# Identification of Mechanical Inclusions

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**Abstract**

Evolutionary Algorithms provide a general approach to inverse problem solving: As optimization methods, they only require the computation of values of the function to optimize. Thus, the only prerequisite to efficiently handle inverse problems is a good numerical model of the direct problem, and a representation for potential solutions.

The identification of mechanical inclusion, even in the linear elasticity framework, is a difficult problem, theoretically ill-posed: Evolutionary Algorithms are in that context a good tentative choice for a robust numerical method, as standard deterministic algorithms have proven inaccurate and unstable. However, great attention must be given to the implementation. The representation, which determines the search space, is critical for a successful application of Evolutionary Algorithms to any problem. Two original representations are presented for the inclusion identification problem, together with the associated evolution operators (crossover and mutation). Both provide outstanding results on simple instances of the identification problem, including experimental robustness in presence of noise.

## 1 Introduction

Evolutionary Algorithms (EAs) are stochastic optimization methods that have been demonstrated useful to solve difficult, yet unsolved, optimization problems. Requiring no regularity of the objective function (or of the constraints), EAs are able to tackle optimization problems on different kinds of search spaces, such as continuous, discrete or mixed spaces, as well as spaces of graphs or lists. The only prerequisite are the definition of evolution operators such as crossover and mutation, satisfying as much as possible heuristically derived requirements. The two main drawbacks of EAs are first the large number of evaluation of the objective function they usually imply before eventually reaching a good, if not optimal, solution; and second, their stochastic aspect, weakening their robustness. Hence, EAs should be used with care, on problems beyond the reach of standard deterministic optimization methods.

In structural mechanics, the non-destructive identification of inclusions is such a difficult problem, resisting to-date numerical methods: in its simplest instance, a structure is known to be made of two different known materials, but their repartition in the structure is unknown. The available data consist of records of the mechanical behavior of the structure under known loadings. The goal is to find the geometrical repartition of both materials from these experimental data. In steel manufacturing plants, for instance, it is of vital importance to check if coal scories are included in steel parts, and if their repartition does not dangerously weaken the whole part. For a given repartition of both materials, the computation of the simulated mechanical behavior of the structure is straightforward (*e.g.* using any Finite Element package). The identification can then be viewed as an inverse problem.

This paper addresses this inverse problem using EAs. A possible objective function for such inverse problems is the difference between the simulated mechanical behavior of a tentative repartition of both materials and the actual experimental behavior. However, the main difficulty is to define the search space in which the EA will be defined. Considering past works on the Optimum Design problem, (a closely related problem, where the goal is to find a partition of a design domain into material and void), the straightforward representation is defined from a fixed mesh of the structure, leading to a fixed-length bitstring well-suited to Genetic Algorithms. However, this approach will not be considered here, as it makes the optimization problem intractable when the underlying mesh is refined. Instead, two non-standard representations (termed the *Voronoï representation* and the *H-representation*) are introduced, independent of any *a priori* discretization, but leading to variable-length "individuals". Hence, specific operators have to be designed and implemented.

Nevertheless, Evolutionary Algorithms working on these representations give outstanding results on the inclusion identification problem in the context of linear elasticity, outperforming previous deterministic numerical solutions. Experimental evidences appear that the quality of the results highly depends of the amount of experimental information the algorithm can rely upon. Furthermore, the robustness against noise in the experimental data is experimentally shown on simulated artificial noisy data.

The paper is organized the following way: In Section 2, a general framework to address inverse problems with EAs is introduced. The mechanical problem of inclusion identification is presented in details in Section 3. It is an instance of the general inverse problem of Section 2, and can thus be solved by Evolutionary Algorithms, once the search space has been skillfully designed: Two original representations for that problem are presented in Section 4, together with their specific evolution operators (crossover and mutations). The first results, using the Voronoï representation, are presented in Section 5, demonstrating outstanding performance on artificial instances of the inclusion identification problem. Comparative results for both representations are presented in Section 5.6, opening the discussion about the *a priori* choice of a representation for a given problem. This discussion leads to propose further directions of research, sketched in Section 6.

# 2 Evolutionary inverse problems solving

This section presents a general framework to solve inverse problems by means of Evolutionary Algorithms, which will be further applied to the Mechanical problem introduced in Section 3.

## 2.1 Direct problems

Consider a process (*e.g.* a physical phenomenon) that produces output experimental results given some inputs (experimental conditions).

The successful resolution of the direct problem consists in building a simulated process able to accurately predict the experimental output of the physical phenomenon from the same experimental input data. Such simulation generally relies on a function (law, algorithm, command, ...) modeling the underlying physical behavior. Figure 1 gives a symbolic representation of a physical process together with its simulated model.

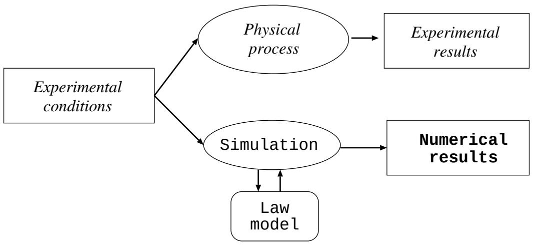



Figure 1: *Direct problem. Physical and simulated process. Simulation is successful (based on an accurate model) iff the error between experimental results and numerical results is small.*

## 2.2 Inverse problems

However, quite often, the model (law, command, ...) is not precisely known: The goal of inverse problems is to find a model such that the numerical simulations based on this model successfully approximates experimentations. The data of inverse problems are experimental conditions together with the corresponding actual experimental results.

Whenever a good simulation of a direct problem exists, Evolutionary Computation can be used to address the inverse problem. The fitness of a candidate solution can be computed as shown in Figure 2: the results of the numerical simulation performed using the individual at hand as the model to identify is compared to original experimental results, and the goal is to reach the smallest possible error.

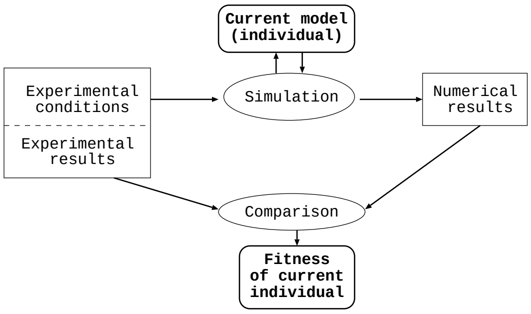



Figure 2: *Evolutionary approach for the inverse problem. The fitness of the individual at hand is achieved by comparing the actual experimental results with the numerical results obtained using the individual in place of the law model.*

### 2.3 The genotype space

The most critical step in evolutionary inverse problem solving is the choice of the representation, which defines the genotype space (*i.e.* the search space). In any case, the EA will find a good – if not the best – solution in the search space. Hence it seems that larger search spaces allow better solutions to be found. However, the larger is the search space, the more difficult is the optimization task, and a trade-off has to be found.

Yet another important issue in inverse problem solving is the *generalization capability* of the solution: How good is the resulting model when used with experimental conditions that are different from those used during the identification process? The usual answer in evolutionary inverse problem solving is to use, during the identification, many different experimental conditions, also termed *fitness cases*. The fitness is then the average of the error over all fitness cases. Needless to say, the total computational cost increases with the number of fitness cases.

## 3 The Mechanical Problem

This section gives a detailed presentation of the mechanical problem of inclusion identification, and states the simplification hypotheses made throughout this paper. The issues raised in Section 2 above will then be addressed on this instance of inverse problem.

### 3.1 Background

Consider an open bounded domain $\Omega \subset \mathbb{R}^N$ $(N = 2, 3)$, with a smooth enough boundary $\partial\Omega$, filled with a linear elastic material. Under the hypothesis of small deformations (linear elasticity) the following equations hold for, respectively, the *strain* and the *stress* tensors:

$$\varepsilon(x) := \frac{1}{2}(\nabla u(x)\nabla u^T(x)), \quad \text{and} \quad \sigma(x) := A(x)\varepsilon(x), \tag{1}$$

where $u(x)$ is the displacement field at point $x$ and $A(x)$ is the elasticity tensor (a fourth order tensor) involved in the *Hooke's law* (1). $A$ is supposed to be *inhomogeneous*, which means that its value depends on the point $x$. $A(x)$ is a positive definite tensor which satisfies some symmetry conditions.

When $A$ is given, one can state two kinds of boundary problems, respectively of *Dirichlet* and *Neuman* type:

$$\left\{\begin{array}{rcll} \mathrm{div}\sigma & = & 0 & \text{in } \Omega, \\ u & = & u_0 & \text{on } \partial\Omega, \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{rcll} \mathrm{div}\ \sigma & = & 0 & \text{in } \Omega, \\ \sigma.n & = & g_0 & \text{on } \partial\Omega, \end{array}\right. \tag{2}$$

where $u_0$ and $g_0$ are respectively a given displacement field and a given external force field on the boundary $\partial\Omega$.

It is well known (see *e.g.* [4]) that each of these problem has a unique solution (for the Neuman's problem, one has to impose an integral condition on $g_0$ to insure existence, and an integral condition on $u$ to eliminate rigid displacements).

In the following, the *inverse problem* will be considered:

$$\text{find } A \text{ such that }, \forall i \in \{1, ..., n\}, \exists u, \left\{\begin{array}{rcll} \mathrm{div}\ \sigma & = & 0 & \text{in } \Omega, \\ u & = & u_i & \text{on } \partial\Omega, \\ \sigma.n & = & g_i & \text{on } \partial\Omega, \\ \sigma & = & A\varepsilon, & \end{array}\right. \tag{3}$$

where $(g_i)_{i=1,..,n}$ and $(u_i)_{i=1,..,n}$ are given.

Problem (3) is a discrete version of the "ideal" inverse problem:

$$\text{find } A, \text{ given application: } \Lambda_A : u_{|\partial\Omega} \longrightarrow \sigma_{|\partial\Omega}. \tag{4}$$

The underlying physical problem is still lacking much more of known data than problem (3) since $\Lambda_A$ is only known through a finite number of *experimental measures* performed at a finite number of points. Hence, the real identification problem treated by mechanical engineers can be stated as:

$$\text{find } A \text{ such that }, \forall 1 \leq i \leq n, \exists u, \left\{\begin{array}{rcll} \mathrm{div}\ \sigma & = & 0 & \text{in } \Omega, \\ u(x^j) & = & u_i^j & \forall 1 \leq j \leq p, \\ \sigma(x^j).n & = & g_i^j & \forall 1 \leq j \leq p, \\ \sigma & = & A\varepsilon, & \end{array}\right. \tag{5}$$

where information on the boundary is only known at a finite number of experimental points $(x^j)_{j=1,...,p}$ for a finite number $(n)$ of experiments. In addition, these data may be known with a certain amount of experimental error or noise.

### 3.2 State of the art

If the aim is the numerical treatment of problem (4) (and *a fortiori* problem (3) or (5)) by "classical" (*i.e.* non-stochastic) methods, two theoretical points are crucial:

- existence and uniqueness of $A$ as a function of $\Lambda_A$,
- continuity of the dependency of $A$ with respect to $\Lambda_A$.

Existence is of course essential to the pertinence of the identification problem, but uniqueness and continuity are only needed to insure the reliability and the stability of deterministic numerical algorithms. On the other hand, EAs can deal with non-continuous functionals and non-unique solutions.

Problem (4) is the elastic equivalent of the so-called *tomography* problem where the elliptic operator is the conductivity operator ($\mathrm{div}(A\nabla u)$, $u$ scalar field) instead of the elasticity one ($\mathrm{div}(A\varepsilon(u))$, $u$ vector field).

The tomography problem as been widely studied. Under some hypothesis, existence and uniqueness have been proved. However, the continuity of the functional is only known in a weak sense, that cannot help for numerical simulations.

The elasticity problem (4) is more difficult. Existence and uniqueness have been proved for isotropic Hooke's laws, but there is no continuity result. For a comprehensive bibliographical discussion on this subject, see [6].

Numerical simulations by classical methods have shown that both tomography [19] and elastic identification problems [6] are ill-posed, and thus EAs are good tentative choice for a robust optimization method.

### 3.3 The direct problem

In this paper, attention has been focused on representation and on specific operators for EAs. To highlight specific problems involved in these algorithms, the mechanical problem (5) was restricted to a two-dimensional simpler class of problems:

Let $A_1$ and $A_2$ be two isotropic elasticity tensors, fully defined by Young's moduli $E_1$ and $E_2$ and Poisson ratios $\nu_1$ and $\nu_2$. The aim is to solve Problems (3) and (5), restricting allowable values of $A(x)$ to

$$A(x) = \begin{cases} A_1 & \text{if } \chi(x) = 0 \\ A_2 & \text{if } \chi(x) = 1 \end{cases} \tag{6}$$

where $\chi$ is a characteristic function defined on $\Omega$: The target is hence a partition of $\Omega$ in two subsets, each made of a material with known characteristics.

These problems, although less general than (3) and (5) are still beyond the capabilities of deterministic algorithms (see [6]). Moreover, the general Problem (5) can be treated in the same way, as well as identification in non-linear elasticity (as discussed in Section 6).

The direct elasticity problems have been solved by a classical finite element method (as described in [13]). All geometrical and mechanical details are specified in Section 5.

# 4 Representations for mechanical inclusions

This section introduces two non-standard representations for the problem described in Section 3. Both are variable-length representations, and use real numbers as main components. Hence, specific operators (*e.g.* crossover and mutation) have to be designed for each representation.

## 4.1 Prerequisites

A solution to the inclusion identification problem is a partition of the domain of the structure into two subsets, each subset representing one of the materials involved. Moreover, all connected components of any subset should have a non-void interior and a regular boundary.

A theoretical framework has been developed by Ghaddar & al. [10] in the context of Structural Optimum Design: The search space is restricted to partitions with polygonal boundaries. Theoretical results are proven, approximation spaces are introduced and corresponding approximation results are obtained. Though the objective function considered in this paper is quite different from the one in [10], the same search space will be used here.

However, a significant difference between the objective functions in [10] and the one to be used here is that the inclusion identification problem requires a Finite Element Analysis on the direct problem (see Section 3.3) to compute the fitness of a point of the search space (*i.e.* a given repartition of both materials), as introduced in the general framework of Section 2, and detailed in Section 5.2. It is well-known that meshing is a source of numerical errors [5]. Hence, for any Evolutionary Algorithm, using a fitness function based on the outputs of two Finite Element Analyses performed on different meshes is bound to failure, at least when the actual differences of behavior becomes smaller than the unavoidable numerical noise due to remeshing. The use of the same mesh for all Finite Element Analyses (at least inside the same generation) is thus mandatory to obtain significant results.

## 4.2 The bitstring representation

Once the decision to use a fixed mesh has been taken, and with even very little knowledge of EAs, the straightforward representation for a partition of the given domain is that of bitstrings: each element of the fixed mesh belongs to either one of the subsets of the partition, which can be symbolically labeled 0 or 1. The resulting representation is a bitstring – or, more precisely, a bitarray, as the pure bitstring point of view can be misleading, see

[18]. Hence, almost all previous works using Genetic Algorithms in Optimum Design did use that representation [11, 3, 17].

However, the limits of this bitstring representation clearly appear when it comes to refining the mesh, in order either to get more accurate results or to solve 3-dimensional problems: this would imply huge bitstring, as the size of the bitstring is that of the underlying mesh. However, the size of the population should increase proportionally to that of the bitstring, according to both theoretical results [2] and empirical studies [27]. Moreover, more generations are also needed to reach convergence, and the resulting algorithm rapidly becomes intractable.

These considerations show the need for other representations, not relying on a given mesh – even if a fixed mesh is used during the computation of the fitness function. Two of such representations have been designed, and successfully used on the Optimum Design problem [23, 24].

## 4.3 The Voronoï representation

A possible way of representing partitions of a given domain comes from computational geometry, more precisely from the theory of Voronoï diagrams. The ideas of Voronoï diagrams are already well-known in the Finite Element community, as a powerful tool to generate good meshes [9]. However, the representation of partitions by Voronoï diagrams aiming at their evolutionary optimization seems to be original.

**Voronoï diagrams**: Consider a finite number of points $V_0, \ldots, V_N$ (the *Voronoï sites*) of a given subset of $\mathbb{R}^n$ (the design domain). To each site $V_i$ is associated the set of all points of the design domain for which the closest Voronoï site is $V_i$, termed *Voronoï cell.* The *Voronoï diagram* is the partition of the design domain defined by the Voronoï cells. Each cell is a polyhedral subset of the design domain, and any partition of a domain of $\mathbb{R}^n$ into polyhedral subsets is the Voronoï diagram of at least one set of Voronoï sites (see [20, 1] for a detailed introduction to Voronoï diagrams).

**The genotype**: Consider now a (variable length) list of Voronoï sites, each site being labeled 0 or 1. The corresponding Voronoï diagram represents a partition of the design domain into two subsets, if each Voronoï cell is labeled as the associated site (here the Voronoï diagram is supposed regular, *i.e.* to each cell corresponds exactly one site). Example of Voronoï representations can be seen in Figure 3. The Voronoï sites are the dots in the center of the cells. Note that this representation does not depend in any way on the mesh that will be used to compute the mechanical behavior of the structure. Furthermore, Voronoï diagrams being defined in any dimension, the extension of this representation to $\mathbb{R}^3$ and $\mathbb{R}^n$ is straightforward.

An important remark is that this representation presents a high degree of *epistasis* (the influence of one Voronoï site on the mechanical structure is highly dependant on all neighbor sites). This will be discussed in more details in Section 6.

**Decoding**: Practically, and for the reasons stated in Section 4.1 the fitness of all structures will be evaluated using the same fixed mesh. A partition described by Voronoï sites is thus mapped on this fixed mesh: the subset an element belongs to is determined from the label of the Voronoï cell in which the center of gravity of that element lies in.

**Crossover operator**: The idea of the crossover operators is to exchange subsets of geometrically linked Voronoï sites. In this respect, it is similar to the specific bitarray crossover described in [18, 17]; moreover, this mechanism easily extends to any dimension [15]. Figure 3 demonstrates an application of this crossover operator.

**Mutation operators**: Different mutation operators have been designed. They are applied in turn, based on user-defined probabilities.

A first mutation operator modifies the coordinates of the Voronoï sites as in the now-standard Gaussian mutation for real-valued variables from Evolution Strategies [25] (i.e. by addition of a Gaussian random variable of mean 0 and user-defined standard deviation) ; Another mutation randomly flips the boolean attribute of some sites; Finally, dealing with variable-length representations, one has to include as mutation operators the random addition and destruction of some Voronoï sites.

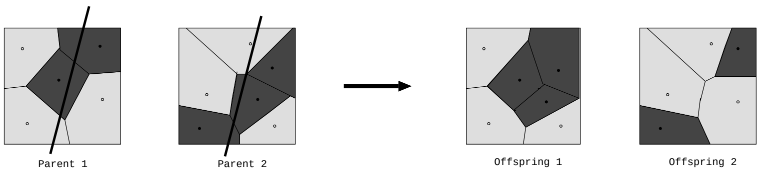



Figure 3: *The Voronoï representation crossover operator. A random line is drawn across both diagrams, and the sites on one side are exchanged*

## 4.4 H-representation

Another representation for partitions is based on an old-time heuristic method in Topological Optimum Design (TOD): from the initial design domain, considered as plain material, remove material at locations where the mechanical stress is minimal, until the constraints are violated. However, the lack of backtracking makes this method useless in most TOD problems. Nevertheless, this idea gave birth to the "holes" representation [7], later termed H-representation.

**The representation**: The design domain is by default made of one material, and a (variable length) list of "holes" describes the repartition of the other material. These holes are elementary shapes taken from a library of possible simples shapes. Only rectangular holes are considered at the moment, though on-going work is concerned with other elementary holes (*e.g.* triangles, circles) [26, 16].

Example of structures described in the H-representation are presented in Figure 4. The rectangles are taken in a domain larger than the design domain, in order not to bias the boundary parts of the design domain toward the default value.

The H-representation, as the Voronoï representation, is independent from any mesh, and hence its complexity does not depend on any required accuracy for the simulation of the mechanical behavior of the structure. Its merits and limitations will be discussed in the light of the experimental results presented in next sections.

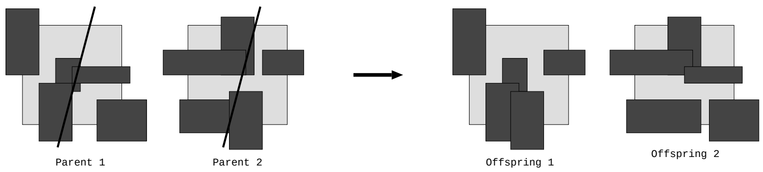



Figure 4: *The H-representation crossover operator. A random line is drawn across both structures, and the holes on one side are exchanged.*

**Decoding**: As for the Voronoï representation, the simulated behavior of the shapes is computed on a given fixed mesh, to limit the numerical noise due to re-meshing. The criterion to decide which subset an element does belong to, is based on whether its center of gravity belongs to a hole (in which case the whole element is void) or not.

**Evolution operators**: The evolution operators are quite similar to those of the Voronoï representation:

• crossover by geometrical (2D or 3D) exchange of holes (see Figure 4 for an example);

• mutation by Gaussian modification of the characteristics (coordinates of the center, width and length) of some holes;

• mutation by random addition or destruction of some holes;

# 5 Numerical Results

This section presents the very first experiments (to the best of our knowledge) on the inclusion identification problem using Evolutionary Algorithms.

## 5.1 Experimental settings

All numerical results for problem (3) have been obtained on a two-dimensional square structure fixed on its left-side, the forces being applied at points of the three other sides.

The aim is to identify the repartition of two materials into a given domain: a hard material ($E = 1.5$ and $\nu = 0.3$) and a soft material ($E = 1$ and $\nu = 0.3$). A fixed mesh of size $24 \times 24$ was used throughout the experiments.

The reference experimental loading cases considered to compute one fitness value (see Section 2) were here actually computed from a known configuration of both materials in the structure. The optimal solution is thus known, which allows a better insight and understanding during the evolution of the population. Moreover, much flexibility was required during the tuning of the overall process, that actual experimental results could not have bought. Finally, considerations about the noise in the experimental reference recording also favor simulated results: there still is a bias (due to numerical error in the Finite Element Analysis), but, as this bias hopefully is the same during the fitness computation, it should not weaken the results of the evolutionary algorithm, as could unpredictable noise in actual measures. Of course, further work will have to consider actual experimental results to fully validate the approach.

## 5.2 The fitness functions

In real world situations, the design of the fitness function should take into account as many loading cases as available, in order to use as much information as possible about the behavior of the structure to be identified. However, the choice made in these preliminary experiments of having simulated "reference experiments" makes it possible to use as many loading cases as needed. In an attempt to have a sampling of the mechanical behavior of the structure as uniform as possible over the domain, 37 different loading cases were used: Each loading case consists in pulling with a given force at one point of the boundary of the structure, following the normal to the boundary of the unloaded structure. The 37 loading points are equidistributed on the free boundary of the square structure.

Another degree of freedom offered by the use of simulated experimental values for the reference displacements addresses the number of points where these reference displacements were available. In real world situations, some gauges are placed to the boundary of the structure, and only the displacements at those points is available. However, it seems clear that the more measure points, the easier the identification task. Hence, three different fitness functions have been used throughout the numerical experiments presented below, all using the 37 loading cases described above, but different in the number of measure points used to compute the error between the reference displacements (the so-called "experimental results") and the displacements of the partition at hand:

• The most informative fitness function, hereafter referred to as the *total fitness*, takes into account the displacements at *all nodes* of the mesh.

• An intermediate fitness function uses only the displacements of all nodes lying at the boundary fo the structure, and is termed the *boundary fitness.*

• The *real-world fitness* uses only 9 measure points equidistributed on the free boundary of the square structure, incorporating much less information to the fitness function, but resembling actual experimental conditions.

## 5.3 The Evolutionary Algorithm

The Evolutionary Algorithm used for all experiments presented in this paper uses a standard GA scheme: linear ranking proportional selection, crossover rate of 0.6, mutation rate of 0.2, all offspring replace all parents. The population size is set to 100, and at most 300 generations of the algorithms are allowed – but it stops whenever 50 generations are run without any improvement of the overall best fitness. Hence, between 10000 and 30000 Finite Element Analyses were performed for each run, requiring around 9h on a middle range HP workstation (715/75).

However, all experiments were run at least 10 times with independent initial populations, to avoid as much as possible stochastic bias (as clearly stated in [14], "You should never draw any conclusion of a single run of any Evolutionary Algorithm").

## 5.4 Results using the Voronoï representation

The very first experiments were performed using the Voronoï representation, described in Section 4.3.

(Corner) (Square) (Double square) (Checker)

Figure 5: *The reference structures.*

They were obtained on the reference partitions represented in Figure 5, where black areas represent the soft material and white areas the harder material (the examples range from the easiest to the most difficult).

The first results on the "corner" example of Figure 5-a were astonishingly good: the exact solution was found 3 times out of 10 runs, in less than 100 generations when using the "total" fitness, and even once (in 250 generations) using the boundary fitness. Figure 6 shows an example of a successful run,

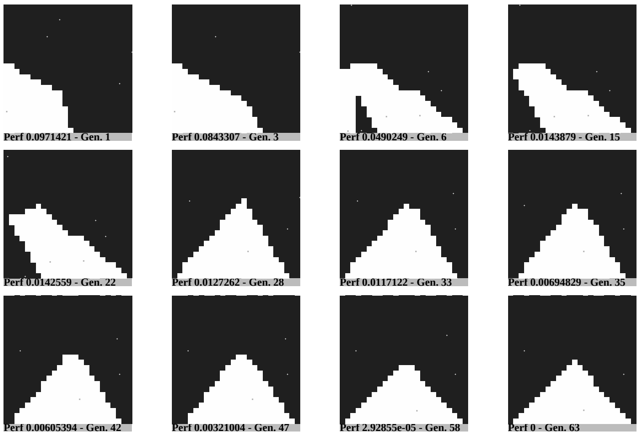



Figure 6: *A successful run on the corner problem, using boundary fitness (Perf). Plots of the best individual at different generations of the evolution*

where the best individual at different stages of evolution is plot together with the error, and the number of Voronoï sites it contains. The Voronoï sites represented on the figure by grey little dots.

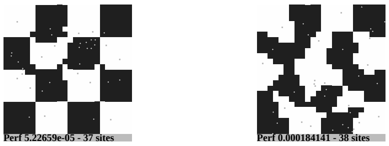
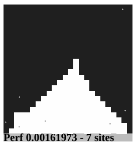
(a)  (b)


Figure 7: *The checker problem: (a): with total fitness. (b): with real fitness.*

On the "square" example of Figure 5-b, some difference began to appear between the different fitnesses, but this example is still an easy problem. This phenomenon was more and more visible as the difficulty of the problem increased. When it came to the "checker" example of Figure 5-d, the total fitness gave much better results than the real-world fitness, as can be seen in Figure 7-a and -b. However, the real-world fitness gave some interesting results, as can be seen on Figure 7-b: the actual values are clearly identified along the boundary, except along the fixed side of the boundary, where too little information is available.

## 5.5 Results in presence of noise

After these first satisfactory results on exact data, further validation of the proposed approach had to take into account possible errors and noise in the data. In order to test the sensibility of the algorithm to noise, artificial noise was purposely introduced in the reference "experimental" displacements. Figure 8 shows the results obtained on the (easy) "corner" example, with 2% and 5% noise (*i.e.* when all reference displacements were multiplied by a term $(1+\varepsilon)$, $\varepsilon$ being a random variable uniformly distributed in $[-0.02, 0.02]$ and $[-0.05, 0.05]$ respectively). The results are – of course ! – degraded, but they demonstrate a fairly good robustness, at least on this example.

(a) 

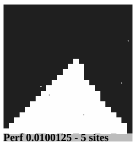

 (b)


Figure 8: *Robustness to noise. (a): with 2% noise. (b): with 5% noise.*

### 5.6 Comparative results

Further experiments were run in order to compare both the Voronoï and the H-representations. In fact, three representations were tested on together on different problems: the Voronoï representation, the H-representation where rectangles represent the soft material (termed H-0), and the H-representation where rectangles represent the hard material (termed H-1).

Regarding the comparison between the two H-representation, their behavior was what could be expected, on reference structures like the "square" example (Figure 5-b): it is by far easier to find rectangles describing the inside square than rectangles approximating the outside of that square, and the H-1 representation consistently outperformed the H-0 representation. Moreover, this phenomenon increases when the size of the inside square decreases.

However, from the limited experiments performed so far, it seems that the Voronoï representation slightly outperforms the H-representations, in contradiction with the situation in the domain of Optimum Design [23]. Figure 9 shows an example of such a situation, on the problem of the "double square" of Figure 5-c with "real" fitness. The plots represent the average over 10 independent runs of the best fitness (*i.e.* the smallest error) along generations for all three representations. Note that the variance of the results for all representation was very small, and all but one runs using the Voronoï representation reached smaller error than the best runs of using the H-representation. The best results for both the H-1 and the Voronoï representation are presented on Figure 10.

## 6 Discussion and further work

Experimental comparisons of different representations encounter the difficulty of designing "fair" experiments: the mutation operators are not the

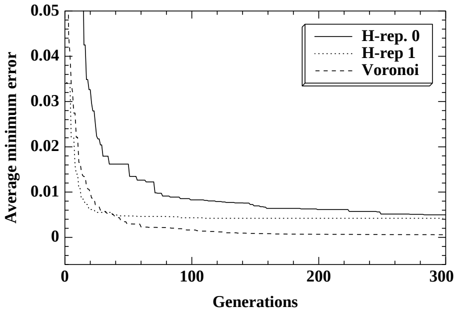



Figure 9: *Comparative results on the "double-square" problem with "real" fitness. The Voronoï representation slightly but consistently outperforms both H-representations.*

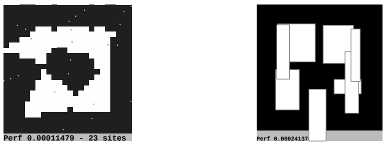
(a)  (b)


Figure 10: *Best results on the "double" example with real fitness for the Voronoï and the H-1 representation.*

same for all representations, forbidding any satisfactory way of comparing the mutation strengths. Moreover, experimental comparative results on one problem can hardly be generalized to too different problems.

Hence, it is essential to design a methodology, or at least some heuristics, to guide a future practitioner of evolutionary inclusion identification in his (or her) choice: Having so many possible representations for partitions os materials (the bitarray of section 4.2, the Voronoï representations, both H-0 and H-1 representations described in section 5.6) makes it more difficult to choose among them when facing a specific instance of a problem. Some promising directions are given in the literature.

The fitness variance theory of Radcliffe [22] studies the variance of the fitness as a function of the order of an extension of schemas called *formae* [21], and, simply put, shows that the complexity and difficulties of evolution increases with the average variance of the fitness as a function of the formae order. But if the formae and their order (or their precision) are well-defined on any binary representation, including the bit-array representation rapidly presented in Section 4.1, it is not straightforward to extend these definitions to variable length representations, as the ones presented in this paper.

Moreover, Radcliffe's fitness variance does not take into account the possible evolution operators. Further step in that direction would be to study the variance of the change of fitness with respect to a given evolution operator (*e.g.* the Gaussian mutation of Voronoï sites for different standard deviations), in the line of the work in [8].

The fitness distance correlation of Jones [12] studies the correlation between the distance to the optimal point and the fitness. Simply put again, the idea is that, the stronger this correlation, the narrower the peak the optimum belongs to. Conjectures based on this remark are experimentally confirmed in the bitstring frame. Nevertheless, the difficulty in variable-length representations is to define a distance which is meaningful for both the representation and the problem at hand. Preliminary work addressing this issue define a purely *genotypic* distance, based on partial matching of items of the variable-length lists representing the two individuals. The first – on-going – studies [16] demonstrate that the results of [12] seem to extend to the variable-length case: the correlation between the distance to optimum and the fitness is a good predictor of the performance of the representation on a given problem. Moreover, in the case where a good correlation exists, equivalent results are obtained when considering either the distance to the actual optimum or the distance to the best individual in the current sample: if this was not true, the method would be of poor practical interest, as the global optimum is usually unknown.

Another direction of research regards the link between the representation and the evolution scheme: as stated and partially demonstrated on the Optimum Design problem in [24], the higher degree of epistasis (*i.e.* interaction among genetic material of the same individual when going from the genotype to the actual mechanical structure) in the representation should favor top-down approaches, *e.g.* ES or EP schemes relying mostly upon mutation operator, rather than the GA bottom-up approach relying upon the gradual assembly of small building blocks to construct the a good solution. As opposed to the Optimum Design problem, the inclusion identification problem is tunable and hence allows precise experiments to be conducted: the optimum partition is known, and can be tailored at will: for instance, the respective amount of both material can be prescribed, as well as the number of connected components of both materials.

Further work will also consider the general problem (3) of Section 3 instead of the boolean simplification (6): the extension of both representations to handle real-valued labels instead of boolean labels is straightforward for the Voronoï representation (replace the boolean label of each site by a real number), and fairly simple to imagine for the H-representation (*e.g.* assign a real-valued label to each “hole”, and, for each element, compute the mean value of the labels of all holes covering the center of the element).

# 7 Conclusion

This paper presents preliminary results using Evolutionary Computation on an important engineering problem: non destructive inclusion identification in Structural Mechanics. In spite of theoretical results on identifiability in the linear elastic case, the standard deterministic numerical methods have to face a ill-conditioned problem, and demonstrated to be both inaccurate and unstable. On the opposite, the evolutionary method demonstrates powerful on the simplified problem of linear elasticity involving two materials of known characteristics.

These results required the design of non-standard representations together with *ad hoc* genetic operators. The main feature of these representation is their independence toward any discretization of the structure: hence the complexity of the algorithm itself, in terms of number of fitness evaluations, only depends on the problem at hand, regardless of the numerical method used to compute the fitness. Moreover, these representations can – and will – be extended easily to identify unknown materials in a given structure.

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