradeoff:

A Formal Information-Theoretic Transmission Scheme (FITTS)

J. Gori, O. Rioul, and Y. Guiard, "Speed-accuracy tradeoff of aimed movement: A formal information-theoretic transmission scheme (FITTS)," ACM Transactions on Computer-Human Interaction (TOCHI), Vol. 25, No. 5, pp. 27:1-27:33, Oct. 2018

- FITTS 2: Scheme for full trajectories with feedback control [arXiv]:1804.05021 `http://www.theses.fr/2018SACLT022`
- EMG Regression pre-print: `https://hal.archives-ouvertes.fr/hal-02111178`