Des relations binaires au treillis de Tamari

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Université Paris-Sud

Séminaire d'équipe GALaC 13 mai 2016

Le tri à bulle sur les permutations

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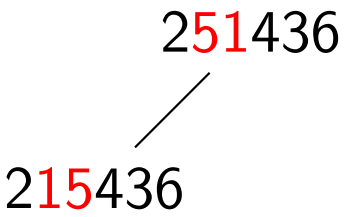
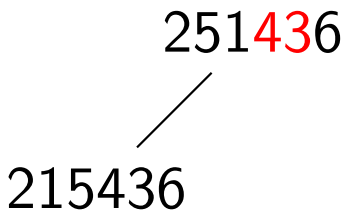


Le tri à bulle sur les permutations



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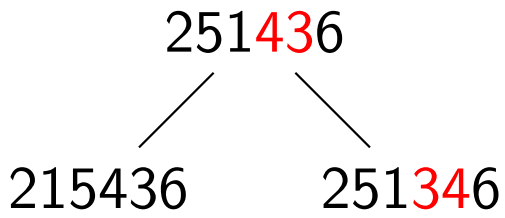
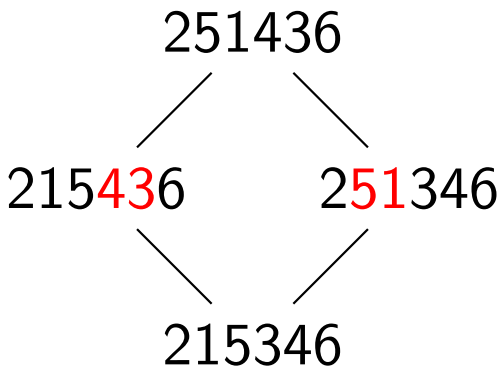
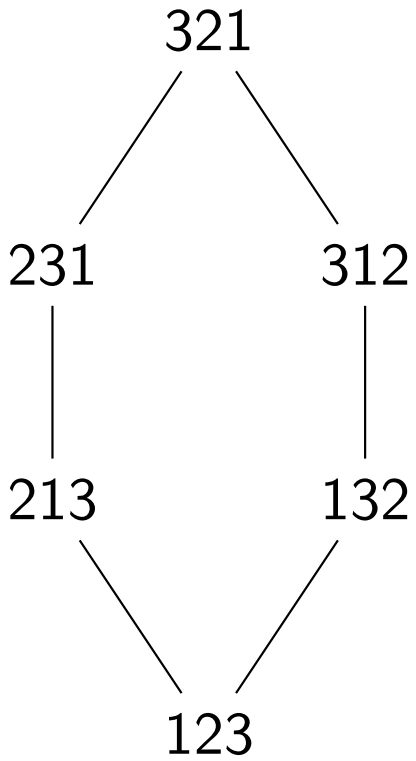
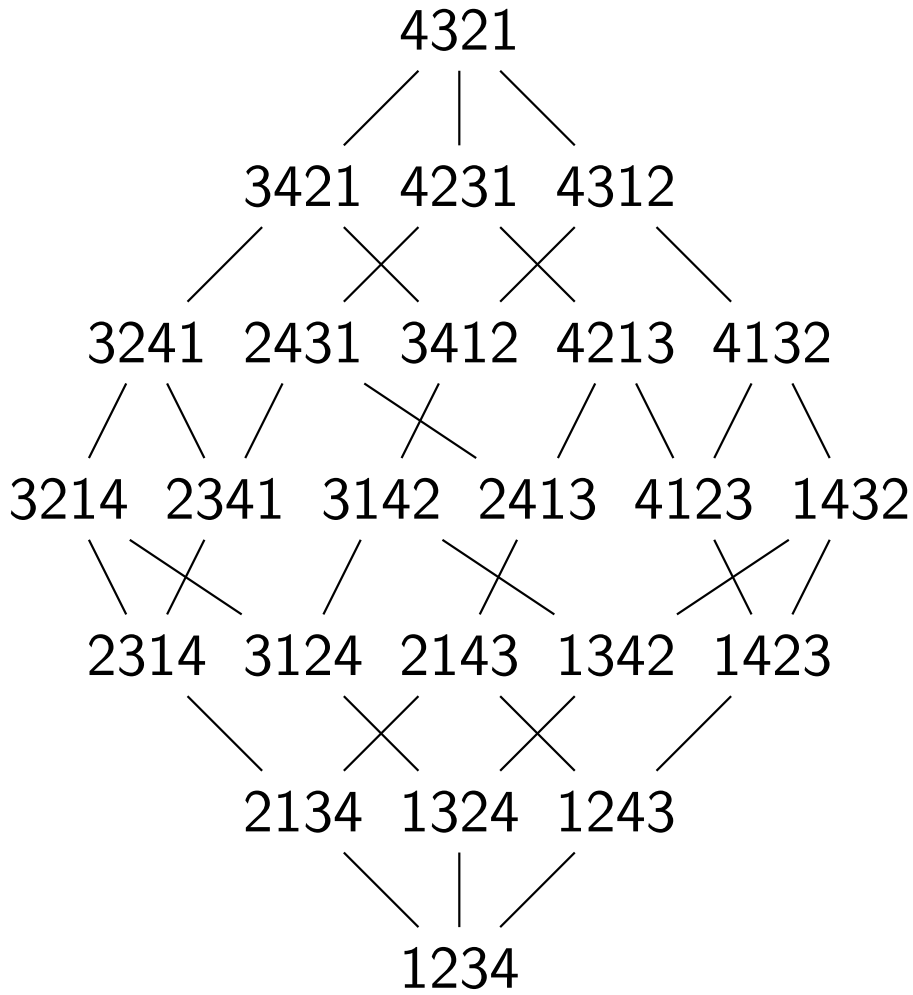


Le tri à bulle sur les permutations



Ordre faible





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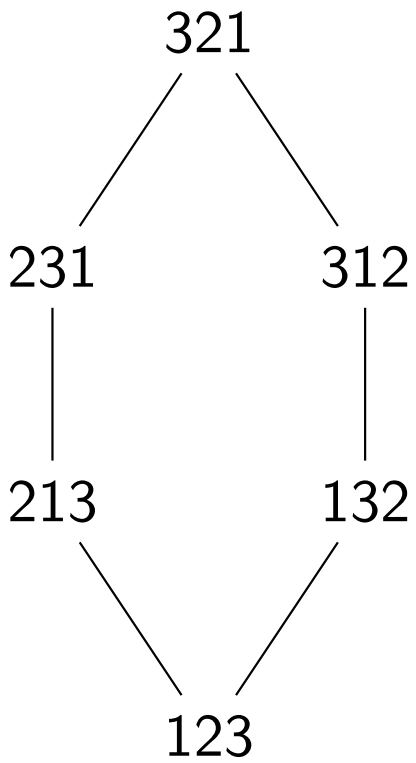

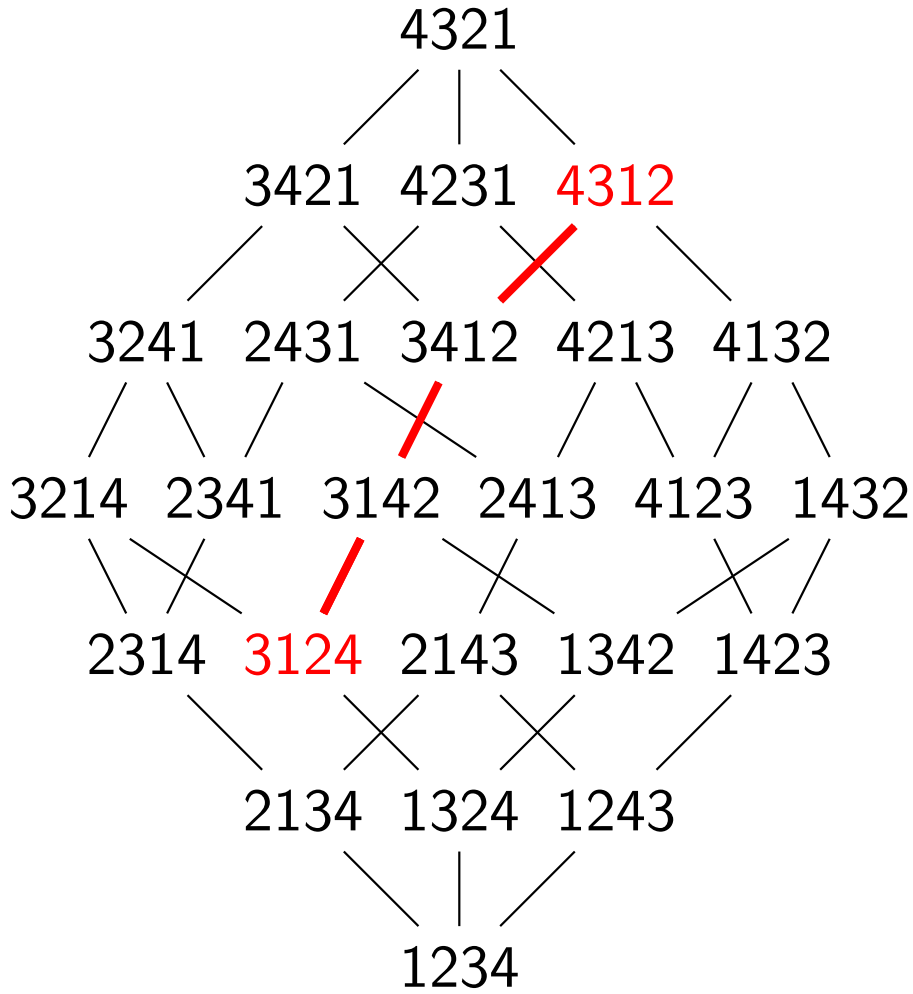

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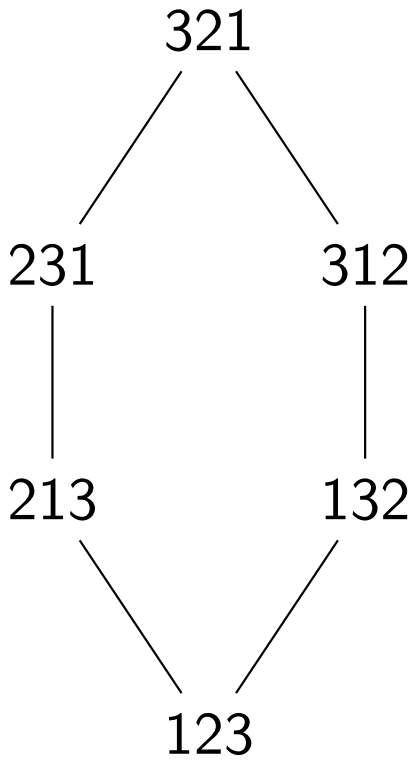

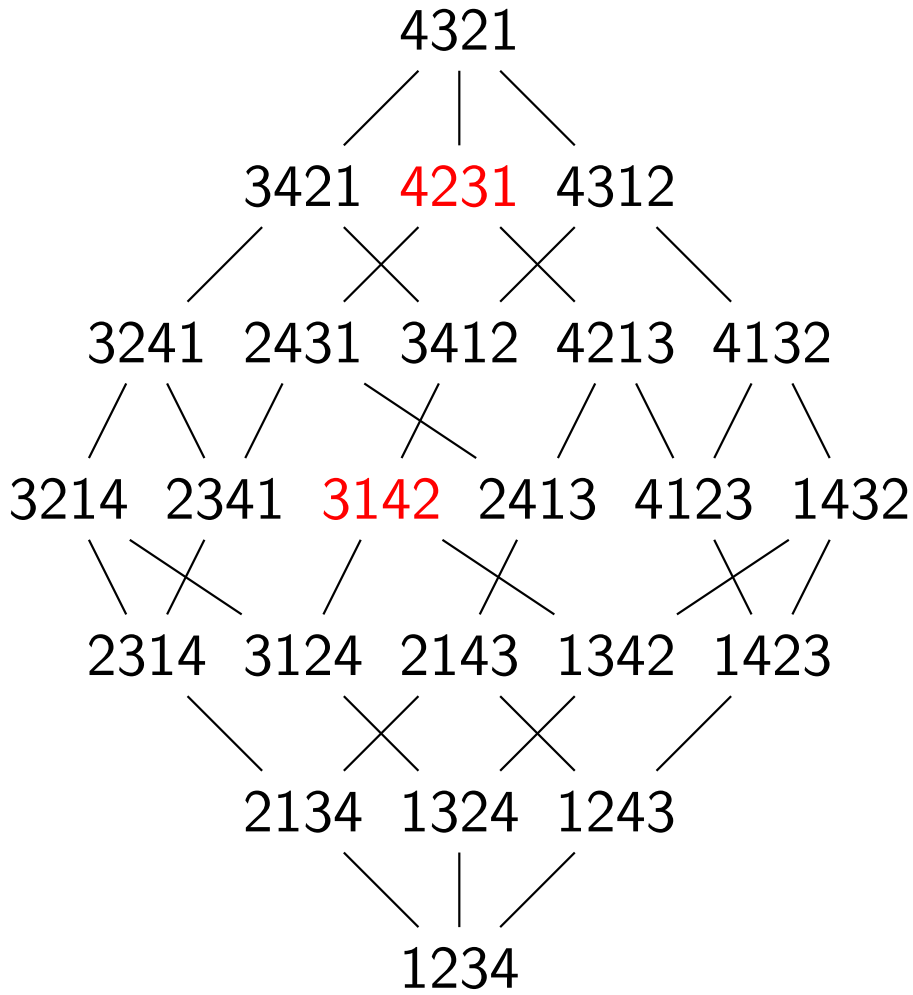

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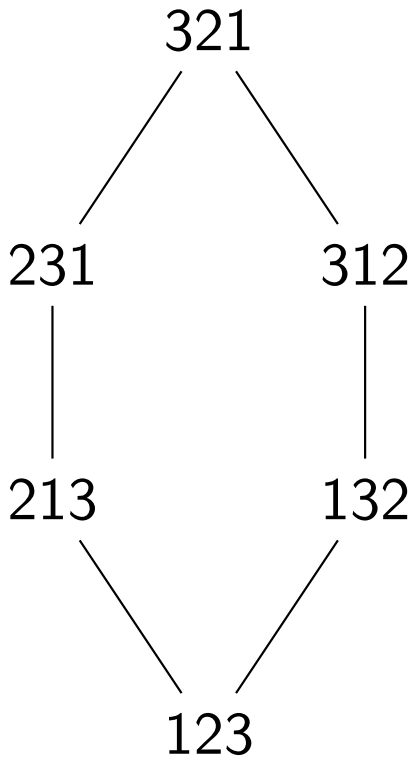
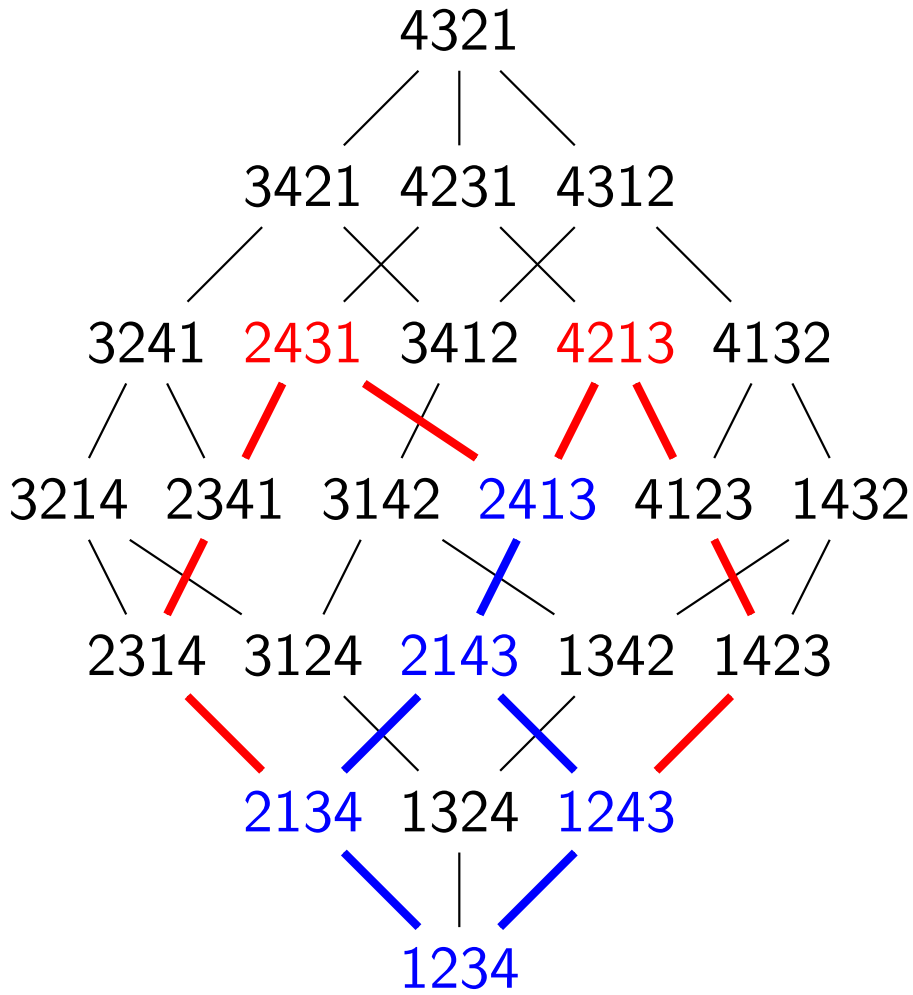
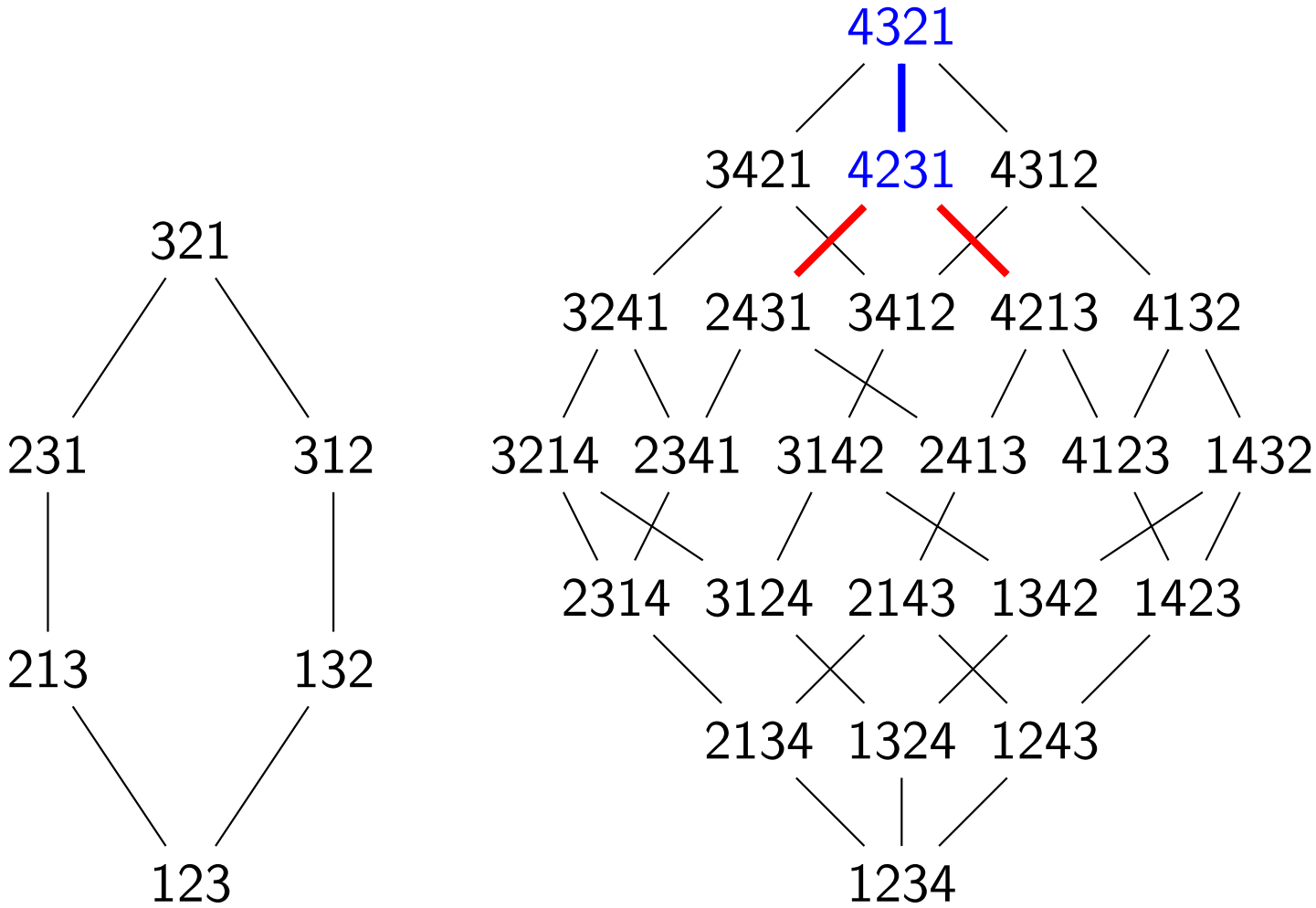




$2413 \wedge 4213 = 2413$

Ordre faible

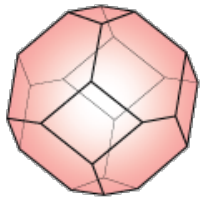


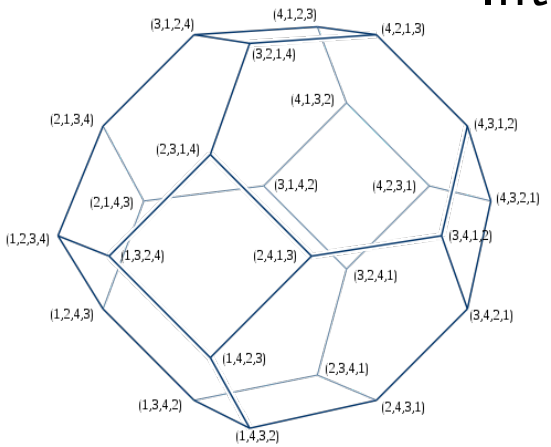
$2413 \wedge 4213 = 2413$

$2413 \vee 4213 = 4231$

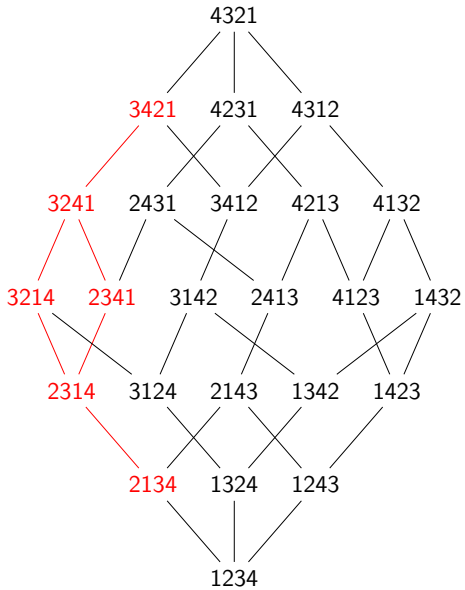
Triple interprétation

Géométrie





Interprétation combinatoire



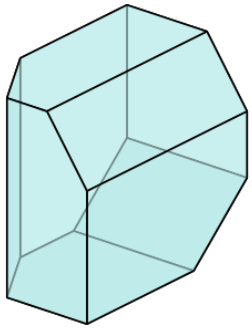
Algèbre

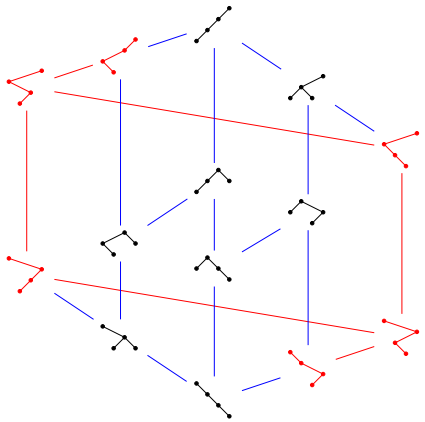
$$\mathbf{F}_{21} \,.\, \mathbf{F}_{12} = \mathbf{F}_{21 \sqcup 12}$$
$$= \mathbf{F}_{2134} + \mathbf{F}_{2314} + \mathbf{F}_{2341} + \mathbf{F}_{3214} + \mathbf{F}_{3241} + \mathbf{F}_{3421}$$

Triple interprétation

Interprétation combinatoire

Géométrie





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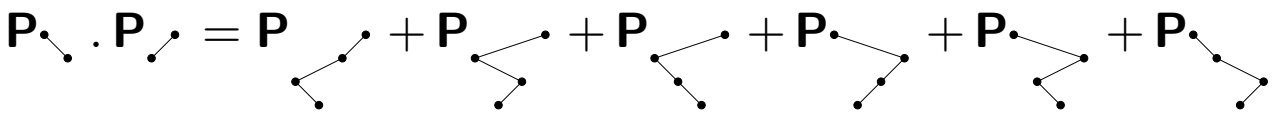

Graphe d'une permutation

4312

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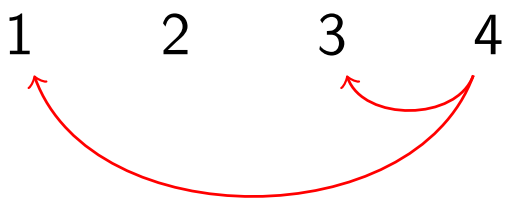
4312



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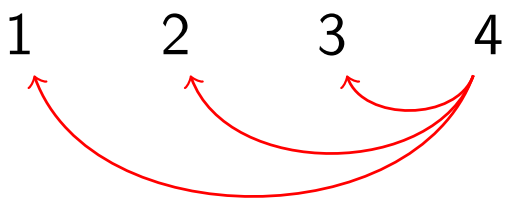
4312



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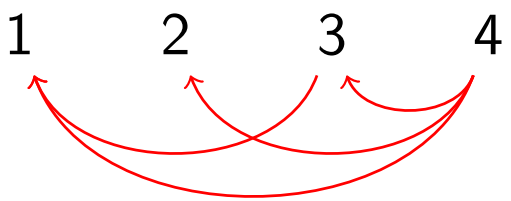

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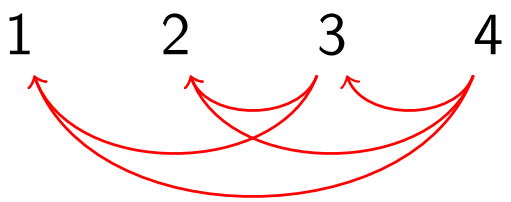

Graphe d'une permutation

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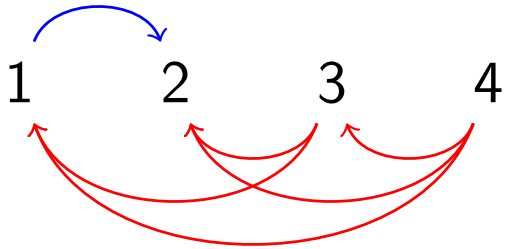

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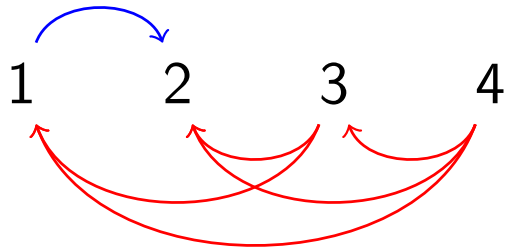

Graphe d'une permutation

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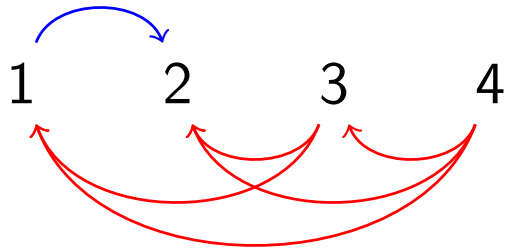
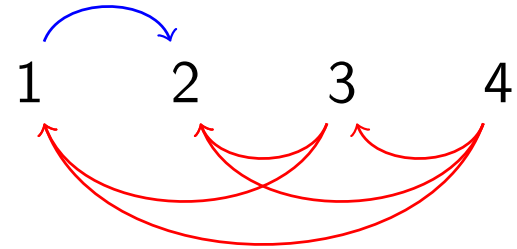


3412

Graphe d'une permutation

4312



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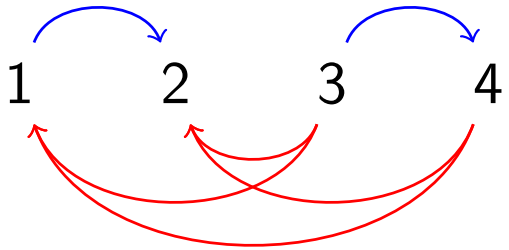

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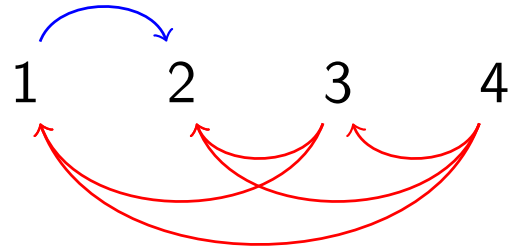

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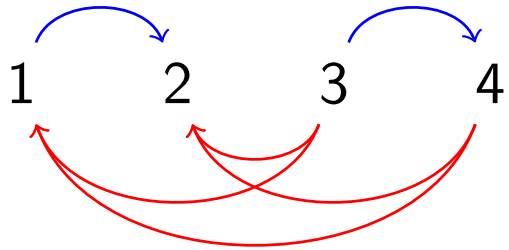

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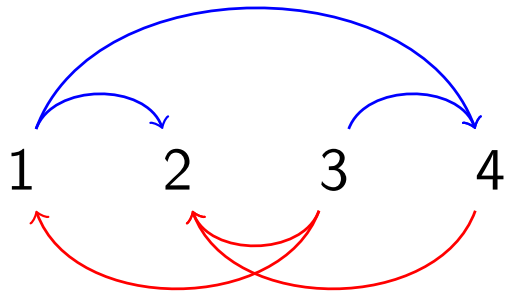

Relations binaires sur les entiers

Soit R une relation de taille n.

$$1 \quad 2 \quad \cdots \quad i \quad \cdots \quad j \quad \cdots \quad k \quad \cdots \quad n$$

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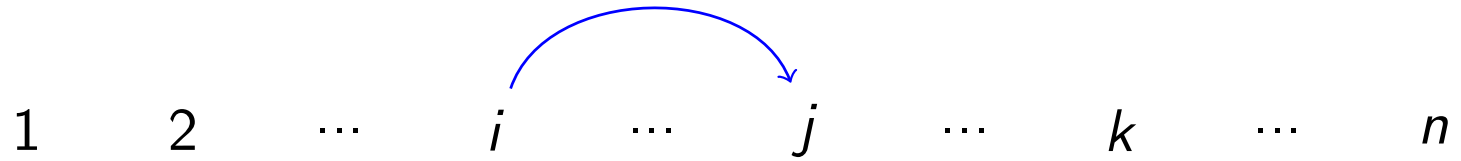

$i \,\mathrm{R}\, j$

Relations binaires sur les entiers

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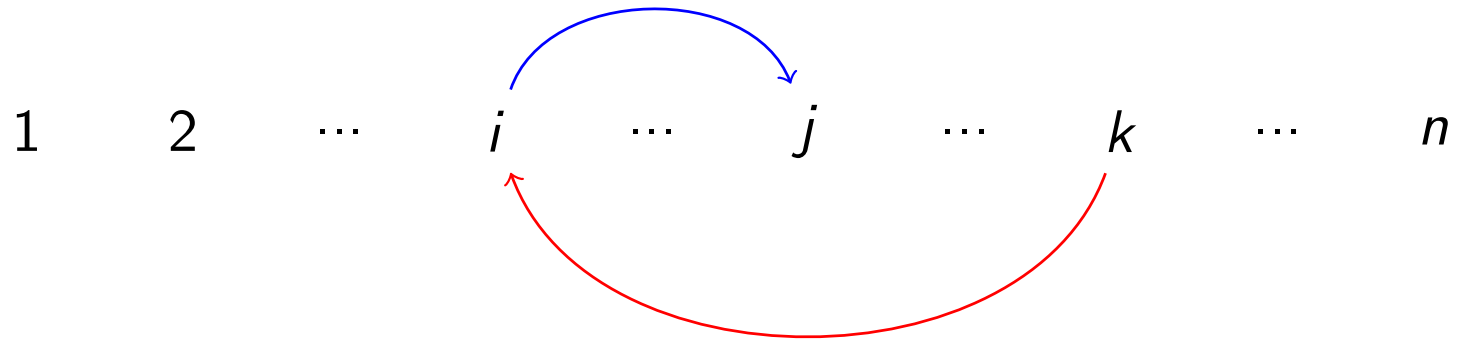

$i \,\mathrm{R}\, j$ $\qquad$ $k \,\mathrm{R}\, i$

Relations binaires sur les entiers

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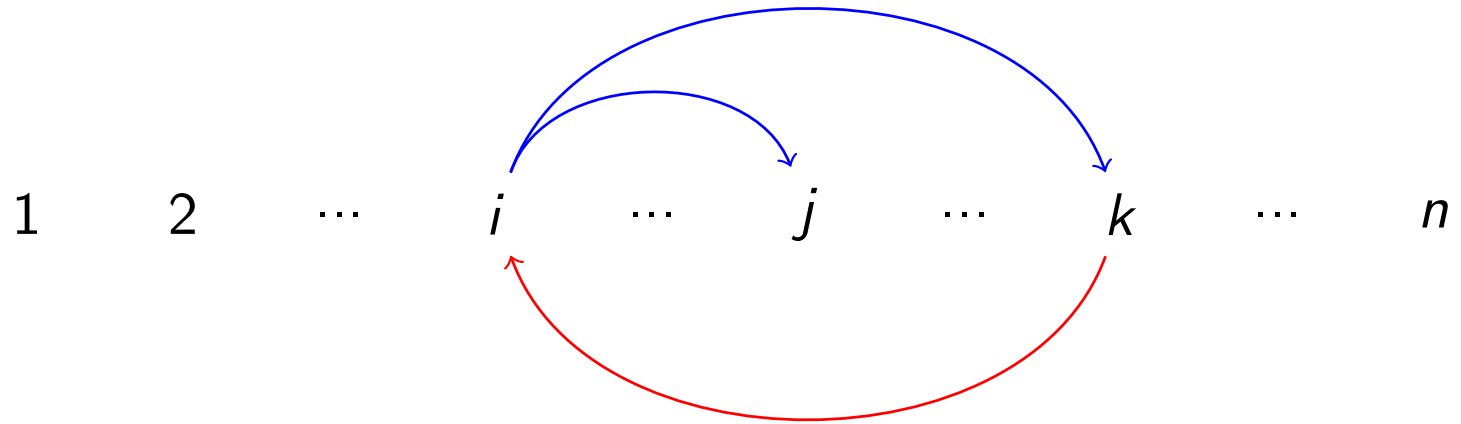

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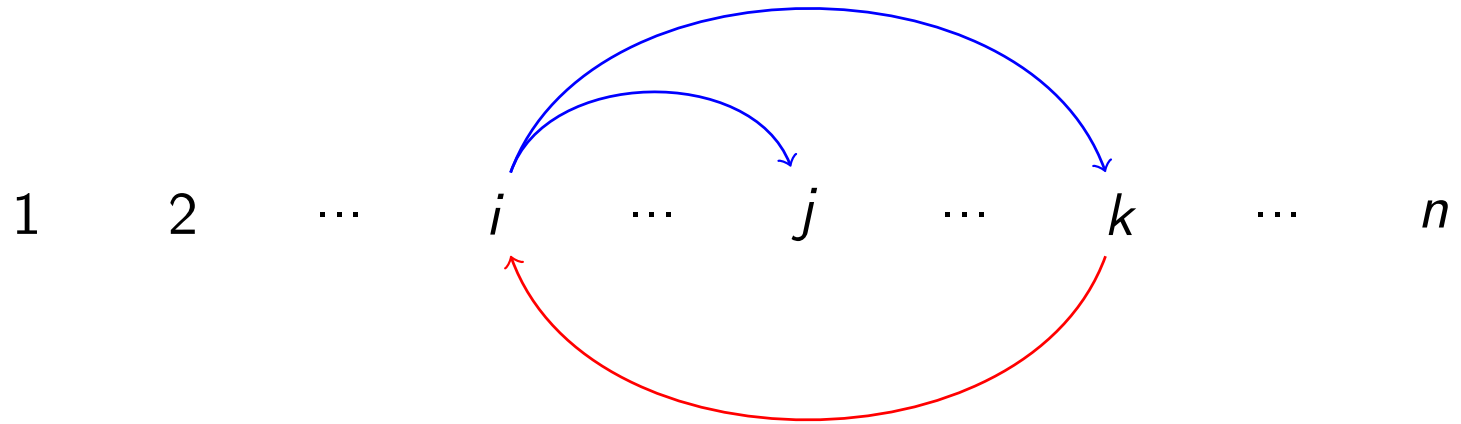

$i \,\mathrm{R}\, j$ $\qquad$ $k \,\mathrm{R}\, i$ $\qquad$ $i \,\mathrm{R}\, k$

En taille n, $2^{n(n-1)}$ relations binaires possibles.

Ordre partiel sur les relations

Soit R une relation binaire

$$\mathrm{R}^{\mathsf{Inc}} = \{i \,\mathrm{R}\, j, i < j\}$$
$$\mathrm{R}^{\mathsf{Dec}} = \{j \,\mathrm{R}\, i, i < j\}$$

Ordre partiel sur les relations

Soit R une relation binaire

$$\mathrm{R}^{\mathsf{Inc}} = \{i \,\mathrm{R}\, j, i < j\}$$
$$\mathrm{R}^{\mathsf{Dec}} = \{j \,\mathrm{R}\, i, i < j\}$$

Soient R et S deux relations binaires,

$$\mathrm{R} \preccurlyeq \mathrm{S} \Leftrightarrow \qquad \mathrm{R}^{\mathsf{Inc}} \supseteq \mathrm{S}^{\mathsf{Inc}} \text{ et } \mathrm{R}^{\mathsf{Dec}} \subseteq \mathrm{S}^{\mathsf{Dec}}$$

Relations de taille 2

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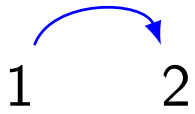
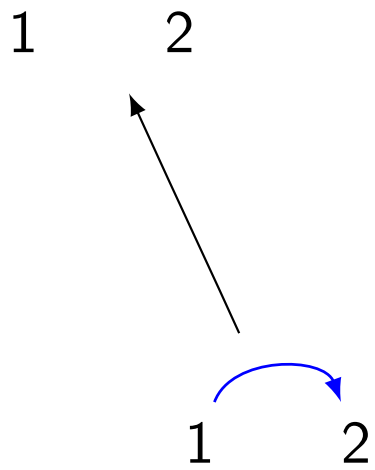
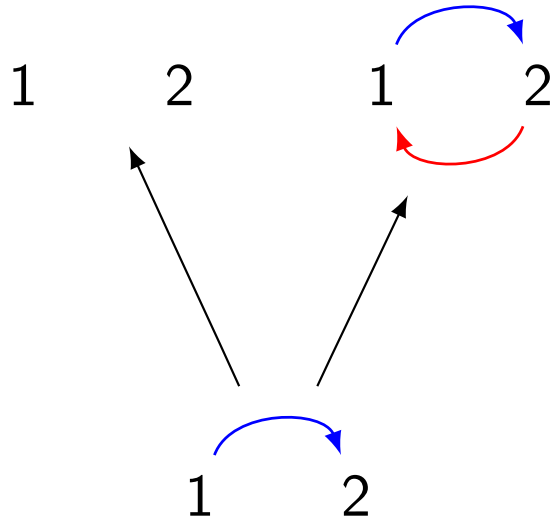


Relations de taille 2



Relations de taille 2



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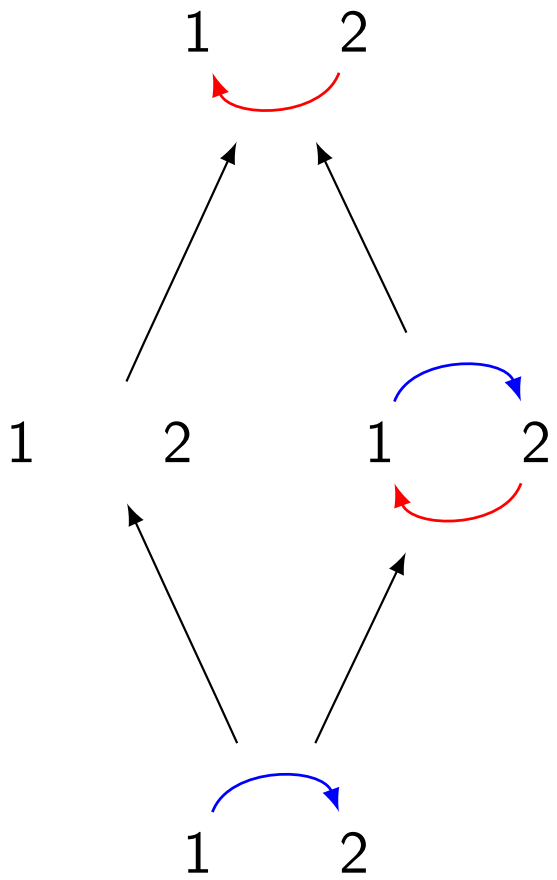
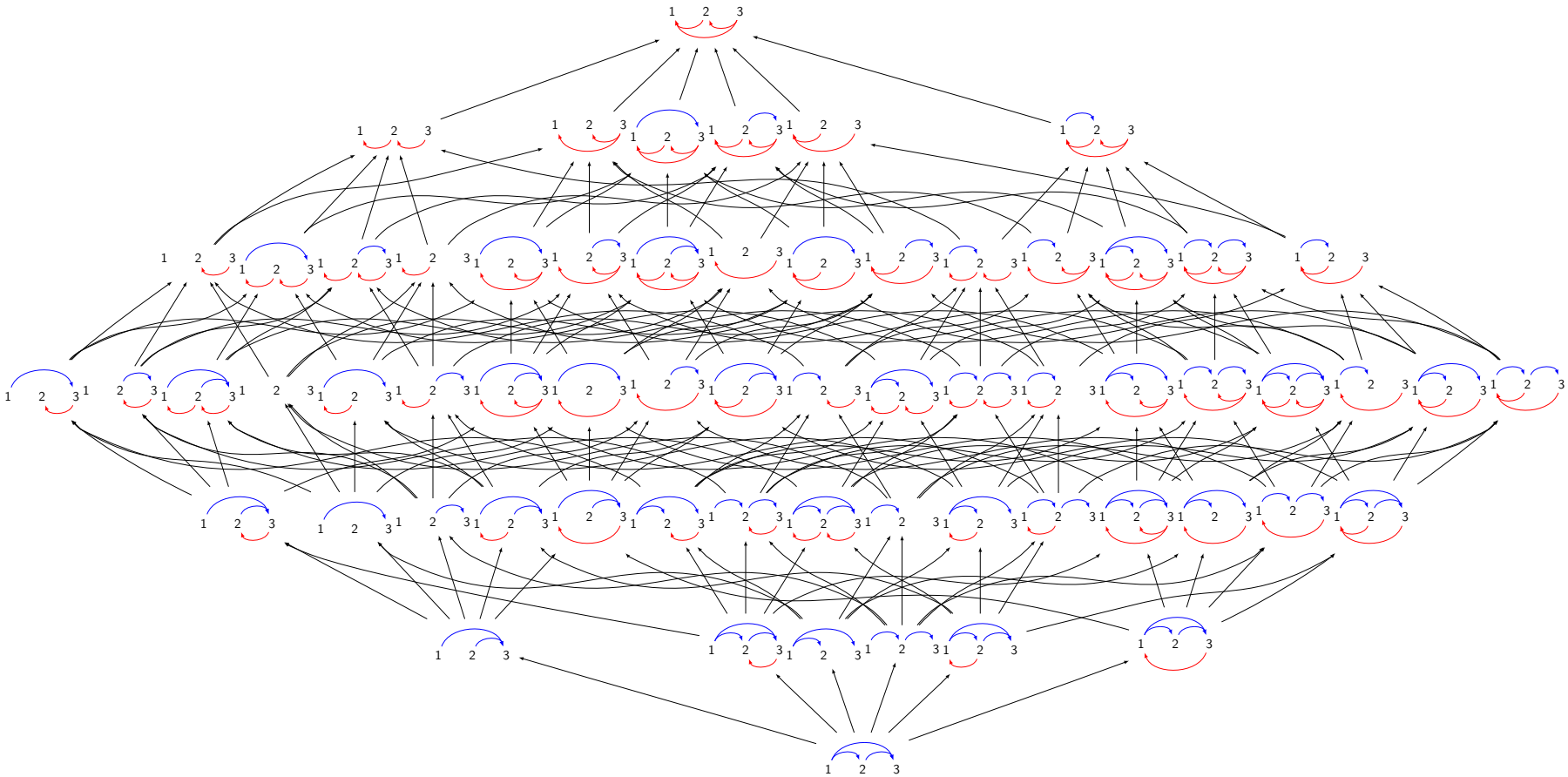
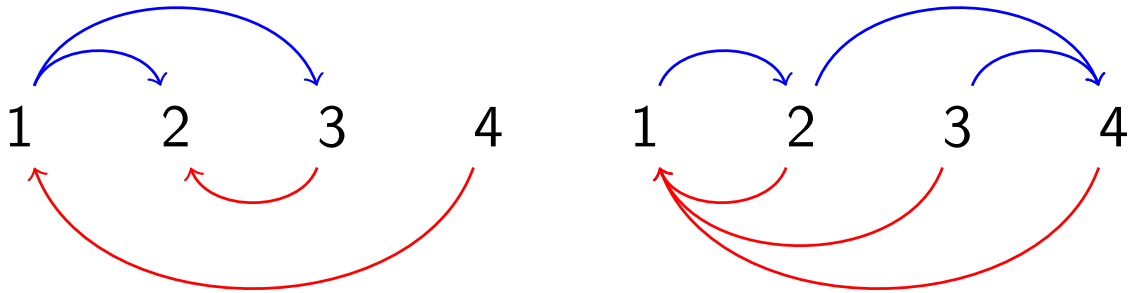




Inf et Sup



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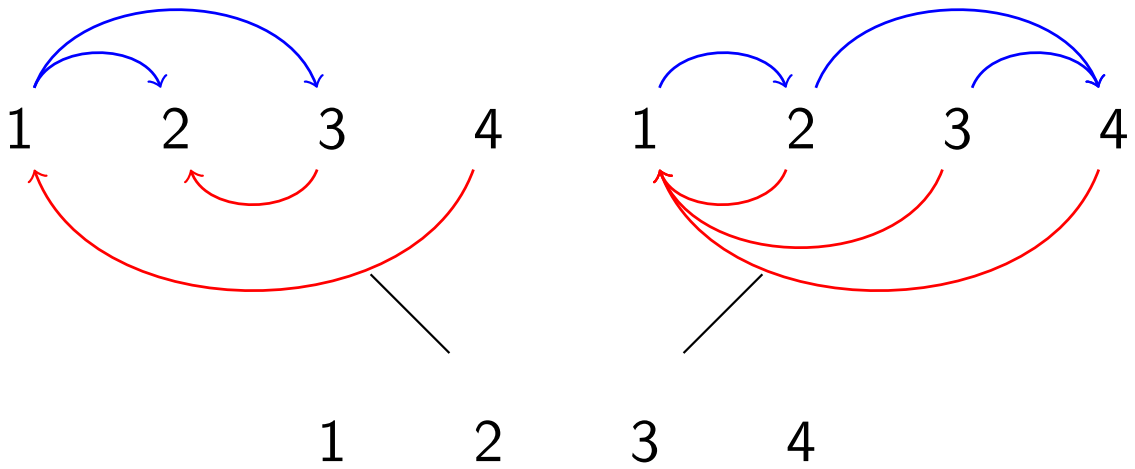

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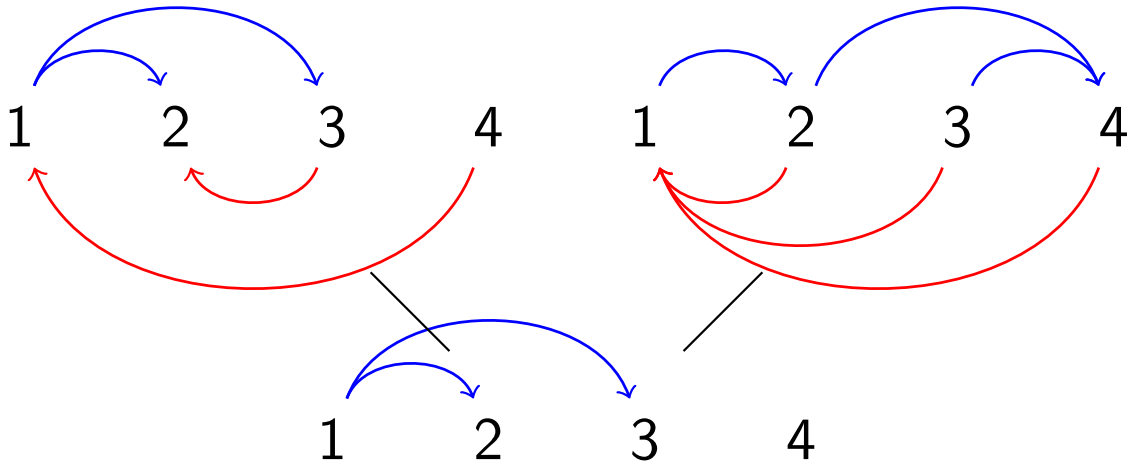

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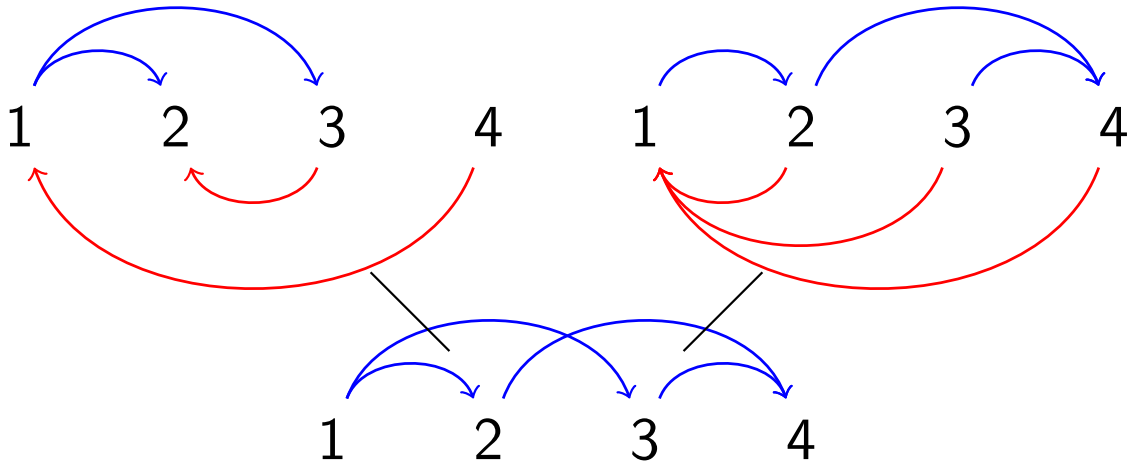

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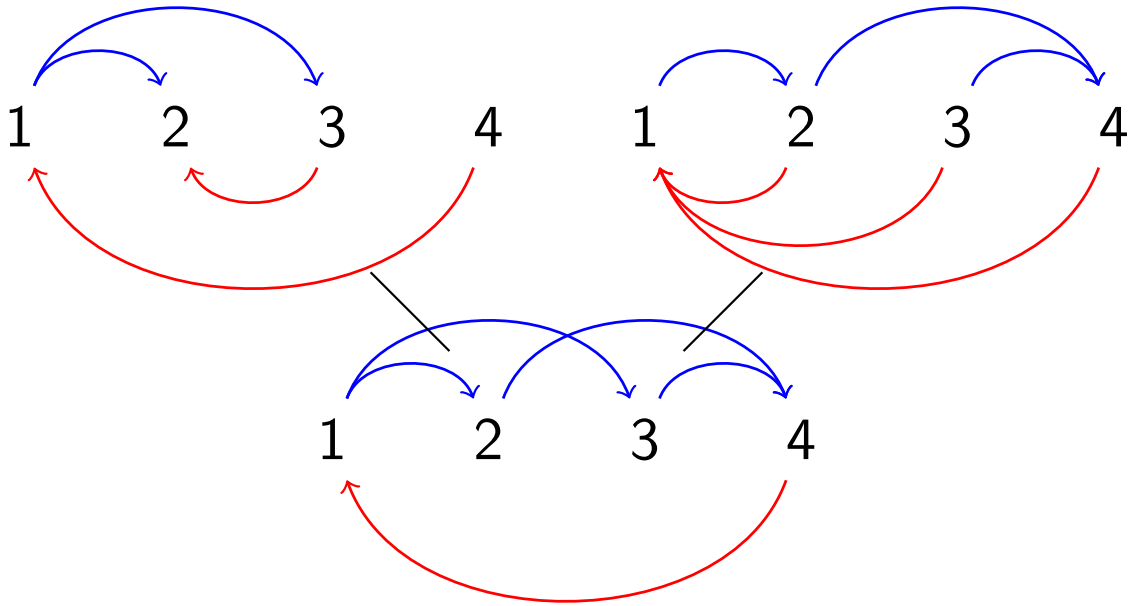

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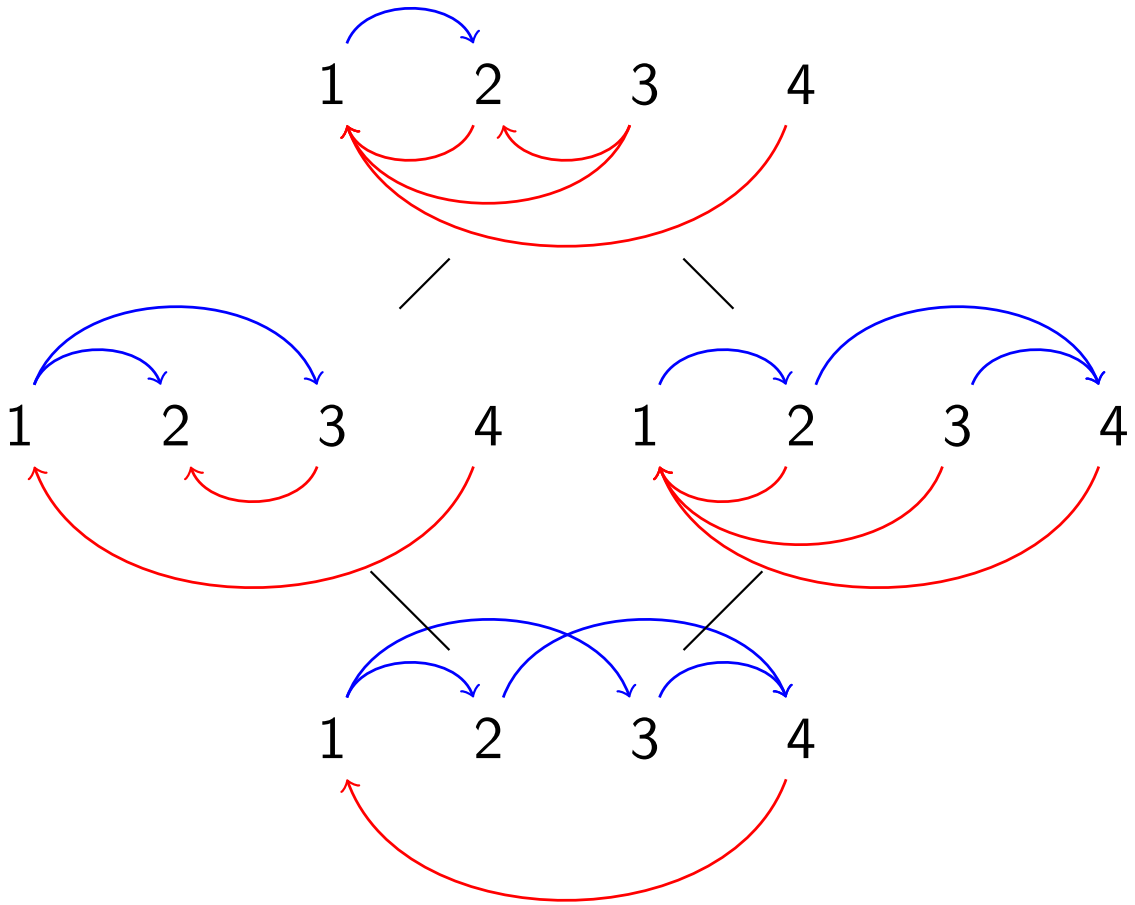
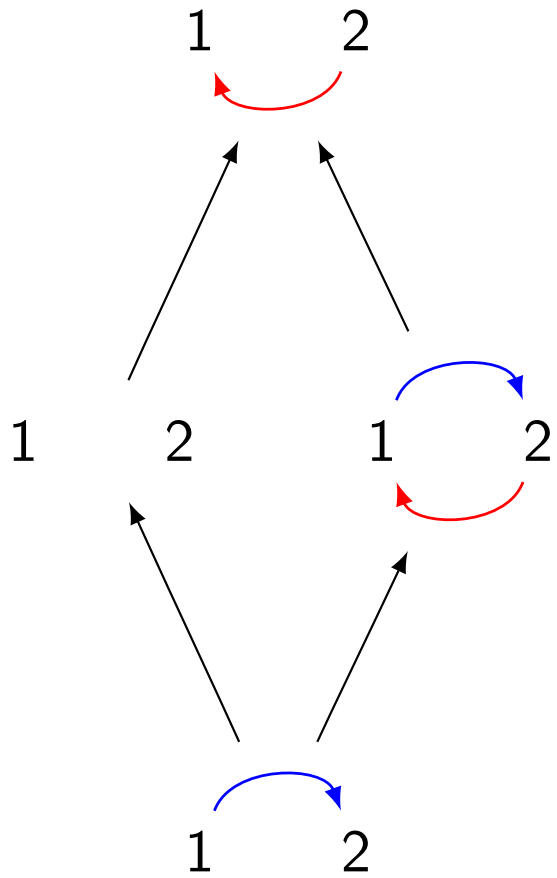


On veut conserver les relations

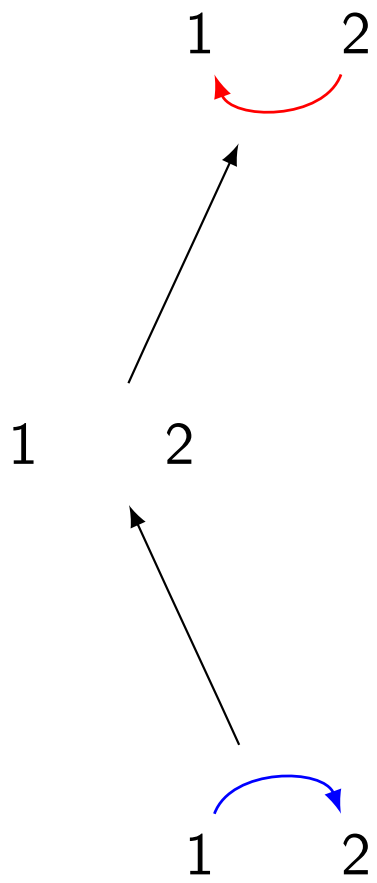
- antisymétriques
- transitives

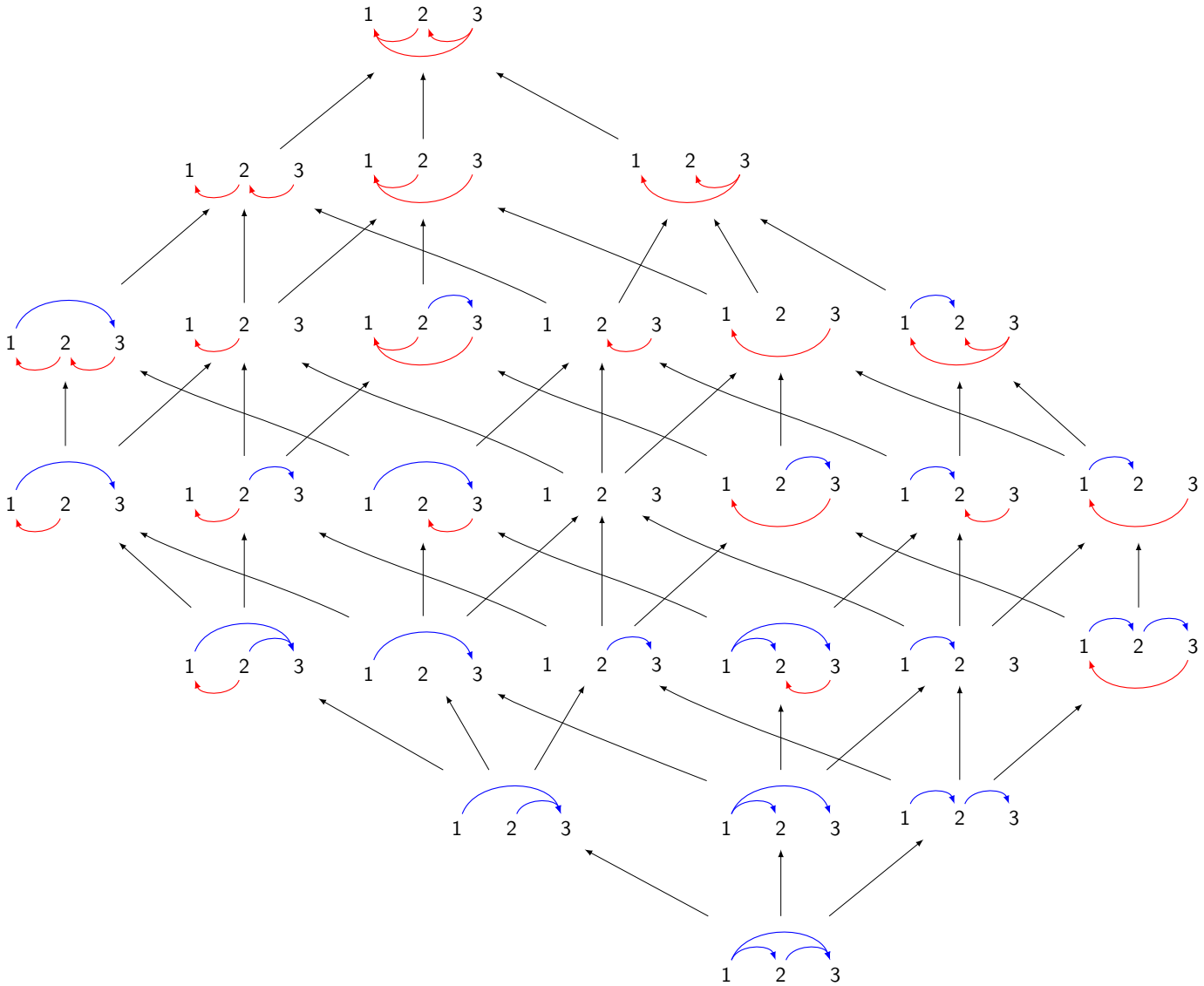
(posets)

Antisymétrie



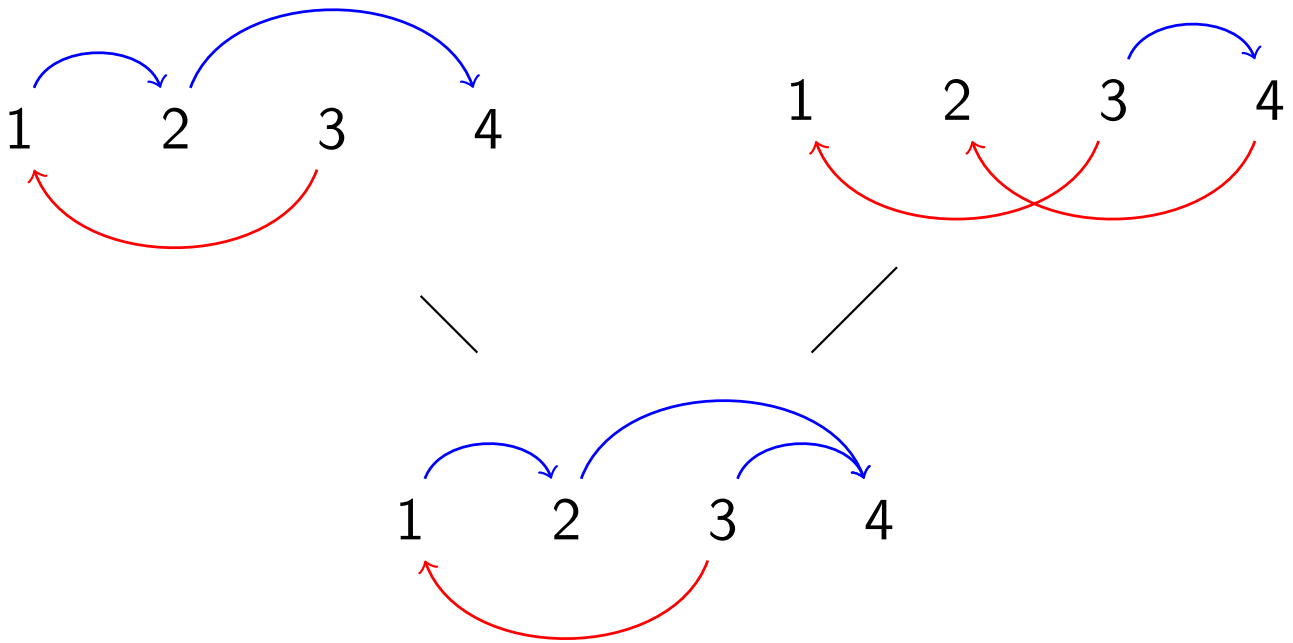
Antisymétrie





Sous treillis ?

Si R et S sont antisymétriques, est-ce que $\mathrm{R} \wedge \mathrm{S}$ l'est aussi ?



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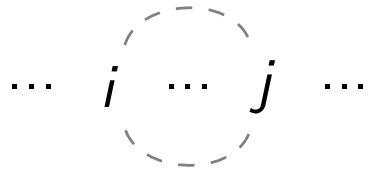

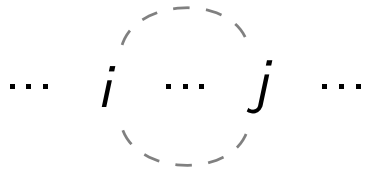

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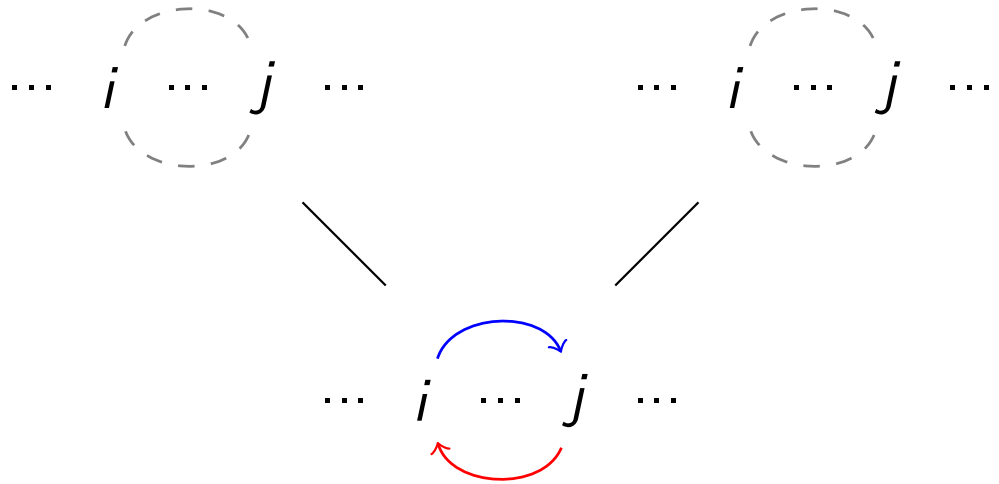
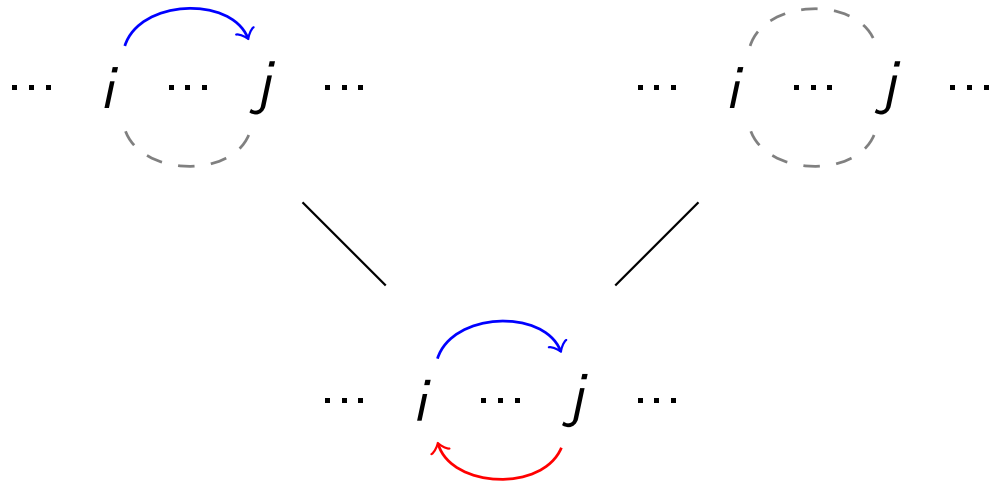


Sous treillis ?

Si R et S sont antisymétriques, est-ce que $\mathrm{R} \wedge \mathrm{S}$ l'est aussi ?



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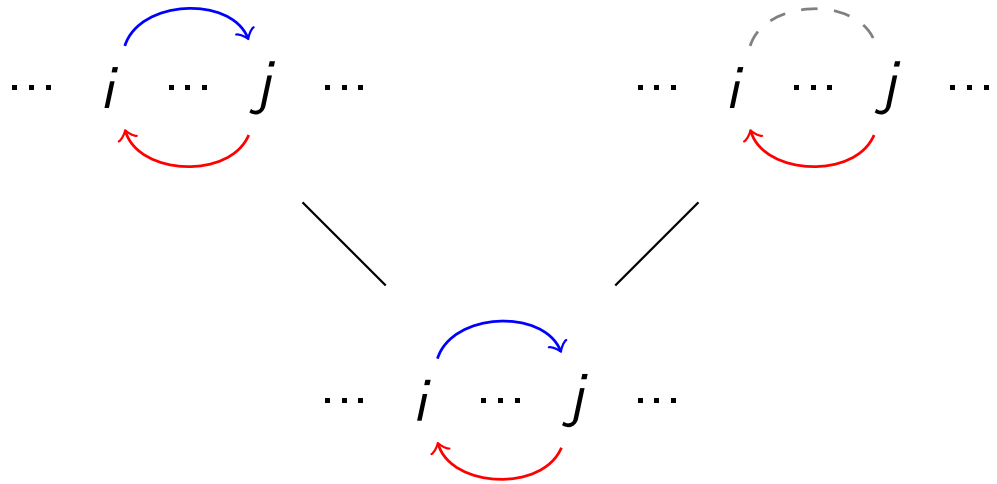
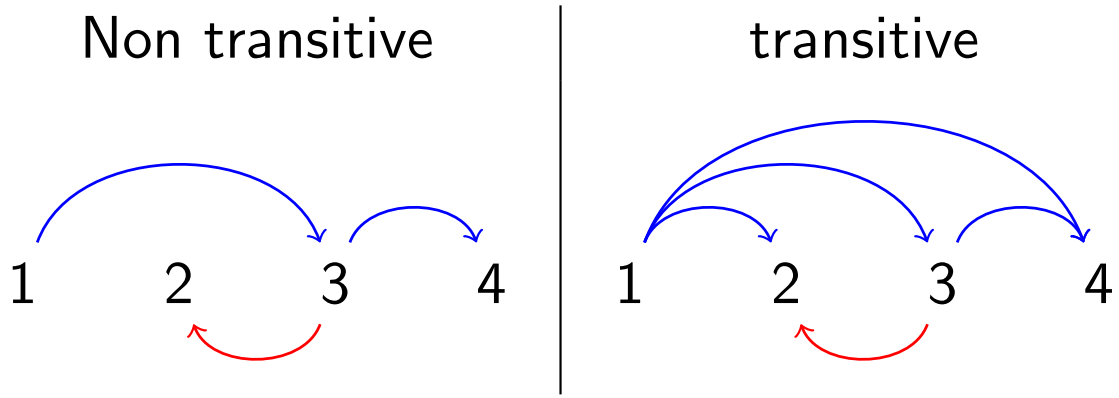
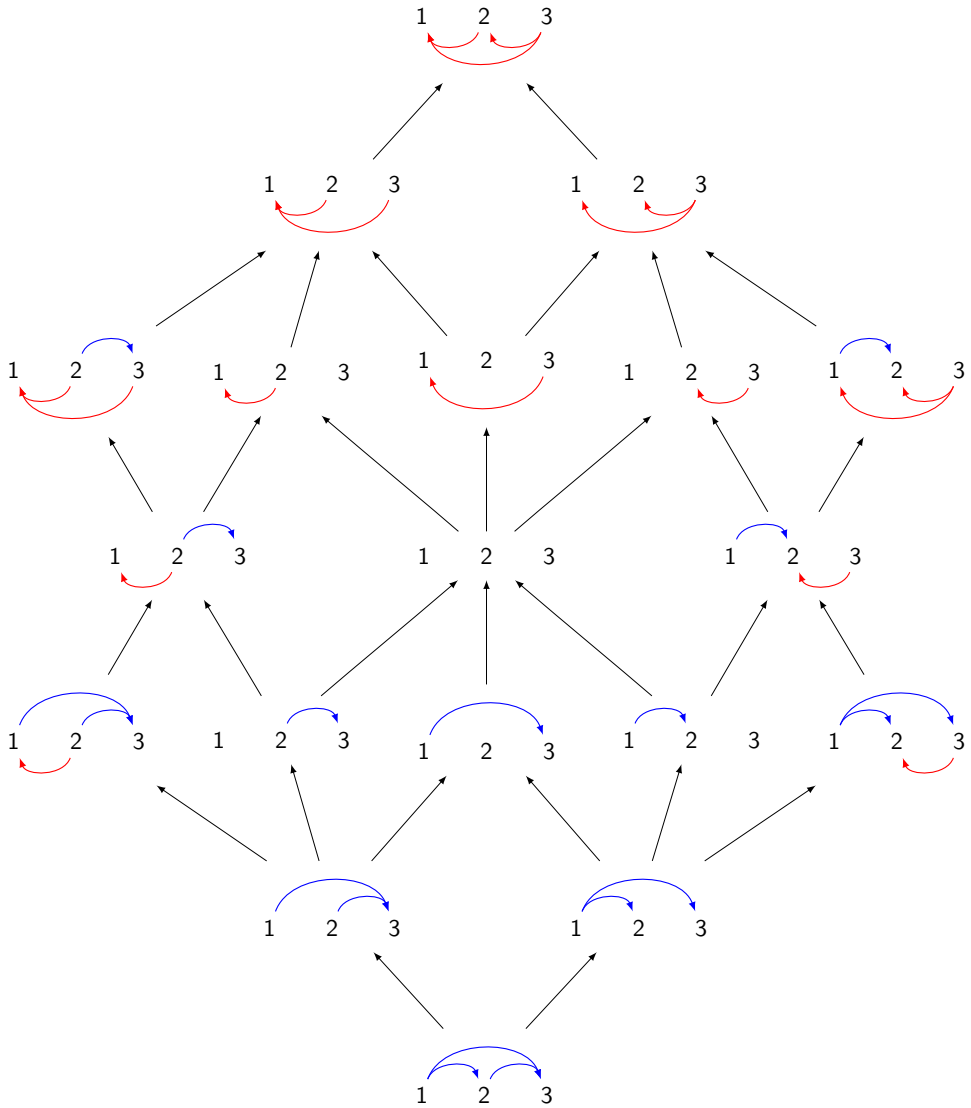


Transitivité

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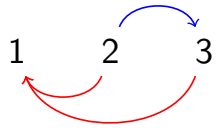
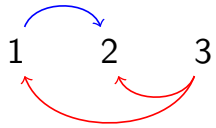
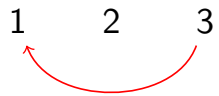
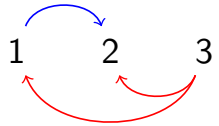
Sous treillis ?

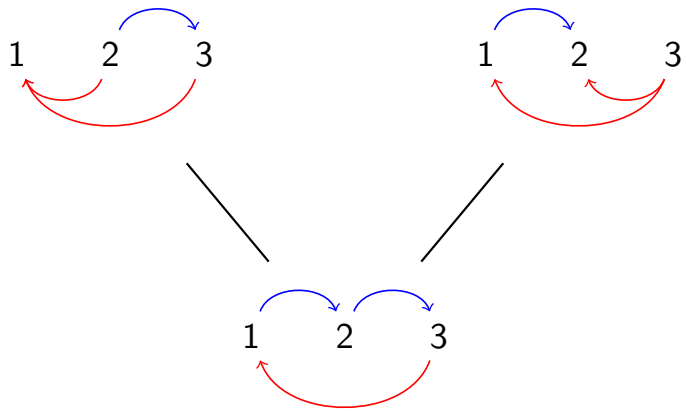


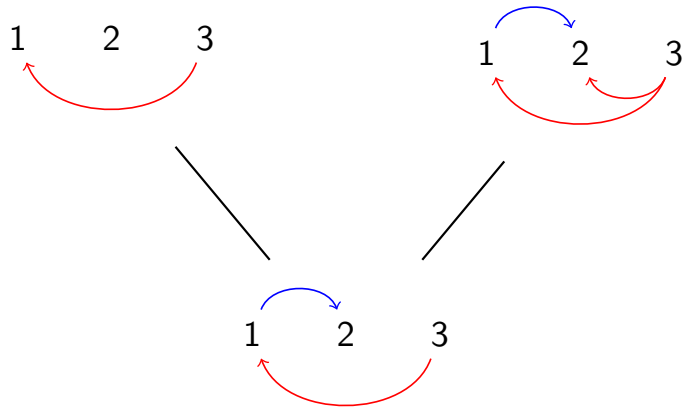




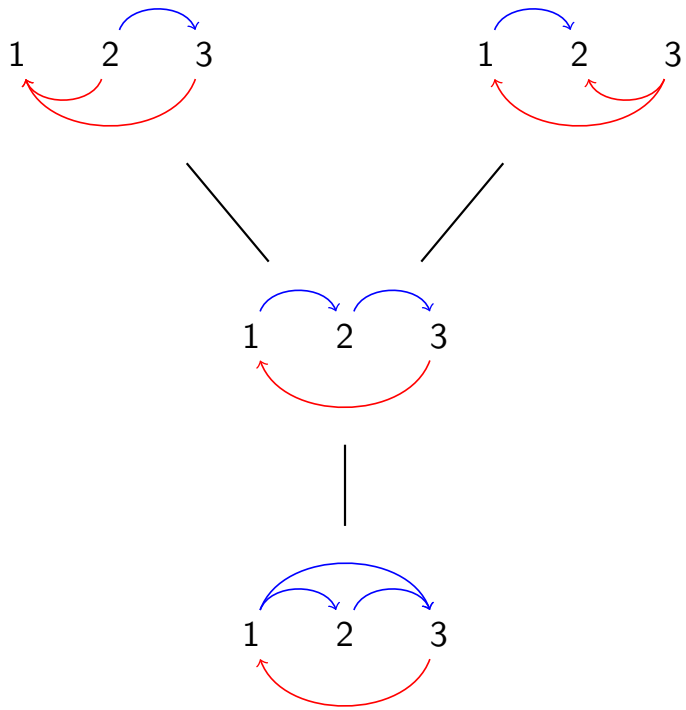


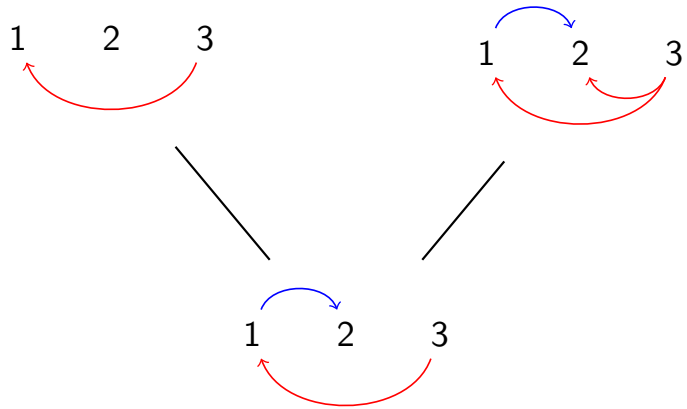
Sous treillis ?



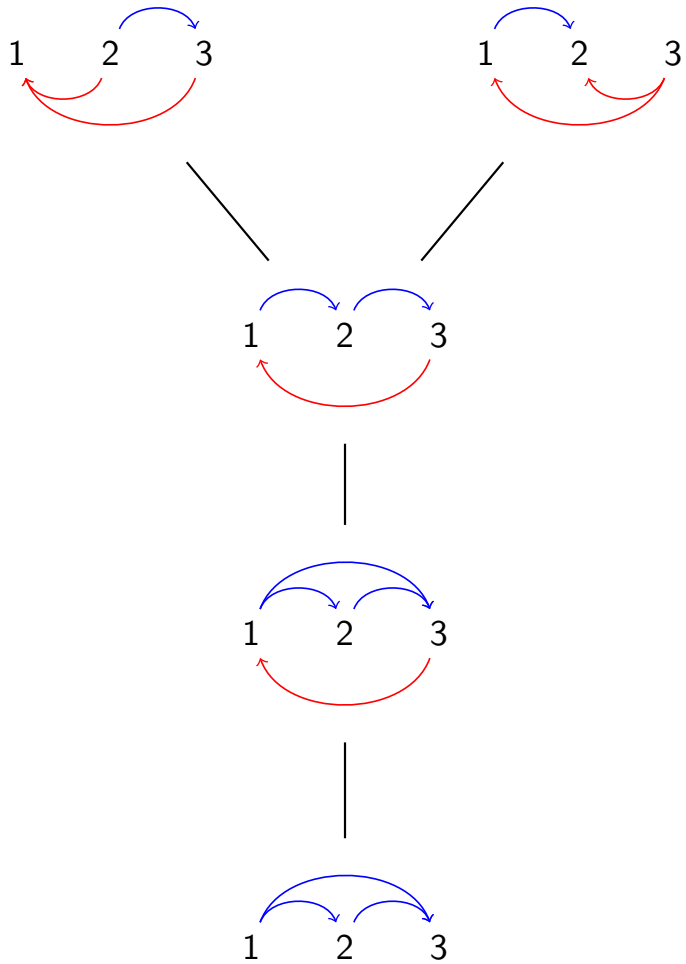


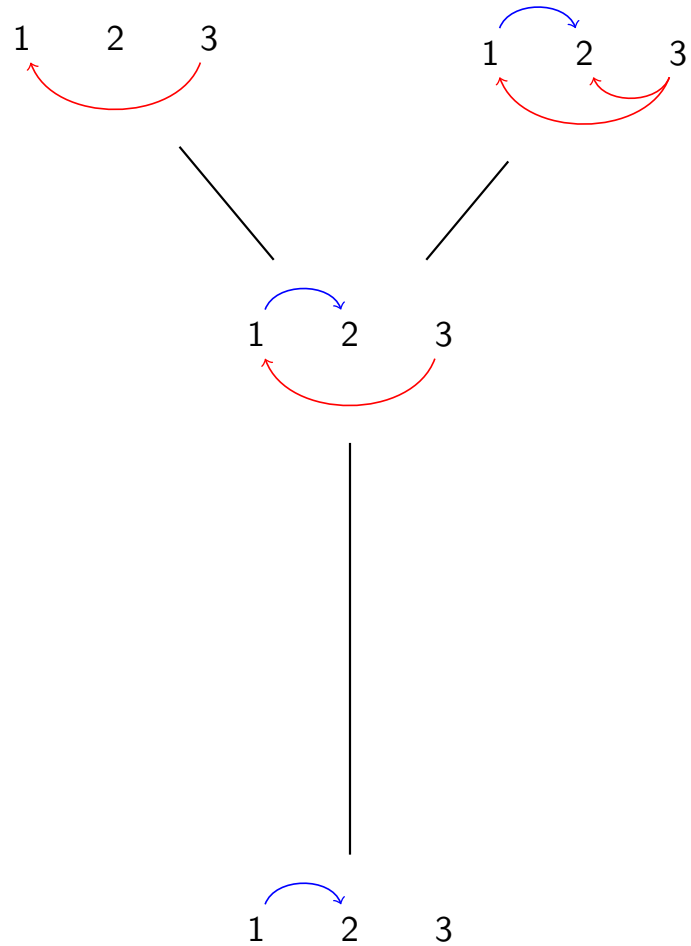
Sous treillis ?



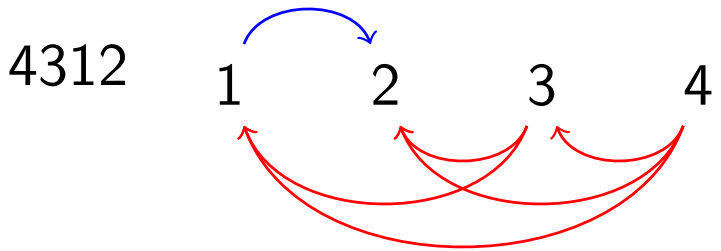


Sous treillis ?





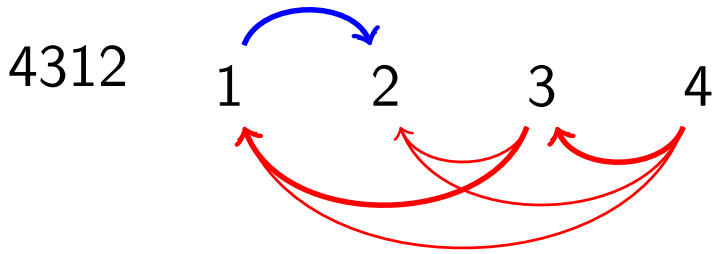
Retour aux permutations



On a $i \,\mathrm{R}\, j$ ssi le nombre i est placé avant le nombre j dans la permutation.
La relation est donc

- antisymétrique
- transitive
- **et totale**

Retour aux permutations



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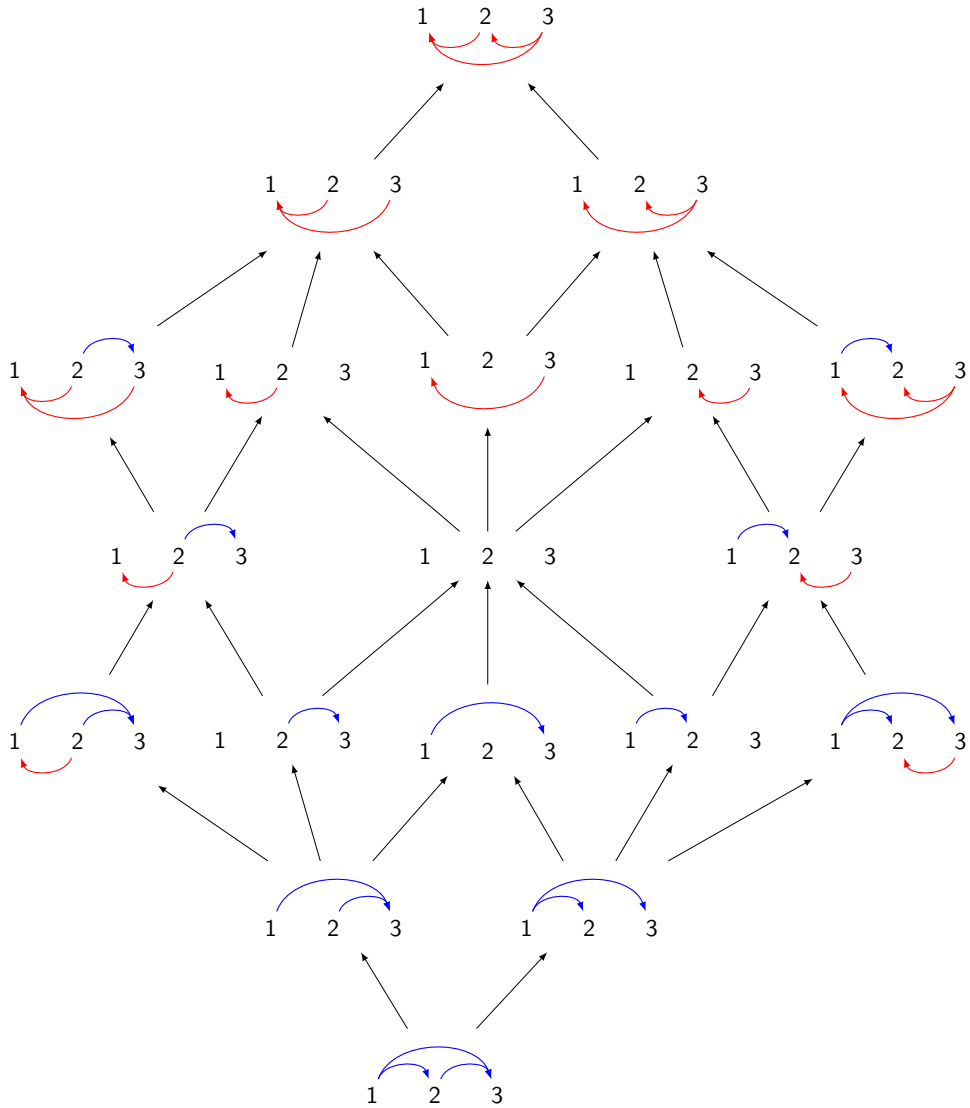
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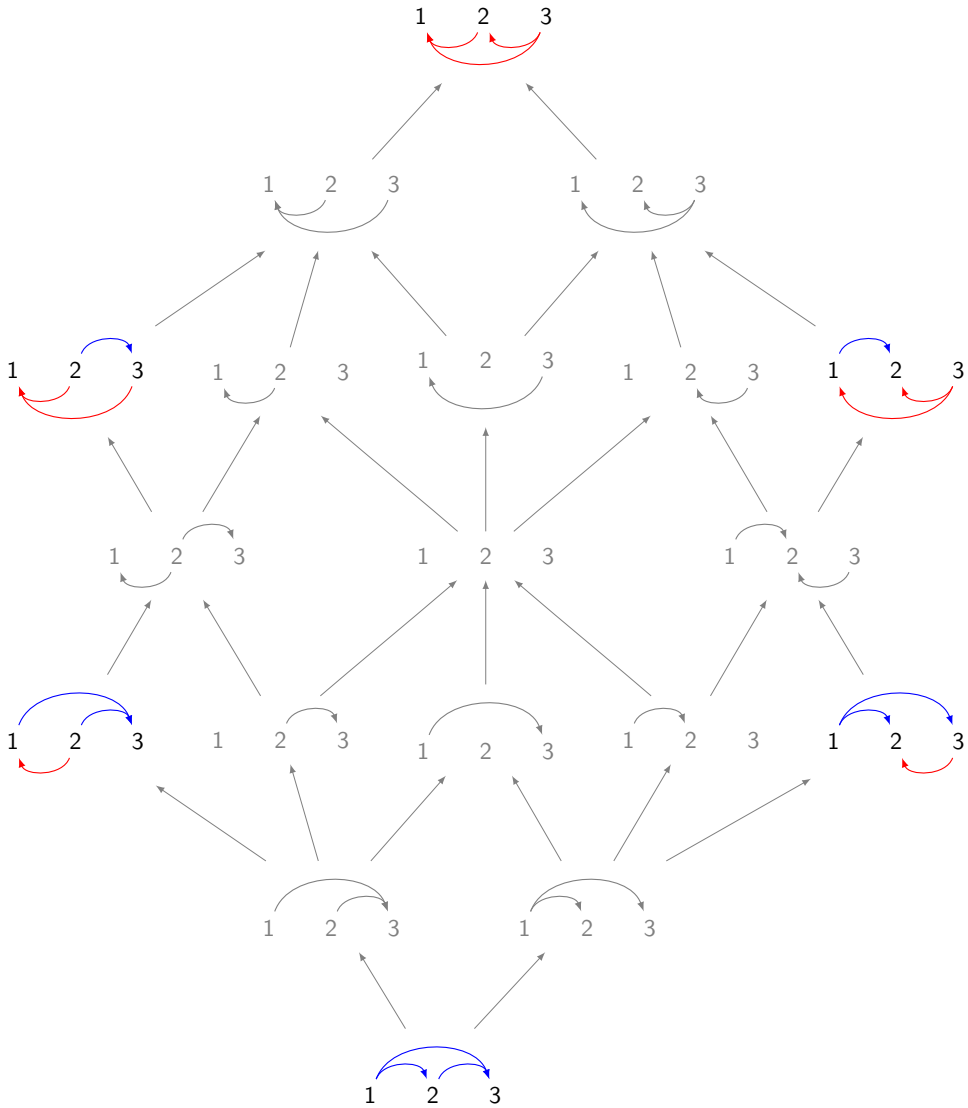
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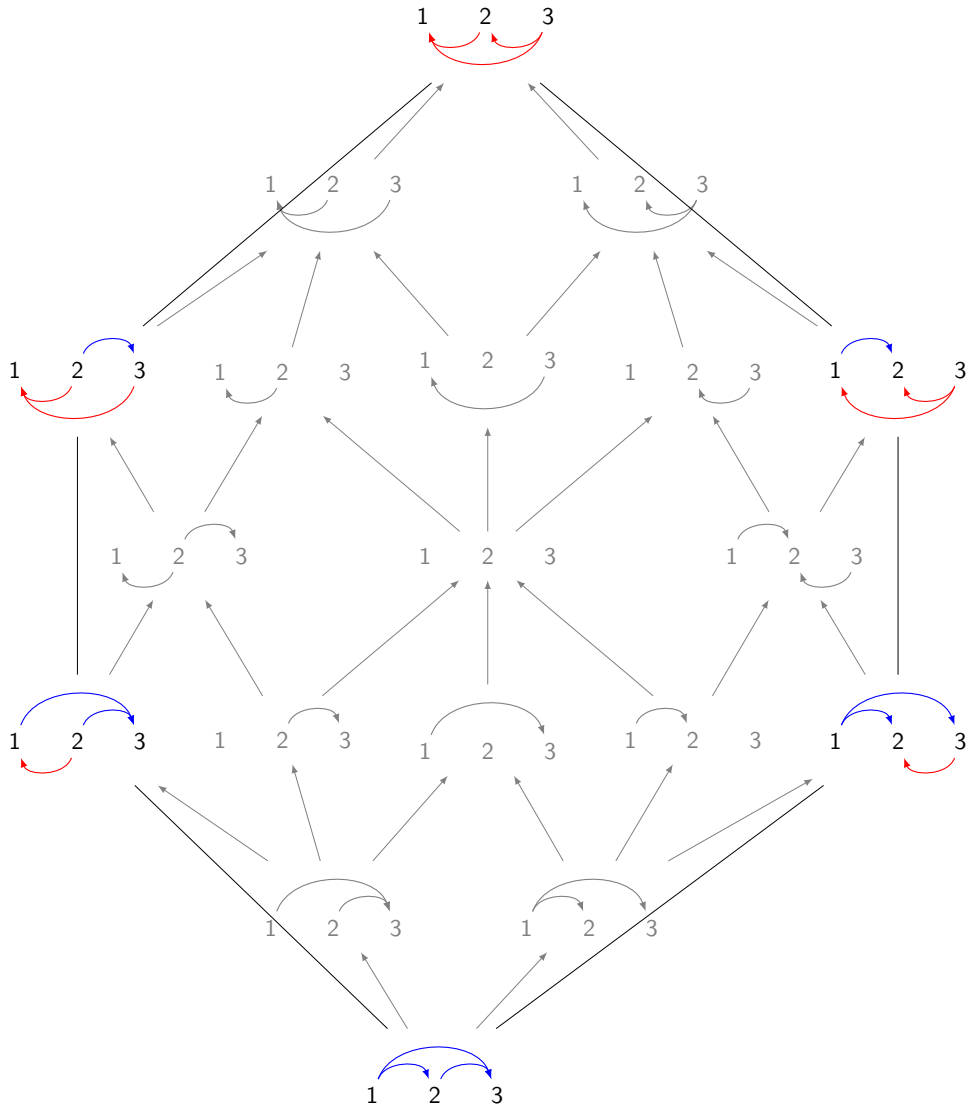

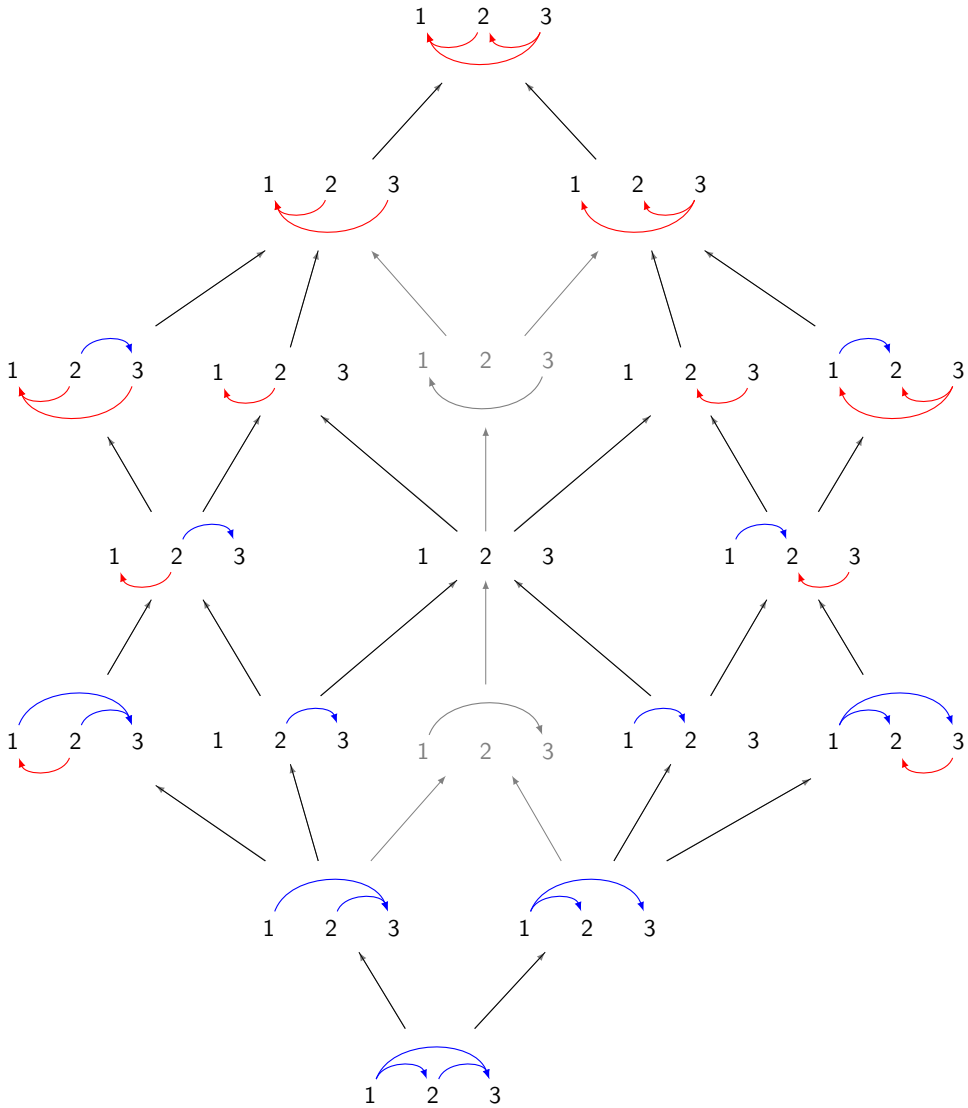

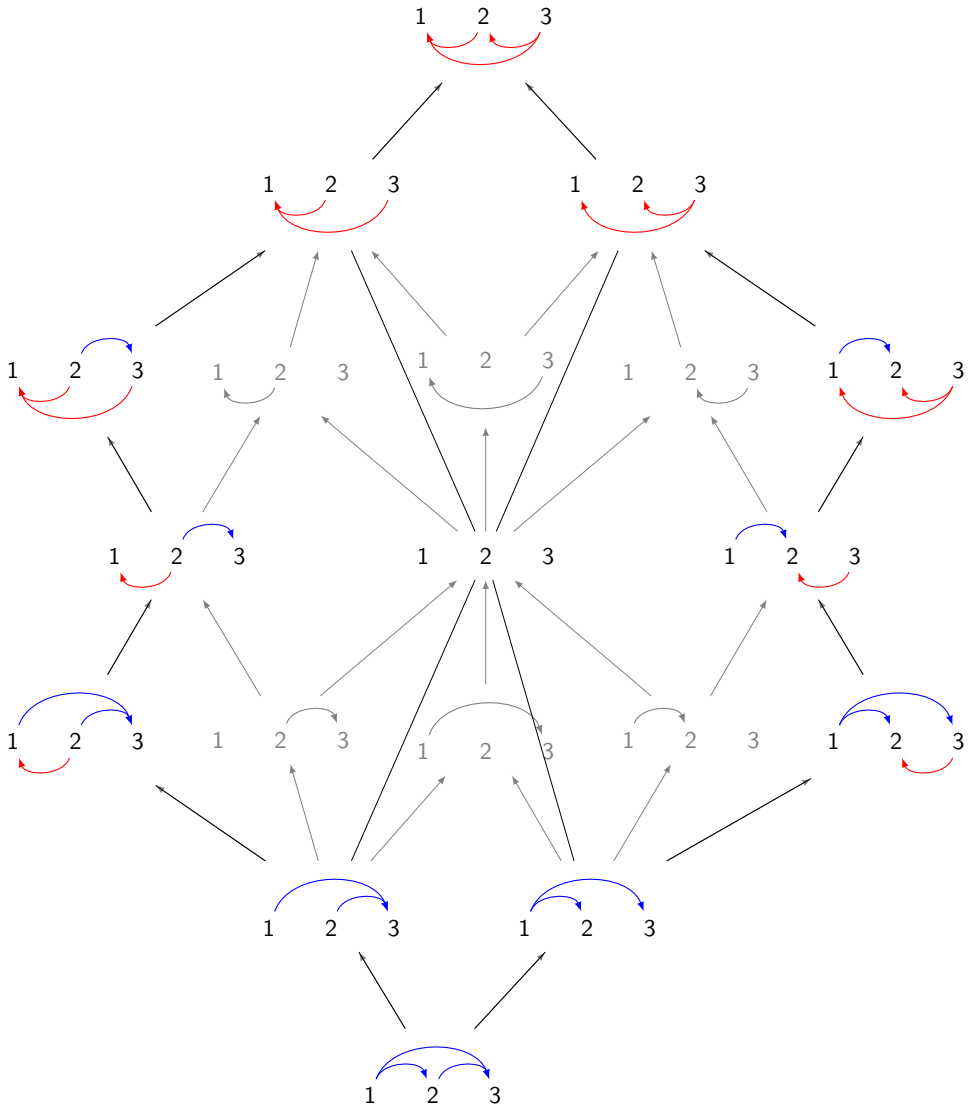

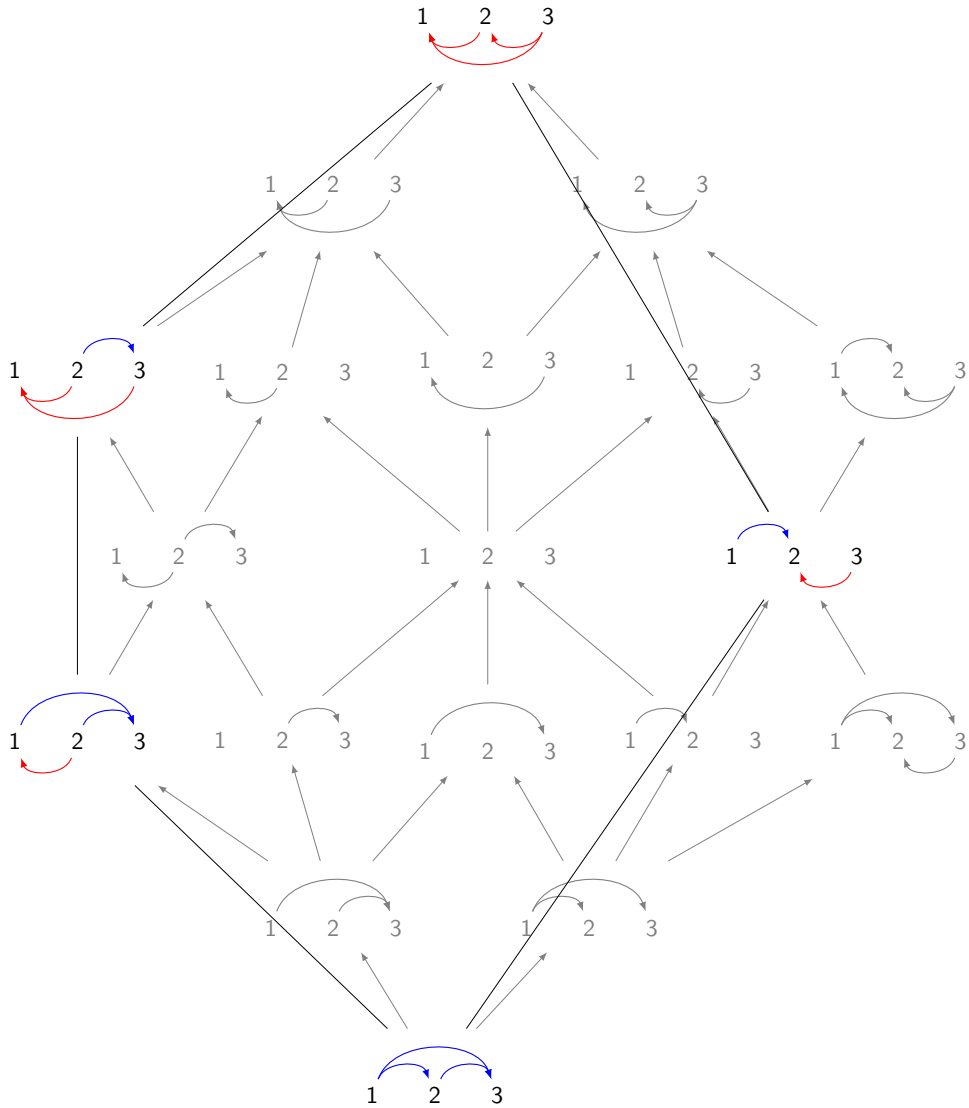

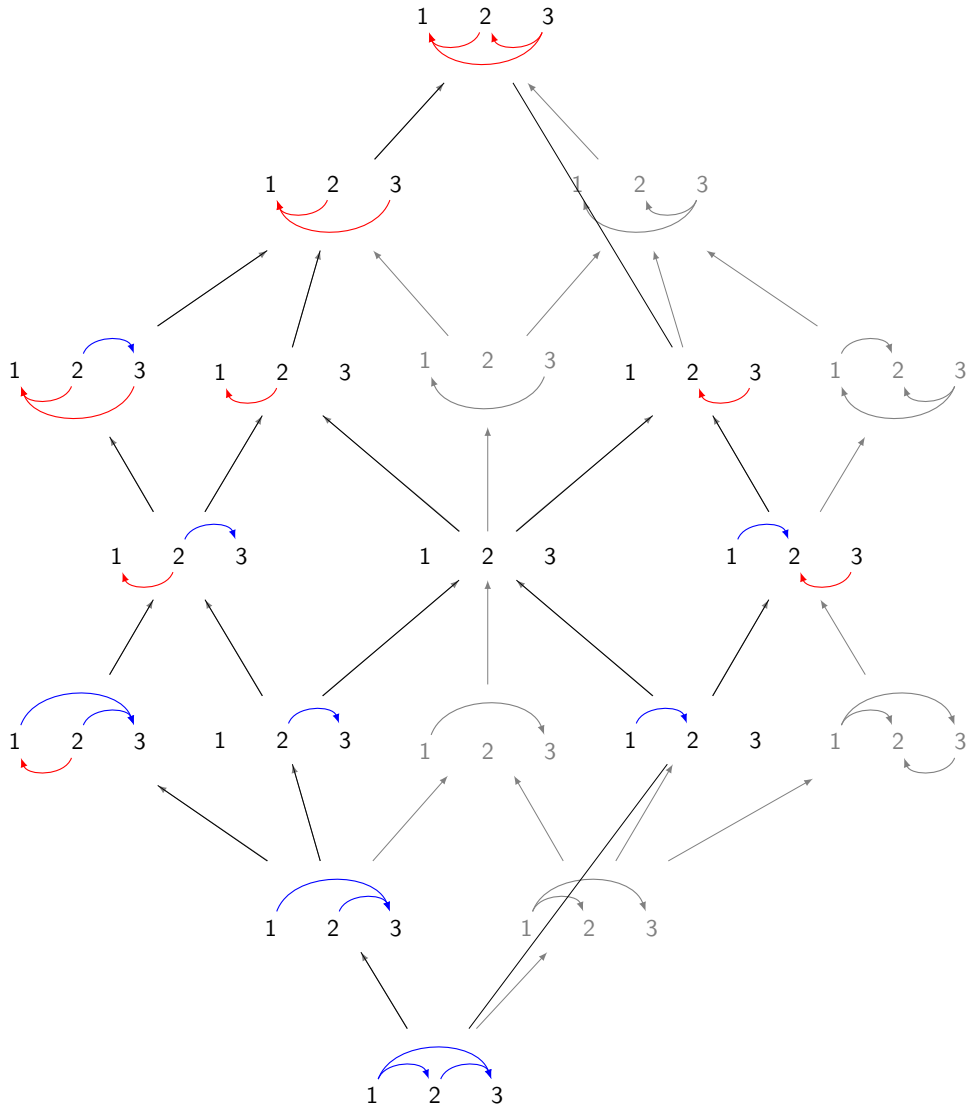
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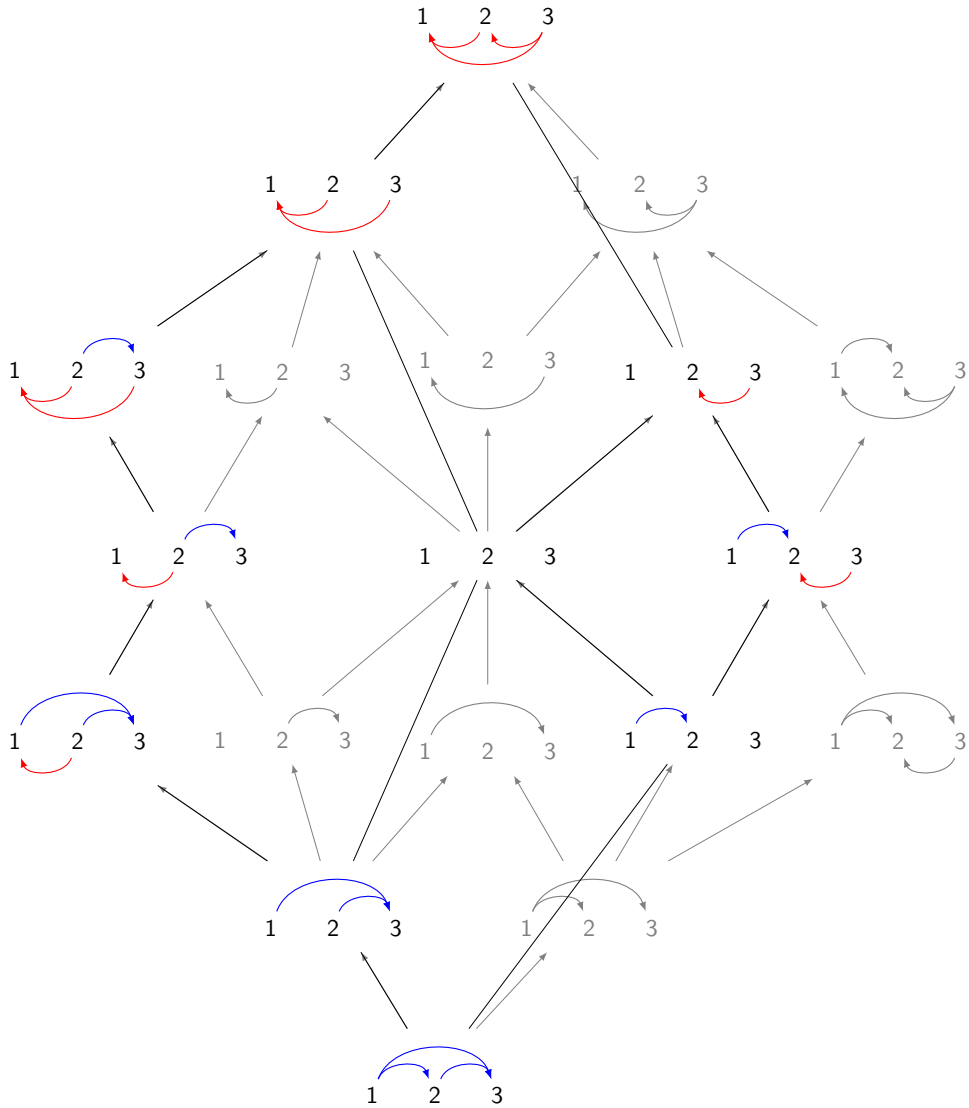

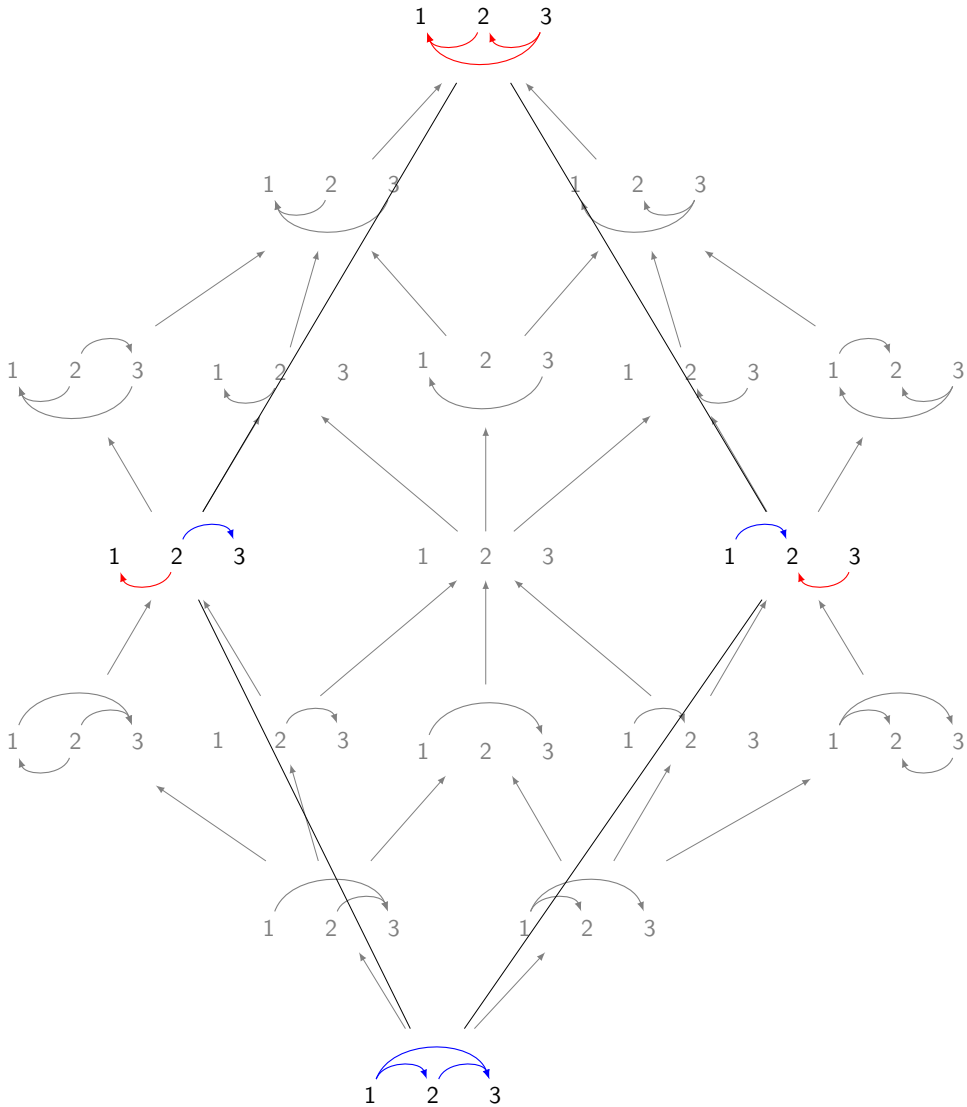

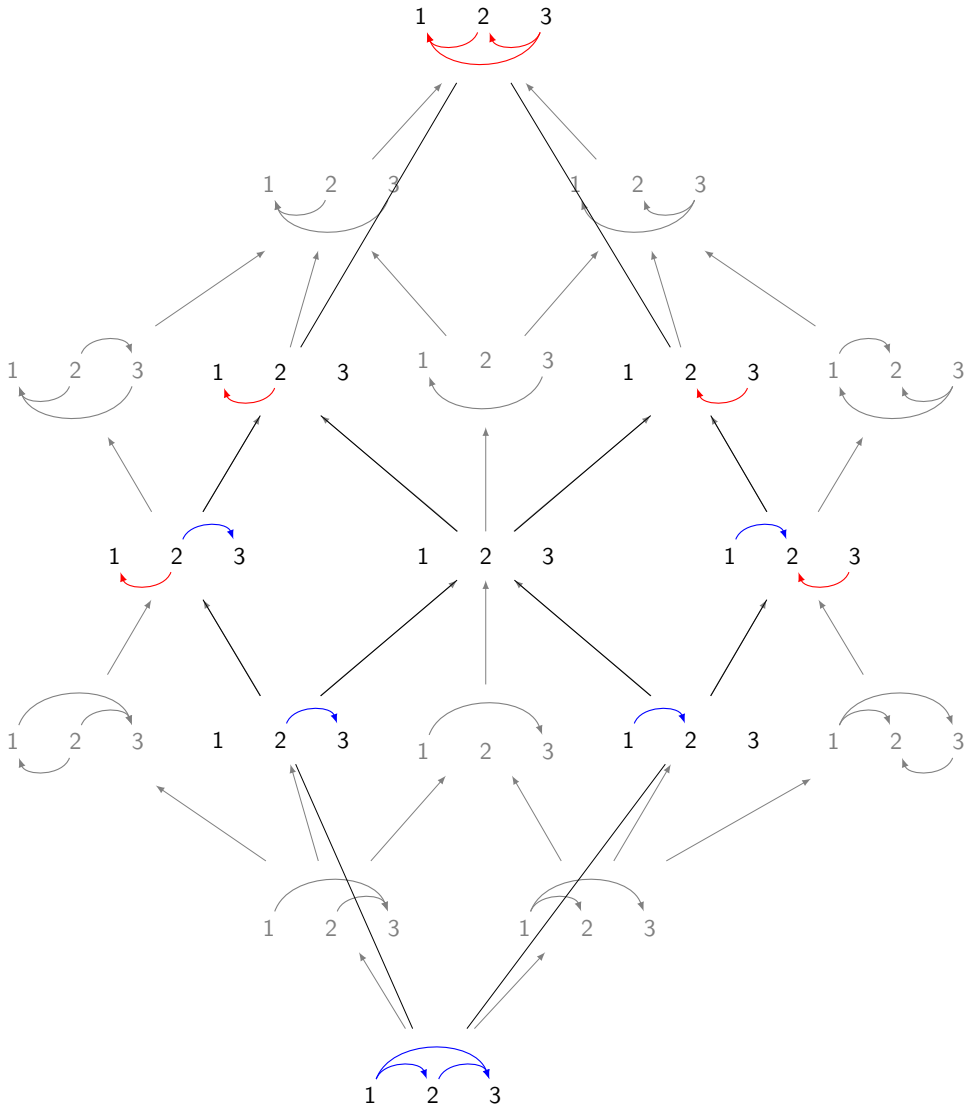

Travail en cours

- Définir des familles de posets qui forment des treillis
- Le cas échéant, prouver que ce sont des sous-treillis.

Et après...

- Étudier les polytopes associés
- Étudier les algèbres associées.