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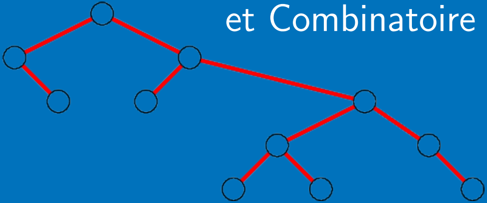
Expérimentation mathématique

et Combinatoire avec Sage



Combinatoire

Étude des ensembles finis ou dénombrables de structures discrètes : graphes, arbres, permutations, mots binaires...

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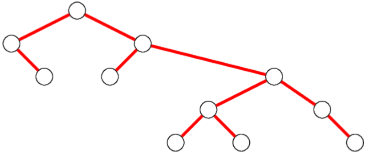
Différentes approches, différentes questions

- Combinatoire énumérative
- Combinatoire bijective
- Combinatoire algébrique
- Combinatoire analytique

Exemple : arbres binaires



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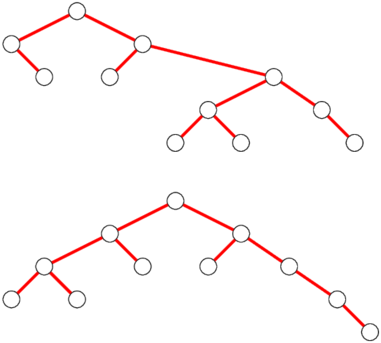

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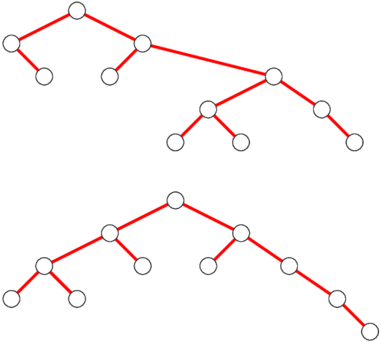
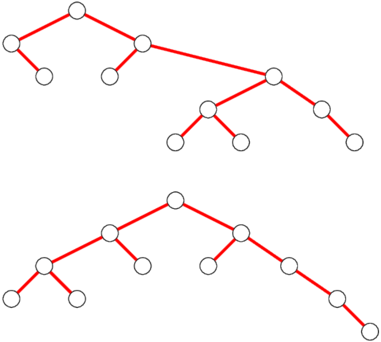


Questions:

- Combien d'arbres binaires à n noeuds?

Exemple : arbres binaires



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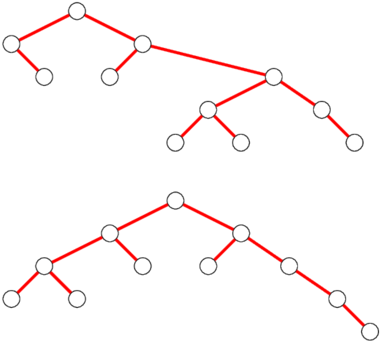
Exemple : arbres binaires



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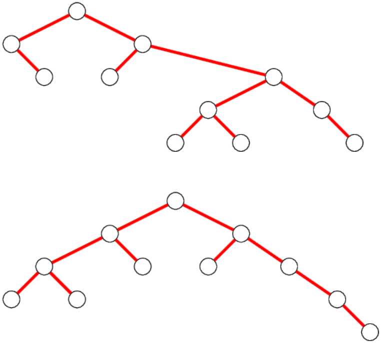
Exemple : arbres binaires



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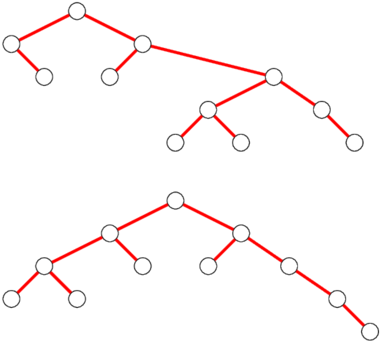

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- A quoi ressemble un arbre binaire aléatoire ?
- Quels autres objets combinatoires sont liés aux arbres binaires ?
- Quelles structures algébriques ?

Quelles motivations ?

- algorithmiques : analyse d'algorithmes, génération aléatoire
- algébriques : géométrie algébrique, théorie des représentations

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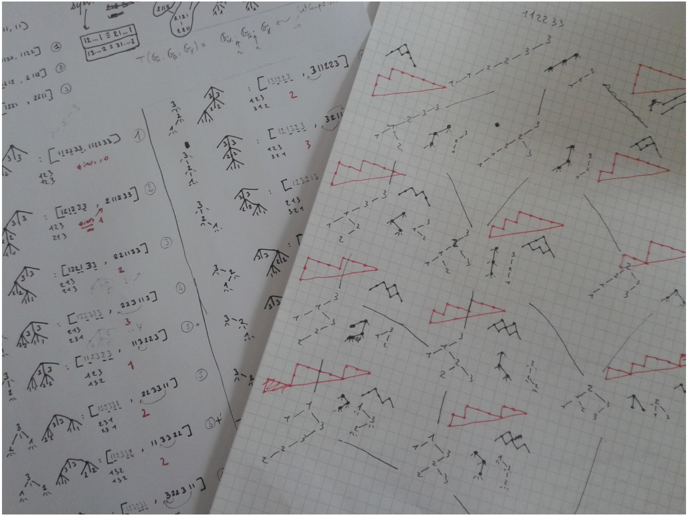
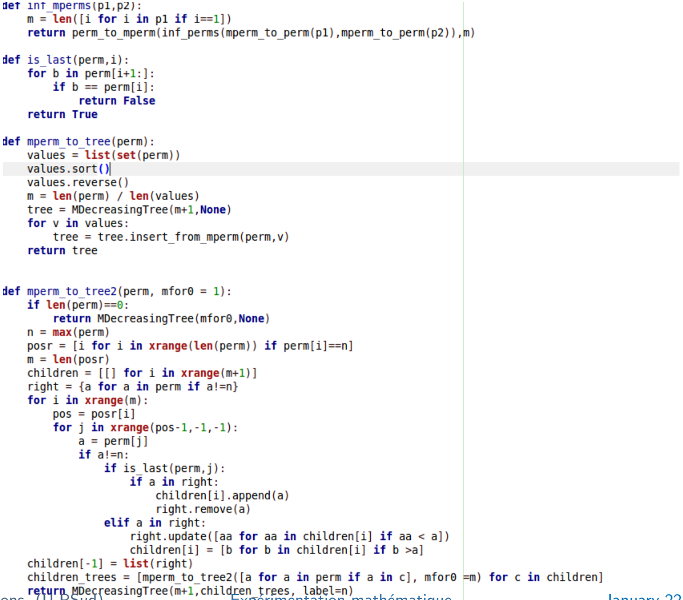
Quelles méthodes ?

Exploration à la main !



Listons les arbres binaires...

Exploration par ordinateur



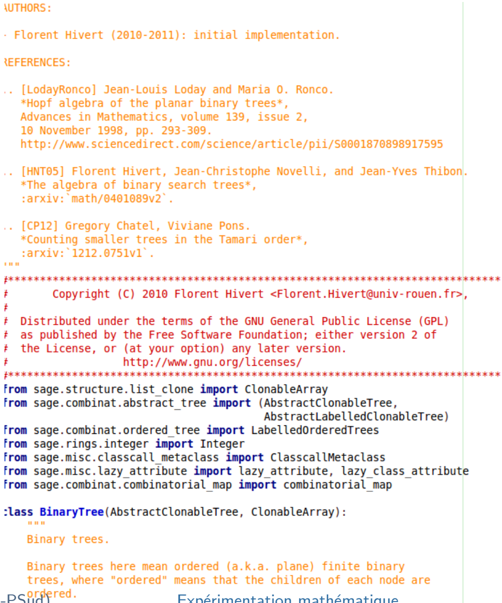
```
def inf_mperms(p1,p2):
    m = len([i for i in p1 if i==1])
    return perm_to_mperm(inf_perms(mperm_to_perm(p1),mperm_to_perm(p2)),m)

def is_last(perm,i):
    for b in perm[i+1:]:
        if b == perm[i]:
            return False
    return True

def mperm_to_tree(perm):
    values = list(set(perm))
    values.sort()
    values.reverse()
    m = len(perm) / len(values)
    tree = MDecreasingTree(m+1,None)
    for v in values:
        tree = tree.insert_from_mperm(perm,v)
    return tree


def mperm_to_tree2(perm, mfor0 = 1):
    if len(perm)==0:
        return MDecreasingTree(mfor0,None)
    n = max(perm)
    posr = [i for i in xrange(len(perm)) if perm[i]==n]
    m = len(posr)
    children = [[] for i in xrange(m+1)]
    right = {a for a in perm if a!=n}
    for i in xrange(m):
        pos = posr[i]
        for j in xrange(pos-1,-1,-1):
            a = perm[j]
            if a!=n:
                if is_last(perm,j):
                    if a in right:
                        children[i].append(a)
                        right.remove(a)
                elif a in right:
                    right.update([aa for aa in children[i] if aa < a])
                    children[i] = [b for b in children[i] if b >a]
    children[-1] = list(right)
    children_trees = [mperm_to_tree2([a for a in perm if a in c], mfor0 =m) for c in children]
    return MDecreasingTree(m+1,children_trees, label=n)
```

Exploration par ordinateur



```
AUTHORS:

- Florent Hivert (2010-2011): initial implementation.

REFERENCES:

.. [LodayRonco] Jean-Louis Loday and Maria O. Ronco.
   *Hopf algebra of the planar binary trees*,
   Advances in Mathematics, volume 139, issue 2,
   10 November 1998, pp. 293-309.
   http://www.sciencedirect.com/science/article/pii/S0001870898917595

.. [HNT05] Florent Hivert, Jean-Christophe Novelli, and Jean-Yves Thibon.
   *The algebra of binary search trees*,
   :arxiv:`math/0401089v2`.

.. [CP12] Gregory Chatel, Viviane Pons.
   *Counting smaller trees in the Tamari order*,
   :arxiv:`1212.0751v1`.
"""
#*****************************************************************************
#       Copyright (C) 2010 Florent Hivert <Florent.Hivert@univ-rouen.fr>,
#
#  Distributed under the terms of the GNU General Public License (GPL)
#  as published by the Free Software Foundation; either version 2 of
#  the License, or (at your option) any later version.
#                  http://www.gnu.org/licenses/
#*****************************************************************************
from sage.structure.list_clone import ClonableArray
from sage.combinat.abstract_tree import (AbstractClonableTree,
                                        AbstractLabelledClonableTree)
from sage.combinat.ordered_tree import LabelledOrderedTrees
from sage.rings.integer import Integer
from sage.misc.classcall_metaclass import ClasscallMetaclass
from sage.misc.lazy_attribute import lazy_attribute, lazy_class_attribute
from sage.combinat.combinatorial_map import combinatorial_map

class BinaryTree(AbstractClonableTree, ClonableArray):
    """
    Binary trees.

    Binary trees here mean ordered (a.k.a. plane) finite binary
    trees, where "ordered" means that the children of each node are
    ordered.
```

SageMath is a free open source mathematics software

- Created in 2005.
- `http://www.sagemath.org/`
- Mission: *Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.*

- the main language of Sage is **python** (but there are many other source languages: cython, C, C++, fortran)
- the source is distributed under the GPL licence.

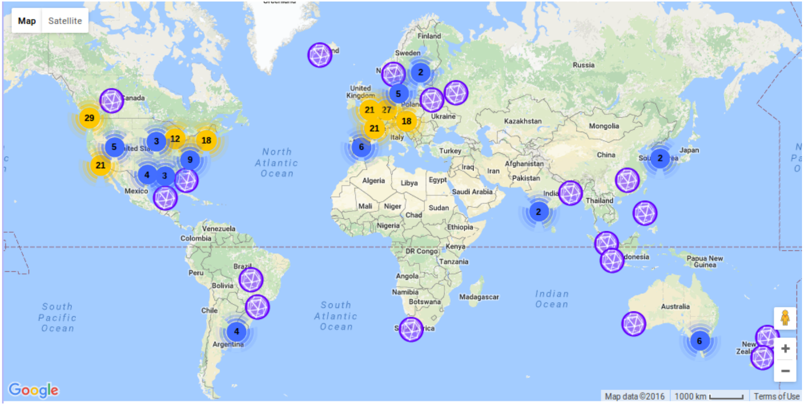
One of the original purpose of Sage was to put together the many existent open source mathematics software programs: Atlas, GAP, GMP, Linbox, Maxima, MPFR, PARI/GP, NetworkX, NTL, Numpy/Scipy, Singular, Symmetrica,...
Sage is **all-inclusive**: it installs all those libraries and gives you a common python-based interface to work on them.
On top of it is the python / cython Sage library it-self.

Development model

- **Sage is developed by researchers for researchers:** the original philosophy is to develop what you *need* for your research and *share* it with the community.
- The first full-time Sage developer has been recruited in January 2016 at Paris-Sud university by the OpenDreamKit project.
- The code is maintained through a git repo with a system of tickets and peer-review for new contributions: `http://trac.sagemath.org/`

The Sage community

268 developers in 182 different places.



Mailing lists, Sage days

Funding?

- Runs without permanent funding
- Benefit from local funding sources
- Since 2015: The **OpenDreamKit** project

OpenDreamKit 2015 – 2019

Open Digital Research Environment Toolkit for the Advancement of Mathematics

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- Pays for full time developers, events, infrastructures

Follow us...

- `http://opendreamkit.org/`
- Twitter: @OpenDreamKit

What can I do with Sage?

Number theory, combinatorics, graph theory,...
Most things: *Calcul Mathématique avec Sage*

How can I use it?

- native install on Linux, Mac, and Windows
- online on **CoCalc.com**

Demo !