Preuves Interactives et Applications

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HOL and its Specification Constructs

Revision: Documents and Commands

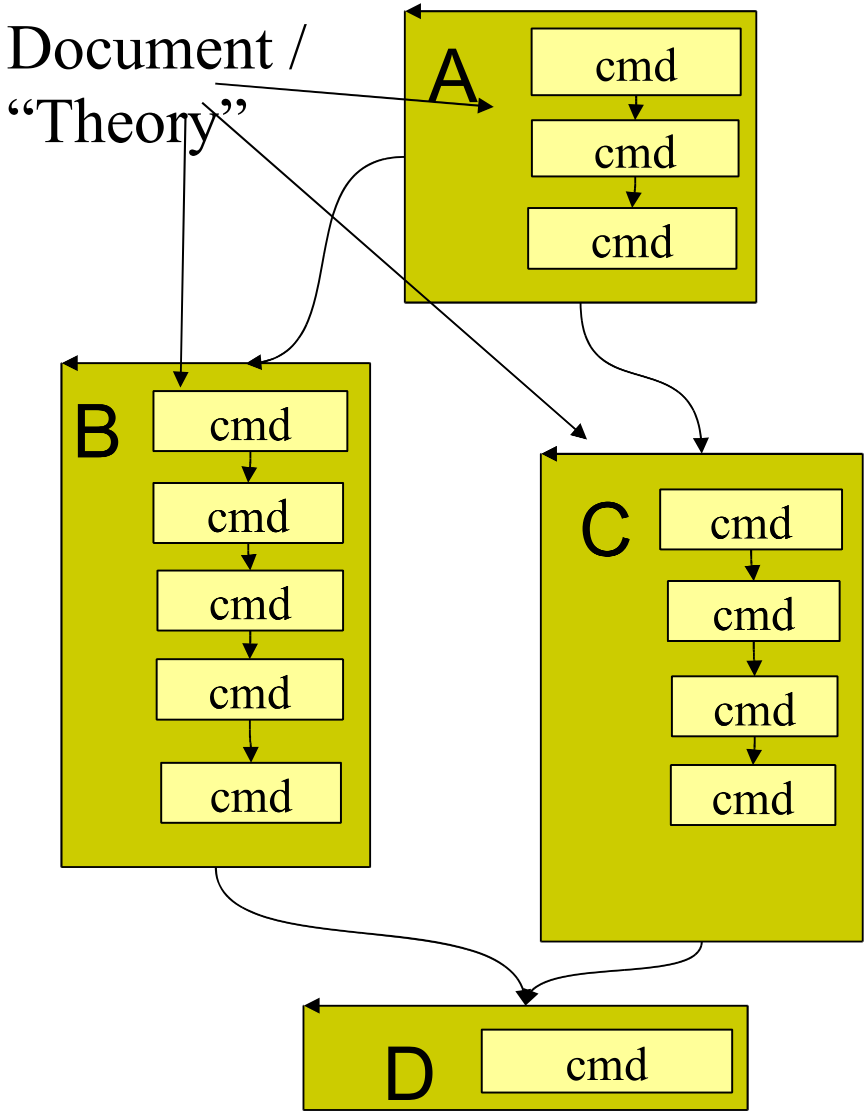
- Isabelle has (similar to Eclipse) a „document-centric" view of development:

  there is a notion on an entire "project" which is processed globally.

- Documents (~ projects in Eclipse) consists of files (with potentially different file-type); .thy files consists of headers commands.

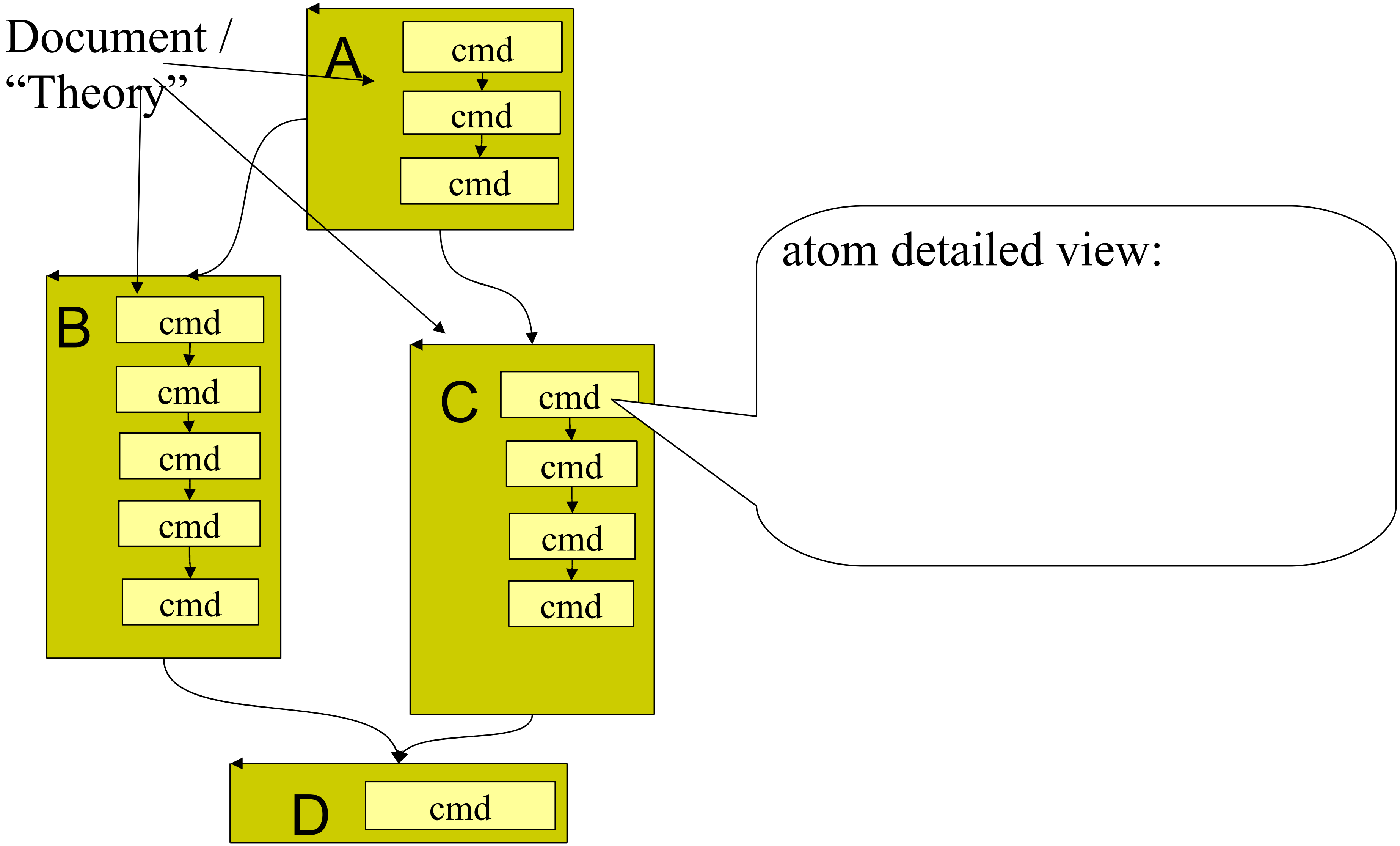
What is Isabelle as a System ?

- Global View of a “session“



What is Isabelle as a System ?

- Global View



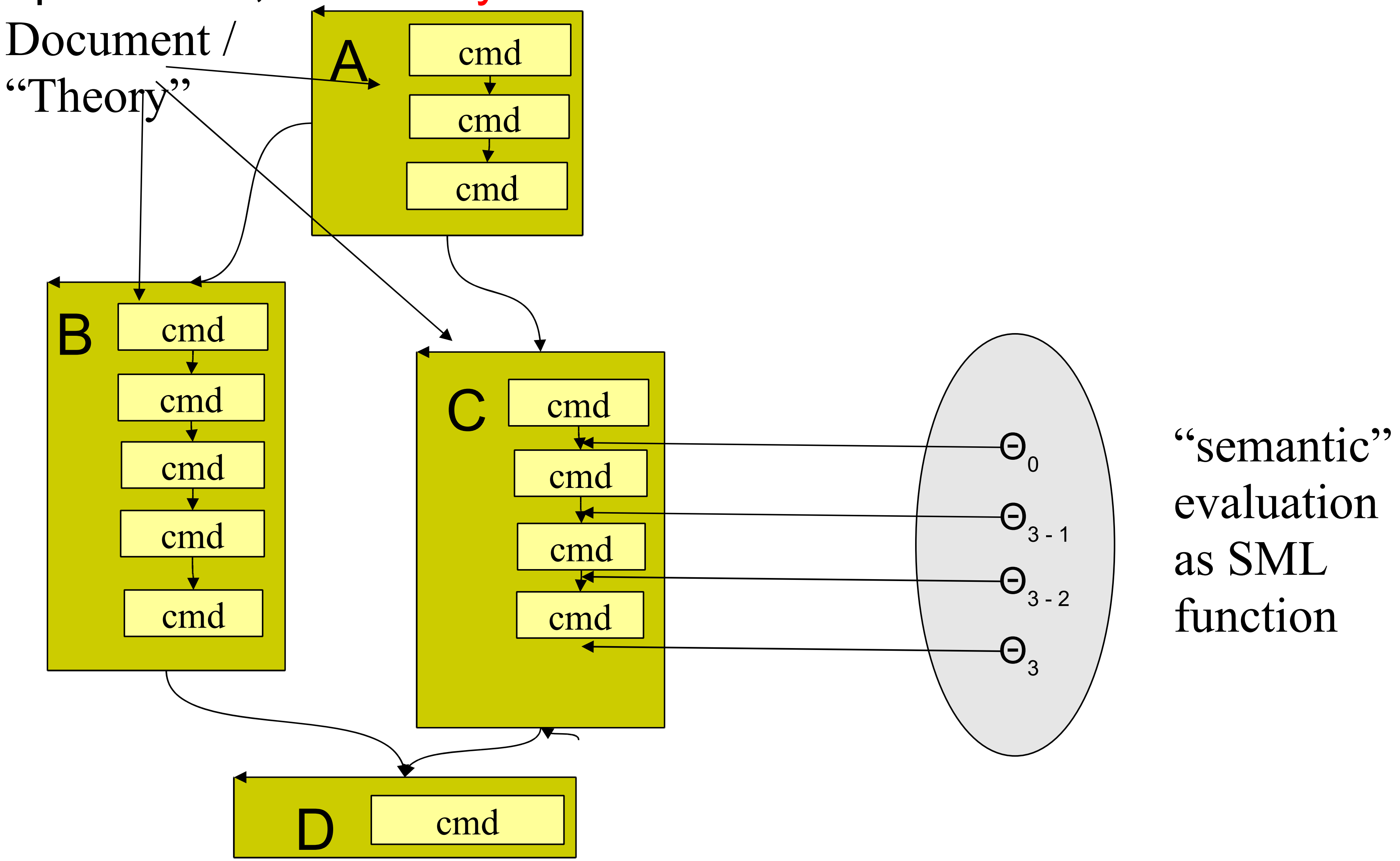
Revision: Documents and Commands

- Each position in document corresponds
  - to a "global context" Θ
  - to a "local context" Θ, Γ

- There are specific „Inspection Commands"
  that give access to information in the contexts
  - thm, term, typ, value, prop : global context
  - print_cases, facts, ... , thm : local context

What is Isabelle as a System ?

- Document "positions" were evaluated to an implicit state, the theory context Θ



Inspection Commands

- Type-checking terms:

term “<hol-term>”

example: term “(a::nat) + b = b + a"

- Evaluating terms:

value “<hol-term>”

example: term “(3::nat) + 4 = 7"

Simple Proof Commands

- Simple (Backward) Proofs:

```
lemma  <thmname> :
  [<contextelem>+ shows] "<phi>"
   <proof>
```

There are different formats of proofs, we concentrate on the simplest one:

apply(<method$_1$>) ... apply(<method$_n$>) done

Exercise demo3.thy

- Examples

lemma X1 : “$A \Longrightarrow B \Longrightarrow C \Longrightarrow (A \wedge B) \wedge C$”

(* output: $[\![A; B; C]\!] \Longrightarrow (A \wedge B) \wedge C$) *)

lemma X2 : assume “A” and “B” and “C”

shows “$(A \wedge B) \wedge C$”

lemma X2 : assume h1: “A” and h2: “B” and h3: “C”

shows “$(A \wedge B) \wedge C$”

Specification Commands

- Simple Definitions (Non-Rec. core variant):

```
definition f::“<τ>”
  where <name> : “f x_1 … x_n = <t>”
```

example: definition C::"bool ⇒ bool"

where "C x = x"

- Type Definitions:

```
typedef ('α_1..'α_n) κ =
  “<set-expr>” <proof>
```

example: typedef even = "{x::int. x mod 2 = 0}

Isabelle Specification Constructs

- Major example:
  The introduction of the cartesian product:

```
subsubsection {* Type definition *}

definition Pair_Rep :: "'a ⇒ 'b ⇒ 'a ⇒ 'b ⇒ bool"
where    "Pair_Rep a b = (λx y. x = a ∧ y = b)"


definition "prod = {f. ∃ a b. f = Pair_Rep (a :: 'a) (b :: 'b)}"

typedef ('a, 'b) prod (infixr "*" 20) = "prod :: ('a ⇒ 'b ⇒ bool) set"
                                     unfolding prod_def by auto

type_notation (xsymbols) "prod" ("(_ ×/ _)" [21, 20] 20)
```

Specification Mechanism Commands

- Datatype Definitions (similar SML):
  (Machinery behind : complex series of const and typedefs !)

```
datatype ('a1..'an) Θ =
 <c> :: "<τ>"  | ... |  <c> :: "<τ>"
```

- Recursive Function Definitions:
  (Machinery behind: Veeery complex series of const and typedefs and automated proofs!)

```
fun <c> ::"<τ>" where
     "<c> <pattern> = <t>"
  | ...
  |  "<c> <pattern> = <t>"
```

Specification Mechanism Commands

- Datatype Definitions (similar SML):
  (Machinery behind : complex !)

```
datatype ('a1..'an) Θ =
  <c> :: "<τ>" | ... | <c> :: "<τ>"
```

- Recursive Function Definitions:
  (Machinery behind: Veeery complex!)

```
fun <c> :: "<τ>" where
    "<c> <pattern> = <t>"
  | ...
  | "<c> <pattern> = <t>"
```

NOTE: Isabelle HOL compiles this internally to axiomatic definitions, i.e. a 'model' in HOL!!!

Specification Mechanism Commands

- Datatype Definitions (similar SML):
  Examples:

```
datatype mynat = ZERO | SUC mynat

datatype 'a list = MT | CONS "'a" "'a list"
```

Specification Mechanism Commands

- Inductively Defined Sets:

```
inductive     <c> [ for <v>:: "<τ>" ]
  where  <thmname> : "<φ>"
          | ...
          | <thmname> = <φ>
```

```
example: inductive path for rel ::"'a ⇒ 'a ⇒ bool"
           where  base : "path rel x x"
               |    step : "rel x y ⟹ path rel y z ⟹ path rel x z"
```

Specification Mechanism Commands

- Extended Notation for Cartesian Products: records (as in SML or OCaml; gives a slightly OO-flavor)

```
record    <c> = [<record> + ]
  tag₁ :: "<τ₁>"
  ...
  tagₙ :: "<τₙ>"
```

- … introduces also semantics and syntax for
  - selectors : $tag_1\ x$
  - constructors : $⦇ tag_1 = x_1, \ldots , tag_n = x_n ⦈$
  - update-functions : $x ⦇ tag_1 := x_n ⦈$

More on Proof-Methods

- Some composed methods (internally based on assumption, erule_tac and rule_tac + tactic code that constructs the substitutions)
  - subst <equation>

    (one step left-to-right rewrite, choose any redex)
  - subst <equation>[symmetric]

    (one step right-to-left rewrite, choose any redex)
  - subst (<n>) <equation>
    (one step left-to-right rewrite, choose n-th redex)

More on Proof-Methods

- Some composed methods (internally based on assumption, erule_tac and rule_tac + tactic code that constructs the substitutions)
  - simp

    (arbitrary number of left-to-right rewrites, assumption or rule refl attepted at the end; a global simpset in the background is used.)
  - simp add: <equation> ... <equation>

More on Proof-Methods

- Some composed methods (internally based on assumption, erule_tac and rule_tac + tactic code that constructs the substitutions)
  - auto
    (apply in exaustive, non-deterministic manner: all introduction rules, elimination rules and
  - auto intro: <rule> ... <rule>
    elim: <erule> ... <erule>
    simp: <equation> ... <equation>

More on Proof-Methods

- Some composed methods (internally based on assumption, erule_tac and rule_tac + tactic code that constructs the substitutions)
  - cases „<formula>“
    (split top goal into 2 cases:
    <formula> is true or <formula> is false)
  - cases „<variable>“
    (- precondition : <variable> has type t which is a data-type)
    search for splitting rule and do case-split over this variable.
  - induct_tac „<variable>“
    (- precondition : <variable> has type t which is a data-type)
    search for induction rule and do induction over this variable.

Screenshot with Examples