Quantum search as a naturally occurring phenomenon

Supervisors. Giuseppe Di Molfetta (MCF), Pablo Arrighi (PR).

Contact details. giuseppe.dimolfetta@lif.univ-mrs.fr, +33646497714, pablo.arrighi@univ-amu.fr, +33611257467.

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Place. Laboratoire d'Informatique et Systèmes (LIS), Natural Computing team (CaNa). Scientific environment : The CaNa research group (Pablo Arrighi, Giuseppe Di Molfetta, Kevin Perrot, Sylvain Sené) seeks to capture at the formal level some of the fundamental paradigms of theoretical physics and biology, via the models and approaches of theoretical computer science and discrete mathematics. The group is located in Luminy, Marseille, France, and benefits from a rich scientific environment with the Cellular Automata experts of I2M (Pierre Guillon, Guillaume Theyssier) and the physicists from CPT (Alberto Verga, Thomas Krajewski).

Theme. Quantum Computing has three main fields of applications for quantum computing : quantum cryptography ; quantum simulation ; and quantum algorithms (*e.g.* Grover, Shor...). Whilst the first two are considered short and mid term applications respectively, the last one, perhaps the most fascinating, is generally considered to be a long term application. This is because of the common understanding that we will need to build scalable implementations of universal quantum gate sets with fidelity 10^{-3} first, and implement quantum error corrections then, in order to finally be able to run our preferred quantum algorithm on the thereby obtained universal quantum computer. This seems feasible, yet long way to go.

This, however, may be a pessimistic view. Science may one day get luckier and find out that nature actually implements some of these quantum algorithms 'spontaneously'. In fact, recent evidence suggests that the Grover search may in fact be a naturally occurring phenomenon, e.g. when fermions propagate in crystalline materials under certain conditions. Amongst all quantum algorithms, the reasons to focus on the Grover search [9] include its remarkable generality, as it speeds up any brute force $O(N)$ problem into a $O(\sqrt{N})$ problem, and its remarkable robustness : the algorithm comes in many variants and has been rephrased in many ways, including in terms of resonance effects [15] and quantum walks [4].

Remember that a quantum walks (QW) are essentially local unitary gates that drive the evolution of a particle on a lattice. They have been used as a mathematical framework for different quantum algorithms [1, 17] but also for quantum simulations e.g. [2, 5, 7]. This is where things get interesting. Indeed, it has been shown that many of these QW admit, as their continuum limit, some well-known PDE of physics, such as the Dirac equation [3, 6, 11, 13]. Recall that the Dirac equation governs the free propagation of the electron. Thus, these Dirac QW provided 'quantum numerical schemes', for the future quantum computers, to simulate the electron. For instance [12] shows that it is possible to describe the dynamics of fermions in graphene using QW. This is great, but now let us turn things the other way round : this also means that fermions provide a natural implementation of these Dirac QW. Could these be useful algorithmically ?

In [10] we provided evidence that these Dirac QW work fine to implement the diffusion step of the Grover search. Thus, free electrons may provide a natural implementation of this step. Similar conclusions were reached over the square grid [14], which we extend to a triangular grid reminiscent of the naturally occurring, graphene-like materials. However, the Grover search is the alternation of a diffusion step, with an oracle step. The later puts on a minus one phase whenever the walker hits the solution of the problem. In [10] we also provided evidence that the mere presence of hole defect suffices to implement an effective oracle step. This second point is, on the practical side, probably more important than the previous one. Indeed, there are several experimental realizations of QW, including 2D QW, often using cold atoms or photons [8, 16], but these had not been considered as scalable substrates for implementing the Grover search so far.

The goal. Your work will be to try variants of [10] in order to see whether they can locate, not just a hole defect, but a particular QR code–like defect, amongst many possible others that could be present on the lattice. This would bring us one step closer to a natural implementation of an unstructured database Grover search. The internship will be funded as legally required.

Prerequisite : Strong programming skills, Quantum theory.

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