$$
\begin{array}{lll}
p \rightsquigarrow \mathsf{Mlistof}\, R\, L & \equiv & \mathsf{match}\, L\, \mathsf{with} \\
& & \mid \mathsf{nil} \;\Rightarrow\; [p = \mathsf{null}] \\
& & \mid X :: L' \;\Rightarrow\; \exists x p'. \quad p \rightsquigarrow \{\!|\mathrm{hd}{=}x;\ \mathrm{tl}{=}p'|\!\} \\
& & \qquad\qquad\qquad\qquad \star\, p' \rightsquigarrow \mathsf{Mlistof}\, R\, L' \\
& & \qquad\qquad\qquad\qquad \star\, x \rightsquigarrow R\, X
\end{array}
$$

Exercise: since $(p : \mathsf{loc})$ and $(x : \mathsf{Val})$ and $(X : A)$ for some A, what is the type of R? What is the type of $\mathsf{Mlistof}$?

$$
\begin{array}{lll}
p \rightsquigarrow \mathsf{MList}\, L & \equiv & \mathsf{match}\, L\, \mathsf{with} \\
& & \mid \mathsf{nil} \;\Rightarrow\; [p = \mathsf{null}] \\
& & \mid x :: L' \;\Rightarrow\; \exists p'. \quad p \rightsquigarrow \{\!|\mathrm{hd}{=}x;\ \mathrm{tl}{=}p'|\!\} \\
& & \qquad\qquad\qquad\qquad \star\, p' \rightsquigarrow \mathsf{MList}\, L'
\end{array}
$$

Exercise: define the identity representation predicate Id such that

$$p \rightsquigarrow \mathsf{Mlistof}\, \mathsf{Id}\, L \;=\; p \rightsquigarrow \mathsf{MList}\, L$$

Exercise: specify functions over queues using a higher-order representation predicate written $p \rightsquigarrow \mathsf{Queueof}\, R\, L$.
Shorthand: just write "$\mathsf{Q}\, R$" instead of "$\mathsf{Queueof}\, R$".

$\{ \qquad \} \; (\mathtt{create()}) \; \{ \qquad \}$

$\{ \qquad \} \; (\mathtt{push}\, x\, p) \; \{ \qquad \}$

$\{ \qquad \} \; (\mathtt{pop}\, p) \; \{ \qquad \}$

$\{ \qquad \} \; (\mathtt{concat}\, p\, p') \; \{ \qquad \}$

Exercise: specify a function $\mathtt{copy}\, f\, p$ that duplicables a mutable queue specified using $\mathsf{Queueof}$, where f is a function to duplicate items.

$$
\begin{array}{ll}
 & \big(\forall x X.\ \{\qquad\qquad\qquad\qquad\} \,(f\,x)\, \{\qquad\qquad\qquad\qquad\}\big) \\
\Rightarrow & \{p \rightsquigarrow \mathsf{Queueof}\, R\, L\} \\
 & (\mathtt{copy}\, f\, p) \\
 & \{\lambda p'.\ p \rightsquigarrow \mathsf{Queueof}\, R\, L \star p' \rightsquigarrow \mathsf{Queueof}\, R\, L\}
\end{array}
$$

$$
\begin{array}{rcl}
p \rightsquigarrow \mathsf{MCellof}\, R_1\, V_1\, R_2\, V_2 & \equiv & \exists v_1 v_2.\quad p \rightsquigarrow \{\!|\mathrm{hd}{=}v_1;\ \mathrm{tl}{=}v_2|\!\} \\
 & & \qquad\qquad \star\ v_1 \rightsquigarrow R_1\, V_1 \\
 & & \qquad\qquad \star\ v_2 \rightsquigarrow R_2\, V_2
\end{array}
$$

Exercise: rewrite the specification of $\mathsf{Mlistof}$ using $\mathsf{MCellof}$.

$$
\begin{array}{rcl}
p \rightsquigarrow \mathsf{Mlistof}\, R\, L & \equiv & \mathsf{match}\, L\, \mathsf{with} \\
 & & \mid \mathsf{nil} \Rightarrow [p = \mathsf{null}] \\
 & & \mid X :: L' \Rightarrow
\end{array}
$$

Exercise: rewrite the specification of $\mathsf{Narytreeof}$ using Nodeof.

$$
\begin{array}{l}
p \rightsquigarrow \mathsf{Narytreeof}\, R\, T \equiv \\
\quad \mathsf{match}\, T\, \mathsf{with} \\
\quad \mid \mathsf{Leaf} \Rightarrow [p = \mathsf{null}] \\
\quad \mid \mathsf{Node}\, X\, L \Rightarrow
\end{array}
$$

Exercise: complete the specification of Bagof using Nodeof.
Hint: chunks are described by the predicate $p' \rightsquigarrow \mathsf{Chunkof}\, R\, E'$.

$$
\begin{array}{l}
p \rightsquigarrow \mathsf{Bagof}\, R\, T \equiv \\
\quad \mathsf{match}\, T\, \mathsf{with} \\
\quad \mid \mathsf{Empty} \Rightarrow [p = \mathsf{null}] \\
\quad \mid \mathsf{Layer}\, E'\, T' \Rightarrow
\end{array}
$$

Exercise: specify the function `miter`, using an invariant of the form $J\,K\,K'$, describing the state before and the state after the iteration.

$$\forall f p R L J.\ \big(\forall x X\ .\ \{x \rightsquigarrow R\,X\}\ (f\,x)\ \{\lambda\_.\}\big) \Rightarrow \{p \rightsquigarrow \mathsf{Mlistof}\,R\,L \star \}\ (\texttt{miter}\,f\,p)\ \{\lambda\_.\}$$

```
let incr_all p =
  miter (fun x -> incr x) p

let example_p =
  { hd = ref 5; tl = { hd = ref 3; tl = null } }
```

$$x \rightsquigarrow \mathsf{Ref}\,X \ \equiv\ x \mapsto X$$

Exercise: using the representation predicates Ref and Mlistof, specify the function `(fun x -> incr x)` and `incr_all`.

$$\{\quad\}\ (\texttt{incr x})\ \{\lambda\_.\quad\}$$

$$\{\quad\}\ (\texttt{incr\_all}\,p)\ \{\lambda\_.\quad\}$$

Exercise: Describe the state at the front of each lines (except 5 and 6). Explicit the instantiation of the existential in the invariant.

```
1   let r = ref 0
2   let s = ref n
3   let p = create_lock()
```

```

let concurrent_step () =
  let () = acquire_lock p in
  incr r;
  decr s;
  release_lock p
```

Exercise: state a conversion rule relating $p \rightsquigarrow \mathsf{Cellsof}\, R\, M$ with a predicate of the form $p \rightsquigarrow \mathsf{Cellsof}\, \mathsf{Id}\, M'$.
Hint: $(R : A \to a \to \mathsf{Hprop})$ and $(M : \mathsf{map}\, \mathsf{int}\, A)$ and $(M' : \mathsf{map}\, \mathsf{int}\, a)$.

$$p \rightsquigarrow \mathsf{Cellsof}\, R\, M \quad =$$