Coq

TYPES Summer School, Bertinoro, Italy

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Coq: outline

- Basic notions
  - Introduction
  - History
  - Architecture
  - Vernacular
  - Basic tactics
- Extra features
  - Modules
  - Computation
  - Tactic language
  - Extraction

Outline

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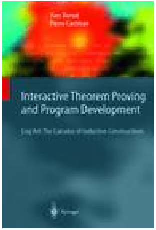
Informations on Coq

- The Coq web site `coq.inria.fr`
  - Official distribution + current version of development publically available
  - Reference manual
  - Library
  - User's contributions
- Reference book : the Coq'art by Yves Bertot and Pierre Castéran



Interactive Theorem Proving and Program Development
Coq'Art: The Calculus of Inductive Constructions
Series: Texts in Theoretical Computer Science.

- Associated web site with exercises:
  `http://www.labri.fr/perso/casteran/CoqArt`

History

See preface of the reference manual

- 85 (Coquand-Huet) type-checker of the pure Calculus of Constructions + hierarchy of universes, tactics
- 88 [V4.10] first version of extraction (Prop/Set) encoded impredicative inductive definitions (Paulin)
- 89-95 [V5.6] primitive inductive definitions with recursor [V5.7] Camllight [V5.8] (co)-inductive definitions with pattern-matching and fixpoints (Paulin,Giménez) new architecture (Chet Murthy)
- 96-01 [V6.1] coercions (A. Saibi), automatic tactics (Ring, S. Boutin), inversion (C. Cornes) [V6.2] Efficient reduction (B. Barras)
- 02-03 [V7] new functional architecture (Filliâtre), Modules (Courant, Chrząszcz), tactic language (Delahaye), local definitions (Herbelin), powerful pattern-matching, new extraction (Letouzey)
- 04- [V8] Coq'art, new syntax, predicative Set, byte-code compilation (B. Grégoire) . . .

Remarkable Developments

- JavaCard architecture modeling (Trusted Logic, EAL7 certification)
- Fundamental theorem of Algebra (Constructive proof, CCorn Nijmegen)
- Four color theorem (Gonthier-Werner)
- Certified compiler for C (Leroy et al)
- Primality checker (Théry et al)
- ...

Architecture

- Implemented in OCAML
- Batch compilation : coqc
- Interfaces : coqide (lablgtk), Proof General (emacs), web ...
- Kernel
  - strong rules for inductive definitions
    complete pattern-matching, guarded definitions
  - modules
  - possible efficient byte-code reduction
- Extensions
  - universe inference
  - coercions, implicit arguments
  - advanced pattern-matching
  - notations, tactic language, functional definitions
  - extraction

Basic terms

```
Prop Set Type

forall x y, B        forall (x y : A) z, B
A->B->C              (A->B)->C

fun x y => t         fun (x y : A) z => t

t x y                t (u x)

match t with .. end

let x:=t in u

fix f (x:A) : B := t
```

Basic notations

- Propositional logic

  `A /\ B` `A \/ B` `A <-> B` `~ A`

- predicate logic

  `exists x, P`

  `x=y`

- data-types

  `A * B` `(x,y)`

  `A + B`

- arithmetic (nat, Z . . .)

  `0 45 n+m n*m`

Basic Vernacular

Enriching the environments

```
Definition name : type := term.
Variable/Hypothesis name : type.
```

(co)inductive definition

```
Inductive/CoInductive name params : type
   := constructors.
Fixpoint name params {struct arg}: type := term.
CoFixpoint name params : type := term.
```

Bottom-up development

```
Lemma/Theorem name : type.
Save/Qed/Defined.
```

Save/Qed: Opaque constants not unfolded in computation.

Libraries

- Section mechanism

```
Section name.
  Variable ...
  Hypothesis ...
  Definition ...
  Lemma ...
End  name.
```

- Loading precompiled libraries

```
Require Export library.
```

- Modules

Basic help

```
Check term.
Print ident.
SearchAbout ident.
Eval compute in term.
Print LoadPath.
```

Basic tactics

- `exact` *term*; `assumption`
- `intros`
- `apply` *term*; `constructor`
- `case` *term*; `elim` *term*
- `change` *term*; `unfold` *term*; `simpl`
- `fix` *n*; `cofix`

Composed tactics

Inductive types

- `inversion` *term*; `induction` *term*
- equality : `rewrite` *term*; `injection` *term*; `discriminate`

Automatic tactics

- Database search: `auto`, `trivial`
- Decision procedure: `tauto`, `firstorder`, `omega`
- Propositional simplification: `intuition`
- Reflexive tactic: `ring`

Tacticals

- $tac_1 ; tac_2 \quad tac; [tac_1 | \ldots | tac_n]$
- `try` *tac*, $tac_1 || tac_2$

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Modules

- Extension of Ocaml modules systems to type theory
  - Modules are not terms
  - Interfaces contains both abstract and explicit declarations.
  - Parameterized Modules (functors)
- Meta-theory by J. Courant (98)
- Implementation by J. Chrząszcz (02)
- Non logical features : notations, Hint databases, Extraction are compatible with the module system.

Demo.

Computation

- Computation is part of type-checking (verification of convertibility)

$$\frac{\Gamma \vdash U : s \qquad \Gamma \vdash t : T \qquad T \equiv U}{\Gamma \vdash t : U}$$

- Internal programming language (functional kernel of CAML)
- Possibility to write complex programs and use them in proofs
  - Four colors theorem
  - Reflexive tactics
- Efficient reduction technics (byte-code compiling, B. Grégoire)
- Ongoing work on proof-irrelevance

Demo.

Reflexive tactics

- We have a function $s2bool : \forall AB, A + B \to bool$ and a proof $strue : \forall p : A + B, s2bool\, p = true \to A$
- If there is a (closed) proof thm of type $\forall x : A, (P\, x) + Q$ then for closed term a:

$$strue\,(thm\, a)\,(refleq\, true) : P\, a$$

  Proof is well-typed when $s2bool\,(thm\, a) \equiv true$
- Problems
  - $a : A$ should be closed (reification), but we also prefer to keep a small
  - thm should be proved and should reduce efficiently
- Application
  - Ring, (R)Omega, Setoid Rewrite...
  - Interfaces between Coq and other systems using traces.

Example of reflection

```
Inductive form : Set :=
    T | F | Var : nat -> form
  | Conj : form -> form -> form.
Def env := list Prop.
Def find_env (e:env) (n:nat) : Prop := ...
Fixpoint interp (e:env)  (f:form) struct f :=
 match f with
    T => True | F => False
  | Conj A B => interp e A /\ interp e B
  | Var n => find_env e n
 end.
Eval compute in
  (interp ((1=1)::(0=0)::nil)
  (Conj (Var 0) (Conj (Var 1) (Var 1)))).
     = 1 = 1 /\ 0 = 0 /\ 0 = 0 : Prop
```

Simplification function

```
Def simplform : form -> form.
Lemma simplcorrect :
    forall e f, interp e (simplform f) -> interp e f.
```

In order to use the simplification

- Find *e* and *f* such that *interp e f* is convertible with the goal (reification)
- Apply *simplcorrect*
- Compute *simplform* and *interp*

Tactic language

Ltac designed by D. Delahaye

- A way to write complex tactics without writing ML code.
- A specific language
  - specific patterns for matching goals (non-linear)

```
match goal with
    id:?A / ?B |- ?A => case id; trivial
    | _ => idtac
end
match goal with |- context[?a+0] => rewrite ...
```

  - specific notion of backtracking
    - patterns are tried until the right-hand side succeeds
  - specific constructions : **fresh** *name*, **type of** *term* ...

Demo

Extraction

- Distinction between Prop and Set given by the user
- Parts in Prop are erased
- Extraction of Ocaml programs (P. Letouzey)
  - Termination problems **fun** $x : \bot \Rightarrow t$
  - Typing problems
    - polymorphism
    - generalised inductive types
    - dependent types : **if** b **then** 0 **else** "*hello*"
  - Efficiency

Demo

Coq-lab

Proposed exercises

http://www.lri.fr/~{}paulin/TypesSummerSchool/exercises.html