1 Encoding Propositions

1. Define an inductive type $\text{formula}$ representing logical formulas ($\land$, $\lor$, $\neg$, $\Rightarrow$) whose atoms are relations ($=$, $\leq$, $<$) between natural numbers.

2. Define a recursive function $\text{interp\_formula}: \text{formula} \rightarrow \text{Prop}$ that converts an inductive formula to the corresponding logical proposition.

3. Define a recursive tactic $\text{reify\_formula}$ that takes a logical proposition and returns an inductive formula whose application to $\text{interp\_formula}$ is $\beta$-convertible to the given proposition.

For instance, the following piece of script has to work fine:

\begin{verbatim}
Goal forall m n : nat, m + n <= n -> m = 0 /
\end{verbatim}

\begin{verbatim}
| |- ?g => let f := reify\_formula g in change (interp\_formula f)
end.
\end{verbatim}

2 Small Scale Reflection

1. Define the three functions $\text{eq\_bool}$, $\text{le\_bool}$, $\text{lt\_bool}$ on natural numbers and prove that they are equivalent to the corresponding relations. For instance, the lemma for $\leq$ is:

\begin{verbatim}
Lemma le\_bool\_correct : forall m n : nat,
\end{verbatim}

\begin{verbatim}
le\_bool m n = true <-> m <= n.
\end{verbatim}

2. Define four functions $\text{and\_bool}$, $\text{or\_bool}$, $\text{not\_bool}$, $\text{imp\_bool}$ and prove that they are equivalent to the corresponding connectors. For instance, the lemma for $\Rightarrow$ is:

\begin{verbatim}
Lemma imp\_bool\_correct : forall p q : bool,
\end{verbatim}

\begin{verbatim}
imp\_bool p q = true <-> (p = true -> q = true).
\end{verbatim}

3. Define a function $\text{bool\_formula}$ such that the following theorem holds:

\begin{verbatim}
Theorem bool\_formula\_correct : forall f : formula,
\end{verbatim}

\begin{verbatim}
bool\_formula f = true <-> interp\_formula f.
\end{verbatim}

3 Classical Logic in a Decidable World

1. Define a tactic that replaces the goal $G$ by the goal $\neg G \Rightarrow G$ (assuming it can be reified).

2. Define a tactic that removes all the $\neg$ and $\Rightarrow$ connectors of the goal (assuming it can be reified).

The negation $\neg$ might still appear in front of equalities.