

# Auto-dualité de WQSym, l'algèbre de Hopf sur les mots tassés

Hugo Mlodecki

**Directeurs de thèse :**

Florent Hivert

Viviane Pons

9 octobre 2018

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- 2 WQSym / mots tassés
- 3 Contributions

# Permutations

## Définition

Une permutation de taille  $n$  est un mot sur l'alphabet  $\{1, 2, \dots, n\}$  où chaque lettre apparaît exactement une fois.

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Une représentation :

			•
	•		
•			
		•	
2	3	1	4

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Une représentation :

			•
	•		
•			
		•	

2 3 1 4

→ transposition →

			•
•			
		•	
	•		

→ inversion →

3 1 2 4

## Produit

	●
●	

1 2

●	
	●

2 1

# Produit de mélange

$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \quad \boxplus \quad \begin{array}{|c|c|} \hline \bullet & \\ \hline & \bullet \\ \hline \end{array} =$$

1 2
2 1

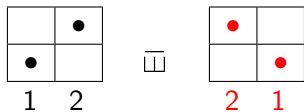
$$\begin{array}{|c|c|c|c|} \hline & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array}$$

1 2 4 3
1 4 2 3
1 4 3 2

$$\begin{array}{|c|c|c|c|} \hline \bullet & & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline \bullet & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline & \bullet & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline \bullet & & & \\ \hline & \bullet & & \\ \hline & & \bullet & \\ \hline & & \bullet & \\ \hline \end{array}$$

4 1 2 3
4 1 3 2
4 3 1 2

# Produit de mélange



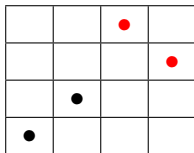
1 2

2 1

=

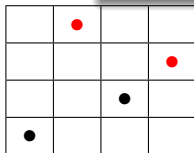
$\mathbb{F}$

$$\mathbb{F}_{12}\mathbb{F}_{21} = \mathbb{F}_{1243} + \mathbb{F}_{1423} + \mathbb{F}_{1432} + \mathbb{F}_{4123} + \mathbb{F}_{4132} + \mathbb{F}_{4312}$$



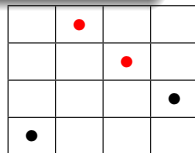
1 2 4 3

+

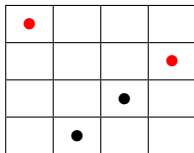


1 4 2 3

+

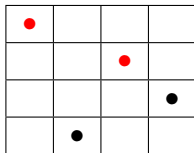


1 4 3 2



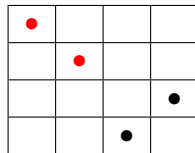
4 1 2 3

+



4 1 3 2

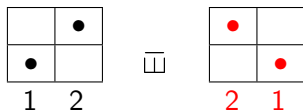
+



4 3 1 2

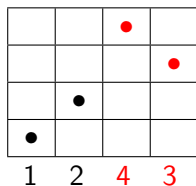


# Produit de mélange

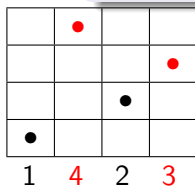


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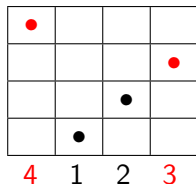
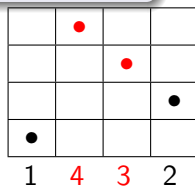
$$\mathbb{F} \mathbb{F}_\sigma \mathbb{F}_\mu := \sum_{\nu \in \sigma \bar{\omega} \mu} \mathbb{F}_\nu$$



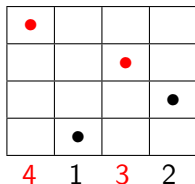
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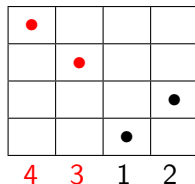
+



+



+



# Produit de mélange sur les valeurs

	●
●	

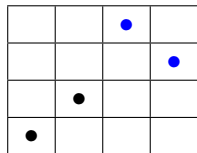
 $\underline{\underline{\square}}$ 

●	
	●

=

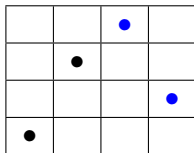
1 2

2 1



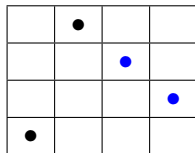
1 2 4 3

+

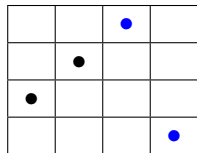


1 3 4 2

+

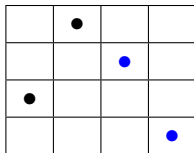


1 4 3 2



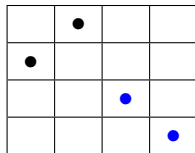
2 3 4 1

+



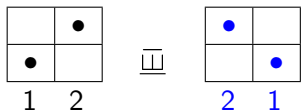
2 4 3 1

+



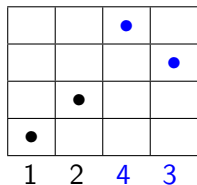
3 4 2 1

# Produit de mélange sur les valeurs

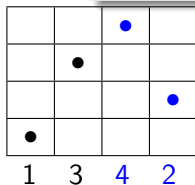


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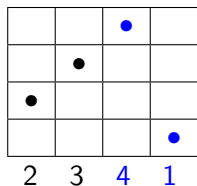
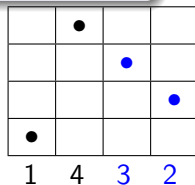
$$\mathbb{G} \mathbb{G} = \mathbb{G}_{12} \mathbb{G}_{21} = \mathbb{G}_{1243} + \mathbb{G}_{1342} + \mathbb{G}_{1432} + \mathbb{G}_{2341} + \mathbb{G}_{2431} + \mathbb{G}_{3421}$$



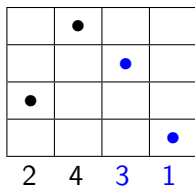
+



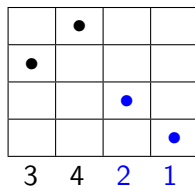
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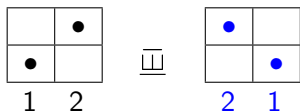
+



+

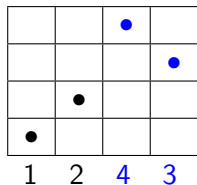


# Produit de mélange sur les valeurs

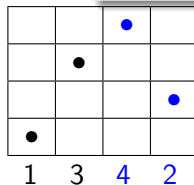


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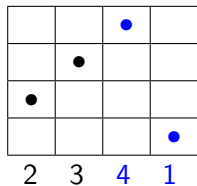
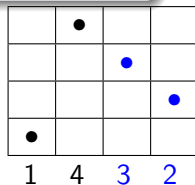
$$\begin{aligned}
 & \mathbb{G} \\
 \mathbb{G}_\sigma \mathbb{G}_\mu & := \sum_{\substack{\nu=uv, \\ \text{std}(u)=\sigma, \\ \text{std}(v)=\mu}} \mathbb{G}_\nu
 \end{aligned}$$



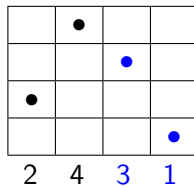
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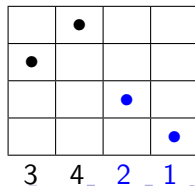
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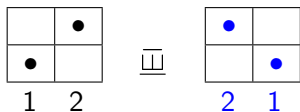
+



+

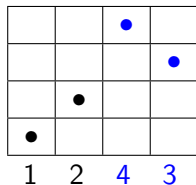


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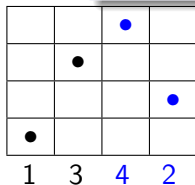


=

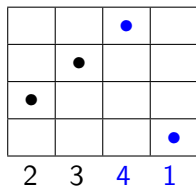
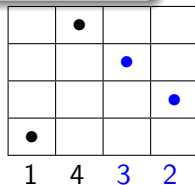
$$G_{\sigma} G_{\mu} := \sum_{\nu \in \sigma \underline{\mu}} G_{\nu}$$



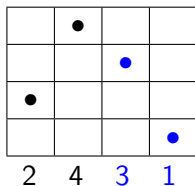
+



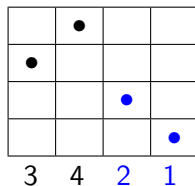
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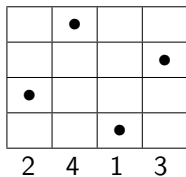
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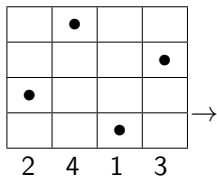
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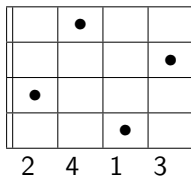
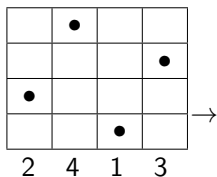
# Désassemblage vertical



# Désassemblage vertical

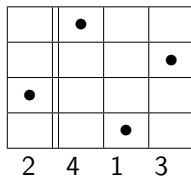
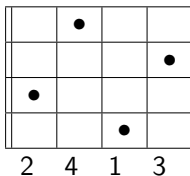
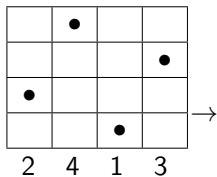


# Désassemblage vertical

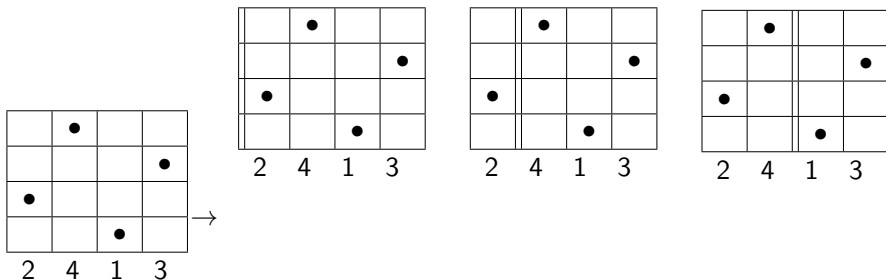




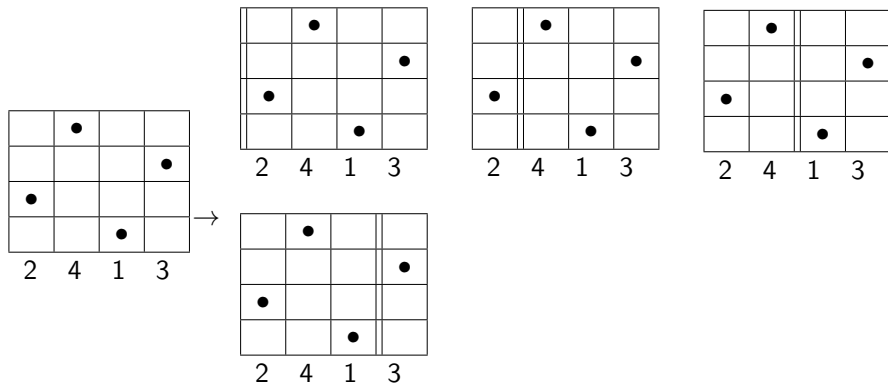
# Désassemblage vertical



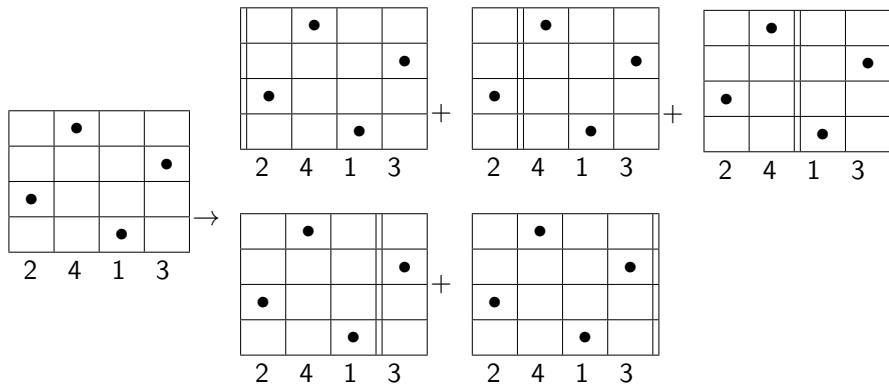
# Désassemblage vertical



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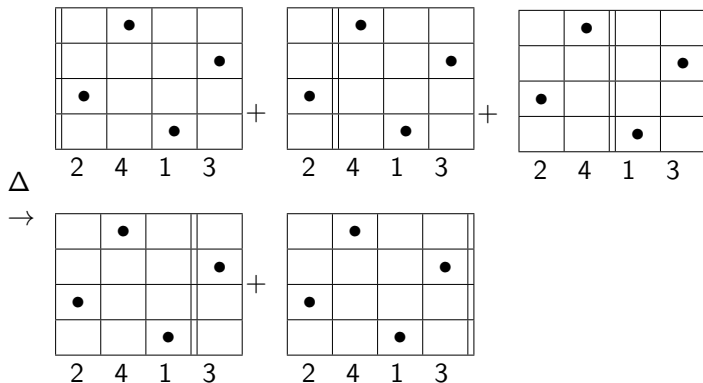


# Désassemblage vertical



# Désassemblage vertical

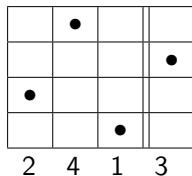
$\mathbb{F}_{2413}$



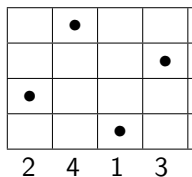
# Désassemblage vertical

$\mathbb{F}_{2413}$

$\Delta$   
→

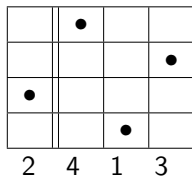


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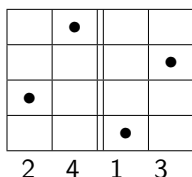


$\mathbb{F}_\epsilon \otimes \mathbb{F}_{2413}$

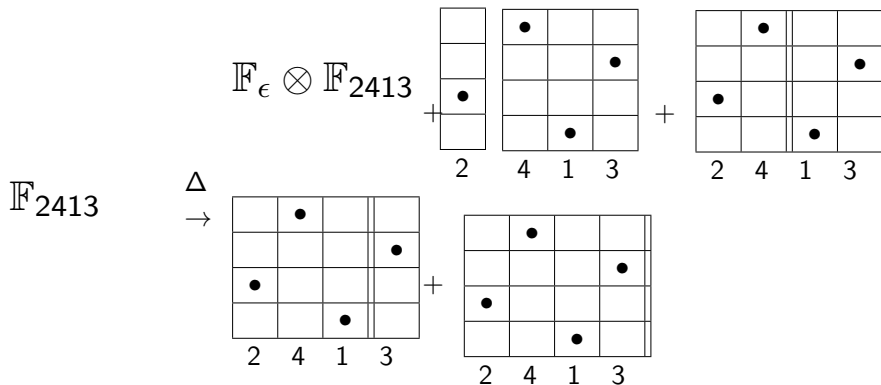
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+



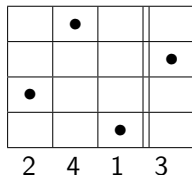
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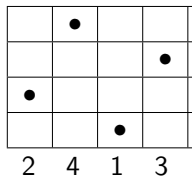
# Désassemblage vertical

$\mathbb{F}_{2413}$

$\Delta$   
→

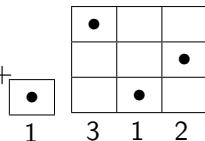


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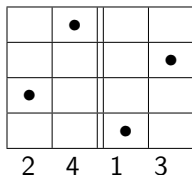


$\mathbb{F}_\epsilon \otimes \mathbb{F}_{2413}$

+

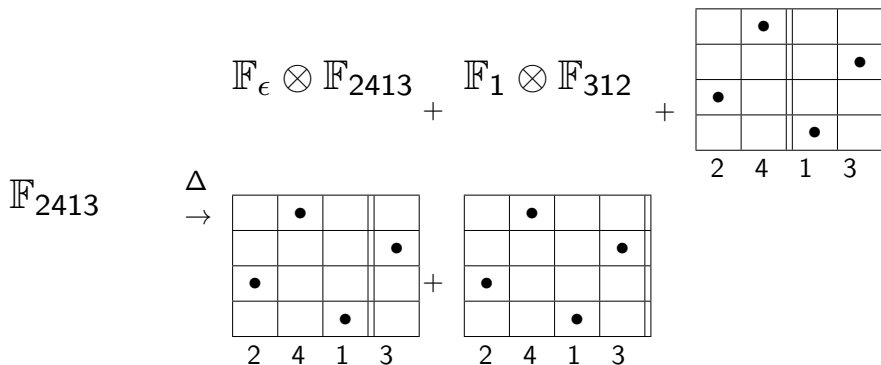


+





# Désassemblage vertical

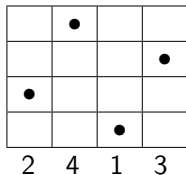


# Désassemblage vertical

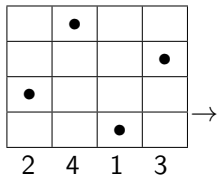
$$\mathbb{F}_\epsilon \otimes \mathbb{F}_{2413} \quad + \quad \mathbb{F}_1 \otimes \mathbb{F}_{312} \quad + \quad \mathbb{F}_{12} \otimes \mathbb{F}_{12}$$

$$\mathbb{F}_{2413} \quad \begin{array}{c} \Delta \\ \rightarrow \end{array} \quad \mathbb{F}_{231} \otimes \mathbb{F}_1 \quad + \quad \mathbb{F}_{2413} \otimes \mathbb{F}_\epsilon$$

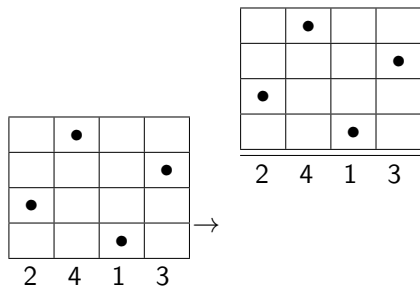
# Désassemblage horizontale



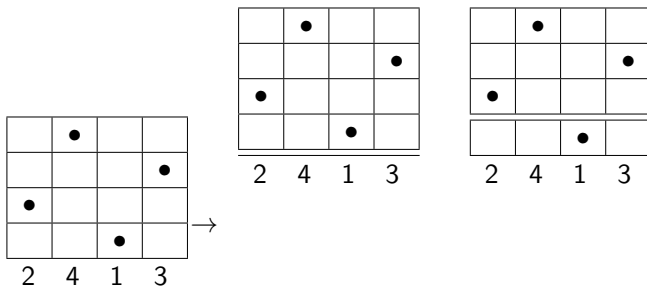
# Désassemblage horizontale



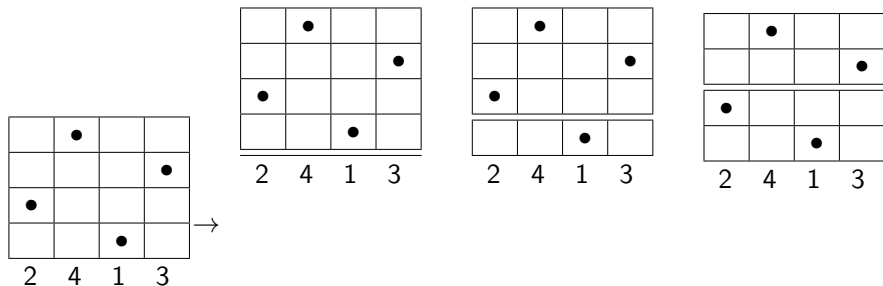
# Désassemblage horizontale



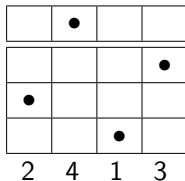
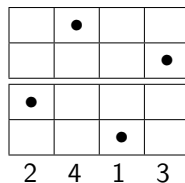
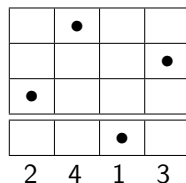
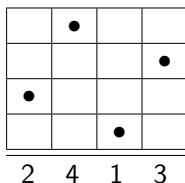
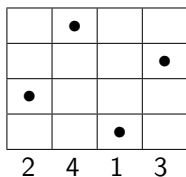
# Désassemblage horizontale



# Désassemblage horizontale

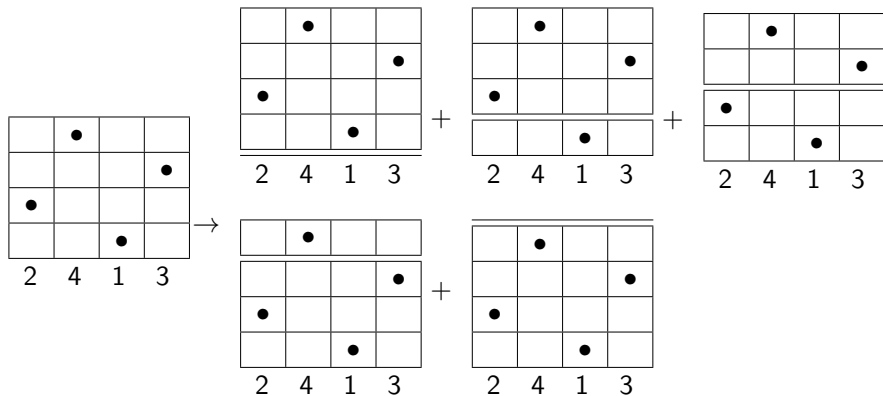


# Désassemblage horizontale



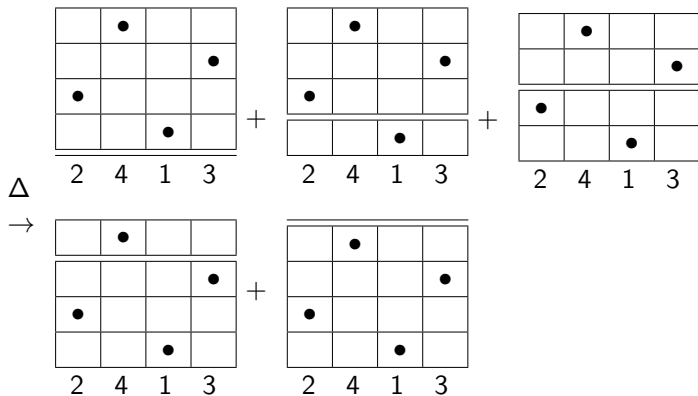


# Désassemblage horizontale



# Désassemblage horizontale

$\mathbb{G}_{2413}$

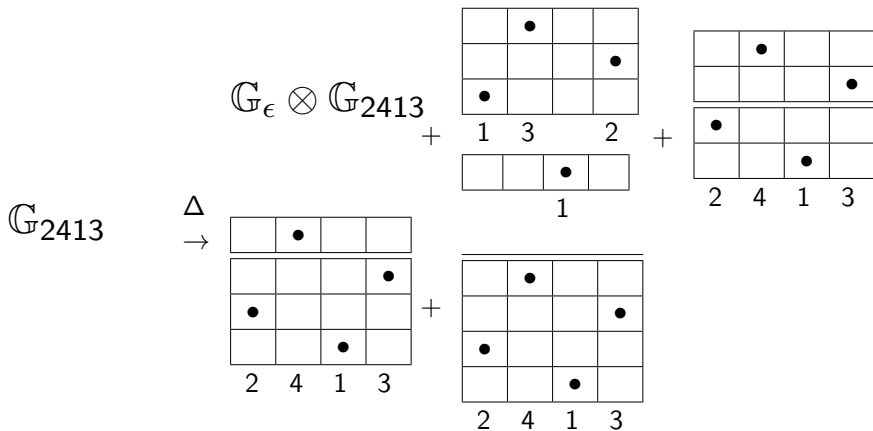


# Désassemblage horizontale

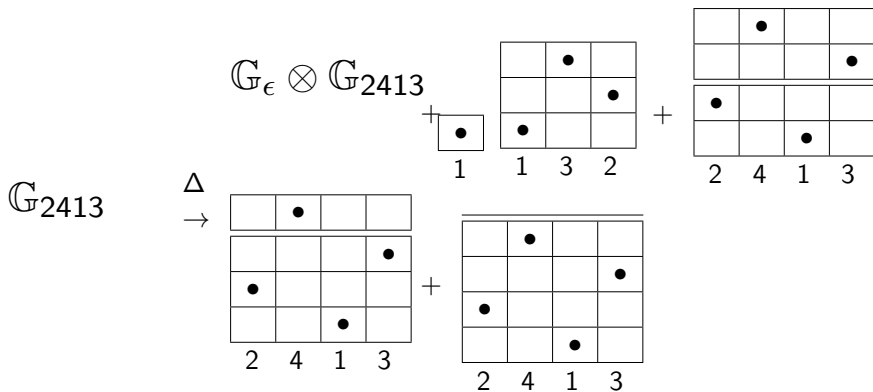
$$\mathbb{G}_{2413} \xrightarrow{\Delta} \mathbb{G}_{\epsilon} \otimes \mathbb{G}_{2413} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array}$$

2 4 1 3
2 4 1 3
2 4 1 3
2 4 1 3
2 4 1 3
2 4 1 3

# Désassemblage horizontale



# Désassemblage horizontale



# Désassemblage horizontale

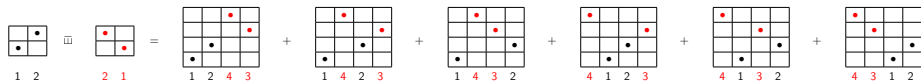
$$\mathbb{G}_{2413} \xrightarrow{\Delta} \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & & \bullet & \\ \hline 2 & 4 & 1 & 3 \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & & \bullet & \\ \hline 2 & 4 & 1 & 3 \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & & \bullet & \\ \hline 2 & 4 & 1 & 3 \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & \bullet & & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline & & \bullet & \\ \hline 2 & 4 & 1 & 3 \\ \hline \end{array}$$

$\mathbb{G}_\epsilon \otimes \mathbb{G}_{2413} + \mathbb{G}_1 \otimes \mathbb{G}_{132}$

# Désassemblage horizontale

$$\begin{array}{ccc}
 & \mathbb{G}_\epsilon \otimes \mathbb{G}_{2413} & \mathbb{G}_1 \otimes \mathbb{G}_{132} & \mathbb{G}_{21} \otimes \mathbb{G}_{21} \\
 & & + & + \\
 \mathbb{G}_{2413} & \xrightarrow{\Delta} & & \\
 & & \mathbb{G}_{213} \otimes \mathbb{G}_1 + & \mathbb{G}_{2413} \otimes \mathbb{G}_\epsilon
 \end{array}$$

# Dualité de FQSym



## Dualité

Soit  $H$  une algèbre de Hopf,

$$\langle \Delta(z), x \otimes y \rangle = \langle z, x \cdot y \rangle$$

$$\langle y \cdot z, x \rangle = \langle y \otimes z, \Delta(x) \rangle$$

$$\forall x, y \in H, z \in H^*,$$

$$\forall x \in H, y, z \in H^*$$



$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \quad \bar{\sqcup} \quad \begin{array}{|c|c|} \hline \bullet & \\ \hline & \bullet \\ \hline \end{array}$$

1   2                      2   1

=

 $\mathbb{F}$ 

Isomorphisme explicite :

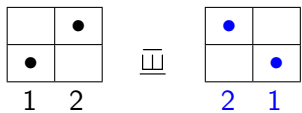
 $\mathbb{F}_\sigma \rightarrow \mathbb{G}_{\sigma^{-1}}$ 

$$\begin{array}{|c|c|c|c|} \hline & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline \bullet & & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline \bullet & & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & & \bullet \\ \hline & & & \\ \hline \bullet & & & \\ \hline \end{array}$$

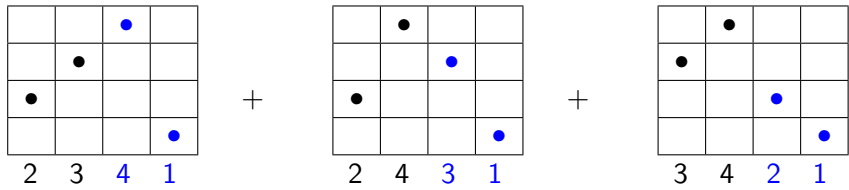
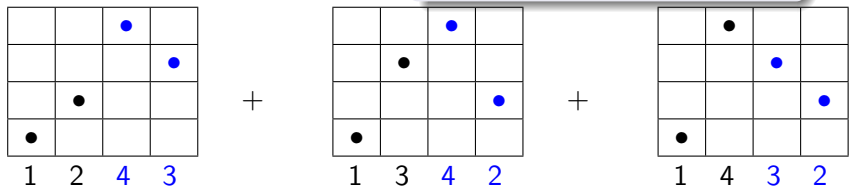
1   2   4   3                      1   4   2   3                      1   4   3   2

$$\begin{array}{|c|c|c|c|} \hline \bullet & & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline \bullet & & & \\ \hline & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline \bullet & & & \\ \hline & \bullet & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline \end{array}$$

4   1   2   3                      4   1   3   2                      4   3   1   2



$\mathbb{G}$   
 Isomorphisme explicite :  
 $\mathbb{F}_\sigma \rightarrow \mathbb{G}_{\sigma^{-1}}$



# Mots tassés

## Définition

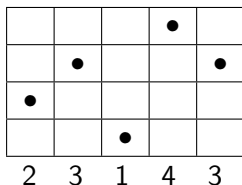
Un mot  $w$  sur l'alphabet  $\{1, 2, \dots, n\}$  est un mot tassé si pour tout nombre  $k > 1$  apparaissant dans  $w$ ,  $k - 1$  apparait aussi dans  $w$ .

# Mots tassés

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Même représentation :  $\# \text{lignes} \leq \# \text{colonnes}$

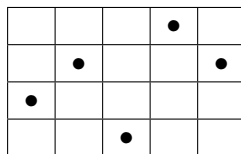


# Mots tassés

## Définition

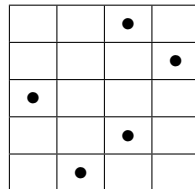
Un mot  $w$  sur l'alphabet  $\{1, 2, \dots, n\}$  est un mot tassé si pour tout nombre  $k > 1$  apparaissant dans  $w$ ,  $k - 1$  apparait aussi dans  $w$ .

Même représentation :  $\# \text{lignes} \leq \# \text{colonnes}$



2 3 1 4 3

→ transposition →

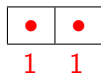
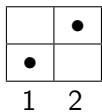


5  
3 1 2 4

→ inversion →

## Produit

## sur les mots tassés



# Produit de mélange sur les mots tassés

$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \quad \square \quad \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} =$$

1 2                      1 1

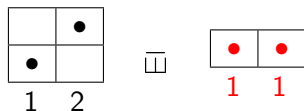
$$\begin{array}{|c|c|c|c|} \hline & & \bullet & \bullet \\ \hline & & & \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \bullet \\ \hline & & \bullet & \\ \hline \bullet & & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array}$$

1 2 3 3                      1 3 2 3                      1 3 3 2

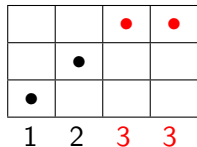
$$\begin{array}{|c|c|c|c|} \hline \bullet & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline \bullet & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline \bullet & \bullet & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline \end{array}$$

3 1 2 3                      3 1 3 2                      3 3 1 2

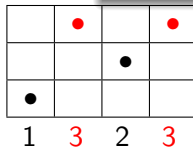
# Produit de mélange sur les mots tassés



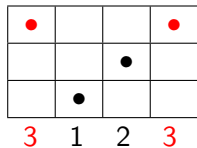
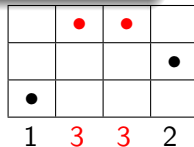
$$\mathfrak{S} \mathfrak{S}_{12} \mathfrak{S}_{11} = \mathfrak{S}_{1233} + \mathfrak{S}_{1323} + \mathfrak{S}_{1332} + \mathfrak{S}_{3123} + \mathfrak{S}_{3132} + \mathfrak{S}_{3312}$$



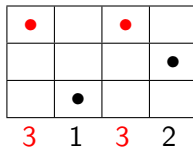
+



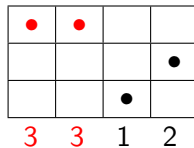
+



+



+





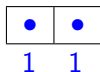
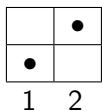
# Produit de mélange sur les mots tassés

$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} = \sum_{\nu \in \sigma \bar{\sqcup} \mu} \mathbb{S}_{\nu}$$

$\mathbb{S}_{\sigma} \mathbb{S}_{\mu} := \sum_{\nu \in \sigma \bar{\sqcup} \mu} \mathbb{S}_{\nu}$

$$\begin{array}{|c|c|c|c|} \hline & & \bullet & \bullet \\ \hline & & & \\ \hline \bullet & & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline & \bullet & & \bullet \\ \hline & & \bullet & \\ \hline \bullet & & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \\ \hline & & & \bullet \\ \hline \bullet & & & \\ \hline \end{array}$$

$\begin{array}{|c|c|c|c|} \hline \bullet & & & \bullet \\ \hline & & \bullet & \\ \hline & \bullet & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline \bullet & & \bullet & \\ \hline & & & \bullet \\ \hline & \bullet & & \\ \hline \end{array} \quad + \quad \begin{array}{|c|c|c|c|} \hline \bullet & \bullet & & \\ \hline & & & \bullet \\ \hline & & \bullet & \\ \hline \end{array}$

Produit de mélange **augmenté** sur les valeurs
 $\mathbb{M}$ 

$$\mathbb{M}_\sigma \mathbb{M}_\mu := \sum_{\substack{\nu=uv, \\ \text{tass}(u)=\sigma, \\ \text{tass}(v)=\mu}} \mathbb{M}_\nu$$

# Produit de mélange **augmenté** sur les valeurs

$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \begin{array}{c} \underline{\underline{1}} \\ \underline{\underline{2}} \end{array} \quad \underline{\underline{1}} \quad \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} \begin{array}{c} 1 \\ 1 \end{array} = \sum_{\substack{\nu=uv, \\ \text{tass}(u)=\sigma, \\ \text{tass}(v)=\mu}} \mathbb{M}_\nu$$
  

$$\begin{array}{|c|c|c|c|} \hline & & \bullet & \bullet \\ \hline & & & \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 3 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & \bullet & \bullet \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \\ 3 \\ 2 \\ 2 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline \bullet & & & \\ \hline & & \bullet & \bullet \\ \hline \end{array} \begin{array}{c} 2 \\ 3 \\ 1 \\ 1 \end{array}$$
  

$$\begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline \bullet & & \bullet & \bullet \\ \hline \end{array} \begin{array}{c} 1 \\ 2 \\ 1 \\ 1 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \bullet \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \\ 2 \\ 2 \\ 2 \end{array}$$

$\mathbb{M}$

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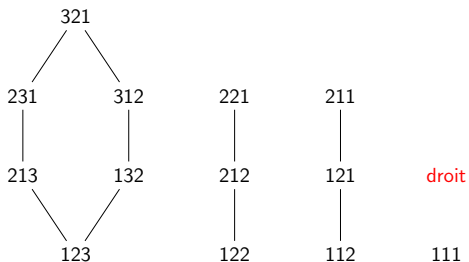
# Produit de mélange **augmenté** sur les valeurs

$$\begin{array}{|c|c|} \hline & \bullet \\ \hline \bullet & \\ \hline \end{array} \begin{array}{c} \underline{\underline{\underline{\quad}}} \\ \underline{\underline{\quad}} \\ \underline{\quad} \end{array} \begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \end{array} = \begin{array}{|c|c|c|c|} \hline & & \bullet & \bullet \\ \hline & & & \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \quad 2 \quad 3 \quad 3 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline & & \bullet & \bullet \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \quad 3 \quad 2 \quad 2 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline \bullet & & & \\ \hline & & \bullet & \bullet \\ \hline \end{array} \begin{array}{c} 2 \quad 3 \quad 1 \quad 1 \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline & \bullet & & \\ \hline \bullet & & \bullet & \bullet \\ \hline \end{array} \begin{array}{c} 1 \quad 2 \quad 1 \quad 1 \end{array} + \begin{array}{|c|c|c|c|} \hline & \bullet & \bullet & \bullet \\ \hline \bullet & & & \\ \hline \end{array} \begin{array}{c} 1 \quad 2 \quad 2 \quad 2 \end{array}$$

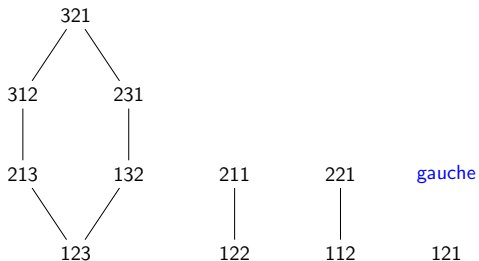
$M$

$$M_{12}M_{11} = M_{1233} + M_{1322} + M_{2311} + M_{1211} + M_{1222}$$

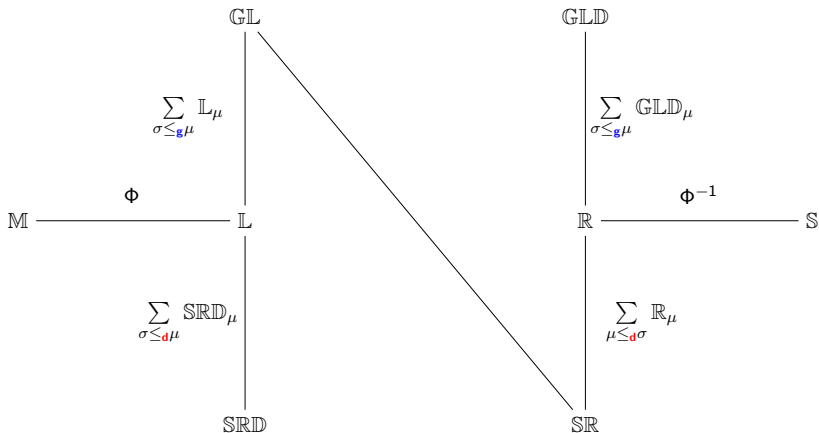
# Ordres partiels



Ordres Partiels :  
 Réflexivité,  
 Transitivité,  
 Antisymétrie.



# From M to S in WQSym



# Quelques matrices

	123	132	213	231	312	321	122	212	221	112	121	211	111
123	.	.	.	.	.	1	.	.	.	.	.	.	.
132	.	-1	1	1	.	.	.	.	.	.	.	.	.
213	.	1	-1	.	1	.	.	.	.	.	.	.	.
231	.	1	.	.	.	.	.	.	.	.	.	.	.
312	.	.	1	.	.	.	.	.	.	.	.	.	.
321	1	.	.	.	.	.	.	.	.	.	.	.	.
122	.	.	.	.	.	.	1	1	1	-1	.	.	.
212	.	.	.	.	.	.	1	1	.	.	.	.	.
221	.	.	.	.	.	.	1	.	.	.	.	.	.
112	.	.	.	.	.	.	-1	.	.	1	1	1	.
121	.	.	.	.	.	.	.	.	.	1	1	.	.
211	.	.	.	.	.	.	.	.	.	1	.	.	.
111	.	.	.	.	.	.	.	.	.	.	.	.	1

Figure: Matrice de changement de base de  $\mathbb{L}$  à  $\mathbb{R}$  pour les mots tassés de taille 3.

# Quelques matrices

	123	132	213	231	312	321	122	212	221	112	121	211	111
123	0	0	0	0	0	1	0	0	1/2	0	0	1/2	1/6
132	0	0	0	1	0	0	0	0	1/2	0	1/2	0	1/6
213	0	0	0	0	1	0	0	1/2	0	0	0	1/2	1/6
231	0	1	0	1	-1	0	1/2	-1/2	1/2	0	1	-1/2	1/6
312	0	0	1	-1	1	0	0	1	-1/2	1/2	-1/2	1/2	1/6
321	1	0	0	0	0	0	1/2	0	0	1/2	0	0	1/6
122	0	0	0	1/2	0	1/2	0	0	3/2	0	1/4	1/4	2/3
212	0	0	1/2	-1/2	1	0	0	7/4	-5/4	1/4	-1/4	1/2	1/6
221	1/2	1/2	0	1/2	-1/2	0	3/2	-5/4	1/4	1/4	1/2	3/4	2/3
112	0	0	0	0	1/2	1/2	0	1/4	1/4	0	0	3/2	2/3
121	0	1/2	0	1	-1/2	0	1/4	-1/4	1/2	0	7/4	-5/4	1/6
211	1/2	0	1/2	-1/2	1/2	0	1/4	1/2	3/4	3/2	-5/4	1/4	2/3
111	1/6	1/6	1/6	1/6	1/6	1/6	2/3	1/6	2/3	2/3	1/6	2/3	13/6

Figure: Matrice de changement de base de  $\mathbb{S}$  à  $\mathbb{M}$  pour les mots tassés de taille 3.



# Contributions

- Développement des mots tassés en Sage, *#25916 implement Packed Words*.

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- Nouveaux résultats et conjectures
  - Stabilité de l'isomorphisme de Vargas sur **FQSym**.
  - Une infinité d'automorphisme de **WQSym**.
  - Généralisation à **PQSym**.