Un auto-morphisme bidendriforme de WQSym

Séminaire IRIF

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Directeurs:
Florent Hivert
Viviane Pons

18 Juin 2020

Introduction

- $\mathbf{abcd} + \mathbf{badc} - 3\mathbf{bcad} - \frac{5}{3}\mathbf{dcba}$

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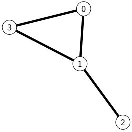
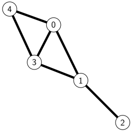
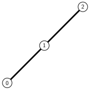
-  $+ \frac{8}{9}$  - 3 

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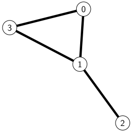
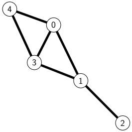
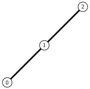
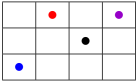
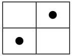
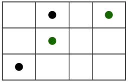
- 

$+ \frac{8}{9}$ 

$- 3$ 

- 3  $+$  $-$ 

Exemples d'algèbres de Hopf

- Arbres binaires, **PBT**, Loday-Ronco
- Fonctions symétriques non-commutatives, **Sym**
- Fonctions quasi-symétriques, *QSym*
- Permutations, **FQSym**, Malvenuto-Reutenauer
- Mots tassés, **WQSym**, Hivert

Mots tassés

Définition

Un mot sur l'alphabet $\mathbb{N}_{>0}$ est dit **tassé** si toutes les lettres de 1 à son maximum m apparaissent au moins une fois.

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Mots tassés de tailles 0, 1, 2 et 3

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- 1
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- 123 132 213 231 312 321
  122 212 221 112 121 211 111

Série des mots tassés

n	1	2	3	4	5	6	7	8
$\mathbf{PW}_n$	1	3	13	75	541	4683	47293	545835

Tassement

Exemple

$24154 \notin \mathbf{PW}$

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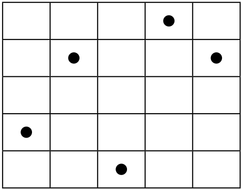
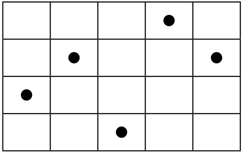
Une représentation : $\#\text{lignes} \leq \#\text{colonnes}$



2 4 1 5 4

retrait lignes vides

→ pack →



2 3 1 4 3

Algèbre de Hopf

Exemple

WQSym

- $3112 + 212 - 3\ 212341 - \frac{5}{3}\ 111$

Algèbre de Hopf

Exemple

WQSym

- $\mathbb{R}_{3112} + \mathbb{R}_{212} - 3\mathbb{R}_{212341} - \frac{5}{3}\mathbb{R}_{111}$

Algèbre de Hopf

Exemple

WQSym

- $\mathbb{R}_{3112} + \mathbb{R}_{212} - 3\mathbb{R}_{212341} - \frac{5}{3}\mathbb{R}_{111}$
- $\mathbb{R}_{12}\mathbb{R}_{11} = \mathbb{R}_{1233} + \mathbb{R}_{1323} + \mathbb{R}_{1332} + \mathbb{R}_{3123} + \mathbb{R}_{3132} + \mathbb{R}_{3312}$

Algèbre de Hopf

Exemple

WQSym

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- $\Delta(\mathbb{R}_{24231}) = \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \mathbb{R}_{121} \otimes \mathbb{R}_{21} + \mathbb{R}_{1312} \otimes \mathbb{R}_{1} + \mathbb{R}_{24231} \otimes \mathbb{R}_{\epsilon}$

Algèbre de Hopf

Exemple

WQSym

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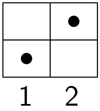
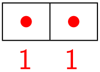
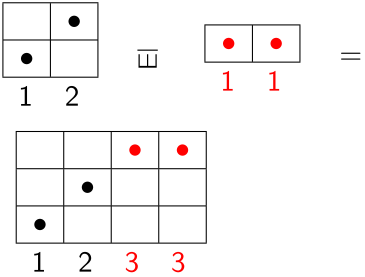
- Un produit associatif unitaire $\cdot$
- Un coproduit coassociatif counitaire Δ
- La relation de Hopf $\Delta(a \cdot b) = \Delta(a) \cdot \Delta(b)$

Produit de mélange sur les mots tassés





Produit de mélange sur les mots tassés



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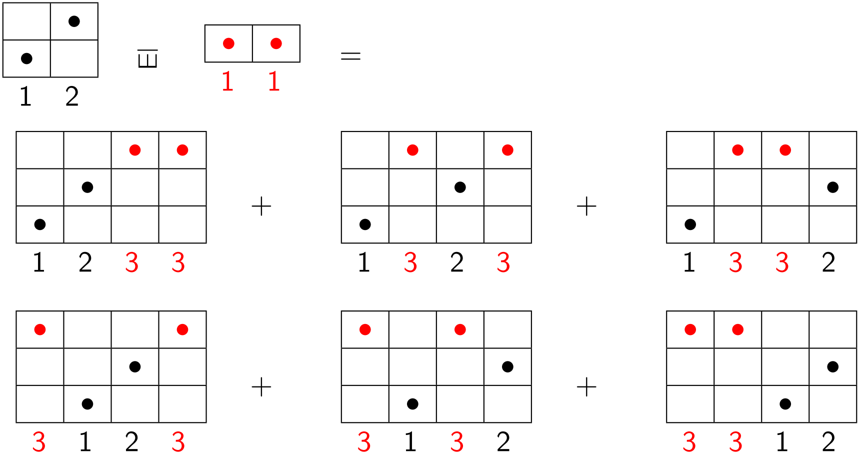

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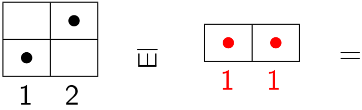
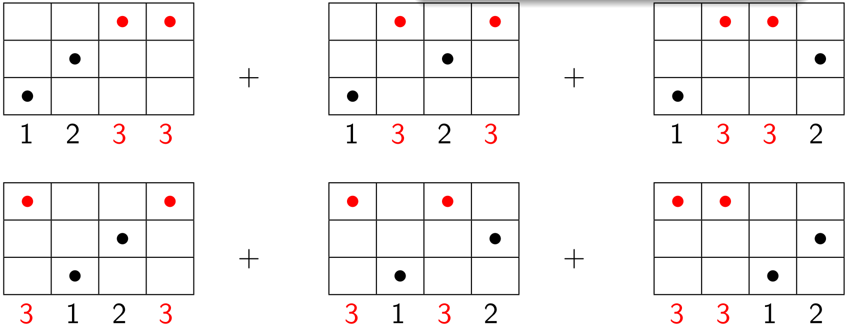
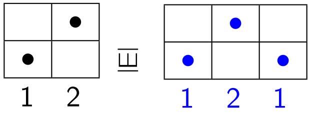


ℝ

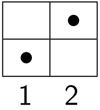
$\mathbb{R}_{12}\mathbb{R}_{11} = \mathbb{R}_{1233} + \mathbb{R}_{1323} + \mathbb{R}_{1332} + \mathbb{R}_{3123} + \mathbb{R}_{3132} + \mathbb{R}_{3312}$



Produit de mélange sur les valeurs



Produit de mélange sur les valeurs



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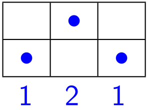

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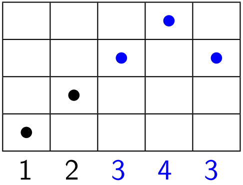
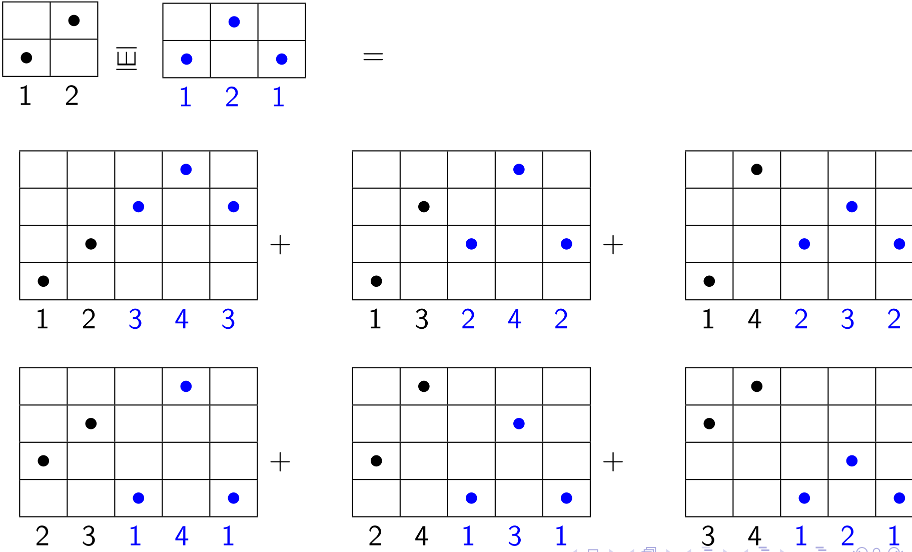


Produit de mélange sur les valeurs



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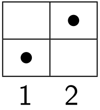
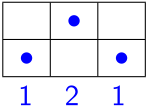


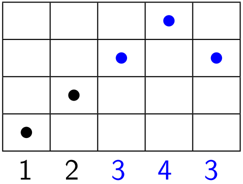
$\shuffle$



=

$\mathbb{Q}$

$\mathbb{Q}_{12}\mathbb{Q}_{121} =$
$\mathbb{Q}_{12343} + \mathbb{Q}_{13242} + \mathbb{Q}_{14232} +$
$\mathbb{Q}_{23141} + \mathbb{Q}_{24131} + \mathbb{Q}_{34121}$



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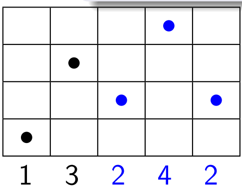

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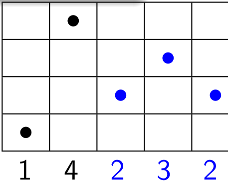

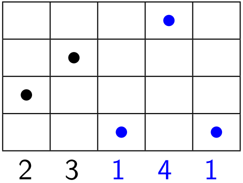

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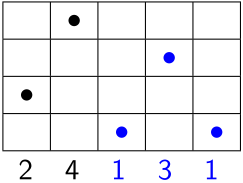

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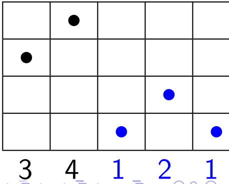
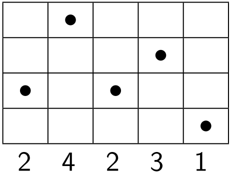


Déconcaténation



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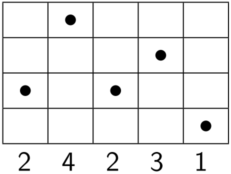

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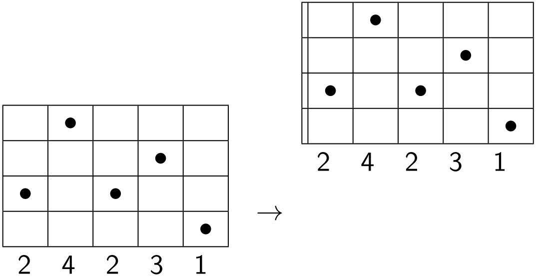

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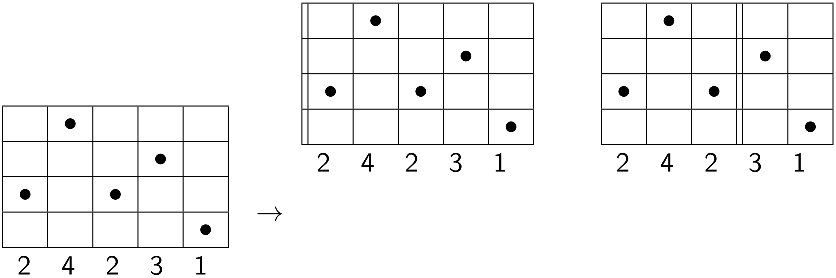

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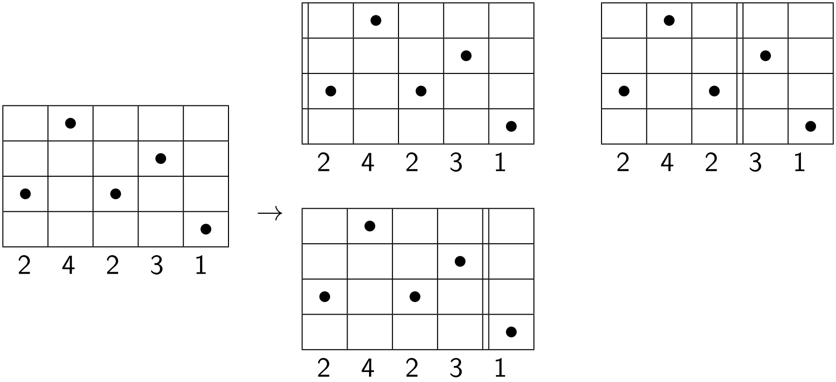

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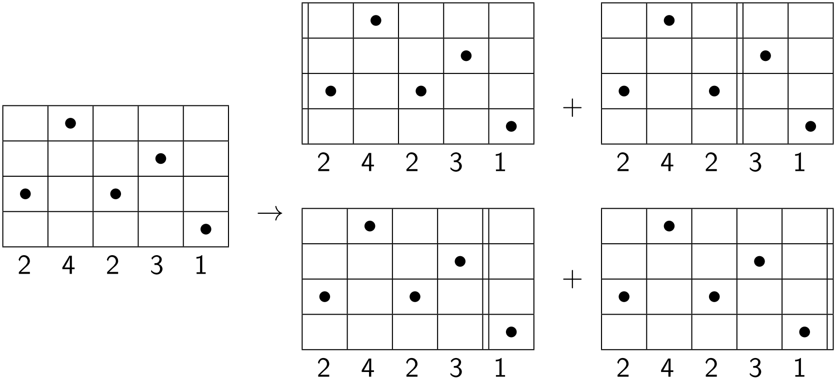
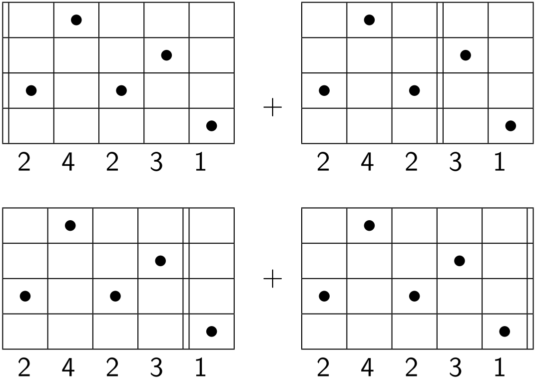


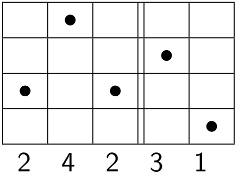
Déconcaténation

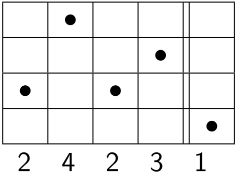
$\mathbb{R}_{24231} \to$



Déconcaténation

$\mathbb{R}_{24231} \xrightarrow{\Delta} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231}$ +





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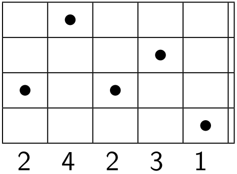
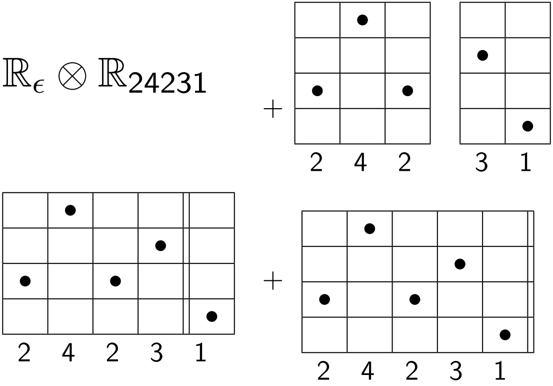


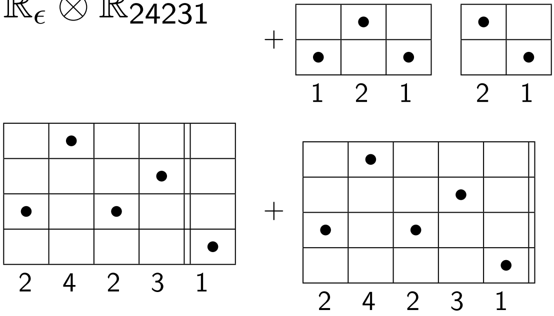
Déconcaténation

$$\mathbb{R}_{24231} \xrightarrow{\Delta} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \ldots + \ldots$$



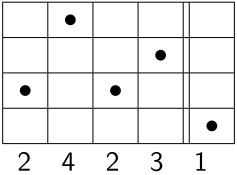
Déconcaténation

$$\mathbb{R}_{24231} \xrightarrow{\Delta} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \ldots$$



Déconcaténation

$$\mathbb{R}_{24231} \xrightarrow{\Delta} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \mathbb{R}_{121} \otimes \mathbb{R}_{21}$$



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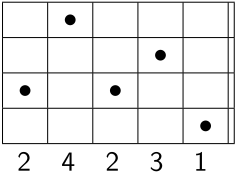
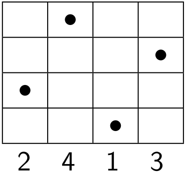


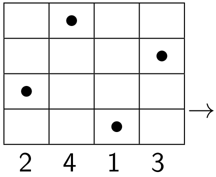
Déconcaténation

$$\mathbb{R}_{24231} \xrightarrow{\Delta} \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \mathbb{R}_{121} \otimes \mathbb{R}_{21} + \mathbb{R}_{1312} \otimes \mathbb{R}_{1} + \mathbb{R}_{24231} \otimes \mathbb{R}_{\epsilon}$$

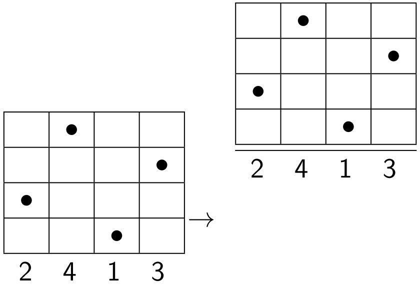
Désassemblage horizontale



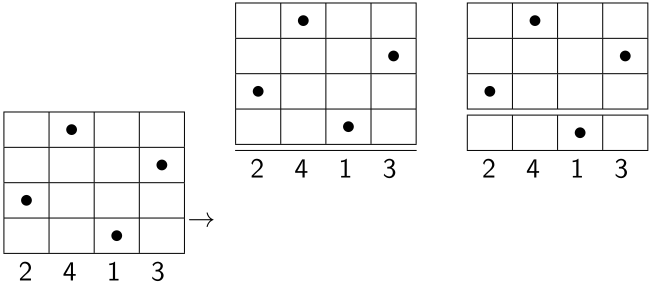
Désassemblage horizontale



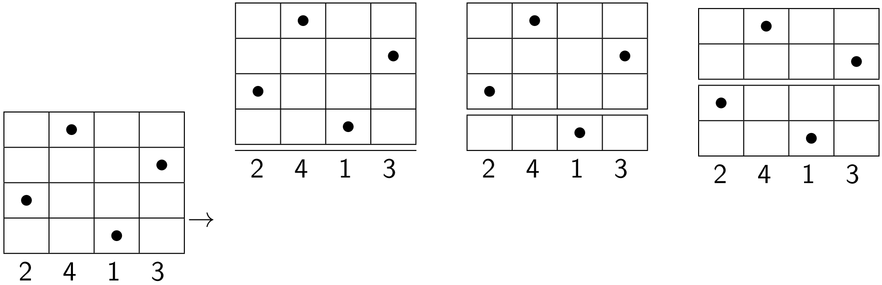
Désassemblage horizontale



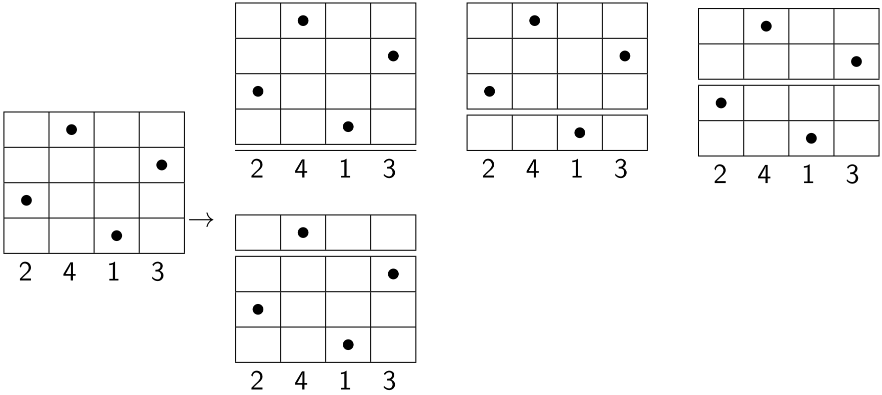
Désassemblage horizontale



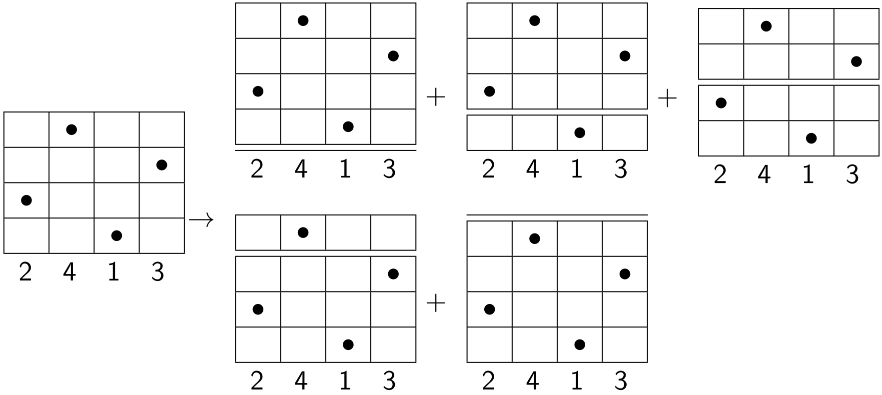
Désassemblage horizontale



Désassemblage horizontale

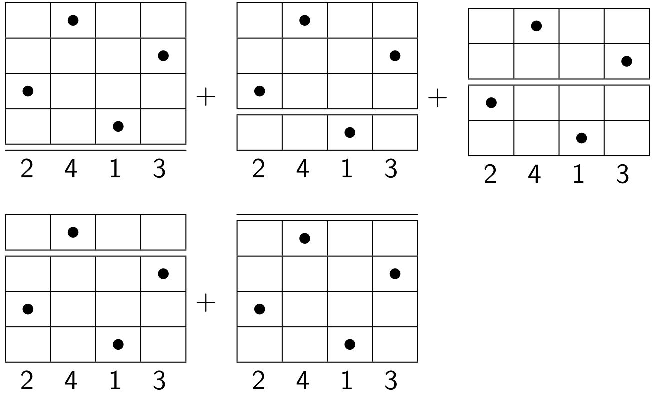


Désassemblage horizontale



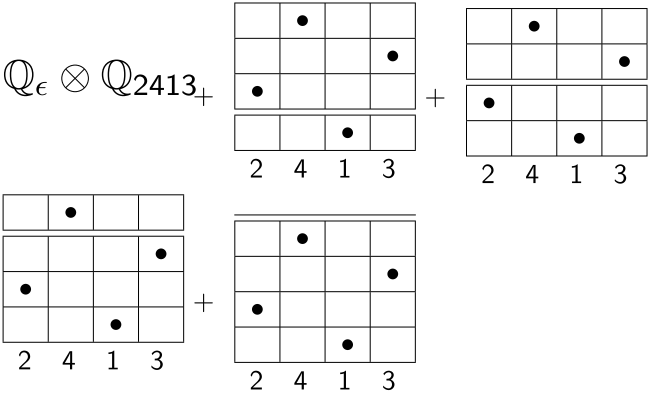
Désassemblage horizontale

$\mathbb{Q}_{2413} \xrightarrow{\Delta}$

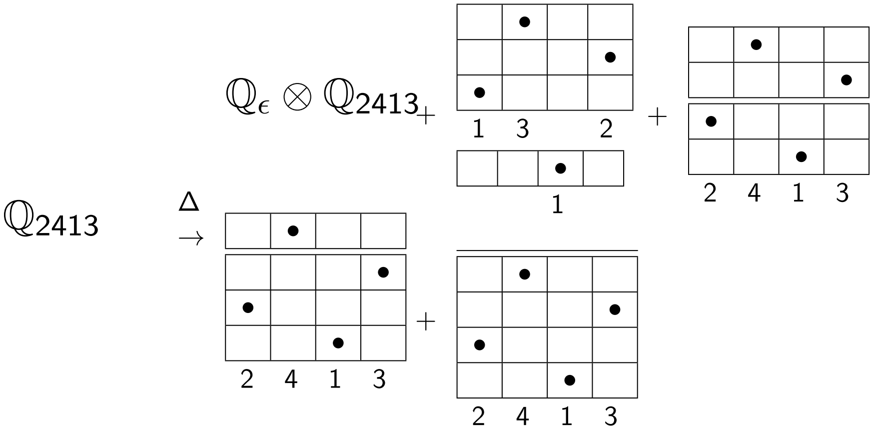


Désassemblage horizontale

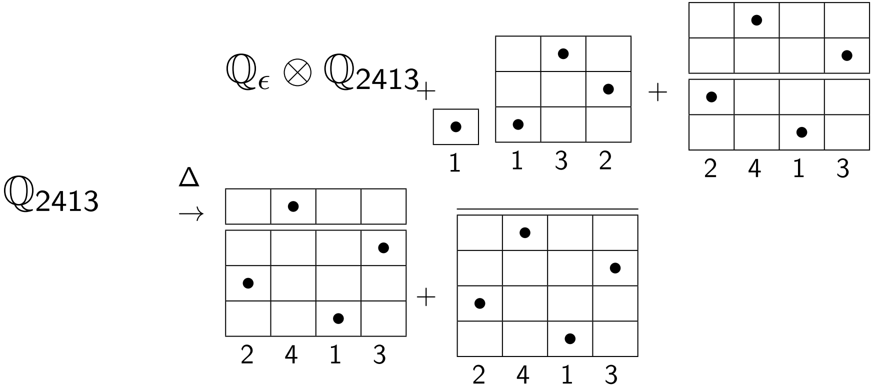
$$\mathbb{Q}_{2413} \xrightarrow{\Delta} \mathbb{Q}_{\epsilon} \otimes \mathbb{Q}_{2413} + \ldots$$



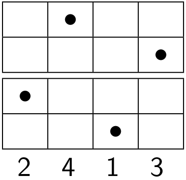
Désassemblage horizontale

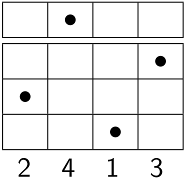


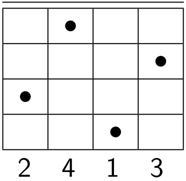
Désassemblage horizontale



Désassemblage horizontale

$\mathbb{Q}_\epsilon \otimes \mathbb{Q}_{2413} + \mathbb{Q}_1 \otimes \mathbb{Q}_{132} +$ 

$\mathbb{Q}_{2413} \xrightarrow{\Delta}$ 

$+$ 

Désassemblage horizontale

$$\mathbb{Q}_{2413} \xrightarrow{\Delta} \mathbb{Q}_{\epsilon} \otimes \mathbb{Q}_{2413} + \mathbb{Q}_{1} \otimes \mathbb{Q}_{132} + \mathbb{Q}_{21} \otimes \mathbb{Q}_{21} + \mathbb{Q}_{213} \otimes \mathbb{Q}_{1} + \mathbb{Q}_{2413} \otimes \mathbb{Q}_{\epsilon}$$

Dualité

$$\begin{aligned}\mathbb{R}_{221}\mathbb{R}_{21} = &\mathbb{R}_{22143} + \mathbb{R}_{22413} + \mathbb{R}_{22431} + \\ &\mathbb{R}_{24213} + \mathbb{R}_{\mathbf{24231}} + \mathbb{R}_{24321} + \\ &\mathbb{R}_{42213} + \mathbb{R}_{42231} + \mathbb{R}_{42321} + \\ &\mathbb{R}_{43221}\end{aligned}$$

Dualité

$$
\begin{aligned}
\mathbb{R}_{221}\mathbb{R}_{21} = & \mathbb{R}_{22143} + \mathbb{R}_{22413} + \mathbb{R}_{22431} + \\
& \mathbb{R}_{24213} + \mathbb{R}_{24231} + \mathbb{R}_{24321} + \\
& \mathbb{R}_{42213} + \mathbb{R}_{42231} + \mathbb{R}_{42321} + \\
& \mathbb{R}_{43221}
\end{aligned}
$$

$$
\begin{aligned}
\Delta(\mathbb{Q}_{24231}) = & \mathbb{Q}_{\epsilon} \otimes \mathbb{Q}_{24231} + \mathbb{Q}_{1} \otimes \mathbb{Q}_{1312} + \\
& \mathbb{Q}_{221} \otimes \mathbb{Q}_{21} + \mathbb{Q}_{2231} \otimes \mathbb{Q}_{1} + \\
& \mathbb{Q}_{24231} \otimes \mathbb{Q}_{\epsilon}
\end{aligned}
$$

Dualité

$$\mathbb{Q}_{121}\mathbb{Q}_{21} = \mathbb{Q}_{12143} + \mathbb{Q}_{13142} + \mathbb{Q}_{14132} + \mathbb{Q}_{23241} + \mathbb{Q}_{\mathbf{24231}} + \mathbb{Q}_{34321}$$

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$$\Delta(\mathbb{Q}_{24231}) = \mathbb{Q}_{\epsilon} \otimes \mathbb{Q}_{24231} + \mathbb{Q}_{1} \otimes \mathbb{Q}_{1312} + \mathbb{Q}_{\mathbf{221}} \otimes \mathbb{Q}_{\mathbf{21}} + \mathbb{Q}_{2231} \otimes \mathbb{Q}_{1} + \mathbb{Q}_{24231} \otimes \mathbb{Q}_{\epsilon}$$

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$$\Delta(\mathbb{R}_{24231}) = \mathbb{R}_{\epsilon} \otimes \mathbb{R}_{24231} + \mathbb{R}_{\mathbf{121}} \otimes \mathbb{R}_{\mathbf{21}} + \mathbb{R}_{1312} \otimes \mathbb{R}_{1} + \mathbb{R}_{24231} \otimes \mathbb{R}_{\epsilon}$$

Auto-dualité

- $\mathbb{R}$ et $\mathbb{Q}$ bases de **WQSym** et **WQSym***

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Demis produits

Définition récursive du **produit de mélange**

- $\epsilon \sqcup\!\sqcup w = w \sqcup\!\sqcup \epsilon = w$
- $ua \sqcup\!\sqcup vb = (u \sqcup\!\sqcup vb)a + (ua \sqcup\!\sqcup v)b$

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Exemple de produits gauche et droit

- $\mathbb{R}_{12}\mathbb{R}_{11} = \mathbb{R}_{1233} + \mathbb{R}_{1323} + \mathbb{R}_{1332} + \mathbb{R}_{3123} + \mathbb{R}_{3132} + \mathbb{R}_{3312}$

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Exemple de coproduits gauche et droit

- $\tilde{\Delta}(\mathbb{R}_{2425531}) = \mathbb{R}_{121} \otimes \mathbb{R}_{3321} + \mathbb{R}_{12133} \otimes \mathbb{R}_{21} + \mathbb{R}_{131442} \otimes \mathbb{R}_{1}$

Demis produits

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- $\Delta_{\prec}(\mathbb{R}_{2425531}) = \mathbb{R}_{12133} \otimes \mathbb{R}_{21} + \mathbb{R}_{131442} \otimes \mathbb{R}_{1}$
- $\Delta_{\succ}(\mathbb{R}_{2425531}) = \mathbb{R}_{121} \otimes \mathbb{R}_{3321}$

Bigèbre bidendriforme

Définition

- Raffinement de l'associativité et la coassociativité
  $(a \sqcup\!\sqcup\, b) \sqcup\!\sqcup\, c = a \sqcup\!\sqcup\, (b \sqcup\!\sqcup\, c)$

Bigèbre bidendriforme

Définition

- Raffinement de l'associativité et la coassociativité
  - $(a \prec b) \prec c = a \prec (b \prec c + b \succ c)$,
  - $(a \succ b) \prec c = a \succ (b \prec c)$,
  - $(a \prec b + a \succ b) \succ c = a \succ (b \succ c)$.

Bigèbre bidendriforme

Définition

- Raffinement de l'associativité et la coassociativité
  - 3 et 3 équations

Bigèbre bidendriforme

Définition

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Théorème [Foissy]

Si A est une bigèbre bidendriforme alors A est généré librement par $\mathrm{TPrim}(A)$ en tant qu'algèbre dendriforme.

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Séries

n	1	2	3	4	5	6	7	8
$\mathbf{WQSym}_n$	1	3	13	75	541	4 683	47 293	545 835
TPrim_n	1	1	4	28	240	2 384	26 832	337 168

Bigèbre bidendriforme

Définition

- Raffinement de l'associativité et la coassociativité
  - 3 et 3 équations
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Corollaire

WQSym est auto-duale.

Définitions

Élément primitif

P est un éléments primitif $\iff \tilde{\Delta}(P) = 0$

Ex : $\mathbb{R}_{1213} - \mathbb{R}_{2321}$

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Élément totalement primitif

P est une élément totalement primitif $\iff \Delta_{\prec}(P) = \Delta_{\succ}(P) = 0$

Ex : $\mathbb{R}_{12443} - \mathbb{R}_{21443} - \mathbb{R}_{23441} + \mathbb{R}_{32441}$

Mon but

Isomorphisme bidendriforme explicite entre **WQSym** et sa duale

Mon but

Isomorphisme bidendriforme explicite entre **WQSym** et sa duale

$\Updownarrow$

Isomorphisme explicite entre TPrim(**WQSym**) et le dual

Mon but

Isomorphisme bidendriforme explicite entre **WQSym** et sa duale

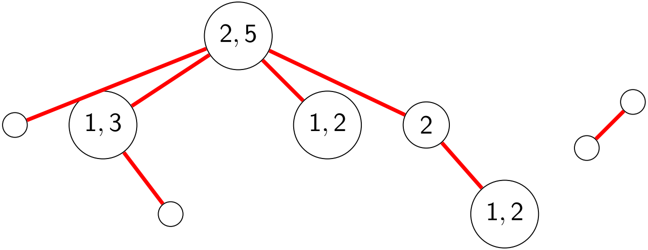
$\Updownarrow$

Isomorphisme explicite entre TPrim(**WQSym**) et le dual

Construction de deux bases de totalement primitif
(dans **WQSym** et **WQSym***)

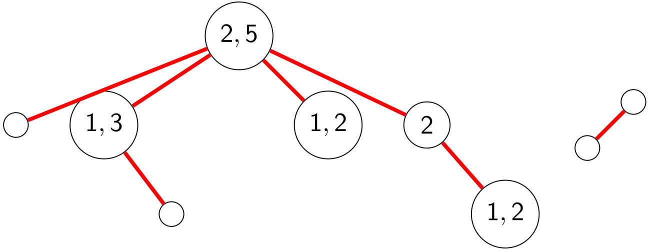
Forêts biplanes décorés

Un exemple de forêt biplane décorée:



Forêts biplanes décorés

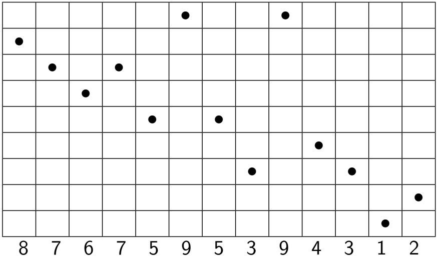
Un exemple de forêt biplane décorée:



En bijection avec: 8767595394312.

Algorithme sur 8767595394312

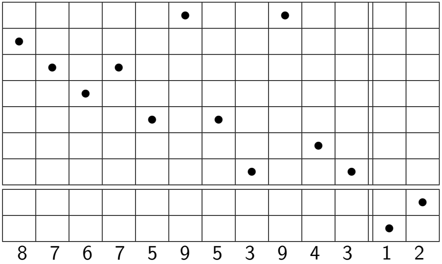
$F(8767595394312)$



Algorithme sur 8767595394312

$F(8767595394312)$

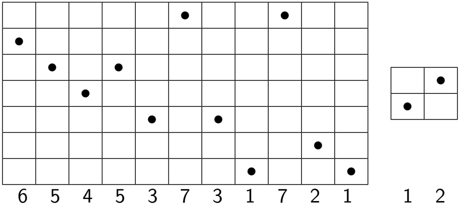
Factorisation en descentes globales



Algorithme sur 8767595394312

$$F(8767595394312) = T(65453731721)T(12)$$

Factorisation en descentes globales + tassement



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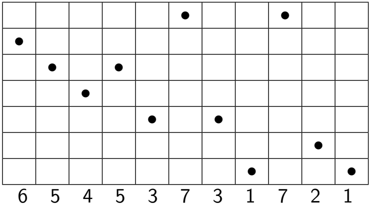
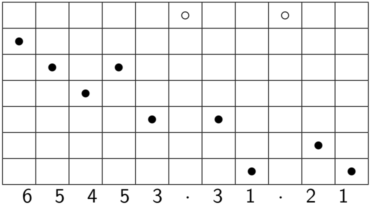


Algorithme sur 8767595394312

$T(65453731721)\,T(12)$

Retrait des lettres de valeur max



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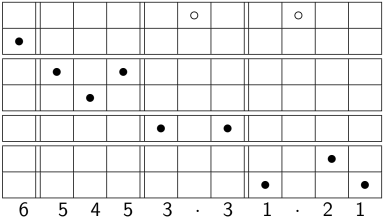
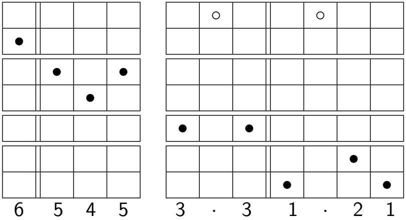
Factorisation en descentes globales



Algorithme sur 8767595394312

$T(65453731721)\,T(12)$

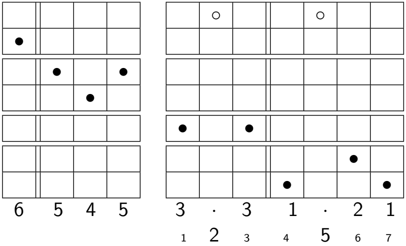
Distinction de deux groupes de facteurs



Algorithme sur 8767595394312

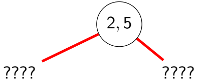
Positions des max

$T(65453731721)\,T(12)$

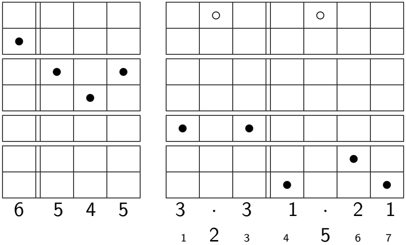


Algorithme sur 8767595394312

$T(65453731721)\,T(12) =$



$T(12)$



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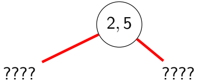
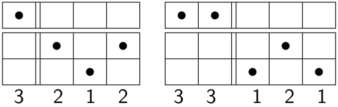


$T(12)$



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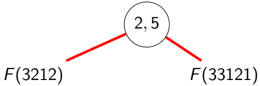
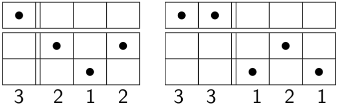


$T(12)$



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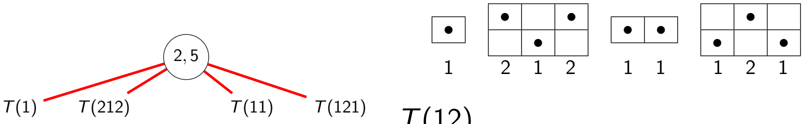

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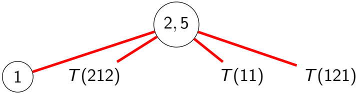

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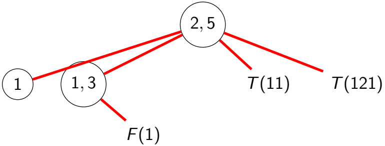

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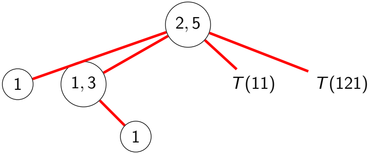

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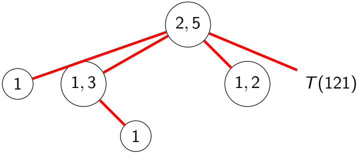

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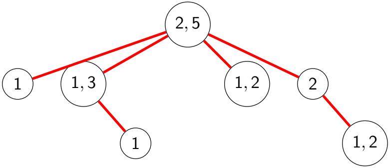

$T(12)$

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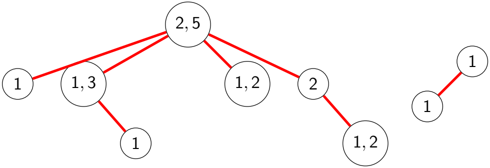

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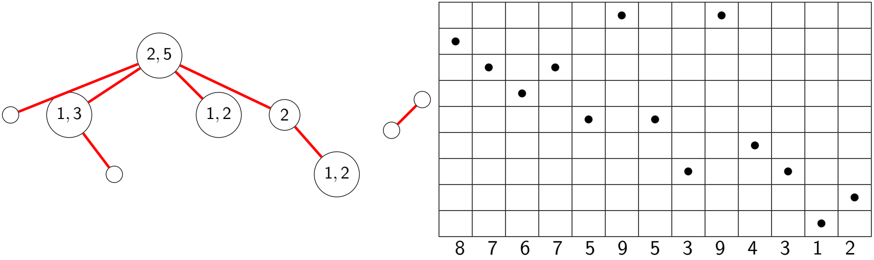
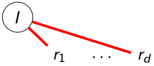


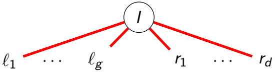
La base $\mathbb{P}$

$$\mathbb{P}_{\circ} := \mathbb{R}_1,$$

$$\mathbb{P}_{t_1,\ldots,t_k} := (\ldots(\mathbb{P}_{t_k} \prec \ldots) \prec \mathbb{P}_{t_2}) \prec \mathbb{P}_{t_1},$$

$\mathbb{P}$ 

$$:= \Phi_I(\mathbb{P}_{r_1,\ldots,r_d}),$$

$\mathbb{P}$ 

$$:= \langle \mathbb{P}_{l_1}, \mathbb{P}_{l_2}, \ldots, \mathbb{P}_{l_k}; \Phi_I(\mathbb{P}_{r_1,\ldots,r_d})\rangle.$$

La base ℙ

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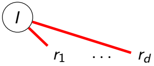
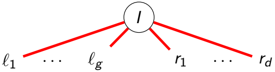
$$\mathbb{P}_{t_1,\dots,t_k} := (\dots(\mathbb{P}_{t_k} \prec \dots) \prec \mathbb{P}_{t_2}) \prec \mathbb{P}_{t_1},$$

$\mathbb{P}$



$$:= \Phi_I(\mathbb{P}_{r_1,\dots,r_d}),$$

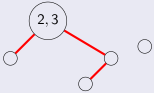
$\mathbb{P}$



$$:= \langle \mathbb{P}_{l_1}, \mathbb{P}_{l_2}, \dots, \mathbb{P}_{l_k}; \Phi_I(\mathbb{P}_{r_1,\dots,r_d}) \rangle.$$

Exemple

$\mathbb{P}$



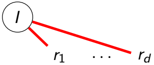
$$= \mathbb{R}_{235541} - \mathbb{R}_{245531} - \mathbb{R}_{244531} - \mathbb{R}_{245431} - \mathbb{R}_{254431} + \mathbb{R}_{325541} - \mathbb{R}_{425531} - \mathbb{R}_{524431} + \mathbb{R}_{352541} - \mathbb{R}_{452531} + \mathbb{R}_{355241} - \mathbb{R}_{455231} + \mathbb{R}_{344521} + \mathbb{R}_{345421} + \mathbb{R}_{354421} + \mathbb{R}_{534421}$$

La base ℙ

$$\mathbb{P}_{\bigcirc} := \mathbb{R}_1,$$

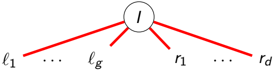
$$\mathbb{P}_{t_1,\dots,t_k} := (\dots(\mathbb{P}_{t_k} \prec \dots) \prec \mathbb{P}_{t_2}) \prec \mathbb{P}_{t_1},$$

$\mathbb{P}$



$$:= \Phi_l(\mathbb{P}_{r_1,\dots,r_d}),$$

$\mathbb{P}$



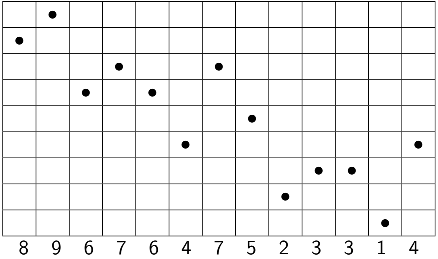
$$:= \langle \mathbb{P}_{l_1}, \mathbb{P}_{l_2}, \dots, \mathbb{P}_{l_k}; \Phi_l(\mathbb{P}_{r_1,\dots,r_d})\rangle.$$

Théorème

- $(\mathbb{P}_f)_{f\in\mathfrak{F}_n}$ est une base de **WQSym**$_n$,
- $(\mathbb{P}_t)_{t\in\mathfrak{T}_n}$ est une base de Prim$_n$,
- $(\mathbb{P}_t)_{t\in\mathfrak{P}_n}$ est une base de TPrim$_n$.

Algorithme sur 8967647523314

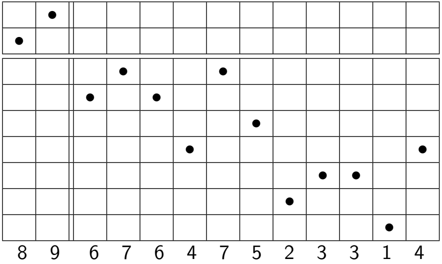
$F^*(8967647523314)$



Algorithme sur 8967647523314

$F^*(8967647523314)$

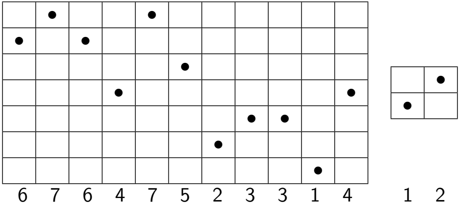
Factorisation en descentes globales



Algorithme sur 8967647523314

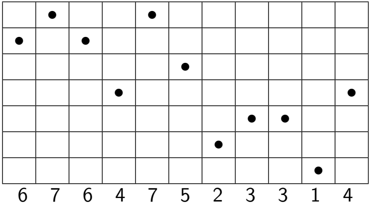
$F^*(8967647523314) =$
$T^*(67647523314)\,T^*(12)$

Factorisation en descentes globales
+ tassement + échange



Algorithme sur 8967647523314

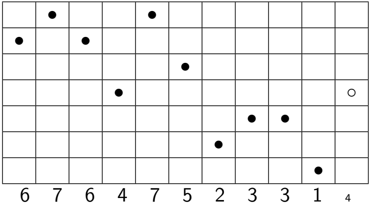
$T^*(67647523314)\,T^*(12)$



Algorithme sur 8967647523314

Retrait de la dernière lettre

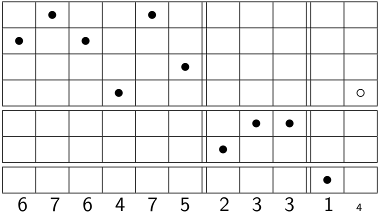
$T^*(67647523314)\,T^*(12)$



Algorithme sur 8967647523314

Factorisation en descentes globales

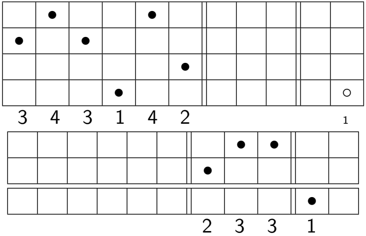
$T^*(67647523314)\,T^*(12)$



Algorithme sur 8967647523314

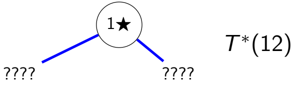
$T^*(67647523314)\,T^*(12)$

Distinction de deux groupes de facteurs

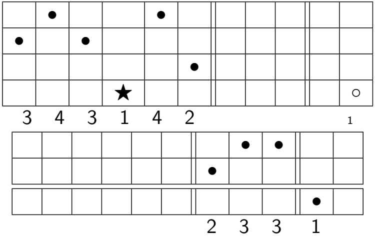


Algorithme sur 8967647523314

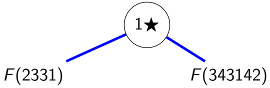
$T^*(67647523314)\,T^*(12) =$



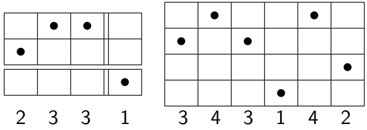
La dernière lettre est-elle présente dans le reste du mot ?



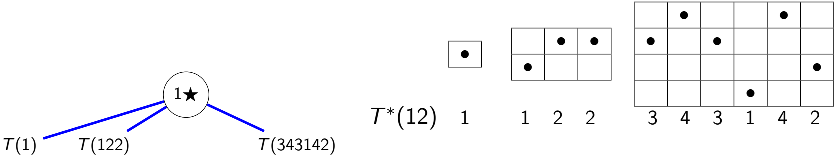
Algorithme sur 8967647523314



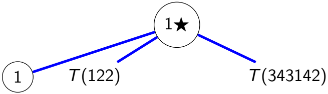
$T^*(12)$



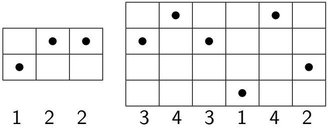
Algorithme sur 8967647523314



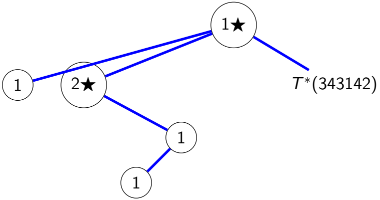
Algorithme sur 8967647523314

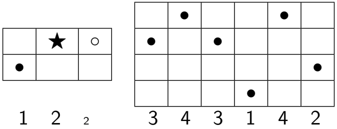


$T^*(12)$



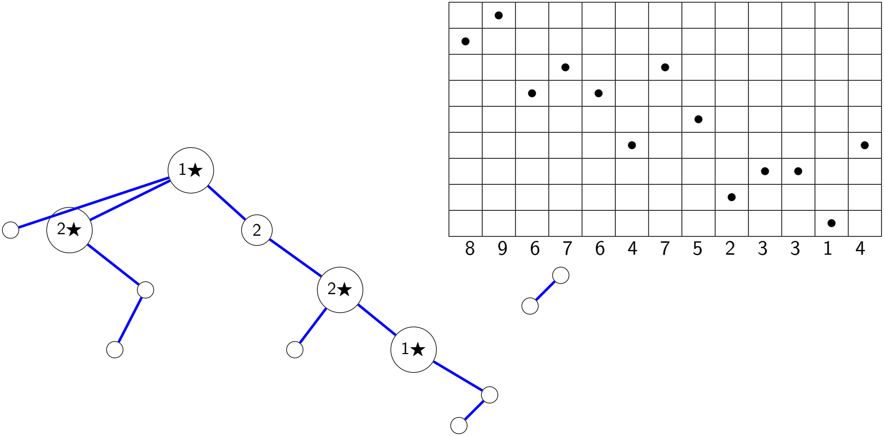
Algorithme sur 8967647523314





$T^*(12)$

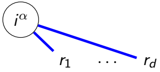
Algorithme sur 8967647523314



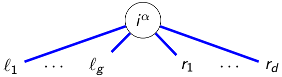
La base 𝕆

$$\mathbb{O}_{\circ} := \mathbb{Q}_1,$$

$$\mathbb{O}_{t_1,\dots,t_k} := (\dots(\mathbb{O}_{t_k} \prec \dots) \prec \mathbb{O}_{t_2}) \prec \mathbb{O}_{t_1},$$

$\mathbb{O}$


$$:= \Psi_i^\alpha(\mathbb{O}_{r_1,\dots,r_d}),$$

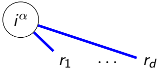
$\mathbb{O}$


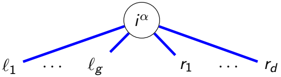
$$:= \langle \mathbb{O}_{l_1}, \mathbb{O}_{l_2}, \dots, \mathbb{O}_{l_k}; \Psi_i^\alpha(\mathbb{O}_{r_1,\dots,r_d})\rangle.$$

La base $\mathbb{O}$

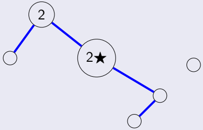
$\mathbb{O}$  $:= \mathbb{Q}_1,$

$$\mathbb{O}_{t_1,\dots,t_k} := (\dots(\mathbb{O}_{t_k} \prec \dots) \prec \mathbb{O}_{t_2}) \prec \mathbb{O}_{t_1},$$

$\mathbb{O}$  $:= \Psi_i^{\alpha}(\mathbb{O}_{r_1,\dots,r_d}),$

$\mathbb{O}$  $:= \langle \mathbb{O}_{l_1}, \mathbb{O}_{l_2}, \dots, \mathbb{O}_{l_k}; \Psi_i^{\alpha}(\mathbb{O}_{r_1,\dots,r_d}) \rangle.$

Exemple

$\mathbb{O}$  $=$

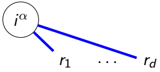
$$\mathbb{Q}_{531442} + \mathbb{Q}_{521443} + \mathbb{Q}_{512443} - \mathbb{Q}_{534142} - \mathbb{Q}_{524143} - \mathbb{Q}_{514243} - \mathbb{Q}_{514432} - \mathbb{Q}_{524431} - \mathbb{Q}_{514423} + \mathbb{Q}_{541432} + \mathbb{Q}_{542431} + \mathbb{Q}_{541423}$$

La base $\mathbb{O}$

$$\mathbb{O}_{\circ} := \mathbb{Q}_1,$$

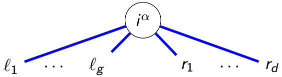
$$\mathbb{O}_{t_1,\dots,t_k} := (\dots(\mathbb{O}_{t_k} \prec \dots) \prec \mathbb{O}_{t_2}) \prec \mathbb{O}_{t_1},$$

$\mathbb{O}$



$$:= \Psi_i^\alpha(\mathbb{O}_{r_1,\dots,r_d}),$$

$\mathbb{O}$

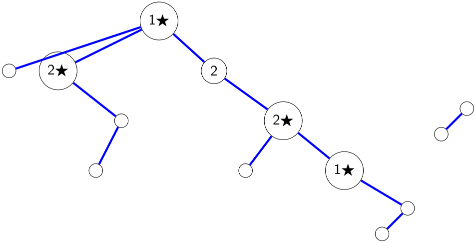


$$:= \langle \mathbb{O}_{l_1}, \mathbb{O}_{l_2}, \dots, \mathbb{O}_{l_k}; \Psi_i^\alpha(\mathbb{O}_{r_1,\dots,r_d}) \rangle.$$

Théorème

- $(\mathbb{O}_f)_{f \in \mathfrak{F}^*{}_n}$ est une base de $\mathbf{WQSym}_n^*$,
- $(\mathbb{O}_t)_{t \in \mathfrak{T}^*{}_n}$ est une base de Prim_n^*,
- $(\mathbb{O}_t)_{t \in \mathfrak{P}^*{}_n}$ est une base de TPrim_n^*.

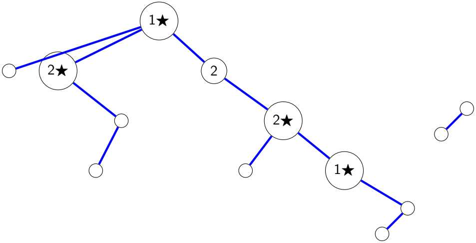
Une bijection



8967647523314

F $[1-9]^{13}$

Une bijection

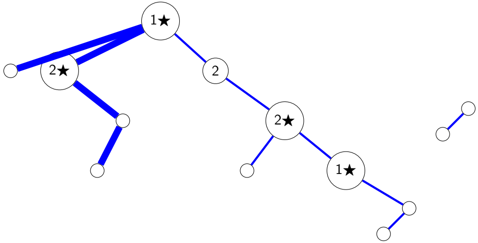


T_1 T_2

89 67647523314

$[3-9]^{11}$ $[\mathbf{1},\mathbf{2}]^{\mathbf{2}}$

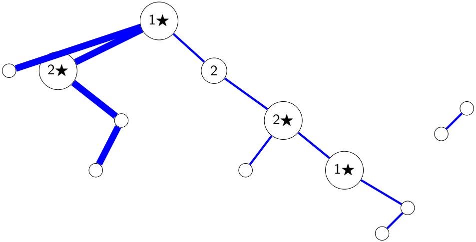
Une bijection



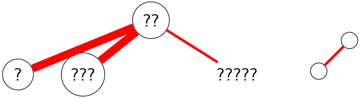
89 676475**2331**4

T_1 T_2 $[3-9]^{11}$ $[1,2]^2$

Une bijection

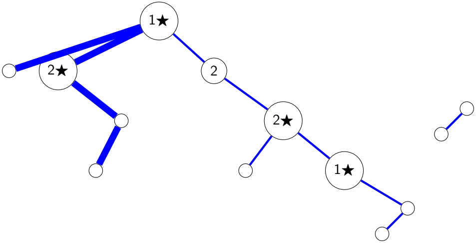


89 676475**2331**4

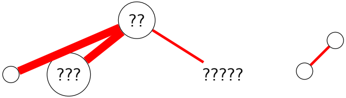


$[\mathbf{6}-\mathbf{8}]^{\mathbf{4}}[3-5,9]^{7}12$

Une bijection

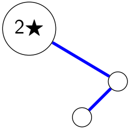


89 676475**233**14



$8[\mathbf{6}, \mathbf{7}]^{\mathbf{3}}[3-5, 9]^{7}12$

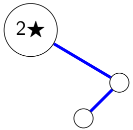
Une bijection



122

$[1, 2]^3$

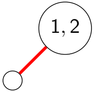
Une bijection



122

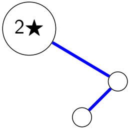
$[1,2]^3$

$[1,2]^3$



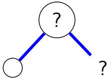
122

Une bijection

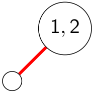


122

$[1,2]^3$

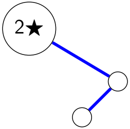


$[1,2]^3$



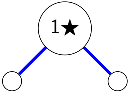
122

Une bijection

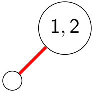


122

$[1, 2]^3$

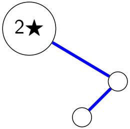


212

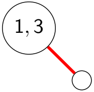


122

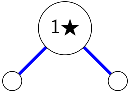
Une bijection



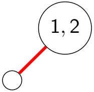
122



212

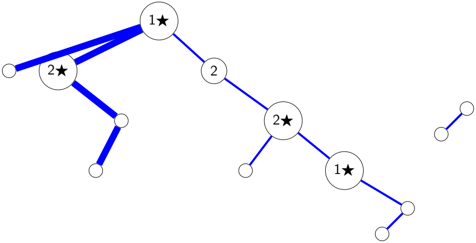


212

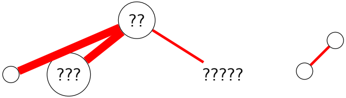


122

Une bijection

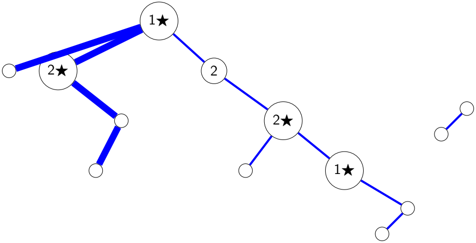


89 676475**233**14

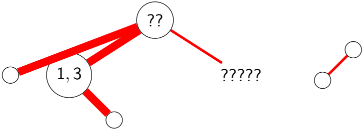


$8[\mathbf{6}, \mathbf{7}]^{\mathbf{3}}[3-5, 9]^{7}12$

Une bijection

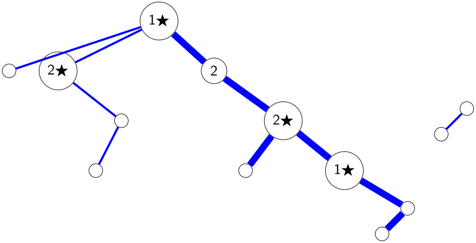


89 676475**233**14

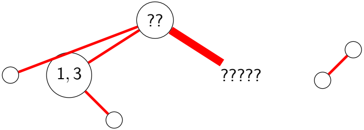


$8\mathbf{767}[3-5,9]^7 12$

Une bijection

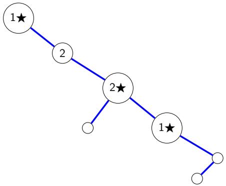


$89 \quad \mathbf{676475}23\,31\,\mathbf{4}$

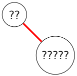


$8767[\mathbf{3}-\mathbf{5},\mathbf{9}]^{\mathbf{7}}12$

Une bijection

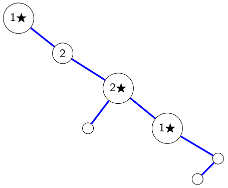


$$3431421 = pack(6764754)$$

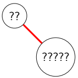


$$[1-4]^7 = pack([3-5,9]^7)$$

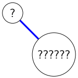
Une bijection



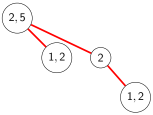
3431421



$[1 - 4]^7$

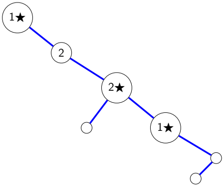


$[1 - 4]^7$

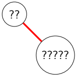


3431421

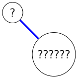
Une bijection



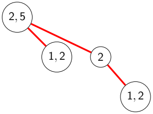
$$3431421 = pack(6764754)$$



$$[1-4]^7 = pack([3-5,9]^7)$$

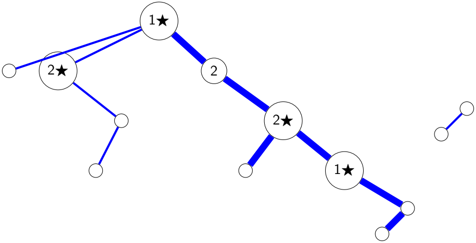


$$[1-4]^7 = pack([4-7]^7)$$

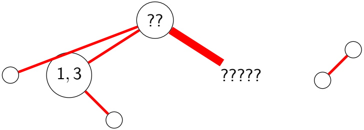


$$3431421 = pack(5953943)$$

Une bijection

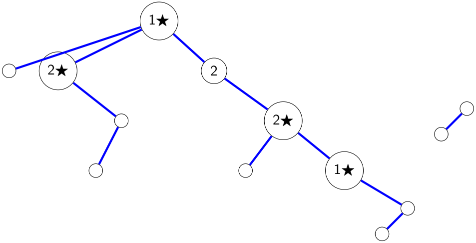


89 **676475**2331**4**

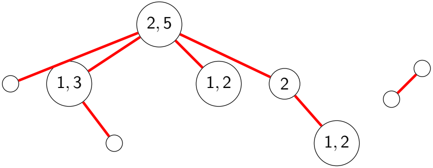


$8767[\mathbf{3} - \mathbf{5}, \mathbf{9}]^{\mathbf{7}}12$

Une bijection



89**676475**2331**4**



8767**595394**312

Théorèmes

Théorème

- $(\mathbb{O}_f)_{f\in\mathfrak{F}^*{}_n}$ base de $\mathbf{WQSym}_n^*$,
- $(\mathbb{O}_t)_{t\in\mathfrak{T}^*{}_n}$ base de Prim_n^*,
- $(\mathbb{O}_t)_{t\in\mathfrak{P}^*{}_n}$ base de TPrim_n^*.

Théorème

- $(\mathbb{P}_f)_{f\in\mathfrak{F}_n}$ base de $\mathbf{WQSym}_n$,
- $(\mathbb{P}_t)_{t\in\mathfrak{T}_n}$ base de Prim_n,
- $(\mathbb{P}_t)_{t\in\mathfrak{P}_n}$ base de TPrim_n.

Bijection

$$\begin{aligned} \mathfrak{F}^* &\leftrightarrow \mathfrak{F} \\ \mathfrak{T}^* &\leftrightarrow \mathfrak{T} \\ \mathfrak{P}^* &\leftrightarrow \mathfrak{P} \end{aligned}$$

Isomorphisme bidendriforme