



# Algorithms for Data Science

## Web Advertising

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**Silviu Maniu**

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M2 Data Science

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# Banner Ads

First iteration: **banner ads** (around 1995)

The screenshot shows the Le Monde website interface. At the top, the Le Monde logo is centered, with navigation links for 'Consultez le journal', 'Se connecter', and 'S'abonner'. Below the logo is a horizontal menu with categories: 'ACTUALITÉS', 'ÉCONOMIE', 'VIDÉOS', 'OPINIONS', 'CULTURE', 'M LE MAG', 'SERVICES', and a search icon. A row of article teasers follows, each with a date, a thumbnail image, and a headline. The main content area is dominated by a large blue banner for 'amazon prime day 13 et 14 oct.' with the text 'Deux jours de Ventes Flash exceptionnelles sur les produits que vous aimez'. Below this banner are several article teasers, including one about the US election system and another about the probability of survival for people infected with COVID-19. The bottom of the page features a 'Newsletter Le Brief du Monde' sign and a small illustration of houses.

Le Monde

Consultez le journal

Se connecter S'abonner

ACTUALITÉS ÉCONOMIE VIDÉOS OPINIONS CULTURE M LE MAG SERVICES Q

13 oct. 12 militaires russes tués

13 oct. Le Japon, modèle de respect de la forme ?

13 oct. **Actualité** Serie : Les faibles de la démocratie américaine, pourquoi ? la disparité des règles de vote risque de entraîner la confusion

13 oct. Le FBI perçoit une rupture « longue et difficile »

14 oct. Grippe : vaccins, aller-vous vous faire vacciner ?

Store >

amazon prime day 13 et 14 oct.

Deux jours de Ventes Flash exceptionnelles sur les produits que vous aimez

amazon prime day 13 et 14 octobre

Voir les ventes flash

Exclusivement pour les membres Amazon Prime

**La faillite de la démocratie aux États-Unis, épisode 2 : la disparité des règles de vote selon les États**

Des chiffres trompeurs montrant la « probabilité de survie » des personnes infectées par le Covid-19

Notre voix, un instrument si fragile

Quand notre voix souffre, elle se fait entendre. De nouveaux outils et exercices permettent de récupérer ces déchirures.

10 ans de lecture

Newsletter Le Brief du Monde

amazon prime day 13 et 14 octobre

Voir les ventes flash

Exclusivement pour les membres Amazon Prime


# Banner Ads

First iteration: **banner ads** (around 1995)

- charging per 1,000 “impressions” (**clicks**)
- **CPM** – cost per thousand impressions (as in TV, print media)
- **untargeted** vs. **demographically targeted**
- low **click through rates** – low return on investment

# Performance-Based Advertising

Second iteration: **ads on search results** (around 2001)



[All](#) [Images](#) [Shopping](#) [Maps](#) [News](#) [More](#) [Settings](#) [Tools](#)

About 37,100,000 results (0.45 seconds)

**Ad** · [www.go-sport.com/running/chaussures](http://www.go-sport.com/running/chaussures)

**Chaussures de Running - GO Sport : Magasins de Sport**

Chaussures de Running - Retrouvez la Sélection de Produits sur GO Sport. Articles & Matériel de Sport : Randonnée, le Fitness, Tennis, Running, Sports collectifs. Paiement sécurisé. Retrait en magasin. Marketplace. Livraison en 24h. Paiement en 3X sans frais.

★★★★★ Rating for go-sport.com: 4.5 · Order accuracy: 95–100%

📍 Les Ulis · 11 locations nearby

**Chaussures Running**

Commandez en ligne les Meilleures Marque de Chaussures de Running

**Opération Automne**

Profitez de Réduction sur une Sélection de Produits

**Vélo, VTT, VTC**

Découvrez notre gamme de vélos en stock et livrés

**Rentré**

Les Ind... une bor

[www.go-sport.com](http://www.go-sport.com) · [running](#) · [chau...](#) · [Translate this page](#)

**Chaussures Running - achat pas cher - GO Sport**

Chaussures Running toujours au meilleur prix sur Go-Sport.com – 24h/24 bénéficiez d'un large choix d'articles de sport – Commande en ligne et Livraison ...

Homme · Chaussures Homme · Femme · Running REVOLUTION 5

[www.decathlon.fr](http://www.decathlon.fr) · Tous les sports · Running route

**Chaussures et baskets running | DECATHLON**

Chaussures et baskets running. Pour courir, rien de plus important qu'une bonne paire de chaussures de course à pied. Vous trouverez ici notre large gamme de ...

★★★★★ Rating: 4.4 · 18,636 reviews

Chaussures de running Homme · Chaussures de Running Femme










[www.i-run.fr](http://www.i-run.fr) · [chaussures\\_homme](#) · [Translate this page](#)

**Chaussures de sport : running, randonnée, fitness pour homme**

Reebok Floatride Run : Idéales pour les hommes ayant une foulée universelle, ces chaussures sont toutes indiquées pour les séances de course à pied sur routes ...

**Ads · See chaussures run...**

[Products](#) [Comparison Sites](#)

|  |  |  |
|--|--|--|
|  <p><b>NIKE Chaussures de running hom...</b></p> <p>€43.99</p> <p>Intersport</p> <p><a href="#">By Google</a></p> |  <p><b>Nike Baskets Revolution 5...</b></p> <p>€54.90</p> <p>La Redoute</p> <p>★★★★★ (1k+)</p> <p><a href="#">By Keyade</a></p>                      |  <p><b>ASICS Gel - Cumulus 21 Gs...</b></p> <p>€49.00</p> <p>outlet.asics.com/...</p> <p>★★★★★ (18)</p> <p><a href="#">By Google</a></p>                        |
|  <p><b>Chaussures Running &amp;...</b></p> <p>€30.00</p> <p>Decathlon.fr</p> <p><a href="#">By Yteo</a></p>       | <p>SALE</p>  <p><b>Nike Nike Air Zoom Pegasus ...</b></p> <p>€59.50 €85</p> <p>Sarenza</p> <p>Free shipping</p> <p><a href="#">By Feed Price</a></p> | <p>SALE</p>  <p><b>Asics Gel Pulse 11 M   Jaune/or   E...</b></p> <p>€65.00 €100</p> <p>i-Run.fr</p> <p>★★★★★ (141)</p> <p><a href="#">By Productcaster</a></p> |
| <p>SALE</p>  <p><b>Reebok Floatride Run</b></p>   | <p>SALE</p>  <p><b>Adidas Ultraboost</b></p>   |  <p><b>New Balance Fresh Foam</b></p>   |

# Performance-Based Advertising

Second iteration: **ads on search results** (around 2001)

- advertisers **bid** on **search keywords**
- on **click** – highest bidder ad is shown
- charging only if add is clicked
- adopted by Google around 2002 – **Adwords**

# Performance-Based Advertising

Part of Web 2.0 – huge industry (several billion \$)

**Problem:** what ads to show for a given query

- another related problem: which search terms should an advertiser bid on, and for how much
- part of computational game theory

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**Data Streams:** limited resources to process data as it comes

## Online algorithms

- decision must be made **immediately** as data comes
- vs. **offline** – data is processed in its entirety

# Greedy Algorithm for Online Optimization Problems

**Optimization problem:** maximizing or minimizing an **objective function** on the data

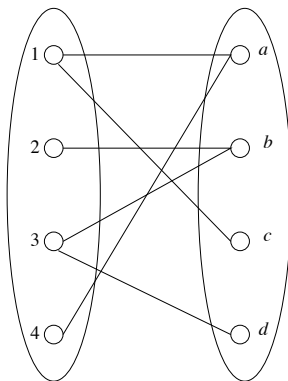
**Greedy algorithm:** take decision locally, by **optimizing** only based on the **current** element and the past

**Not always optimal vs. offline algorithms:**

- **competitive ratio:** the ratio between the offline solution and the online solution **over all inputs**  $c = \max_G \frac{|M_g|}{|M_o|}$

# Matching Problem

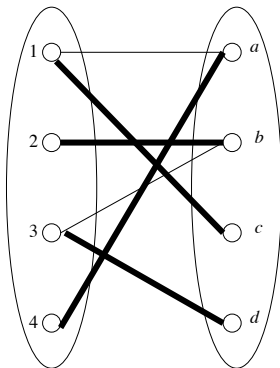
**Bipartite Graph:** a graph  $G(V_1 \cup V_2, E)$  having two disjoint sets of nodes  $V_1$  and  $V_2$  and edges **only** havins one endpoint in  $V_1$  and one in  $V_2$ , i.e.,  $E \subseteq V_1 \times V_2$



# Matching Problem

**Matching:** choosing a **subset of the edges** in the bipartite graph s.t. **no node has more than two edges** in the matching

- **perfect** – every node is in the matching
- **maximal** – has the largest number of edges possible



# Greedy Algorithm for Matching

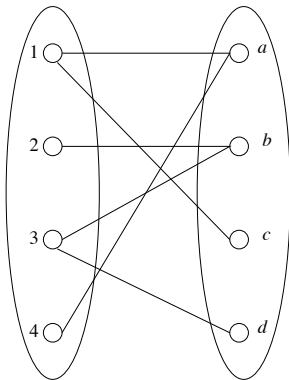
**Offline case:** algorithms for finding maximal matchings are  $\mathcal{O}(n^2)$ , where  $n = |E|$

**Online case:** can use the **greedy** algorithm:

1. consider the edges in the order they arrive
2. add edge  $(x, y)$  only if neither  $x$  nor  $y$  are endpoints

## Example of Greedy Matching

Edges arrive in the order:  $(1, a)$ ,  $(1, c)$ ,  $(2, b)$ ,  $(3, b)$ ,  $(3, d)$ ,  $(4, a)$



Result of greedy matching:  $(1, a)$ ,  $(2, b)$ ,  $(3, d)$  – **not maximal**

# Competitive Ratio of Greedy Matching

$M_o$  – maximal matching,  $M_g$  – greedy matching

$L$  – left nodes matched in  $M_o$  but not in  $M_g$

$R$  – right nodes connected to any node in  $L$

**Claim:** every node in  $R$  is matched in  $M_g$

- **prove by contradiction:** assume it is not the case
- then there will exist edge  $(l, r)$ ,  $l \in L$
- then, it should be matched (neither is added to the matching)
- contradiction!

# Competitive Ratio of Greedy Matching

**Claim:** every node in  $R$  is matched in  $M_g$

- $|M_o| \leq |M_g| + |L|$  – only nodes in  $L$  can be matched in  $M_o$
- $|L| \leq |R|$  – in  $M_o$ , all nodes in  $L$  are matched
- $|R| \leq |M_g|$  – every node in  $R$  is matched in  $M_g$

– this gives us  $|M_g| \geq \frac{|M_o|}{2}$  – **lower bound** on the competitive ratio

But  $1/2$  is also an **upper bound** – can find a counter example

**Competitive ratio is then exactly  $1/2$**



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# Adwords Problem

**Problem:** match queries in a search engines with advertisers

We have:

- a set of **bids** by advertisers for search queries
- **click-through rate** for each advertiser-query pair
- **budget** for each advertiser (time, money, etc.)
- **limit on the number of ads** to be displayed

# Adwords Problem

**Problem:** match queries in a search engines with advertisers

Restrictions on the set of advertisers:

- **the size is under the limit** of number of ads
- each advertiser in the set **has bid on the query**
- each advertiser **has enough budget left over**

# Adwords Setting

1. **stream of queries** arrives at search engines  $q_1, q_2, \dots$
2. advertisers **bid** on each query
3. when  $q_i$  arrives search engine picks a subset of advertisers

**Objective:** maximize search engine revenue

If we consider queries as being the “left” side and advertisers the “right” side in a bipartite graph – **online bipartite matching**

- **weighted case:** the matching depends on the CTR and the budget

# Adwords in Practice

In practice: combine CTR and bid – **expected revenue**

- **value** of an ad – expected revenue
- **revenue** to the search engine – sum of values of matched ads

| Advertiser | CTR  | Bid | CTR $\times$ Bid |
|------------|------|-----|------------------|
| <b>A</b>   | 0.02 | 7.5 | 0.15             |
| <b>B</b>   | 0.05 | 5.0 | 0.25             |
| <b>C</b>   | 0.01 | 1.0 | 0.01             |

# Measuring CTR

**Value** of an ad is directly linked to the CTR rate

- high bid is useless if the CTR is very low

**Click-through rate** is measured historically – **difficult problem**

- **explore**: do we try an ad to measure the CTR rate for future campaigns?
- **exploit**: do we use the current known CTR rate, even if they could be outdated?

# Greedy Algorithm

## Setting:

- there is one ad shown for each query
- all advertisers have the same budget  $B$
- all ads have same CTR
- value is then the same

## Greedy algorithm:

- pick any advertiser who has bid for that query
- same **competitive ratio** as in online matching –  $1/2$

# Worst-case Greedy

**Advertiser A:** bids on query  $x$ , budget 4 **Advertiser B:** bids on queries  $x$  and  $y$ , budget 4

**Stream:**  $x\ x\ x\ x\ y\ y\ y\ y$

**Greedy choice:**

- worst case:  $B\ B\ B\ B\ \dots$
- **optimal:**  $A\ A\ A\ A\ B\ B\ B\ B$
- **competitive ratio:**  $1/2$



# BALANCE algorithm [Mehta et al., 2007]

## BALANCE Algorithm:

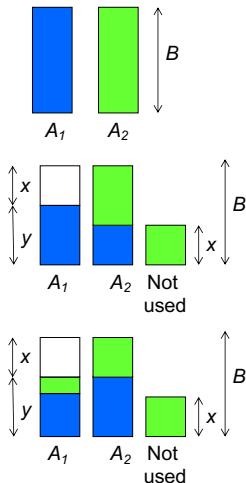
- assign query to the bidder having the most budget left
- **competitive ratio**  $3/4$
- **tie breaking**: must be deterministic

## Previous example:

- if  $A$  is preferred to  $B$ :  $A B A B B B \dots$
- establishes an **upper bound** on competitive ratio for 2 bidders

## BALANCE – Lower Bound for 2 Bidders

**Assumption:** advertisers  $A_1, A_2$  budget  $B$  (consumed by the optimal algo), revenue  $2B$



BALANCE must **exhaust the budget of at least one bidder**, e.g.,  $A_2$

Case of assigned bids ( $x + y = B$ ):

- **at least half of the queries are assigned to  $A_1$ :**  $y \geq B/2$ , so  $y \geq x$
- **more than half of the queries are assigned to  $A_2$ :** remaining budget of  $A_2$  is less than  $B/2$ , so  $x \leq B/2$ , so  $y \geq x$

**Minimal BALANCE revenue** at  $x = y = B/2$ ,  
revenue  $3B/2$  **competitive ratio**  $\frac{3B/2}{2B} = 3/4$

## BALANCE – Multiple Bidders

In the general case, BALANCE competitive ratio is not much lower than the simple case:

- competitive ratio:  $1 - 1/e = 0.63\dots$
- no online algorithm has a better competitive ratio

## BALANCE – Worst Case for Multiple Bidders

**Advertisers:**  $N - A_1, \dots, A_N$ , each having budget  $B > N$

**Queries:**  $N$  rounds of  $B$  queries

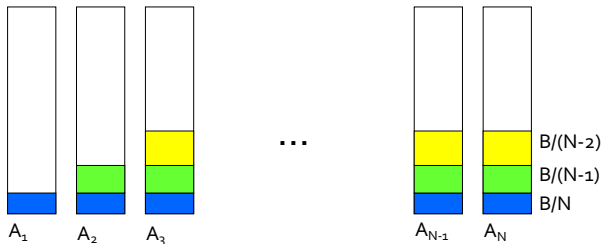
**Bids:** round  $i$  - bidders  $A_i, \dots, A_n$

**Optimum allocation:** allocate round  $i$  queries to  $A_i$

- revenue  $N \cdot B$

# BALANCE – Worst Case for Multiple Bidders

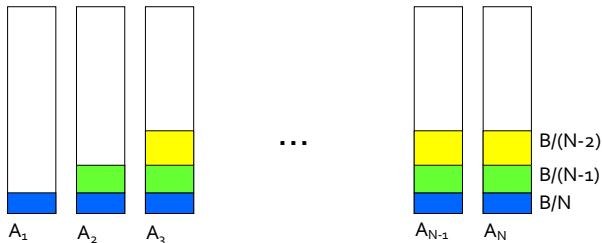
## BALANCE allocation



- BALANCE assigns queries in round  $k$  to  $N - (k - 1)$  advertisers
- sum of allocation to each advertiser  $A_k, \dots, A_N$ :  $S_k = \sum_{i=1}^{k-1} \frac{B}{N-(i-1)}$
- the smallest  $k$  at which  $S_k \geq B$  is the point after which no advertisers can be allocated  $k = N(1 - 1/e)$

# BALANCE – Worst Case for Multiple Bidders

## BALANCE allocation



- after  $k = N(1 - 1/e)$  we cannot get any revenue
- **total revenue:**  $B \cdot N(1 - 1/e)$
- upper bound on **competitive ratio:**  $1 - 1/e$

# BALANCE with Arbitrary Bids

## BALANCE works well when bids are 0 or 1

- if arbitrary bids, it can fail and have competitive ratio 0

### Example:

- advertisers  $A_1, A_2$ , one query  $q$  arriving 10 times
- $A_1$ : bids 1, budget 110
- $A_2$ : bids 10, budget 100
- **optimal**: assign all queries to  $A_2$ , revenue 100
- **BALANCE**: assigns all queries to  $A_1$ , revenue 10

# Generalized BALANCE

BALANCE can be generalized to arbitrary bids:

- bid  $x_i$ , budget  $b_i$ , amount spend so far  $m_i$
- **fraction of leftover budget**  $f_i = 1 - m_i/b_i$
- for a query  $q$ , use  $\psi_i(q) = x_i(1 - e^{-f_i})$

**Decision:**

- allocate query  $q$  to bidder  $i$  having largest value of  $\psi_i(q)$

**Same competitive ratio:**  $1 - 1/e$



# Adwords Implementation

## In practice

- advertisers **bid of sets of words**
- if a search query contains exactly those words – the advertiser becomes a bidder
- can use **distributed hash tables**
- queries can be distributed on **several machines** also – multiple streams

## Another applications:

- Google also matches **ads to emails** – much harder problem (mails are much larger)

# Acknowledgments

The contents follows Chapter 8 of [Leskovec et al., 2020]. Figures in slides 11, 12, 14, 26, 29, and 30 are taken from <https://www.mmds.org/>

# References i



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