Une base des éléments totalement primitifs de WQSym

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   - Coproduit
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Mots tassé

Définition

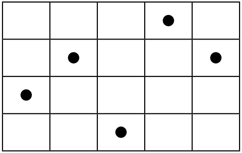
Un mot w sur l'alphabet $\{1, 2, \ldots n\}$ est un mot tassé si pour tout nombre $k > 1$ apparaissant dans w, $k - 1$ apparaît aussi dans w.

Mots tassé

Définition

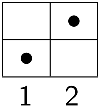
Un mot w sur l'alphabet $\{1, 2, \dots n\}$ est un mot tassé si pour tout nombre $k > 1$ apparaissant dans w, $k - 1$ apparaît aussi dans w.

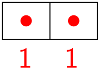
Une représentation : $\#\text{lignes} \leq \#\text{colonnes}$



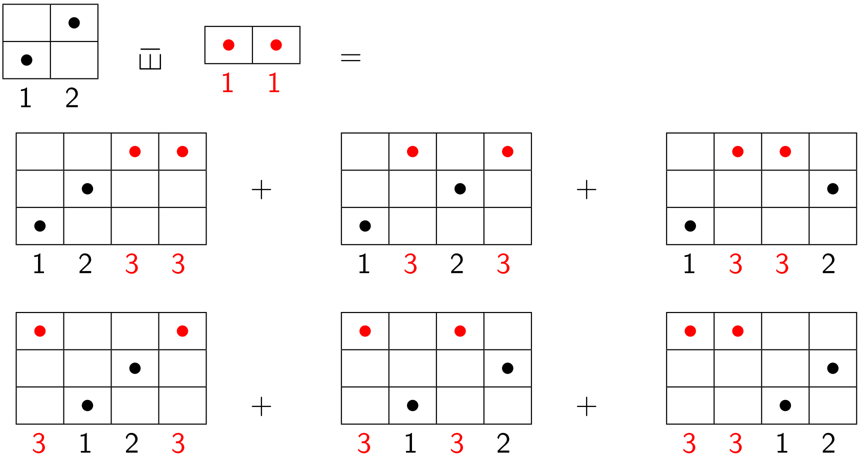
2 3 1 4 3

Produit de mélange sur les mots tassés

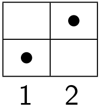




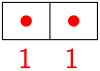
Produit de mélange sur les mots tassés



Produit de mélange sur les mots tassés



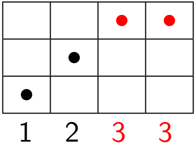
$\overline{\sqcup\!\sqcup}$



$=$

$\mathbb{R}$

$\mathbb{R}_{12}\mathbb{R}_{11} = \mathbb{R}_{1233} + \mathbb{R}_{1323} + \mathbb{R}_{1332} + \mathbb{R}_{3123} + \mathbb{R}_{3132} + \mathbb{R}_{3312}$



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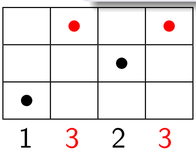

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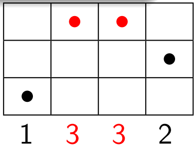

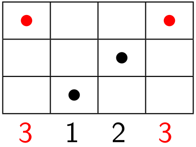

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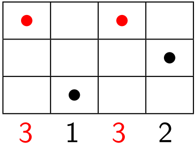

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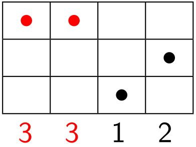
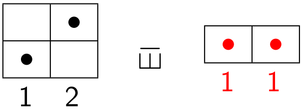
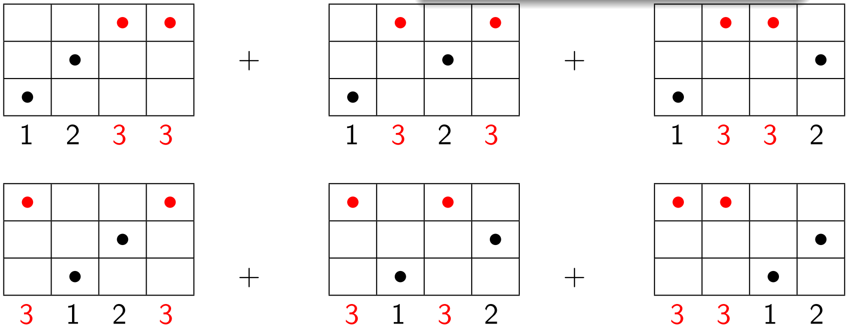


Produit de mélange sur les mots tassés

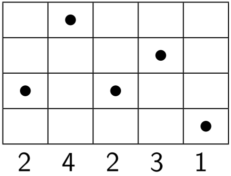


ℝ

$$\mathbb{R}_\sigma \mathbb{R}_\mu := \sum_{\nu \in \sigma \overline{\sqcup\!\sqcup} \mu} \mathbb{R}_\nu$$



Déconcaténation



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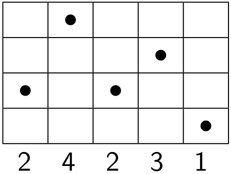
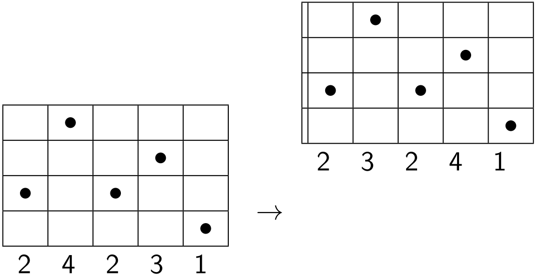


$\rightarrow$

Déconcaténation



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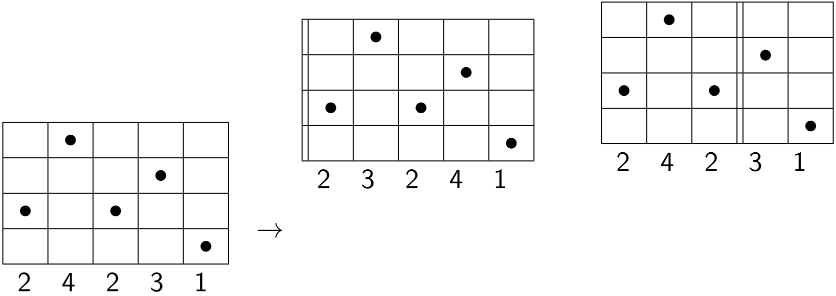

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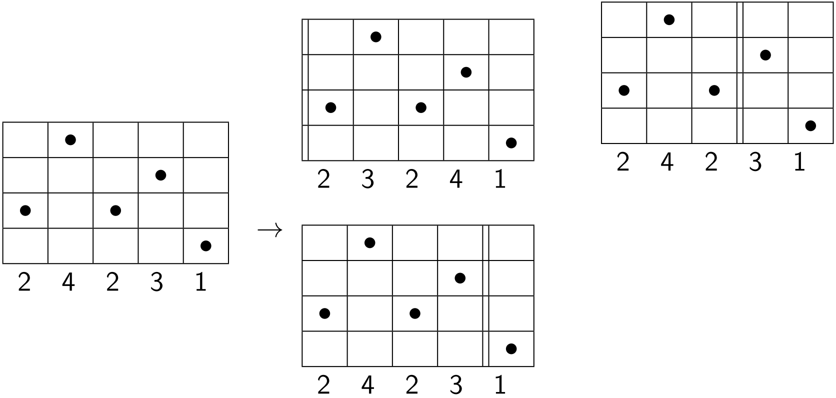
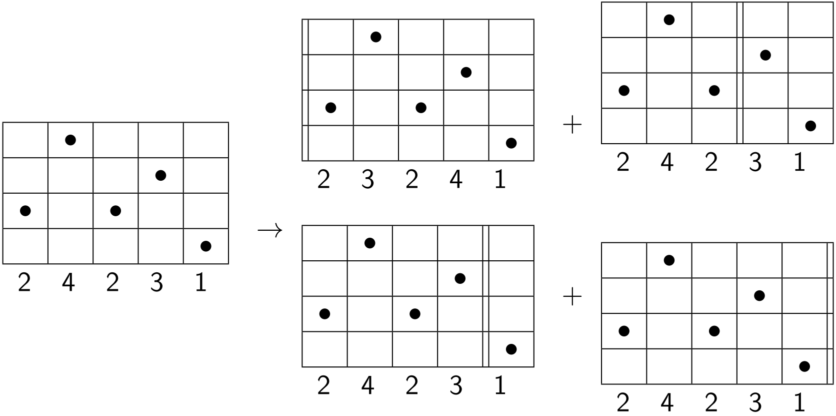
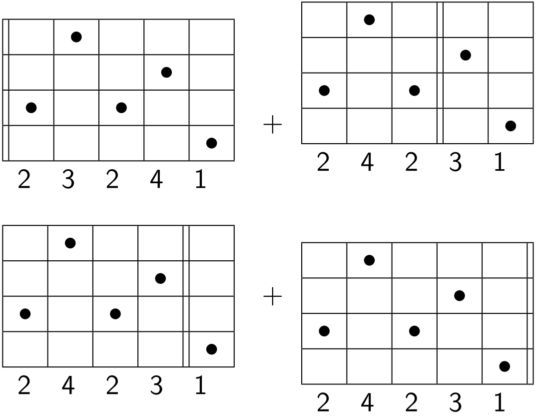


Déconcaténation



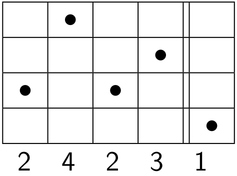
Déconcaténation

$\mathbb{R}_{24231} \rightarrow$



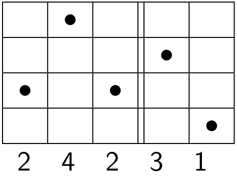
Déconcaténation

$\mathbb{R}_{24231} \xrightarrow{\Delta}$

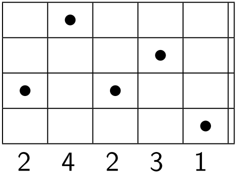


$+ \ \mathbb{R}_\epsilon \otimes \mathbb{R}_{24231}$

$+$



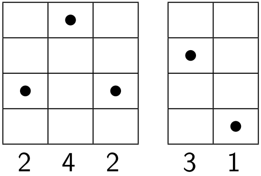
$+$



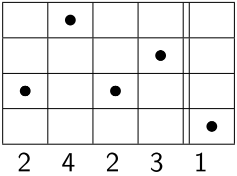
Déconcaténation

$$\mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231}$$

$+$



$$\mathbb{R}_{24231} \xrightarrow{\Delta}$$



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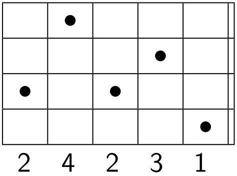
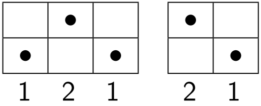
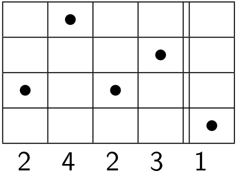


Déconcaténation

$\mathbb{R}_{24231} \xrightarrow{\Delta}$ $\mathbb{R}_\epsilon \otimes \mathbb{R}_{24231}$ $+$





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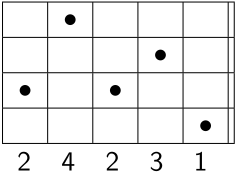
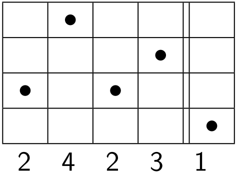


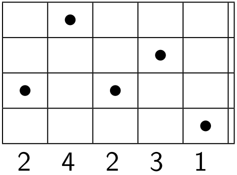
Déconcaténation

$$\mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \mathbb{R}_{121} \otimes \mathbb{R}_{21}$$

$$\mathbb{R}_{24231} \xrightarrow{\Delta}$$



$+$



Déconcaténation

$$\mathbb{R}_{24231} \xrightarrow{\Delta} \begin{array}{ll} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} & + \ \mathbb{R}_{121} \otimes \mathbb{R}_{21} \\ \mathbb{R}_{1312} \otimes \mathbb{R}_{1} & + \ \mathbb{R}_{24231} \otimes \mathbb{R}_{\epsilon} \end{array}$$

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$$ua \shuffle vb = (u \shuffle vb)a + (ua \shuffle v)b$$

$$deconcat(w_1 w_2 ... w_n) = 1 \otimes w_1 ... w_n + w_1 \otimes w_2 ... w_n + ... + w_1 ... w_n \otimes 1$$

$$\begin{aligned} ua \sqcup\!\sqcup\, vb &= (u \sqcup\!\sqcup\, vb)a + (ua \sqcup\!\sqcup\, v)b \\ &= ua \prec vb + ua \succ vb \end{aligned}$$

$$deconcat(w_1 w_2 \dots w_n) = 1 \otimes w_1 \dots w_n + w_1 \otimes w_2 \dots w_n + \dots + w_1 \dots w_n \otimes 1$$

$$\mathbb{R}_{ua}\mathbb{R}_{vb} = \mathbb{R}_{ua} \prec \mathbb{R}_{vb} + \mathbb{R}_{ua} \succ \mathbb{R}_{vb}$$

$$deconcat(w_1 w_2 \dots w_n) = 1 \otimes w_1 \dots w_n + w_1 \otimes w_2 \dots w_n + \dots + w_1 \dots w_n \otimes 1$$

$$\mathbb{R}_{ua}\mathbb{R}_{vb} = \mathbb{R}_{ua} \prec \mathbb{R}_{vb} + \mathbb{R}_{ua} \succ \mathbb{R}_{vb}$$

$$\begin{aligned}
deconcat(w_1 w_2 \ldots w_n) =& 1 \otimes w_1 \ldots w_n + w_1 \otimes w_2 \ldots w_n + \ldots + w_1 \ldots w_n \otimes 1 \\
=& \sum_{\substack{i \in [|0,k|], \\ w_k = max(w)}} w_1 \ldots w_i \otimes w_{i+1} \ldots w_n + \\
& \sum_{\substack{i \in [|k+1,n|], \\ w_k = max(w)}} w_1 \ldots w_i \otimes w_{i+1} \ldots w_n \\
=& deconcat(w_1 \ldots w_k) + deconcat(w_{k+1} \ldots w_n)
\end{aligned}$$

$$\mathbb{R}_{ua}\mathbb{R}_{vb} = \mathbb{R}_{ua} \prec \mathbb{R}_{vb} + \mathbb{R}_{ua} \succ \mathbb{R}_{vb}$$

$$\Delta(\mathbb{R}_w) = \Delta_{\prec}(\mathbb{R}_w) + \Delta_{\succ}(\mathbb{R}_w) + 1 \otimes \mathbb{R}_w + \mathbb{R}_w \otimes 1$$
$$\bar{\Delta}(\mathbb{R}_w) = \Delta_{\prec}(\mathbb{R}_w) + \Delta_{\succ}(\mathbb{R}_w)$$

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Définitions

Élément primitif

P est un élément primitif $\iff \bar{\Delta}(P) = 0$

Ex : $\mathbb{R}_{1213} - \mathbb{R}_{2321}$

Élément totalement primitif

P est un élément totalement primitif $\iff \Delta_{\prec}(P) = \Delta_{\succ}(P) = 0$

Ex : $\mathbb{R}_{1232} + \mathbb{R}_{1231} - \mathbb{R}_{2132} - \mathbb{R}_{2131}$

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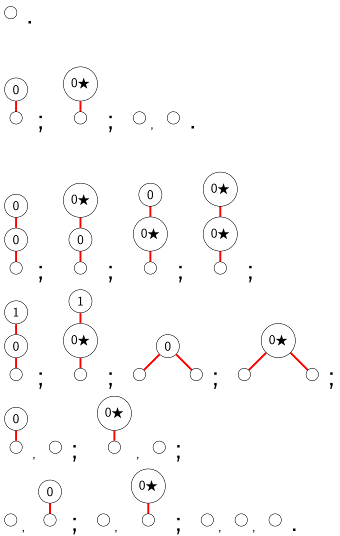
Arbres décorés

Arbres décorés

Arbres enracinés ordonnés dont les noeuds internes sont décorés.

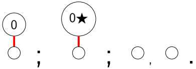
- i un entier entre 0 et $n-1$ avec n est la taille du sous arbre le plus à gauche,
- $*$ qui peut être présente si l'une des propositions suivante est vraie :
  - le noeud a plusieurs fils,
  - l'entier de l'étiquette est inférieur ou égal à celle du seul fils.

Les premiers arbres

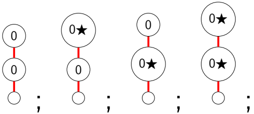


Les premiers arbres

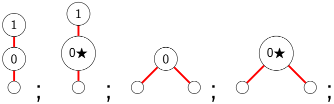
1



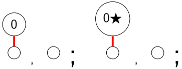
12; 11; 21



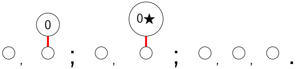
123; 122; 112; 111;



132; 121; 213; 212;

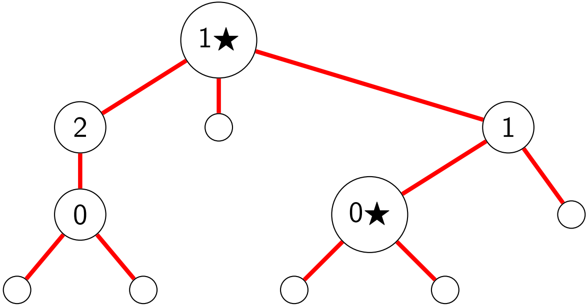


312; 211;



231; 221; 321

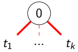
Un exemple



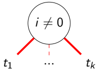
Changement de base

$$\mathbb{P}_{\circ} := \mathbb{R}_1,$$

$$\mathbb{P}_{t_1,\dots,t_k} := \mathbb{P}_{t_k} \overline{\prec} (\mathbb{P}_{t_{k-1}} \overline{\prec} (\dots \overline{\prec} \mathbb{P}_{t_1})\dots),$$

$\mathbb{P}$ 

$$:= < \mathbb{P}_{t_k}, \mathbb{P}_{t_{k-1}}, \dots, \mathbb{P}_{t_1}; \mathbb{P}_{\circ} >,$$

$\mathbb{P}$ 

$$:= < \mathbb{P}_{t_k}, \mathbb{P}_{t_{k-1}}, \dots, \mathbb{P}_{t_2}; \Phi_{i,m}(\mathbb{P}_{t_1}) > \textit{avec } m = n_F(t),$$

$\mathbb{P}$ 

$$:= \sum_{l=n_F(t)-n_F(tk)+1}^{n_F(t)} \Phi_{i,l}(\mathbb{P}_{t_1,\dots,t_k}).$$